



# **Application of AFMM/RCWA for bent waveguides**

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IMEP-LAHC, Grenoble

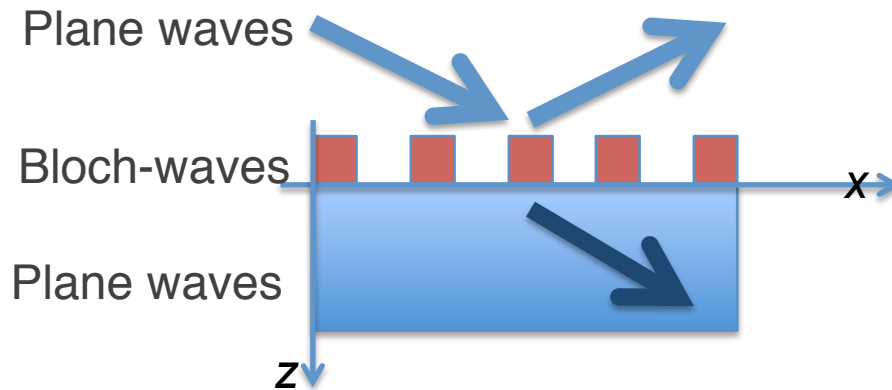


# Outline

- Introduction/historical
- Developments of the method
- Test and comparison with FDTD
- Conclusion

# History of RCWA/AFMM

- ~1980: Rigorous Coupled Wave Analysis



Fourier series of E, H,  $\epsilon$ ,  $\mu$   
 [K. Knop, JOSA. (1978)]

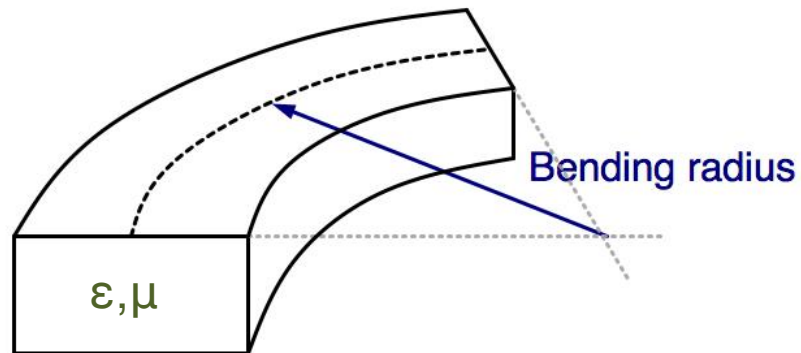
Moharam, Gaylord, Nevière...

TE: OK, problems in TM

- 1996: TM convergence issue solved  
 [Lalanne, Morris, JOSA A(1996)] [Li, JOSA A(1996)]
- 2001: Application to integrated optics:  
 Aperiodic Fourier Modal Method  
 [Silberstein et al., JOSA A 2001]

# Bent waveguides

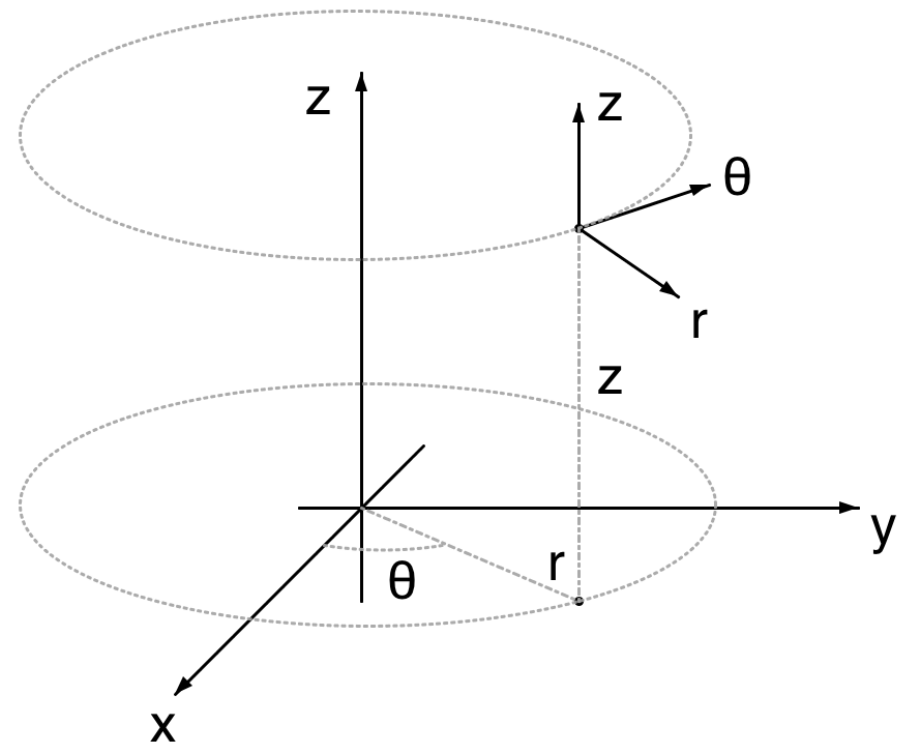
- Basic structure



Invariant cross-section

- Some classical results
  - Conformal mapping
  - Perturbation
  - ...

- Coordinate system



# Cylindrical coordinates

- Maxwell eq.

$$\begin{cases} \nabla \wedge \vec{E} = -j\omega\bar{\mu}\vec{H} \\ \nabla \wedge \vec{H} = j\omega\bar{\epsilon}\vec{E} \end{cases}$$

- Propagation axis:  $\theta$

- Transverse components

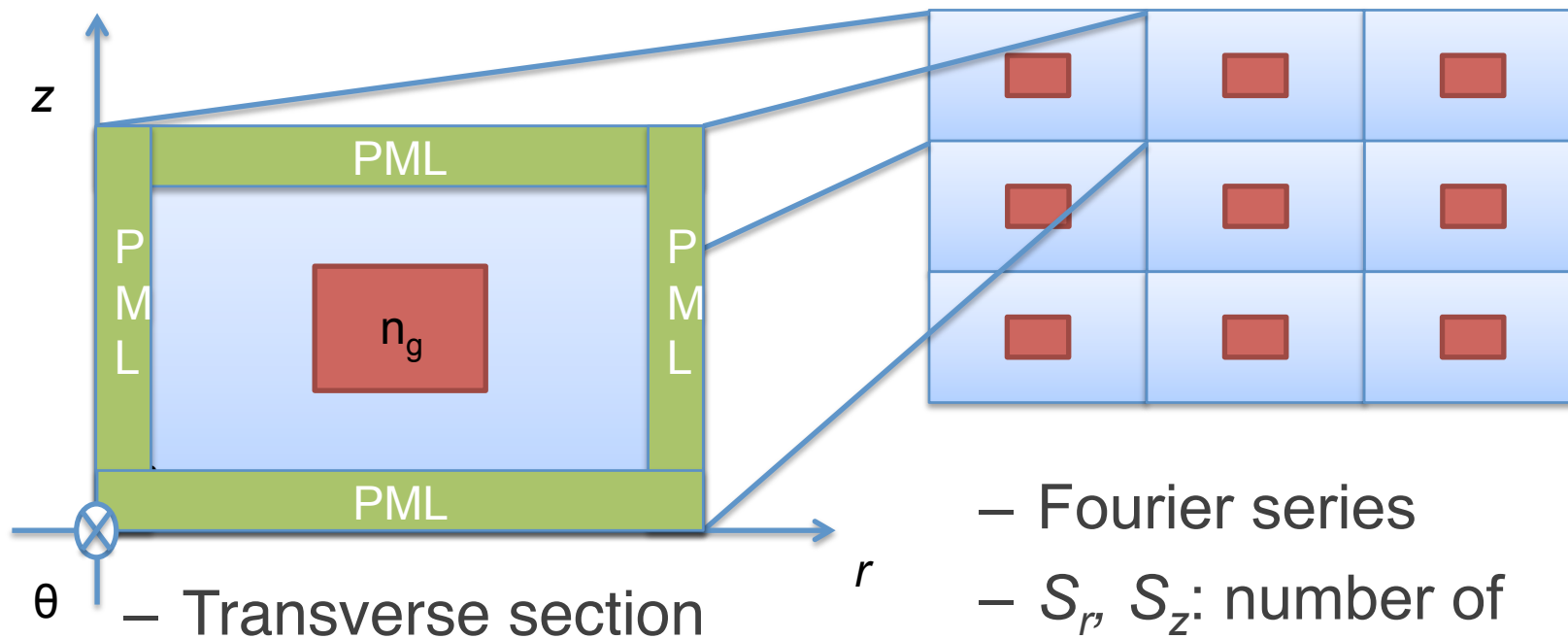
$$\begin{cases} \frac{\partial E_r}{\partial \theta} = \frac{\partial}{\partial r} \left[ \frac{r}{\omega\epsilon_\theta} \left( \frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \right) \right] + \omega\mu_z r H_z \\ \frac{\partial E_z}{\partial \theta} = r \frac{\partial}{\partial z} \left[ \frac{1}{\omega\epsilon_\theta} \left( \frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \right) \right] - \omega\mu_r r H_r \\ \frac{\partial H_r}{\partial \theta} = \frac{\partial}{\partial r} \left[ -\frac{r}{\omega\mu_\theta} \left( \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \right) \right] - \omega\epsilon_z r E_z \\ \frac{\partial H_z}{\partial \theta} = r \frac{\partial}{\partial z} \left[ -\frac{1}{\omega\mu_\theta} \left( \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \right) \right] + \omega\epsilon_r r E_r. \end{cases}$$

$$\nabla \wedge \vec{E} = \hat{r} \left( \frac{1}{r} \frac{\partial E_z}{\partial \theta} - \frac{\partial E_\theta}{\partial z} \right) + \hat{\theta} \left( \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \right) + \hat{z} \left( \frac{\partial(rE_\theta)}{\partial r} - \frac{\partial E_r}{\partial \theta} \right)$$

# 2D periodization

- Original structure

- Artificial periodization



[E. Silberstein et al., JOSA A (2001)]

# Matrix operators

$$\frac{\partial H_r}{\partial \theta} = \frac{\partial}{\partial r} \left[ -\frac{r}{j\omega\mu_\theta} \left( \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \right) \right] - j\omega\epsilon_z r E_z$$

$$\frac{\partial}{\partial \theta} \rightarrow -j\beta_s$$

$$-j\beta_s H_r = \frac{\partial}{\partial r} \left[ -\frac{r}{j\omega\mu_\theta} \left( \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} \right) \right] - j\omega\epsilon_z r E_z$$

Fourier coeff.

Deriv.

Convol.

(naïve approach)  $\left[ \frac{1}{\epsilon_z} \right]^{-1} [E_z]$

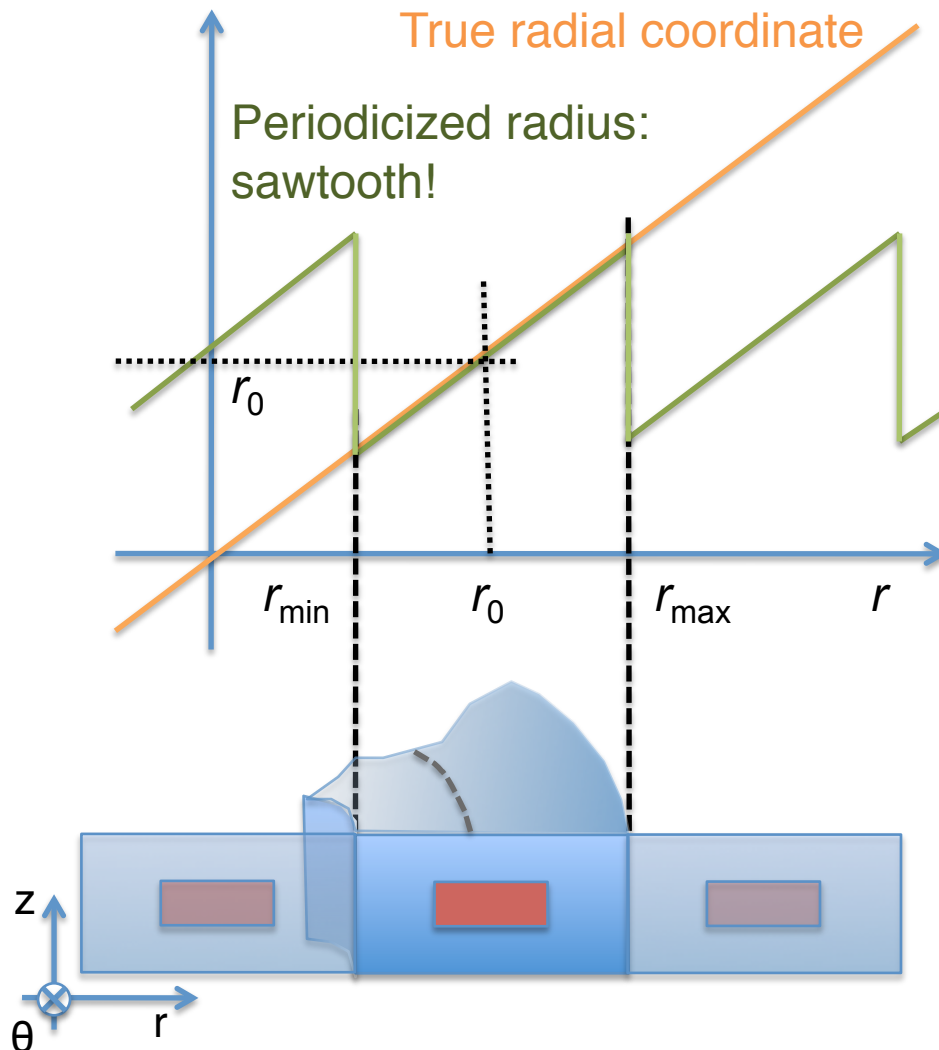
$$-j\beta_s [H_r] = K_r \left[ -\frac{1}{j\omega} [[r]] [[\mu_\theta^{-1}]] (K_z [E_r] - K_r [E_z]) \right] - j\omega [[\epsilon_z]] [[r]] [E_z]$$

Blue: vector

Red: (block) diagonal matrix

Green: (block) Toeplitz matrix

# Calculation of $[[r]]$



- Fourier coefficients  $r_k$ :

$$r_k = \begin{cases} r_0 & \text{if } k = 0, \\ j \frac{r_{\max} - r_{\min}}{2\pi k} & \text{if } k \neq 0. \end{cases}$$

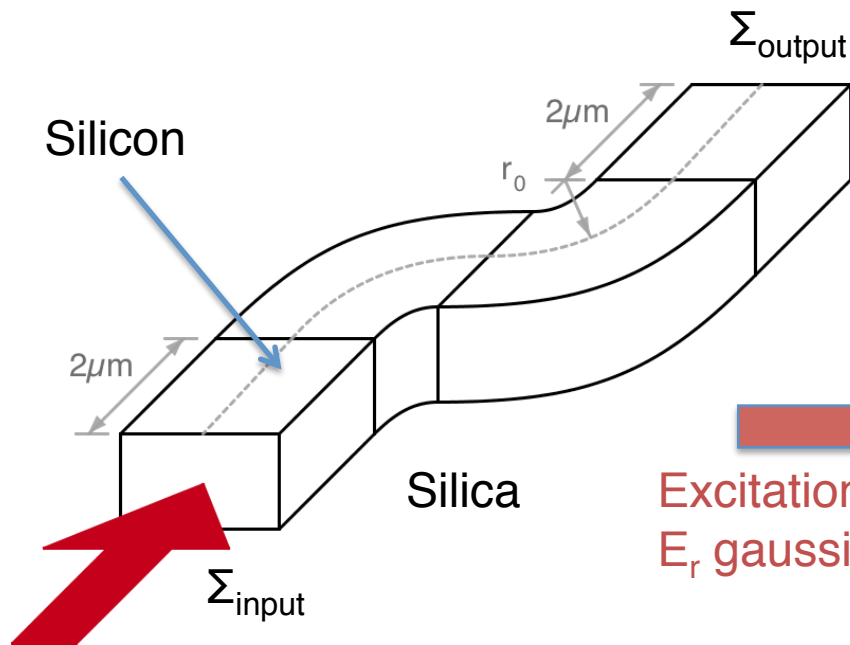
$[[r]]$ : Toeplitz matrix  
formed by  $r_k$ .

➔ Matrix operator



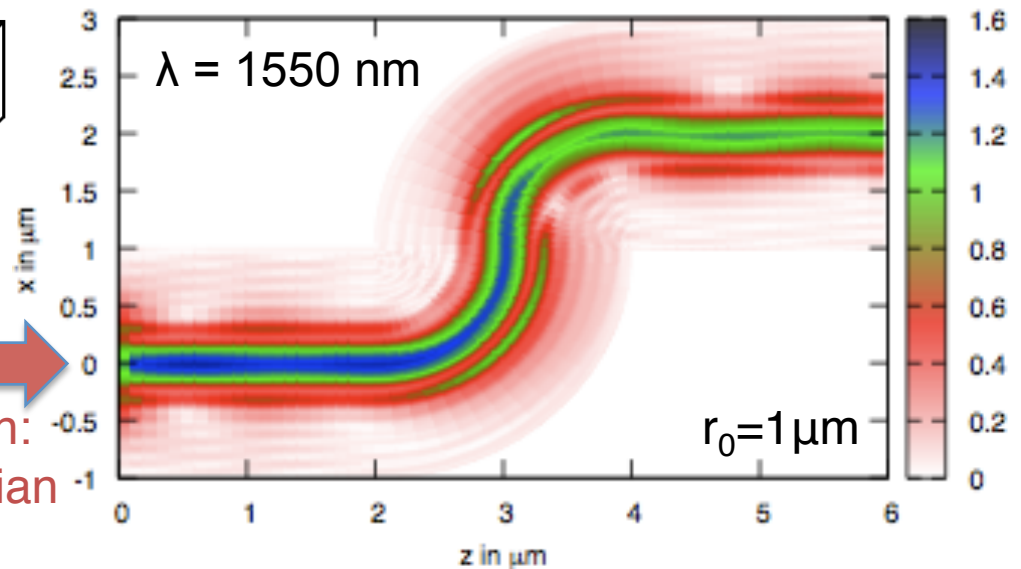
# 3D propagation: S-bend

- Structure:



$n_c = 3.48$ ,  $n_s = 1.44$   
 $w = 500 \text{ nm}$ ,  $h = 200 \text{ nm}$   
 $r_0 = 0.5 \mu\text{m} \rightarrow 2 \mu\text{m}$   
 4 sections: S matrix approach

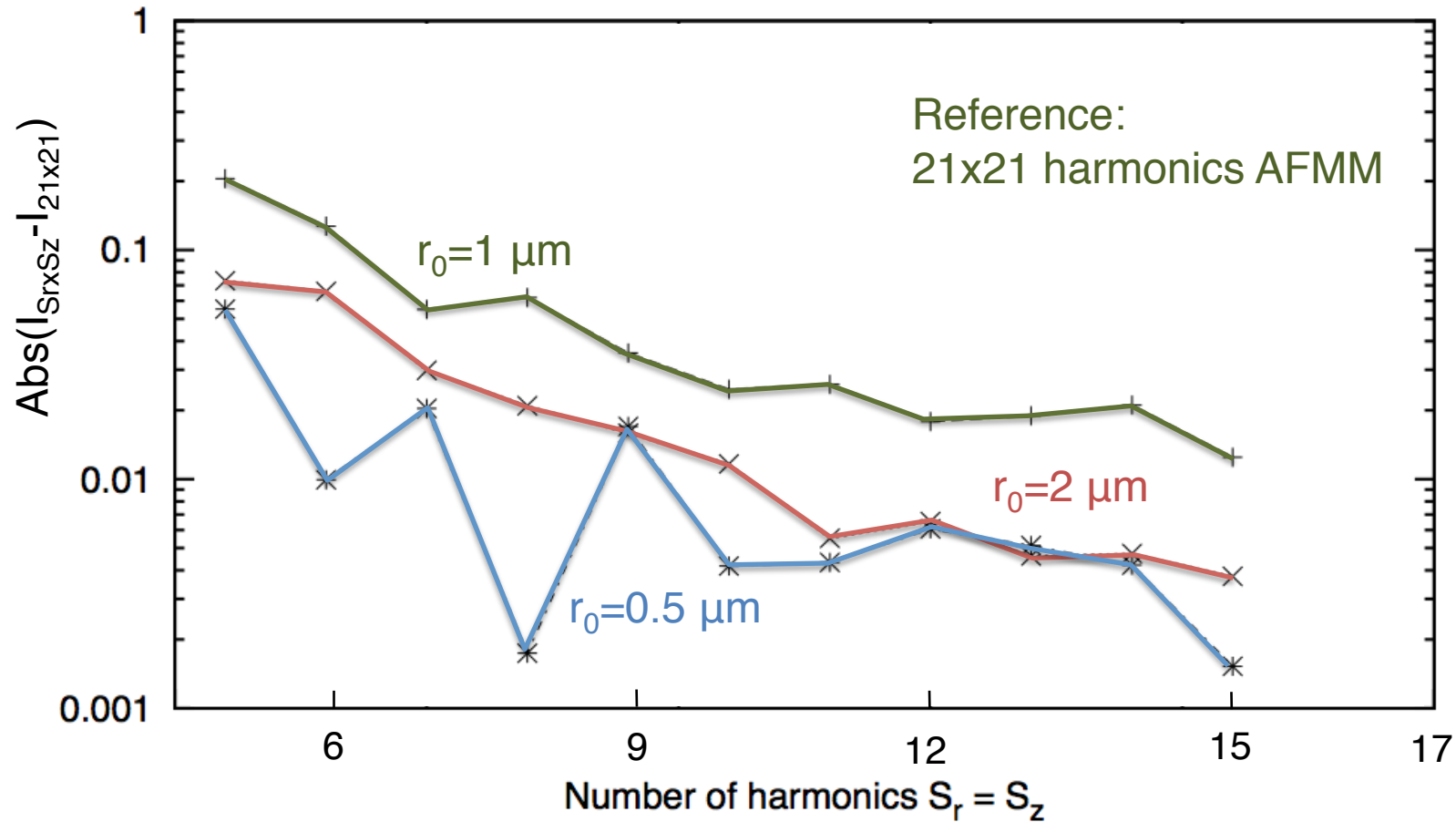
- Field  $|E_r|$  (xz cut):



- Estimation of losses:

$$I = \frac{\int_{\Sigma_{\text{input}}} |E_r(r, z, \theta_{\text{input}})|^2 d\Sigma}{\int_{\Sigma_{\text{output}}} |E_r(r, z, \theta_{\text{output}})|^2 d\Sigma}$$

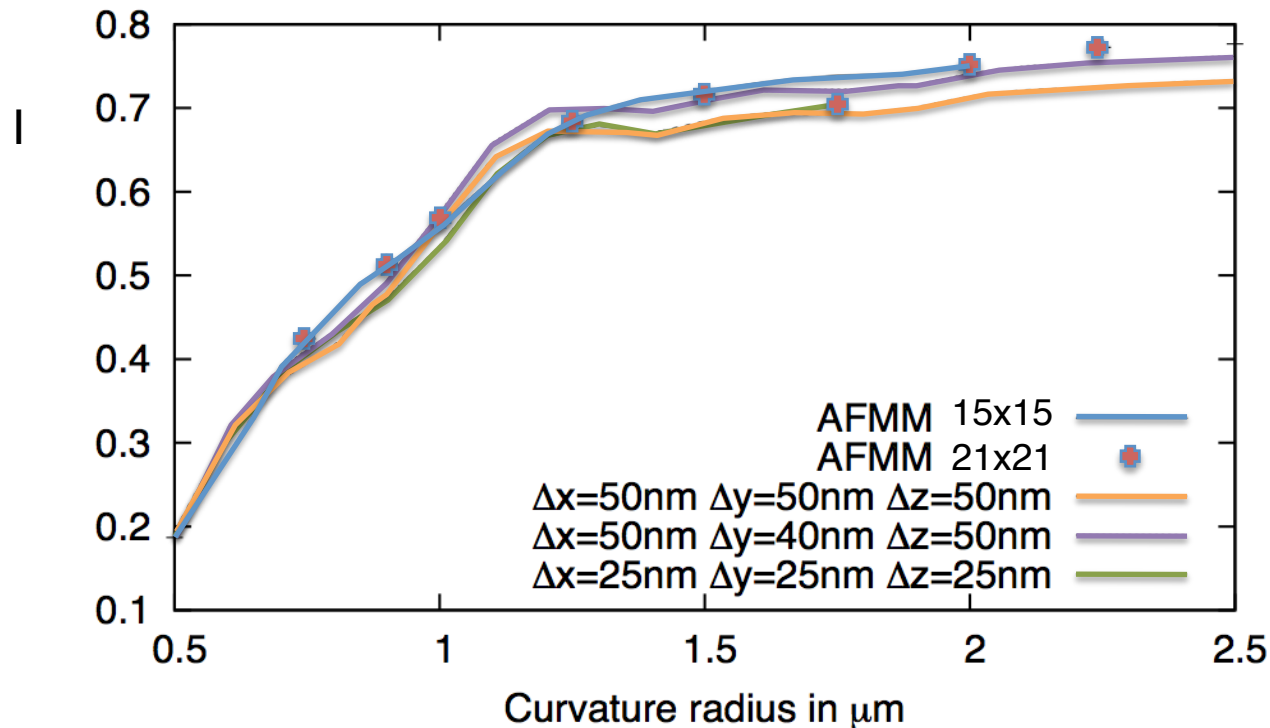
# AFMM Convergence



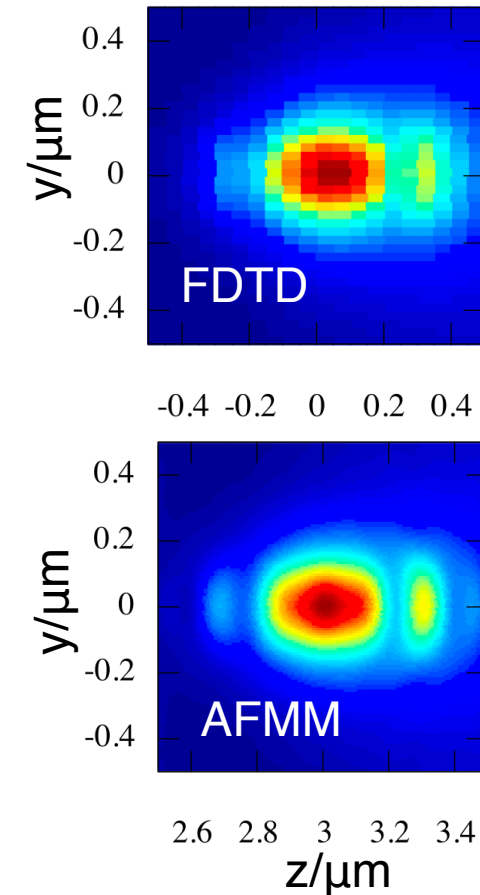
Semi-empirical matrix developments  
[Lalanne, JOSA A 1997]

Is  $I_{21 \times 21}$  correct?

# AFMM vs. FDTD



[Bucci, Martin, Morand, JOSA A 2012]



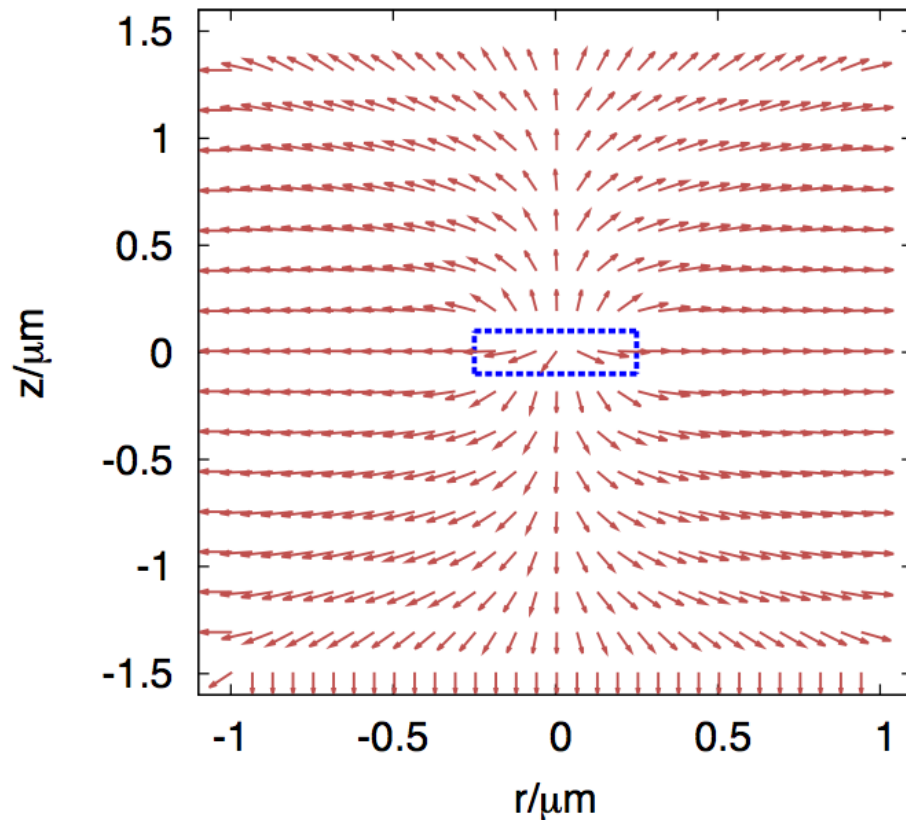
AFMM: constant calculation time  
 FDTD: calculation time increases with  $r_0$

Good agreement FDTD-AFMM

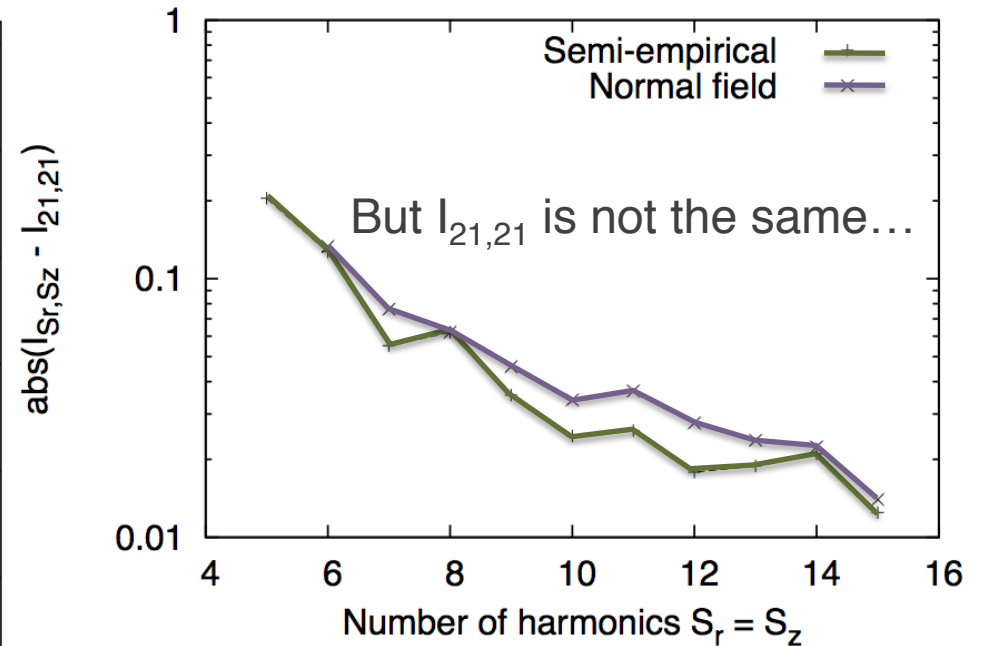
# Current work...

- “Normal field” vectors

[Schuster et al., JOSA A 2007]



- Convergence ( $r_0=1 \mu\text{m}$ )



... still some test to do...

# Conclusion

- Perspectives:
    - Convergence issue
    - Asymmetric PMLs
    - Mathematical implications
    - Symmetry
    - Parallelization
    - Testing...
  - To conclude...
    - Advantages:
      - Rigorous
      - Full vectorial approach
      - Complementary to FDTD and BPM
    - Disadvantages:
      - Black box approach
      - Memory requirements
- ...

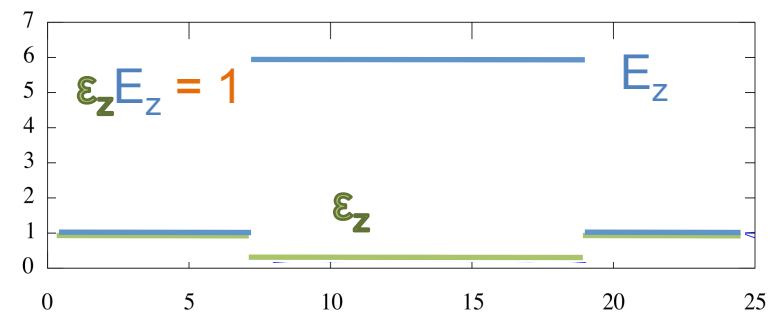


Thank you!

# Convergence issues for discontinuous field

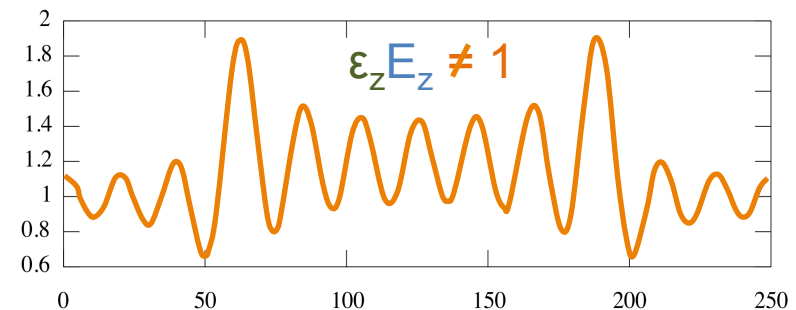
$$-j\beta_s[H_r] = K_r \left[ -\frac{1}{j\omega} \llbracket r \rrbracket \llbracket \mu_\theta^{-1} \rrbracket (K_z[E_r] - K_r[E_z]) \right] - j\omega \llbracket \epsilon_z \rrbracket \llbracket r \rrbracket [E_z]$$

- If  $\epsilon_z$  is discontinuous:
  - Field  $E_z$  discontinuous
  - $D = \epsilon_z E_z$  continuous!



- Pb. with truncated Fourier series!

$$\llbracket \epsilon_z \rrbracket [E_z]$$



# Solution: inverse rule (Lalanne, Li, 1996)

- Direct (Laurent) rule:

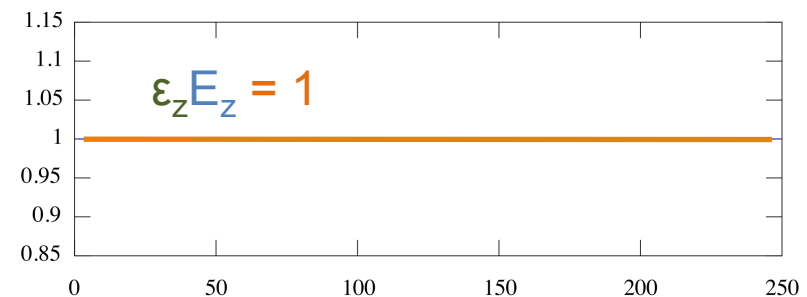
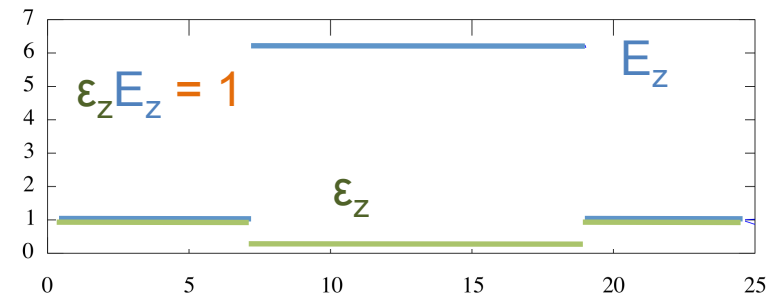
$$\left[ \epsilon_z \right] \left[ E_z \right]$$

- Inverse (Li) rule:

$$\left[ \frac{1}{\epsilon_z} \right]^{-1} \left[ E_z \right]$$

- The same for an infinite number of harmonics

- Not the same for truncated series:

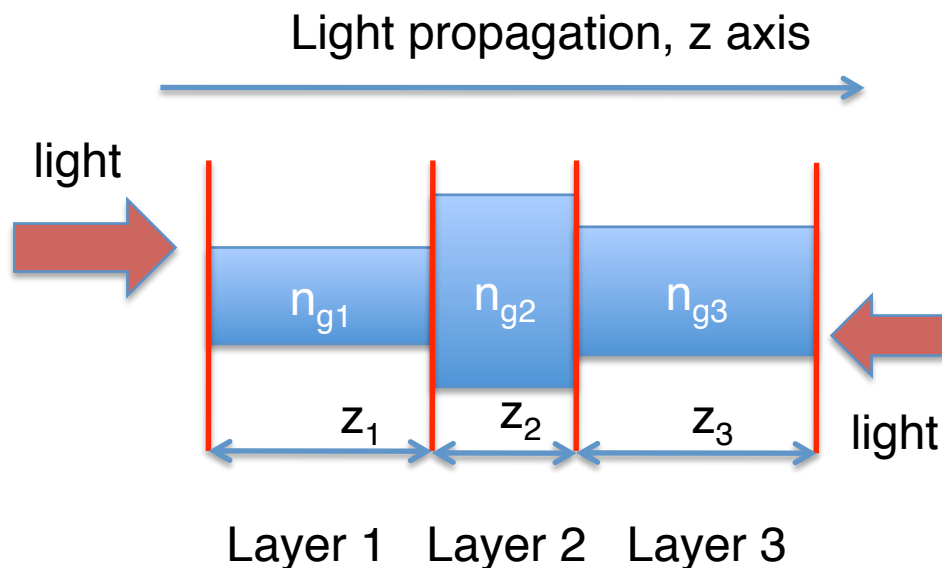


Need to know where the field is discontinuous!



# Propagation

- Structure: sequence of “layers”



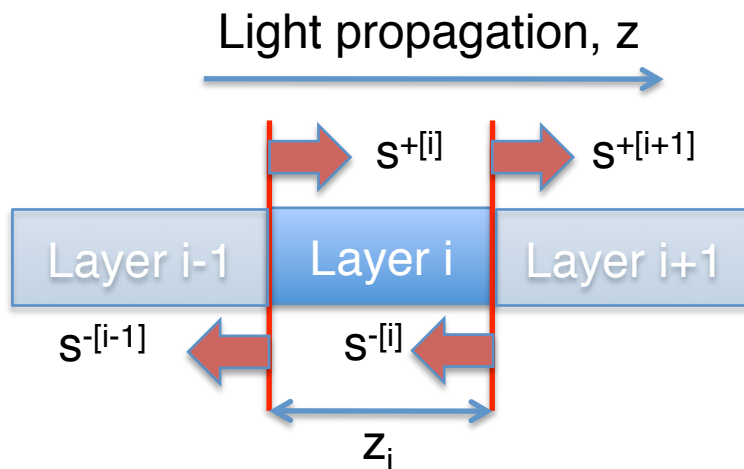
- Each layer:
  - Invariant in the z axis
  - Can be treated as seen before!

$A_{zi}$ : propagator operator layer  $i$

Eigenvalues/vectors  $A_{zi}$ :  
Diagonalization via  $W$ !

# S-matrix formalism

- Representation



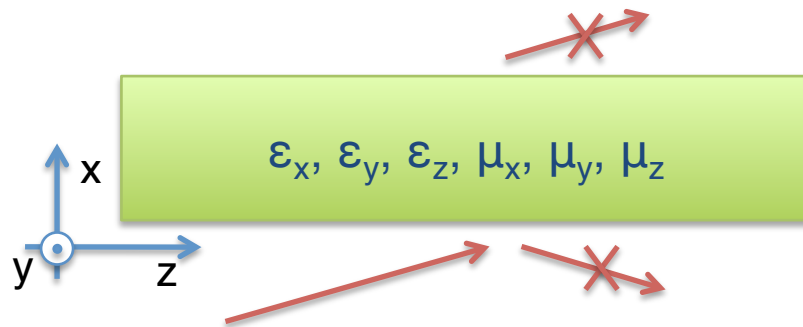
$$\begin{pmatrix} s^{+[i+1]} \\ s^{-[i-1]} \end{pmatrix} = S_i \begin{pmatrix} s^{+[i]} \\ s^{-[i]} \end{pmatrix}$$

- Steps:

1. E.vectors + E, H continuity:  $S_i$  for each layer
2. Iterative calculation: total S matrix
3. Calculation of  $s^{+,-}$  for each layer
4. Field propagation inside each layer

# PML strategies

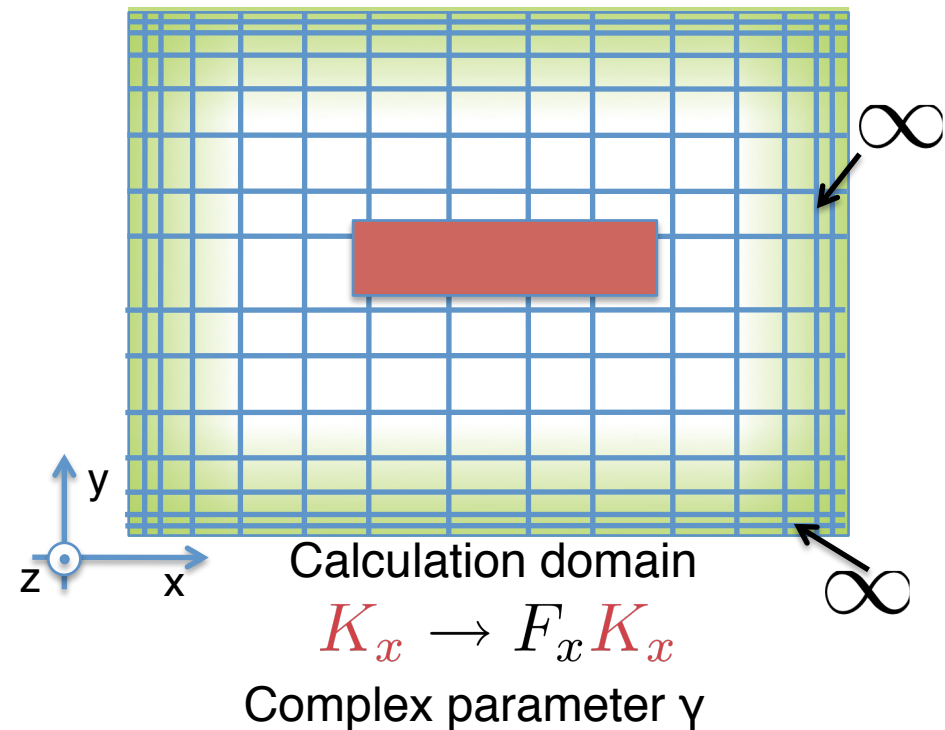
## I. Anisotropic material



- Th. no reflection
- Absorption

[E. Silberstein et al., JOSAA (2001)]

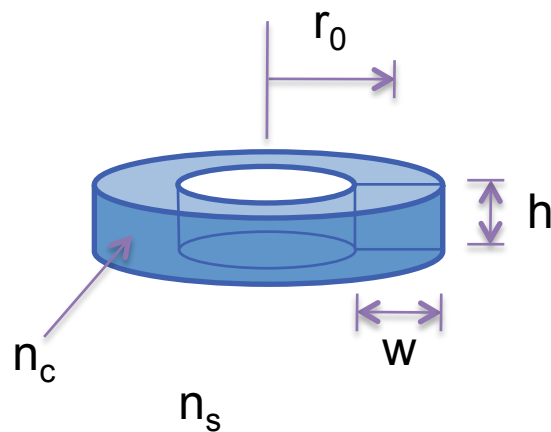
## II. Coordinate transform



[J.P Béranger, J. Comput. Physics (1994)]

# Ring resonator: full vectorial 2D

- Silicon micro-ring in silica
  - [Armaroli et al., JOSA A 2008]
  - $n_c = 3.48$ ,  $n_s = 1.44$ ,  
 $r_0 = 1.1 \mu\text{m}$
  - $w = 200 \text{ nm}$ ,  $h = 300 \text{ nm}$



- Goals
  - Mode solver:  $n_{\text{eff}}$
  - Find resonance; integer azimuthal order  $m$

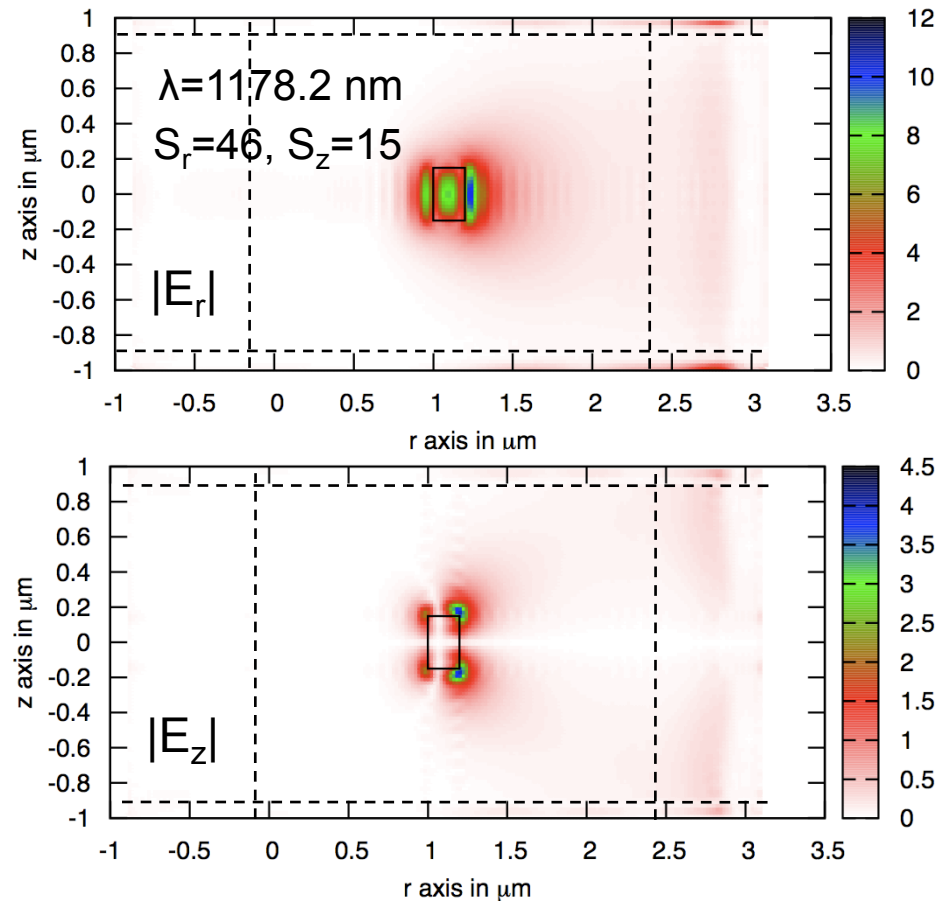
$$m = \frac{2\pi}{\lambda} r_0 \mathbb{R}\{n_{\text{eff}}\}$$

- Quality factor at the resonance:

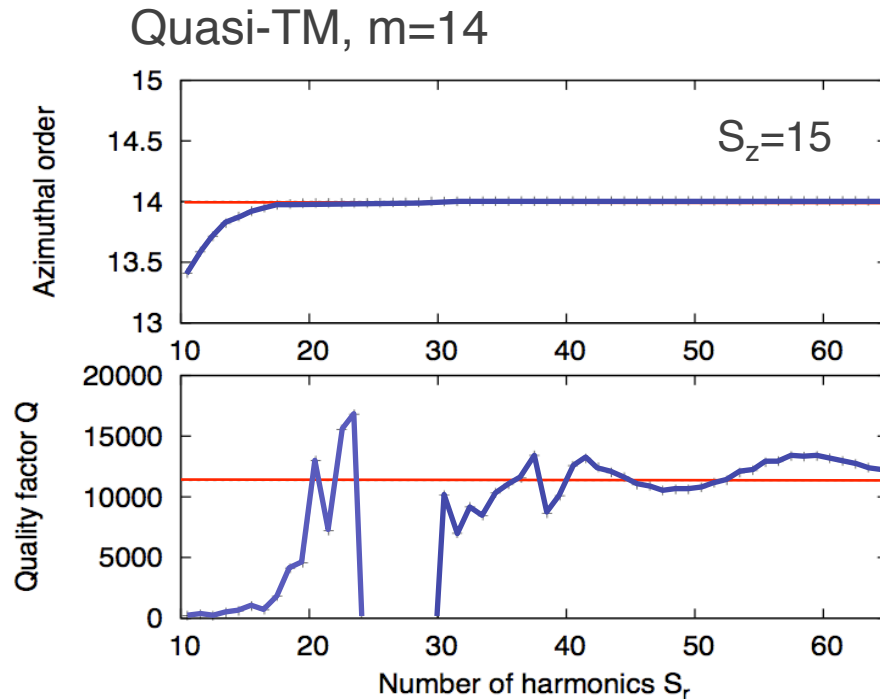
$$Q = -\frac{\mathbb{R}\{n_{\text{eff}}\}}{2\mathbb{I}\{n_{\text{eff}}\}}$$

# Mode calculation

- General setup
  - Calculation window:  
 $4 \mu\text{m} \times 2 \mu\text{m}$
  - Coordinate transform  
PMLs (each side):  
750 nm on r  
100 nm on z  
 $\gamma = 0.5 - j0.5$   
[Hugonin and Lalanne, JOSA A 2005]
  - Convergence study
  - “naïve” matrix  
developments



# High Q resonances



- Observations:

- Trade off:  
calculation window size



- number of harmonics

- PMLs are critical!

- Q factors of ~12,000  
attainable

[Bucci, Martin, Morand SPIE PW2010]

**3D-FDTD[\*]**

**AFMM**

$\lambda = 1.1770 \mu\text{m}$      $\lambda = 1.7929 \mu\text{m}$

Q = 11369            Q = 12856

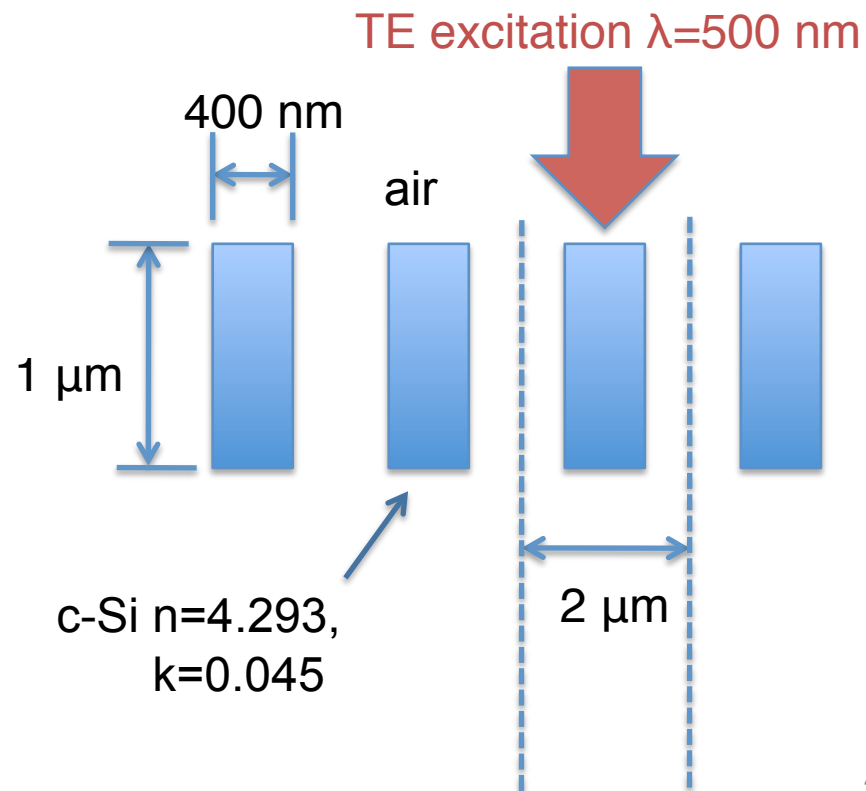
\*[Armaroli et al., JOSAA 2008]

# 2D example: solar cells

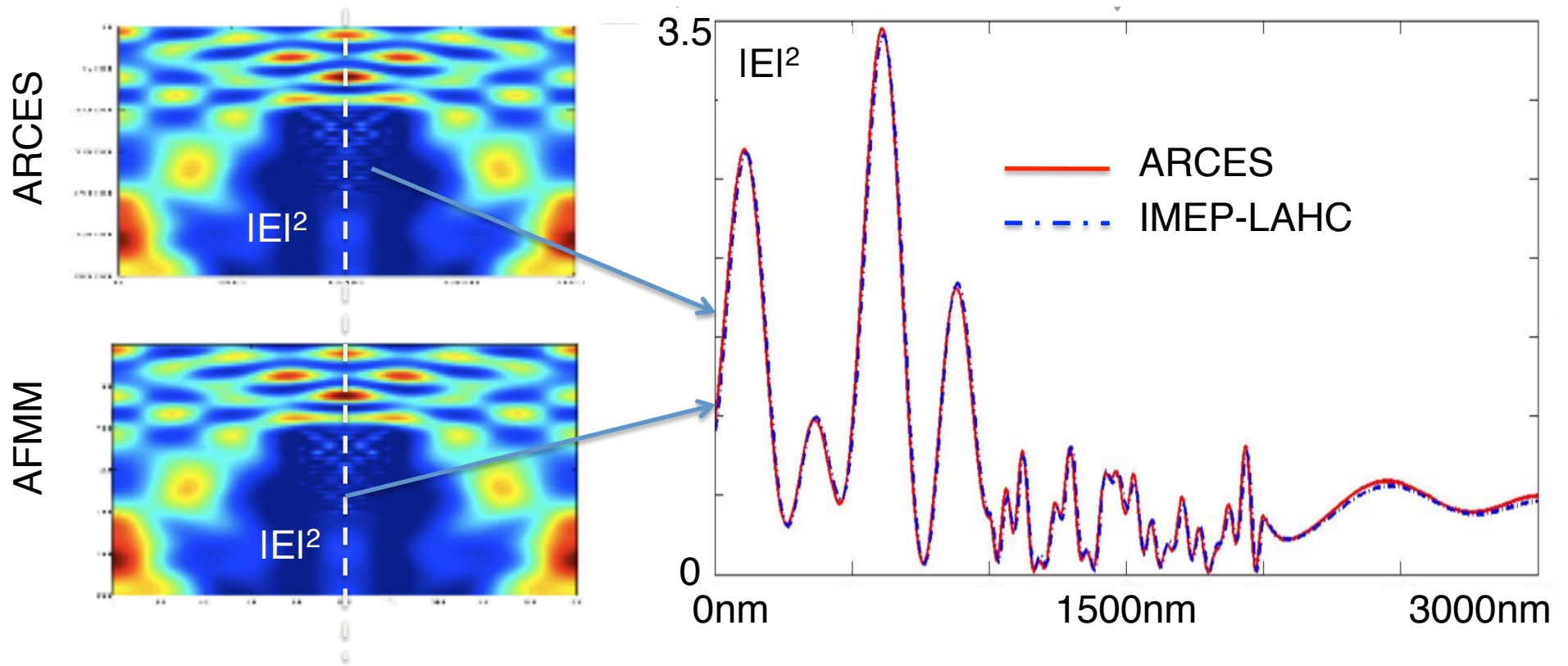
- IMEP-LAHC
  - A. Kaminski
  - D. Roulier (Phelma)
  - J Michallon

- ARCES-UNIBO  
(RCWA+finite diff.)
  - M. Zanucoli
  - C. Fiegna

- Microstructured solar cell (2D nanowires)



# Comparisons



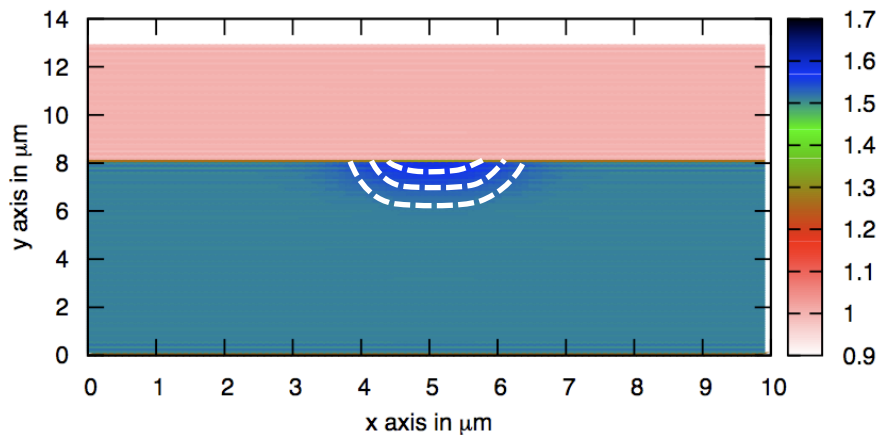
[Courtesy of M. Zanucoli]

- Good agreement!



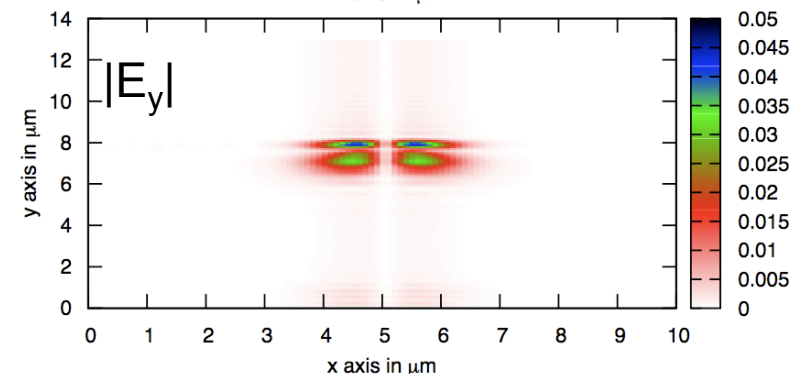
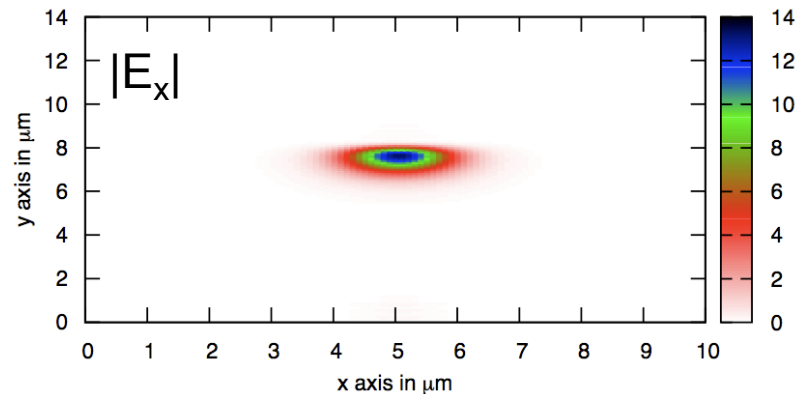
# Exemple: mode solver for ion-exchange on glass

- Diffusion => smooth



- At  $\lambda = 600$  nm,
  - 3 quasi-TE modes
  - 3 quasi-TM modes

- Fundamental: quasi TE



$\Delta n_{\text{eff}} 10^{-3}$  to  $10^{-4}$  with  
OptiBPM solver