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A New Extraction of Pion Parton Distributions in the Statistical Model

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Abstract

We present a new analysis to extract pion's parton distribution functions (PDFs) in the framework of the statistical model. Starting from the statistical model framework first developed for the spin-1/2 nucleon, we apply appropriate modifications taking into account the spin-0 nature of pion and the isospin and charge-conjugation symmetry properties. This results in a significant reduction of the number of parameters compared to a recent work to extract pion's PDFs. Using π^--induced Drell-Yan data to determine the parameters of this statistical model approach, we show that a good description of these experimental data with Next-to-Leading order QCD calculations can be obtained. Good agreement between the calculations and the π^+/π^- Drell-Yan cross section ratio data, not included in the global fit, has confirmed the predictive power of these new pion PDFs.

Key words: Pion parton distributions, Statistical approach, Drell-Yan process
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The first determination of the proton parton distribution functions (PDFs), based on the framework of the statistical model, was proposed about 20 years ago by Bourrely, Buccella, and Soffer [1]. Some salient features of the statistical approach include the natural connection between the polarized and unpolarized parton distributions, as well as the relationships between the valence and sea quark distributions. These important features of the statistical approach allow many predictions for the flavor and spin structures of proton's PDFs, which are usually not possible for the conventional global fits without adding theoretical constraints. Some notable successes of the statistical model include the prediction of x distribution of the flavor asymmetry of unpolarized sea, $\bar{d}(x) - \bar{u}(x)$, and the prediction of $\Delta\bar{u}(x) > 0 > \Delta\bar{d}(x)$ [2]. These predictions were confirmed in recent unpolarized Drell-Yan experiment [3] and single-spin asymmetry measurement of W-boson production [4]. A review of the major results on describing the nucleon parton distributions in the statistical model can be found in [5].

The statistical approach for extracting proton's PDFs can be naturally extended to other hadrons. Of particular interest are the PDFs for pions. Pion has the dual roles of being the lightest quark-antiquark bound state and a Goldstone boson due to the spontaneous breaking of the chiral symmetry. Many theoretical models have explored the partonic structures of the pion [6]. Recent advance [7] in lattice QCD has also led to the first calculations on the x distribution of the meson PDFs in the Large-Momentum Effective Theory (LaMET) [8, 9]. New experimental data relevant to the pion PDFs have been collected in the COMPASS experiment with pion-induced dimuon production [10]. The interesting prospects of probing pion's PDFs with tagged deep-inelastic scattering are being pursued at the Jefferson Laboratory and considered for the future Electron-Ion Collider (EIC) [11]. The interest in pion's partonic structure is reflected in several recent publications [12, 13, 14] where pion's PDFs were extracted via global fits to existing data.

The first extraction of the pion's PDFs based on the statistical model approach was reported in [13]. In this paper we adopt a much simpler parametrization for the functional forms of the parton distributions by imposing some constraints based on symmetry principles. We first define the parametrizations of the pion's quark, antiquark and gluon distributions based on the statistical model. We then show that the existing pion-induced Drell-Yan data E615 [15], NA10 [16] and E326 [17] can be very well described with these new pion PDFs. A prediction for the ratio π^+/π^- is also shown, followed by conclusion.

In the previous analysis to extract pion's PDFs in the statistical model [13], no assumption was made on the flavor and spin structures of the quark and antiquark distributions in pion. While this led to a significant flexibility in determining pion's PDFs, the limited amount of relevant experimental data would limit the ability to determine these flexible parametrizations in an unambiguous fashion. It is advantageous to reduce the number of parameters, taking into account some symmetry principles and other plausible assumptions. However, it was unclear whether the experimental data can be well described by the statistical model when the number of parameters is significantly reduced. The findings of this work, to be discussed later, are that an excellent description of the data can be obtained in the statistical model approach when the parameters are reduced in a judicious fashion.

We begin by defining the notations of the various parton distribution functions for pions. After imposing the particle-antiparticle charge-conjugation (C) symmetry for the parton distributions in charged pions, we can define the PDFs in π^+ and

π^- as follows:

$$U(x) \equiv u_{\pi^+}(x) = \bar{u}_{\pi^-}(x); \quad D(x) \equiv \bar{d}_{\pi^+}(x) = d_{\pi^-}(x) \,. \quad (1)$$

$$\bar{U}(x) \equiv \bar{u}_{\pi^+}(x) = \bar{d}_{\pi^-}(x); \quad \bar{D}(x) \equiv d_{\pi^+}(x) = u_{\pi^-}(x) \,. \quad (2)$$

$$S(x) \equiv s_{\pi^+}(x) = \bar{s}_{\pi^-}(x); \quad \bar{S}(x) \equiv \bar{s}_{\pi^+}(x) = s_{\pi^-}(x) \,. \quad (3)$$

$$G(x) \equiv g_{\pi^+}(x) = g_{\pi^-}(x) \,. \quad (4)$$

In Eqs. (1)-(4), we define 7 PDFs, namely, $U(x)$, $D(x)$, $\bar{U}(x)$, $\bar{D}(x)$, $S(x)$, $\bar{S}(x)$ and $G(x)$. We can further require charge symmetry (CS), which is a weaker form of the isospin symmetry, to reduce the number of independent PDFs. The CS refers to the invariance under a rotation by 180° along the second axis in the isospin space. We note that in the previous statistical model analysis [13], the C symmetry was imposed but the CS was not required. While it is of great interest to test the validity of CS in pion PDFs, the existing data are not sensitive to violation of CS, as discussed in a review of the theories and experiments on CS at the partonic level [18]. Therefore, the CS is now required for the present work. This requirement can be relaxed in the future when precision data sensitive to CS in pion PDFs become available.

It is well known that the requirements of C and CS invariance would imply $U(x) = D(x)$ and $\bar{U}(x) = \bar{D}(x)$. As shown in Eq. (1), C symmetry leads to $u_{\pi^+}(x) = \bar{u}_{\pi^-}(x)$. Invariance under the rotation in the isospin space by 180°, i.e., CS invariance, would give $\bar{u}_{\pi^-}(x) = \bar{d}_{\pi^+}(x)$. Therefore, invariance under the combined operations of C and CS implies $U(x) = D(x)$. In a similar fashion, it can be readily shown that $\bar{U}(x) = \bar{D}(x)$.

The absence of valence strange quark in pion only implies that the first moments of $S(x)$ and $\bar{S}(x)$ are the same, namely,

$$\int_0^1 [S(x) - \bar{S}(x)]dx = 0; \quad \int_0^1 S(x)dx = \int_0^1 \bar{S}(x)dx. \quad (5)$$

Using C and CS invariance, it can be shown that $S(x) = \bar{S}(x)$ in Eq. (3). First, the C invariance implies $s_{\pi^+}(x) = \bar{s}_{\pi^-}(x)$. A subsequent rotation in the isospin space would give $\bar{s}_{\pi^-}(x) = \bar{s}_{\pi^+}(x)$, because s and $\bar{s}$ are isoscalar particles and invariant under isospin rotation. Therefore, we obtain

$$S(x) = \bar{S}(x) \quad (6)$$

as a result of the invariance under a combined operation of C and CS. In Ref. [13] the strange quark contents in pion were ignored. In this work, we include the contributions from strange sea, as they must be present and contribute to the momentum sum rule.

In the statistical approach for proton's PDFs, there are positive and negative helicity distributions for each quark and antiquark flavor. Unlike the spin-1/2 proton, pion has zero spin. Hence, it is no longer necessary to define the positive and negative helicity distributions for pion's PDFs. This further simplifies the analysis compared with the earlier work [13], and it is reflected in Eqs. (1)-(3) which only define a single parton distribution for each quark or antiquark flavor.

Based on the framework of the statistical model, we adopt the following functional forms for pion's PDFs:

$$xU(x) = xD(x) = \frac{A_U X_U x^{b_U}}{\exp[(x - X_U)/\bar{x}] + 1} + \frac{\tilde{A}_U x^{\tilde{b}_U}}{\exp(x/\bar{x}) + 1} \,. \quad (7)$$

$$x\bar{U}(x) = x\bar{D}(x) = \frac{A_U (X_U)^{-1} x^{b_U}}{\exp[(x + X_U)/\bar{x}] + 1} + \frac{\tilde{A}_U x^{\tilde{b}_U}}{\exp(x/\bar{x}) + 1} \,. \quad (8)$$

$$xS(x) = x\bar{S}(x) = \frac{\tilde{A}_U x^{\tilde{b}_U}}{2[\exp(x/\bar{x}) + 1]} \,. \quad (9)$$

$$xG(x) = \frac{A_G x^{b_G}}{\exp(x/\bar{x}) - 1} \,. \quad (10)$$

Following the formulation developed for proton's PDFs, the x distributions for fermions (quark and antiquark) have Fermi-Dirac functional form, while gluon has a Bose-Einstein x distribution. The two separate terms for $U(x)$ and $\bar{U}(x)$ in Eqs. (7) and (8) refer to the non-diffractive and diffractive contribution, respectively. In the previous analysis [13], the diffractive term was neglected for simplicity. As shown in the analysis of proton's PDFs in the statistical model [2], the presence of the diffractive term is important for describing the data at the low x region. Therefore, we have added the diffractive term in this new analysis for pion's PDFs.

A key feature of the statistical model is that the chemical potential, X_U, for the quark distribution $U(x)$ becomes $-X_U$ for the antiquark distribution $\bar{U}(x)$. The parameter $\bar{x}$ plays the role of the effective "temperature". For the strange-quark distribution $S(x)$, the requirement that the S and $\bar{S}$ have identical x distribution implies that the chemical potential in the non-diffractive term must vanish. This implies that the non-diffractive and diffractive terms for $S(x)$ have a similar functional form, and we make the simple assumption that $S(x)$ is equal to half of the diffractive part of $\bar{U}(x)$ due to the heavier strange quark mass.

Equations (7)-(10) contain a total of 8 parameters, namely, A_U, X_U, b_U, $\bar{x}$, $\tilde{A}_U$, $\tilde{b}_U$, A_G, and b_G. In contrast, the number of parameters for Ref. [13] is significantly larger, at 14, even without including strange-quark sea in the pion PDFs.

Among these 8 parameters, only 6 are truly free, due to the constraints from two sum rules. The quark-number sum rule requires

$$\int_0^1 [U(x) - \bar{U}(x)]dx = 1 \,, \quad (11)$$

and the momentum sum rule implies

$$\int_0^1 x[2U(x) + 2\bar{U}(x) + 2S(x) + G(x)]dx = 1 \ . \quad (12)$$

Since the Drell-Yan data are not sensitive to pion's gluon distribution, it is mainly through this momentum sum rule that $G(x)$ is determined. The strong correlation between A_G and b_G can be removed by requiring

$$b_G = 1 + \tilde{b}_U \ . \quad (13)$$

Equation (13) has the interesting consequence that $G(x)$ has the same x dependence as the diffractive part of the quark distributions when $x \to 0$. The dominance of the gluon and sea-quark distributions at $x \to 0$ and the strong interplay among them make Eq. (13) quite a reasonable assumption. Equation (13) further reduces the number of parameters by one. The significant reduction in the total number of free parameters in the statistical model, resulted from the application of symmetry constraints and Eq. (13) discussed above, allows a stringent test of the statistical model for describing pion's PDFs. It is not evident *a priori* that existing data can be well described by the statistical model with very limited number of parameters.

In order to obtain the parameters for pion's PDFs according to the parametrizations listed in Eqs. (7) - (10), we have performed a NLO QCD fits of π^--induced Drell-Yan dimuon production data on tungsten targets from E615 [15] at 252 GeV, E326 [17] at 225 GeV, and NA10 [16] at 194 GeV and 286 GeV. Details of the NLO calculations were described in [13]. The QCD evolution was done using the HOPPET program [19], and the χ^2 minimization was performed utilizing the CERN MINUIT program [20].

Table 1 lists the number of data points and the values of χ^2 obtained from the best fit to these data sets. Note that the normalizations for the absolute cross sections from various experiments contain systematic uncertainties on the order of ~ 10 percents. In the global fit, the normalizations for various data sets are allowed to vary, as listed in Table 1, in order to achieve improved consistency among various data sets.

	P(GeV)	K	N_{data}	χ^2
E615	252	1.066	91	123
E326	225	1.223	50	75
NA10	286	1.2928	23	8
NA10	194	1.2928	44	17
Total			208	223

Table 1: Results of the K factor and χ^2 for each data set from the global fit. P is the beam momentum, K the normalization factor for the Drell-Yan cross sections, N_{data} the number of data points.

Table 1 shows that good χ^2 values can be obtained from the best fits to the Drell-Yan data. The best-fit parameters, obtained at an initial scale $Q_0^2 = 1$ GeV^2, are:

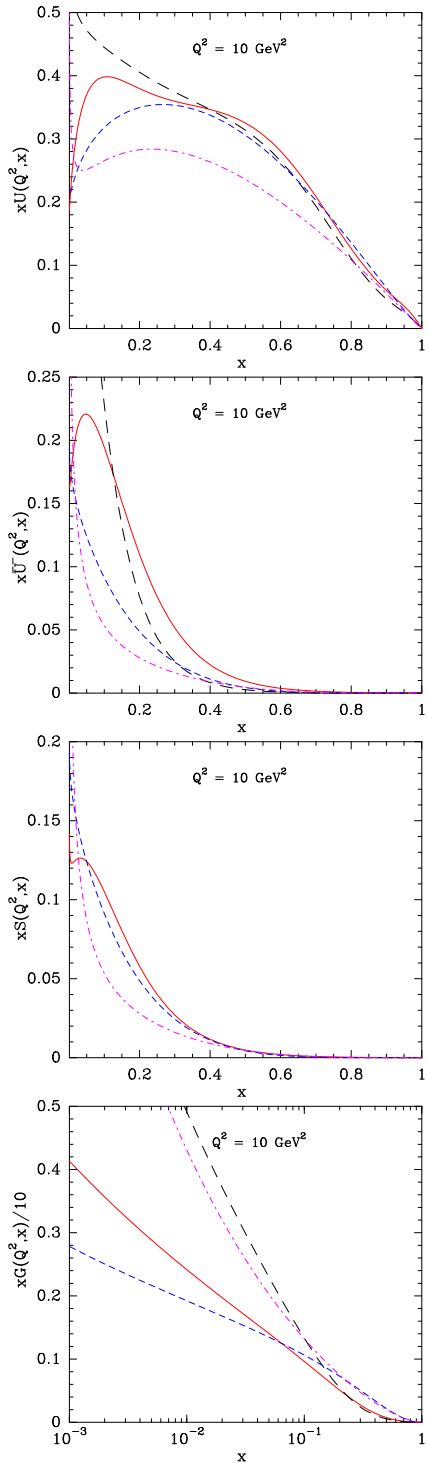

Figure 1: Different π^- parton distributions versus x, after NLO QCD evolution at $Q^2 = 10$ GeV^2. Present statistical model (solid), previous statistical model [13] (long dashed), SMRS PDFs from Ref. [21] (dashed), GRV PDFs from Ref. [22] (dotted-dashed), are shown. For SMRS and GRV $x\bar{S}(x) = x\bar{U}(x)$.

$$
\begin{aligned}
A_U &= 0.80536 \pm 0.10 & b_U &= 0.5161 \pm 0.02 \\
X_U &= 0.7551 \pm 0.01 & \bar{x} &= 0.10614 \pm 0.004 \\
\tilde{A}_U &= 2.2773 \pm 0.324 & \tilde{b}_U &= 0.4911 \pm 0.0092 \\
A_G &= 31.0019 \pm 1.68 & b_G &= 1 + \tilde{b}_U \ . \qquad (14)
\end{aligned}
$$

It is worth noting that the temperature, $\bar{x} = 0.106$, found for pion is very close to that obtained for proton, $\bar{x} = 0.090$ [5], indicating a common feature for the statistical description for baryons and mesons. On the other hand, the chemical potential of the valence quark for pion, $X_U = 0.7551$, is significantly large than that for proton, $X_U \sim 0.39$ [5]. This reflects the fact that baryons contain three valence quarks while mesons only consist of two valence quarks.

Figure 1 displays $xU(x)$, $x\bar{U}(x)$, $xS(x) = x\bar{S}(x)$, and $xG(x)$ at $Q^2 = 10$ GeV^2 obtained in the present analysis. Comparisons with the distributions from the previous analysis in the statistical model [13] and global fits of SMRS [21] and GRV [22] are also shown in Fig. 1. The shape and magnitude of the pion PDFs obtained in the statistical model analysis are significantly different from that of SMRS and GRV. This reflects the very different functional forms for the PDFs in the statistical model compared with that of the conventional global fits.

In Figures 2 - 4, we show the fits to the E615 and NA10 data using the current result on pion's PDFs. We note that the data are well described by the statistical model with a parametrization much simpler than the previous analysis [13]. A good agreement is also obtained for the E326 data at 225 GeV [17].

To check the predictive capability of the current pion PDFs, we show in Figure 5 the calculations for the ratios of π^+/π^- Drell-Yan cross section ratios at 200 GeV on hydrogen and platinum targets. Reasonable agreement with the NA3 data [23] is found. It was suggested some time ago that a comparison of the π^+ and π^- induced Drell-Yan data on an isoscalar target such as deuteron or ^{12}C can probe sensitively the sea-quark distribution in pion [24]. Such measurement is indeed being planned in a future experiment at CERN [25].

The Drell-Yan cross section at the NLO QCD contains the contribution from the quark-gluon fusion process, which leads to some sensitivity to the gluon distribution in pion. Nevertheless, it is important to consider other processes which are sensitive to gluon distributions at the LO QCD level. A recent analysis suggests that existing pion-induced J/Ψ production can probe the gluon distribution in pion at large x sensitively [26]. The prospect for including the π^+ induced Drell-Yan data as well as the pion-induced J/Ψ production data to extract the pion's PDFs in a future global fit with the statistical model is being considered. An extension of this approach to extract the kaon's PDFs would also be of great interest [27].

In conclusion, we have performed a new analysis to extract pion's PDFs in the statistical model using a parametrization containing fewer number of parameters than an earlier analysis. This significant reduction in the number of parameters is largely due to symmetry considerations. This new analysis with reduced number of parameters shows that a good description of

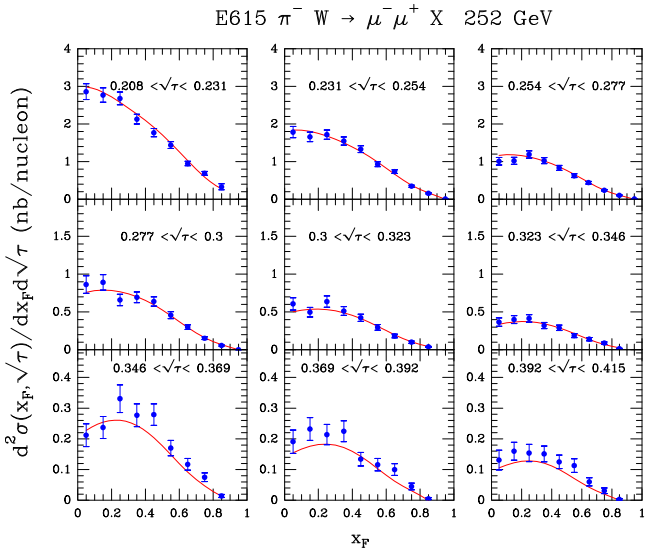

Figure 2: Drell-Yan data from the E615 experiment $\pi^- W$ at $P^{\pi}_{lab} = 252$ GeV [15]. $d^2\sigma/d\sqrt{\tau}dx_F$ versus x_F for several $\sqrt{\tau}$ intervals are compared with the results of our global fit (solid curves).

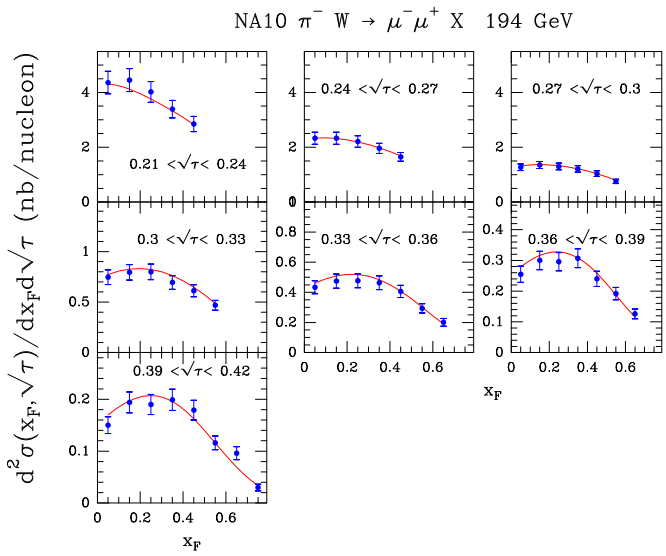

Figure 3: Drell-Yan data from the NA10 experiment $\pi^- W$ at $P^{\pi}_{lab} = 194$ GeV [16]. $d^2\sigma/d\sqrt{\tau}dx_F$ versus x_F for several $\sqrt{\tau}$ intervals are compared with the results of our global fit (solid curves).

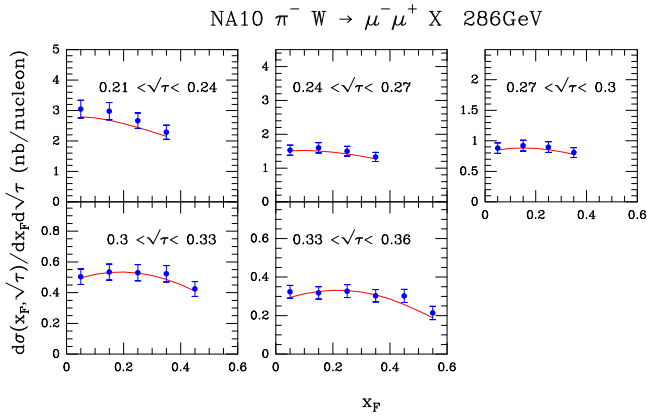

Figure 4: Drell-Yan data from the NA10 experiment $\pi^- W$ at $P^{\pi}_{lab} = 286$ GeV [16]. $d^2\sigma/d\sqrt{\tau}dx_F$ versus x_F for several $\sqrt{\tau}$ intervals are compared with the results of our global fit (solid curves).

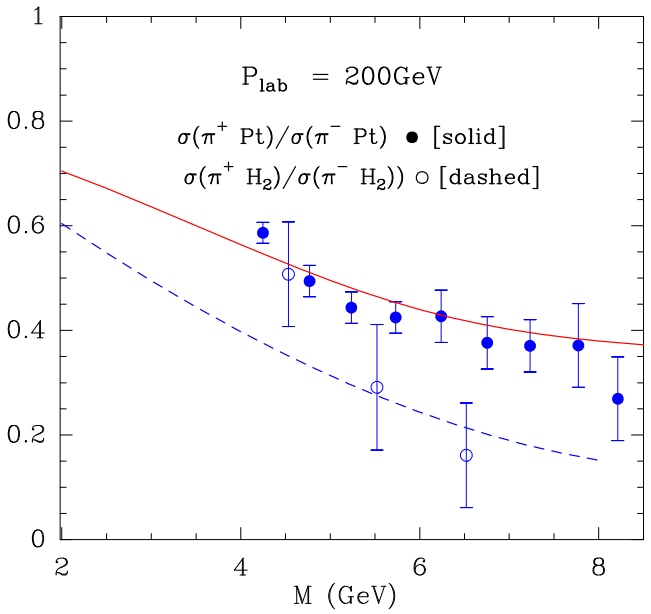

Figure 5: Drell-Yan π^+/π^- cross section ratio data on hydrogen and platinum targets at 200 GeV from the NA3 experiment [23]. Calculations using the pion PDFs obtained in the current analysis are compared with the data.

the existing π^--induced Drell-Yan data can be obtained in the statistical model approach.

A comparison between the proton and pion PDFs in the statistical model approach shows that very similar temperature parameters are found for both cases, suggesting the consistency of this appraoch for different hadronic systems. The higher value of the valence-quark chemical potential found for pion than proton reflects the different number of valence quarks in mesons and baryons. The predictive power of the pion PDFs obtained in this analysis has been illustrated from the good agreement between the calculation and the π^+/π^- Drell-Yan cross section ratio data. New pion-induced Drell-Yan data anticipated from COMPASS would provide further tests of the pion PDFs obtained in the statistical approach.

This statistical model approach can be extended in the future by enlarging the data sets to include the π^+-induced Drell-Yan data as well as the pion-induced J/Ψ production data. These data further constrain the sea-quark and gluon distributions in pion. The prospect to extend this analysis to extract kaon's PDFs is also being considered.

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