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# Equilibrium shape of core(Fe)-shell(Au) nanoparticles as a function of the metals volume ratio 

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#### Abstract

The equilibrium shape of nanoparticles is investigated to elucidate the various core-shell morphologies observed in a bimetallic system associating two immiscible metals, iron and gold, that crystallize respectively in the bcc and fcc lattices. Fe-Au core-shell nanoparticles present a crystalline Fe core embedded in a polycrystalline Au shell, with core and shell morphologies both depending on the $\mathrm{Au} / \mathrm{Fe}$ volume ratio. A model is proposed to calculate the energy of these nanoparticles as a function of the Fe volume, $\mathrm{Au} / \mathrm{Fe}$ volume ratio, core shape and shell shape, using the DFT-computed energy densities of the metal surfaces and of the two possible Au/Fe interfaces. Three driving forces leading to equilibrium shapes were identified: the strong adhesion of Au on Fe , the minimization of the $\mathrm{Au} / \mathrm{Fe}$ interface energy that promotes one of the two possible interface types, and the Au surface energy minimization that promotes a 2D-3D Stranski-Krastanov like transition of the shell. For low $\mathrm{Au} / \mathrm{Fe}$ volume ratio, the wetting is the dominant driving force and leads to the same polyhedral shape for the core and the shell, with an octagonal section. For large Au/Fe ratio, the surface and interface energy minimizations can act independently to form an almost cube-shaped Fe core surrounded by six Au pyramids. The experimental nanoparticles shapes are well reproduced by the model, for both low and large $\mathrm{Au} / \mathrm{Fe}$ volume ratios.


[^0]
## I. INTRODUCTION

Bimetallic nanoparticles (NPs) can be synthesized in different shapes, sizes and structures. For two immiscible metals, the chemical order can be core-shell, Janus or multi-shell [1-3]. These morphologies combined to size effects are at the origin of new or exalted properties compared to monometallic NPs. Bimetallics NPs are thus widely investigated for a variety of applications including catalysis, optics, magnetic recording, hyperthermia, magnetically driven drug delivery, bactericidal actions [1]. In the Fe-Au system, a core-shell order with iron at the core preserves the magnetic properties of the iron and protects it from oxidation thanks to the gold shell. The latter also ensured the NP biocompatibility. The preservation of the magnetic properties of the core requires to control its crystallinity, its size and the thickness of the protecting shell. As the adsorption of molecules depends of the crystalline orientation of gold [4], biological applications based on molecule binding also requires to manage the crystalline quality and orientation of the shell facets.

Predicting the NPs shapes on a rational basis is thus desirable, however it is challenging. Although the usual concepts of the crystal growth remain relevant, as those related to epitaxy, adhesion or elasticity, new models are required to understand the specific geometry of core@shell NPs, that cannot be directly deduced from the structural properties of the two components in their own phase or from the growth modes on a 2D substrate. The case where both core and shell are fcc crystals was deeply investigated |5, 6]. This contrasts with Fe@Au NPs, a study case where the Fe core and the Au shell crystallize respectively in the body-centered cubic lattice (bcc) lattice and facecentered cubic (fcc) lattice. Whereas remaining in a core-shell geometry, both core and shell morphologies of Fe@Au NPs strongly evolve as a function of the respective volumes of the two metals [7]. In this article, we aim at uncovering the mechanisms responsible for this important morphological evolution. As for some other nanostructures [8], the continuous approach adopted here provides a theoretical frame to explore the energy landscape as a function of the core and shell morphologies for NPs of any size, particularly in the typical experimental range (from 1 to 50 nm [7, 9-11]), and as such is a complementary approach to atomistic simulations.

In previous studies [7, 9] nanometric AuFe based NPs were formed at high temperature $\left(600^{\circ} \mathrm{C}-800^{\circ} \mathrm{C}\right)$ in an ultra-high vacuum growth process by the sequential deposition of Fe then Au . They exhibit a regular core@shell shape where Fe forms a single crystal with a polyhedral shape, entirely covered by a polycrystalline Au shell. Two morphologies are observed (in the following, we will systematically refer crystallographic notations to either Fe or Au nanocrystal):

In the NPs as that presented in cross-section in Fig. 1(a), the Fe core is close to a perfect <001>Fe oriented cube, and the Au shell consists essentially in six single crystals located on the
$\{001\} \mathrm{Fe}$ faces of the cube. Each of these six crystals is $\{001\} A u$ oriented and is constituted by a thin wetting layer surmounted by a truncated pyramid with 4 \{111\}Au sides. The dominant interfaces are $\{001\} \mathrm{Au} /\{001\} \mathrm{Fe}$ (green dashed lines in the crystallographic scheme).

In Fig. 1(b) the Fe core appears in cross-section as an octagon. The core can thus be described as a polyhedron limited by $\{001\} \mathrm{Fe}$ and $\{110\} \mathrm{Fe}$ faces. The shell has a homogeneous thickness without pyramids. Two interfaces coexist: the same $\{001\} \mathrm{Au} /\{001\} \mathrm{Fe}$ interface as in Fig. 1(a), and the $\{111\} \mathrm{Au} /\{110\} F e$ interface (red dashed lines in the scheme). The shell is thus formed by 18 different crystallites, $12<111>A u$ oriented on the $\{110\} F e$ faces and $6<001>A u$ oriented on the $\{001\} F \mathrm{Fe}$ faces (respectively filled in red and yellow in the scheme).


FIG. 1. Typical NPs for large (a) and small (b) shell/core volume ratio. High resolution transmission electron microscopy images (left) and corresponding crystallographic models (right). (c) In-plane epitaxial relationships for both interfaces.
The $\{001\} \mathrm{Au} /\{001\}$ Fe interface [7] presents a perfect epitaxial relationship thanks to a $45^{\circ}$ rotation of $\{001\} A u$ with respect to $\{001\} F \mathrm{Fe}$ (Fig. 1(c)). This orientation reduces the effective misfit to $0.6 \%$ (the lattice constant of bcc Fe and fcc Au being 0.28665 nm and 0.40784 nm respectively [12]) leading to a pseudomorphic growth of Au on Fe , without dislocations. For the $\{111\} \mathrm{Au} /\{110\} \mathrm{Fe}$ interface, both theoretical [13] and experimental [14] results agree for the so-called NishiyamaWasserman relationship [13] described in Fig. 1(c). This interface presents a high misfit of $23 \%$ along the $<112>A u / /<110>F e$ direction, while in the $<110>A u / /<001>F e$ direction it is only $0.6 \%$.
The shell facets are $\{001\} \mathrm{Au}$ and $\{111\} \mathrm{Au}$ (respectively green and red lines in Fig. 1), which are the most stable facets in fcc crystals as $\mathrm{Au}[15,16]$.
These two morphologies are highly reproducible and several NPs of various nanometric sizes similar to the ones displayed in Fig. 1(a) and 1(b) can be found in [7] (and supplemental material) and in [9]. The two morphologies are not size distributed, but NPs with well-formed pyramids as in Fig. 1(a) are associated to a much larger Au amount compared to Fe amount than the NPs with an octagonal section as in Fig. 1(b). The Au/Fe volume ratios are for instance estimated around 1.6 and 0.65 respectively for the NPs displayed in Fig. 1(a) and Fig. 1(b).
Core@shell NPs with the same crystallographic structure and combining an almost cubeshaped core topped by pyramids as in Fig. 1(a) were also reported in the literature. Thanks to in situ heating, spherical Fe@Au NPs grown at room temperature transformed into an almost cube-shaped core topped by Au pyramids which are more truncated than in our experiments [10]. A core with a shape close to a Fe cube topped by thin Au pyramids was also observed in large FeCo@Au NPs [11]. Note also that while the elastic strain relaxation can be at the origin of a strong asymmetry of the core position in the shell [5], this mechanism seems inoperant here due to the small misfit [9].
The deposition of pure Fe in our experimental setup leads to roughly rounded single crystals [7]. This shape is the Wulff equilibrium shape of bcc iron [17, 18]: $\{110\}$ and $\{100\}$ facets are dominant and the ratio of their surface energy densities is close to 1 , but they are truncated by differently oriented small facets, leading to an iron nanocrystal appearing rounded rather than faceted. We cannot access the detailed scenario of the NPs formation, but we can suppose that the Au shells grow on these nanocrystals.
To elucidate the Fe@Au NP morphology evolution, we investigate the energy landscape of the NPs as a function of their shape and of the $\mathrm{Au} / \mathrm{Fe}$ volume ratio. For this purpose, in section II we develop a model based on the observed morphologies and we present DFT calculations performed to access the energy densities of all surfaces and interfaces involved in the model. In particular, the energy of the complex $\{111\} \mathrm{Au} /\{110\} F \mathrm{Fe}$ interface, which is not available in the literature to our
knowledge, is computed. The wetting in the $\mathrm{Au} / \mathrm{Fe}$ system is predicted in section III in a theoretical frame and we compare the predicted growth mode of Au on a 2D infinite Fe substrate and on a 3D finite Fe nanosubstrate. In section IV, the equilibrium shape (ES) at constant NP volume is explored for a large Au/Fe volume ratio and a small one, and the driving forces leading to the ES, or in some cases to a metastable shape, are discussed. For this purpose, we analyze the pathways allowed in our geometrical model by the core and shell shape transformations. In section V, the ES evolution with $\mathrm{Au} / \mathrm{Fe}$ volume ratio is discussed and compared to experimental observations.

## II. MODEL

A geometrical model describing the NPs morphologies with only 4 independent parameters is proposed. As exploring the energy landscape of a NP requires the consideration of the energies of all surfaces and interfaces, their areas are calculated and DFT calculations of their energy densities are presented.

Note that the elastic energy due to the misfit and the grain boundary energy are not included (this will be justified later on in section V. E.) and that the NPs are considered as free of any external stress (In our experimental setup, the NPs are formed at the surface of an amorphous $\mathrm{Al}_{2} \mathrm{O}_{3}$ substrate with which they interact, leading to the Volmer-Weber growth mode of the NPs on the substrate [7]. As for Au on alumina [19], the adhesion of Fe@Au NPs on alumina is weak. It thus can be neglected in the determination of the NP equilibrium shape.)

## A. Geometrical model of Fe@Au NPs

The geometrical model (displayed in 3D in Fig. 2(a) and in cross-section in Fig. 2(b)) is designed to allow progressive transitions between the different observed shapes.

As only $\{110\} F e$ and $\{001\} F e$ faces are involved at interfaces, we modelled the core by a polyhedron limited by 12 hexagonal \{110\} and 6 square $\{001\}$ faces (truncated rhombic dodecahedron, in blue in Fig. 2). The respective extension of $\{110\} \mathrm{Fe}$ and $\{001\} \mathrm{Fe}$ faces can be modulated, respecting the cubic symmetry. The Fe core can thus be fully described by two parameters: the Fe volume $V_{F e}$, and the aspect ratio

$$
\begin{equation*}
R_{F e}=h_{110}^{F e} / h_{001}^{F e} \tag{1}
\end{equation*}
$$

where $h_{110}^{F e}$ and $h_{001}^{F e}$ are the distances from the core center to the $\{110\} F e$ and $\{001\} F e$ faces, respectively (Fig. 2(b)).


FIG. 2. Generic model of a Fe@Au NP. (a) 3D view of the Fe core and Au shell surfaces. (b) Cross-section. The Fe core shape (in blue) is defined by $R_{F e}=h_{110}^{F e} / h_{001}^{F e}$ (Eq. (1)). The shell is decomposed into a part homothetic to the Fe core (limited by the blue dash-dotted line) and 6 pyramids. The shell aspect ratio is defined by $f=1+\left(h_{111}^{A u} / h_{110}^{F e}\right)=1+\left(h_{001}^{A u} / h_{001}^{F e}\right)$. The platelets filled in red on the $\{110\} F e$ faces are $\mathrm{Au}<111>$ oriented crystals. The $\mathrm{Au}<001>$ oriented crystals are formed by the platelets on the $\{001\} \mathrm{Fe}$ surfaces (dark yellow) topped by square based pyramids (light yellow). The NP is built at the scale for $R_{F e}=1.2, V_{F e}=176 \mathrm{~nm}^{3}, f=1.16$ and $V_{A u} / V_{F e}=1.40$. The corresponding sizes are $h_{001}^{F e}=2.92 \mathrm{~nm}, h_{110}^{F e}=3.50 \mathrm{~nm}, h_{001}^{A u}=0.58 \mathrm{~nm}, h_{111}^{A u}=0.70 \mathrm{~nm}$ and $h_{p y r}^{A u}=1.19$ nm ( or $t=0.66$ ).

The Au shell is geometrically decomposed in a part homothetic to the core (limited in Fig. 2(b) by the interfaces on one side and by the blue dash-dotted line on the other side) and 6 square based pyramids. The Au shell consists in 18 crystallites distributed on the core according to the cubic symmetry: (i) Twelve are <111>Au oriented platelets grown on the \{110\}Fe faces with thickness $h_{111}^{A u}$ (in red in Fig. 2(b)). (ii) The six crystallites grown on the $\{001\} F e$ faces are $<001>A u$ oriented (in yellow in Fig. 2(b)); each of them consists in a platelet of thickness $h_{001}^{A u}$ topped by one square based pyramid with four \{111\}Au facets. The pyramid height is denoted $h_{p y r}^{A u}$.

The whole shell can be fully described by two parameters, the Au volume $V_{A u}$, and one aspect ratio to express its distribution between the homothetic shell and the pyramids. For convenience, we chose the aspect ratio $f$ that expresses the expansion from the Fe core (in blue) to the homothetic part of the shell (blue dash-dotted line). $f$ is defined by

$$
\begin{equation*}
f=1+\left(h_{111}^{A u} / h_{110}^{F e}\right)=1+\left(h_{001}^{A u} / h_{001}^{F e}\right) \tag{2}
\end{equation*}
$$

The Fe@Au NP can thus be fully described with only four independent parameters: $V_{F e}$ and three dimensionless parameters $V_{A u} / V_{F e}, R_{F e}$ and $f$.

## B. Useful geometrical quantities

Let us now display geometrical quantities derived from these four parameters. These quantities will be useful to describe a NP and to calculate all surface and interface energies involved in the NP energy.
(i) Core (face areas, volume)

The areas of one $\{001\} \mathrm{Fe}$ and one $\{110\} \mathrm{Fe}$ core faces (see Appendix A) are respectively

$$
\begin{gather*}
S_{001}^{F e}=4\left(h_{001}^{F e}\right)^{2}\left(\sqrt{2} R_{F e}-1\right)^{2}  \tag{3}\\
S_{110}^{F e}=4\left(h_{001}^{F e}\right)^{2}\left(\sqrt{2}-R_{F e}\right)\left[(3 \sqrt{2} / 4) R_{F e}-1 / 2\right] \tag{4}
\end{gather*}
$$

The Fe volume is found by decomposition of the core in 18 pyramids converging at its center:

$$
\begin{equation*}
V_{F e}=\left(6 h_{001}^{F e} S_{001}^{F e}+12 h_{110}^{F e} S_{110}^{F e}\right) / 3 \tag{5}
\end{equation*}
$$

It comes

$$
\begin{equation*}
h_{001}^{F e}=V_{F e}{ }^{1 / 3}\left(8-12 \sqrt{2} R_{F e}\left(\sqrt{2}-R_{F e}\right)^{2}\right)^{-1 / 3} \tag{6}
\end{equation*}
$$

Equation (6) allows to calculate $S_{001}^{F e}$ and $S_{110}^{F e}$ as a function of $V_{F e}$ and $R_{F e}$.
(ii) Part of the Au shell homothetic to the core (areas and volume)

The areas of the top facet of a <001>Au and of a <111>Au oriented platelet are respectively

$$
\begin{equation*}
S_{001}^{A u}=f^{2} S_{001}^{F e} \quad \text { and } \quad S_{111}^{A u}=f^{2} S_{110}^{F e} \tag{7}
\end{equation*}
$$

The volume of the homothetic part of the shell is

$$
\begin{equation*}
V_{A u}^{\text {homo }}=V_{F e}\left(f^{3}-1\right) \tag{8}
\end{equation*}
$$

(iii) Au pyramids (volume, facet areas, height and truncation rate)

To calculate the pyramid facet areas, we introduce for convenience the dimensionless truncation rate $t$ defined by

$$
\begin{equation*}
t=1-2 h_{p y r}^{A u} /\left(b_{001}^{A u} \tan \beta\right)=b_{001 p y r}^{A u} / b_{001}^{A u} \tag{9}
\end{equation*}
$$

where $\beta=54.7^{\circ}$ is the angle between the $\{001\} A u$ planes and the pyramid facets $\{111\}(\tan \beta=\sqrt{2})$, $h_{p y r}^{A u}$ is the pyramid height, $b_{001}^{A u}$ and $b_{001 p y r}^{A u}$ are the edges of the pyramid basis and of the top facet, respectively (Fig. 2(b)). Note that $t$ varying from 0 (complete pyramid) to 1 (no pyramid) provides a direct description of the pyramid shape. $b_{001}^{A u}$ is directly related to the basis area $S_{001}^{A u}$ by

$$
\begin{equation*}
b_{001}^{A u}=S_{001}^{A u^{1 / 2}}=2 f h_{001}^{F e}\left(\sqrt{2} R_{F e}-1\right) \tag{10}
\end{equation*}
$$

The volume of one pyramid $V_{A u}^{p y r}$ can be written as a function of $t$

$$
\begin{equation*}
V_{A u}^{p y r}=S_{001}^{A u^{3 / 2}}\left(1-t^{3}\right) /(3 \sqrt{2}) \tag{11}
\end{equation*}
$$

$t$ can therefore be written as

$$
\begin{equation*}
t=\left[1-6 V_{A u}^{p y r} /\left(\sqrt{2} S_{001}^{A u^{3 / 2}}\right)\right]^{1 / 3} \tag{12}
\end{equation*}
$$

As $V_{A u}^{p y r}$ is easy to write as a function of $V_{F e}, V_{A u} / V_{F e}$ and $f$

$$
\begin{equation*}
V_{A u}^{p y r}=\left(V_{A u}-V_{A u}^{\text {homo }}\right) / 6=\left[\left(V_{A u} / V_{F e}\right)+1-f^{3}\right] V_{F e} / 6 \tag{13}
\end{equation*}
$$

it comes from Eq. (3), (6) and (7) that $t$ only depends on $R_{F e}, V_{A u} / V_{F e}$ and $f$. The areas of one lateral $\{111\} A u$ facet $S_{111 p y r}^{A u}$ and of the top $\{001\} A u$ facet $S_{001 p y r}^{A u}$ are

$$
\begin{equation*}
S_{111 p y r}^{A u}=\left(1-t^{2}\right) S_{001}^{A u} /(4 \cos \beta) \quad \text { and } \quad S_{001 p y r}^{A u}=t^{2} S_{001}^{A u} \tag{14}
\end{equation*}
$$

It is important to keep in mind that once $R_{F e}, V_{A u} / V_{F e}$ and $f$ are fixed, there exists a unique distribution of the gold in the homothetic shell and the pyramids (there is a unique correspondence between $f$ and $t$ ). All the geometrical quantities defined here (distances, areas and volumes) depend on $V_{F e}, V_{A u} / V_{F e}, R_{F e}$ and $f$.

## C. Limits of the aspect ratios

As the core is a polyhedron limited by $\{001\} \mathrm{Fe}$ and $\{110\} \mathrm{Fe}$ faces within the cubic symmetry, $R_{F e}$ varies from $1 / \sqrt{2}$ (perfect rhombic dodecahedron built with $12\{110\} F$ lozenges), to $\sqrt{2}$ (perfect cube built with 6 \{001\}Fe squares) (Fig. 3(a)).

The limits of $f, f_{\min }$ and $f_{m a x}$, depend on a $V_{A u} / V_{F e}$ and $R_{F e}$ (see Appendix B). We also calculate the limits $t_{\min }$ and $t_{\max }$ of the pyramid truncation $t$, which is a monotonous and increasing function of $f$. When Au is entirely distributed in the homothetic part of the shell, $f$ and $t$ reach their maxima $f_{\max }$ and $t_{\max }$ :

$$
\begin{equation*}
f_{\max }=\left[1+\left(V_{A u} / V_{F e}\right)\right]^{1 / 3} ; t_{\max }=1 \tag{15}
\end{equation*}
$$

When the Au pyramids are as complete as possible, $f$ and $t$ reach their minima $f_{\text {min }}$ and $t_{\text {min }}$. As illustrated in Fig. 3(b) for a particular $R_{F e}$ and 3 different volume ratios, these minima are:
(i) $\quad 1<f_{\min }<f_{\max }$ and $t_{\text {min }}=0$ if the wetting layer still exists when the pyramids are complete.
(ii) $\quad f_{\text {min }}=1$ and $t_{\text {min }}=0$ when the completion of the pyramids coincides with an empty wetting layer
(iii) $\quad f_{\text {min }}=1$ and $0<t_{\text {min }}<1$ if pyramids cannot be complete even without wetting layer.

The exact calculation of $f_{\min }$ and $t_{\min }$ is given in Appendix B.


FIG. 3. (a) Cross-section (top) and top-view (bottom) of the core for various aspect ratios $R_{F e}$ from $1 / \sqrt{2}$ to $\sqrt{2}$. (b) Geometrical limits of the shell, illustrated in cross-section with $R_{F e}=1.2$ for various volume ratios $V_{A u} / V_{F e}$. The scale is for $V_{F e}=175.6 \mathrm{~nm}^{3}$ (same order of magnitude as the average experimental volumes in [7]).

## D. Experimental analysis with the geometrical model

Average values of $R_{F e}, h_{001}^{F e}, h_{111}^{A u}$ and $h_{p y r}^{A u}$ of a single NP can be measured from high resolution transmission electron microscopy (HRTEM) images. $V_{F e}, f$ and $V_{A u} / V_{F e}$ are then deduced through Eq. (2), (6) and (13). The models represented in Fig. 1 are built with the measures from the experimental images and are displayed at the same scale.

For Fig. 1(a), the measured values used to build the model are $R_{F e}=1.35+/-0.04, h_{001}^{F e}=2.2$ $+/-0.1 \mathrm{~nm}, h_{111}^{A u}=0.41+/-0.1 \mathrm{~nm}$ and an average $t=0.66$ (or $h_{p y r}^{A u}=1.1 \mathrm{~nm}$ ). We thus estimate $V_{F e} \approx$ $84 \mathrm{~nm}^{3}, f \approx 1.14$ and $V_{A u} / V_{F e} \approx 1.6$ in this single NP. Estimates of $V_{A u} / V_{F e}$ in several other NPs from [7, 9] (and supplemental material of ref. [7]) with the same morphology as in Fig. 1(a) are in the range $1.6-2.3$. The $\{111\} \mathrm{Au} /\{110\} \mathrm{Fe}$ interfaces barely even exist and the core corners are covered by 2 or more Au MLs.

For Fig. 1(b) the measured values used to build the model are $R_{F e}=0.98+/-0.02, h_{001}^{F e}=3.26$ $+/-0.1 \mathrm{~nm}, h_{111}^{A u}=0.58+/-0.1 \mathrm{~nm}$ and $h_{p y r}^{A u} \approx 0$, giving directly $t \approx 0$ and $f \approx 1$. The deduced Fe volume is $168+/-25 \mathrm{~nm}^{3}$ and the deduced volume ratio $V_{\mathrm{Au}} / V_{\mathrm{Fe}}$ in this single NP is $0.65+/-0.15$.

These estimations indicate that the Au/Fe ratio in NPs with well-formed pyramids is 3 to 4 times larger than in NPs as in Fig. 1(b).

## E. Surface and interface energy densities

To investigate the energy landscape of a NP with the geometrical model, the surface and interface energies densities, defined as excess energies compared to bulk phases, are necessary. They were calculated with the density functional theory (DFT) with the methodology described in $[20,21]$ and are reported in Table I.

## 1. DFT calculations

Spin-polarized DFT calculations have been performed using the VASP code, with PAW pseudopotentials and the PBE functional. The cutoff energy has been set to 600 eV for all calculations and a Methfessel-Paxton smearing with $\sigma 0.05 \mathrm{eV}$ was used. A Monkshorst-Pack mesh of special kpoints has been determined in order to achieve the convergence of the energy up to $2 \mathrm{meV} /$ atom for each investigated system. Periodic boundary conditions are used in the $x, y$ and $z$ directions and the two slabs are separated from their images by adding 1.5 nm of vacuum in the z -direction. The atomic positions were relaxed until the forces reaches a value lower than $10^{-2} \mathrm{eV} . \AA^{-1}$.

As discussed in [20] the (001)Au and (111)Au surface energies, respectively $\gamma_{001}^{A u}$ and $\gamma_{111}^{A u}$, are underestimated with respect to the experimental values (around $1.5 \mathrm{~J} / \mathrm{m}^{2}$ [15]). ). This is due to the use of the PBE exchange and correlation functional which, although giving excellent results in reproducing the properties of iron, is not very performing for gold. However, the ratio $\gamma_{111}^{A u} / \gamma_{001}^{A u}$ is in excellent agreement with the experimental estimates; for instance, a 0.84 ratio is deduced applying
the Wulff theorem to the Au nano-island from ref. [22]. For (001)Fe and (110)Fe, the surface energies, respectively $\gamma_{001}^{F e}$ and $\gamma_{110}^{F e}$, are in very good agreement with experiments [18, 23].

For the energy density of the $\{001\} \mathrm{Au} /\{001\} \mathrm{Fe}$ interface, $\gamma_{(001) \mathrm{Au} /(001) \mathrm{Fe}}^{\text {interf }}$, we used the same model of coherent interface as in $[20,21]$. For the energy density of the $\{111\} \mathrm{Au} /\{110\} \mathrm{Fe}$ interface, $\gamma_{(111) A u /(110) F e}^{\text {interf }}$, we developed a new model reproducing the epitaxial NishiyamaWasserman relationship [7,13] with periodic boundary conditions. As a first approximation, we chose to model this interface using a semi-coherent approach in the [101] direction as proposed by Lu et al. for the (111) $\mathrm{Ag} /(110)$ Fe interface [24]. The coincidence between the two lattices along the [110] direction is 5 Fe cells for 4 Au cells ( $5 \times 4$ ). After optimization, the system exhibits an interface dislocation enabling to accommodate a large part of the high lattice misfit in this direction. A more detailed study will be reported elsewhere. For both interfaces, the energies were converged within +/-10 mJ/m² for a slab with 12 Fe MLs and more than 4 Au MLs.

|  | DFT-PBE $\left(\mathrm{J} / \mathrm{m}^{2}\right)$ | Experimental $\left(\mathrm{J} / \mathrm{m}^{2}\right)$ |
| :--- | :--- | :--- |
| $\gamma_{001}^{A u}$ | 0.873 |  |
| $\gamma_{111}^{A u}$ | 0.734 | $\approx 1.5[15]$ |
| $\gamma_{111}^{A u} / \gamma_{001}^{A u}$ | 0.84 | $0.84(\mathrm{a})$ |
| $\gamma_{001}^{F e}$ | 2.478 |  |
| $\gamma_{110}^{F e}$ | 2.428 | $\approx 1[17]$ |
| $\gamma_{110}^{F e} / \gamma_{001}^{F e}$ | 0.98 |  |
| $\gamma_{(001) \mathrm{Au} /(001) \mathrm{Fe}}^{\text {interf }}$ | 0.378 |  |
| $\gamma_{(111) \mathrm{Au} /(110) \mathrm{Fe}}^{\text {interf }}$ | 0.700 |  |

Table I: Surface and interface energy densities obtained from DFT calculations with the PBE functional and experimental data. (a) is found by application of the Wulff theorem to the Au nano-island of ref. [22].

## 2. Dependence with the Au thickness

The excess energy experienced by an iron surface covered by Au is due to the Au surface energy and interface energy. However, to explore the case of small $\mathrm{Au} / \mathrm{Fe}$ volume ratios (Fig. 3(b)), it is mandatory to consider the possibility for Fe surfaces to be bare (the excess energy is then due to
$\gamma_{\text {surface }}^{F e}$ ) and to introduce a progressive transition from the bare Fe surface to the full wetting by Au. By sake of simplicity, we chose to monitor the full wetting condition with a single parameter, a critical Au thickness. In the following, for Au thicknesses larger than this critical value, the surface and interface energies are calculated with the densities $\gamma_{\text {surface }}^{A u}$ and $\gamma_{\text {interface }}$ from Table I. Below the critical thicknesses, we impose a linear increase in $\gamma_{\text {surface }}^{A u}$ and $\gamma_{\text {interface }}$ as the Au thickness decreases until $\gamma_{\text {surface }}^{A u}+\gamma_{\text {interface }}$ becomes equal to $\gamma_{\text {surface }}^{F e}$ when the Au thickness is null, as described in Fig. 4. We fixed the critical thicknesses at $2 \mathrm{MLs}(0.46 \mathrm{~nm}$ for $\{111\} A u$ and 0.40 nm for $\{001\} \mathrm{Au})$ corresponding to the minimal Au coverage experimentally observed in our NPs.


FIG. 4. $\gamma_{\text {surface }}^{A u}+\gamma_{\text {interface }}$ as a function of the gold thickness. If the Au layer exceeds a critical thickness, chosen here to be $2 \mathrm{MLs}, \gamma_{\text {surface }}^{A u}$ and $\gamma_{\text {interface }}$ are the DFT data computed (Table I). Below the critical thickness, a linear increase is imposed to $\gamma_{\text {surface }}^{A u}$ and $\gamma_{\text {interface }}$ so that for the bare iron, $\gamma_{\text {surface }}^{A u}+\gamma_{\text {interface }}$ is equal to $\gamma_{\text {surface }}^{F e}$ (DFT data from Table I). The illustrations are NPs with $R_{F e}=1.2$. For $V_{F e}=175.6 \mathrm{~nm}^{3}$ the $\mathrm{Au} / \mathrm{Fe}$ volume ratios correspond from left to right to: bare iron, Au thickness around the critical thickness, and larger Au thickness, here around 1.3 nm ( 6 MLs ).

## III. GROWTH MODES OF GOLD ON INFINITE 2D AND FINITE 3D IRON SUBSTRATES

The growth mode of the $\mathrm{Au} / \mathrm{Fe}$ system can be predicted on the basis of our DFT calculated surface and interface energies. Beyond the classical case of the growth on a 2D infinite substrate, our aim in this section is to discuss the growth mode when the substrate is a 3D nanocrystal of finite size.

## A. Growth mode of Au on an infinite 2D Fe substrate

The wetting factor $\Phi_{A u / F e}$ that expresses the balance of surface and interface energies involved in the formation of a 2D Au layer on a 2D Fe substrate is [13]

$$
\begin{equation*}
\Phi_{A u / F e}=\gamma_{\text {surface }}^{A u}+\gamma_{\text {interface }}-\gamma_{\text {surface }}^{F e} \tag{16}
\end{equation*}
$$

The growth mode (in near equilibrium conditions) is predicted from the sign of $\Phi_{A u / F e}$ : 2D Frank van der Merwe (full wetting) if negative or 3D Volmer-Weber (partial wetting) if positive. It is also interesting to calculate the adhesion energy of the $\mathrm{Au} / \mathrm{Fe}$ system

$$
\begin{equation*}
W_{a d}=\gamma_{\text {surface }}^{A u}-\gamma_{\text {interface }}+\gamma_{\text {surface }}^{F e} \tag{17}
\end{equation*}
$$

Thanks to the DFT calculated energies of both $\{111\} A u /\{110\} F e$ and $\{001\} \mathrm{Au} /\{001\} F e$ interfaces, we are able to predict and compare quantitatively their growth modes. Table II reports the adhesion energies and the wetting factors of Au on Fe calculated with the surface and energies DFT computed data of Table I.

| interface |  | DFT-PBE (J/m²) |
| :--- | :--- | :--- |
| (111)Au / (110)Fe | $W_{a d}$ | 2.462 |
| (001) $\mathrm{Au} /(001) \mathrm{Fe}$ | $W_{a d}$ | 2.973 |
| (111)Au on (110)Fe | $\Phi_{(111) \mathrm{Au} /(110) \mathrm{Fe}}$ | -0.994 |
| (001)Au on (001)Fe | $\Phi_{(001) \mathrm{Au} /(001) \mathrm{Fe}}$ | -1.227 |
| (110)Fe on (111)Au | $\Phi_{(110) \mathrm{Fe} /(111) \mathrm{Au}}$ | 2.394 |
| (001)Fe on (001)Au | $\Phi_{(001) \mathrm{Fe} /(001) \mathrm{Au}}$ | 1.983 |

Table II. Adhesion energies $W_{a d}$ of the $\mathrm{Au} / \mathrm{Fe}$ interfaces and wetting factors $\Phi_{A u / F e}$ of Au on Fe, and $\Phi_{\mathrm{Fe} / \mathrm{Au}}$ of Fe on Au, calculated from the DFT computed data of Table I.

Both interfaces have a high adhesion energy. The largest adhesion energy of $\{001\} \mathrm{Au} /\{001\} \mathrm{Fe}$ compared to $\{111\} \mathrm{Au} /\{110\} F \mathrm{Fe}$ is due for one third to the difference in $\gamma_{111}^{A u}$ and $\gamma_{001}^{A u}$ and for two
thirds to the difference in $\gamma_{(111) A u /(110) F e}^{\text {interf }}$ and $\gamma_{(111) A u /(110) F e}^{\text {interf }}$. The negative wetting factors in Table II induced that for the two considered Fe orientations, the growth mode of Au on Fe system predicted by DFT is 2D Frank van der Merwe, i.e. the 2D Au layer is formed even for null sursaturation. In addition, the absolute values of the wetting factors are comparable to the corresponding Au surface energies, which indicates a strong tendency to Au on Fe wetting.

Table Il also reports the wetting factors of iron on gold. They are much larger than for gold on iron, positive values even suggesting a Volmer-Weber growth mode of iron on gold (i.e. partial wetting). Combined to the low miscibility of the two metals, here this fully preserves the initial iron NPs as cores for the further growth of gold shell. More generally, this clearly favors a Fe-core@Aushell chemical order in this system.

## B. Growth of Au on a finite 3D Fe nano-substrate: from a 2D growth mode to a 3D Stranski-Krastanov like growth mode

Within our geometrical model, we can calculate precisely the variation of free energy $\Delta F$ needed to form an Au shell on a Fe nano-substrate of predefined shape and finite size. To condensate n atoms from Au vapor as a shell with a homogeneous thickness, $\Delta F$ is

$$
\begin{equation*}
\Delta F=E_{\text {surface }}^{A u}+E_{\text {interface }}-E_{\text {surface }}^{F e}-n \Delta \mu \tag{18}
\end{equation*}
$$

where $E_{\text {surface }}^{A u}$ and $E_{\text {interface }}$ are the costs in Au surfaces and $A u / F e$ interfaces to form an Au shell, ( $-E_{\text {surface }}^{F e}$ ) is the gain provided by the full wetting of Fe by Au , and $\Delta \mu$ is the difference of chemical potential of gold between the vapor and the solid phase. To ensure the constraint on the homogeneous shell thickness, we consider here that the shell adopts the same shape as the core so that $f$ is fixed to $f_{\max }$ (defined by Eq. (15) in section II. C.). It comes

$$
\begin{gather*}
E_{\text {surface }}^{F e}=12 \gamma_{110}^{F e} S_{110}^{F e}+6 \gamma_{001}^{F e} S_{001}^{F e}  \tag{19}\\
E_{\text {surface }}^{A u}=12 \gamma_{111}^{A u} S_{111}^{A u}+6 \gamma_{001}^{A u} S_{001}^{A u}  \tag{20}\\
E_{\text {interface }}=12 \gamma_{(111) A u /(110) F e}^{\text {interf }} S_{110}^{F e}+6 \gamma_{(001) A u /(001) F e}^{\text {interf }} S_{001}^{F e}  \tag{21}\\
\Delta F=12\left(\gamma_{(111) A u /(110) F e}^{\text {interf }}+f_{\text {max }}{ }^{2} \gamma_{111}^{A u}-\gamma_{110}^{F e}\right) S_{110}^{F e}+6\left(\gamma_{(001) A u /(001) F e}^{\text {interf }}+f_{\text {max }}{ }^{2} \gamma_{001}^{A u}-\right. \\
\left.\gamma_{001}^{F e}\right) S_{001}^{F e}-n \Delta \mu
\end{gather*}
$$

It is interesting to define $\Phi_{A u /(3 D) F e}$ by dividing the surface and interface contributions by the core area:

$$
\begin{gather*}
\Phi_{A u /(3 D) F e}=\frac{E_{\text {surface }}^{A u}+E_{\text {interface }}-E_{\text {surface }}^{F e}}{12 S_{110}^{F e}+6 S_{001}^{F e}} \\
=\frac{\left(\gamma_{(111) A u /(110) F e}^{\text {interf }}+f_{\max }{ }^{2} \gamma_{111}^{A u}-\gamma_{110}^{F e}\right)\left(\sqrt{2}-R_{F e}\right)\left(3 \sqrt{2} R_{F e}-2\right)+2\left(\gamma_{(001) A u /(001) F e}^{\text {interf }}+f_{\max }{ }^{2} \gamma_{001}^{A u}-\gamma_{001}^{F e}\right)\left(\sqrt{2} R_{F e}-1\right)^{2}}{\left(\sqrt{2}-R_{F e}\right)\left(3 \sqrt{2} R_{F e}-2\right)+2\left(\sqrt{2} R_{F e}-1\right)^{2}} \tag{23}
\end{gather*}
$$

Note that $\Phi_{A u /(3 D) F e}$ is nothing else than the wetting factor adapted to the 3D morphology of the nanosubstrate. In contrast with a 2D infinite substrate, here the wetting factor depends on the Au deposited volume. Indeed $f_{\max }$ is related to $V_{A u} / V_{F e}$ through Eq. (15). $\Phi_{A u /(3 D) F e}$ is displayed in Fig. 5 as a function of $V_{A u} / V_{F e}$ for various $R_{F e}$ from $1 / \sqrt{2}$ to $\sqrt{2}$. For small volume ratio, it comes from Eq. (15) that $f_{\max }$ tends to 1 , so that $\Phi_{A u /(3 D) F e}$ reduces to the average value of $\Phi_{A u / F e}$ weighted by the respective surface areas of the two kinds of crystallites and is negative whatever the Fe nanocrystal shape ${ }^{1}$. The wetting factor $\Phi_{A u /(3 D) F e}$ then increases linearly with $f_{\max }^{2}$ with a $R_{F e}$ dependency ${ }^{2}$. There exists a critical value $f_{\text {critical }}$ and a critical volume ratio for which $\Phi_{A u /(3 D) F e}$ becomes null:
$f_{\text {critical }}$
$=\left[-\frac{\left(\gamma_{(111) A u /(110) F e}^{\text {interf }}-\gamma_{110}^{F e}\right)\left(\sqrt{2}-R_{F e}\right)\left(3 \sqrt{2} R_{F e}-2\right)+2\left(\gamma_{(001) A u /(001) F e}^{\text {interf }}-\gamma_{100}^{F e}\right)\left(\sqrt{2} R_{F e}-1\right)^{2}}{\gamma_{111}^{A u}\left(\sqrt{2}-R_{F e}\right)\left(3 \sqrt{2} R_{F e}-2\right)+2 \gamma_{100}^{A u}\left(\sqrt{2} R_{F e}-1\right)^{2}}\right]^{1 / 2}$

[^1]\[

$$
\begin{equation*}
\left(V_{A u} / V_{F e}\right)_{\text {critical }}=\left(f_{\text {critical }}\right)^{3}-1 \tag{25}
\end{equation*}
$$

\]



FIG. 5. Wetting factor $\Phi_{A u /(3 D) F e}$ adapted to the 3D morphology of the nanosubstrate, calculated as a function of the $\mathrm{Au} / \mathrm{Fe}$ volume ratio for various Fe core shapes. The energies densities used are those calculated by DFT-PBE (table I). The illustrations are for $R_{F e}=0.98$ and 1.2 and the two volume ratios 0.47 and 2.5.

The critical volume ratio is between 2.6 and 2.7 depending on $R_{F e}$ (Fig. 5). Below the critical ratio, the shell formation with a full wetting provides a gain in energy as in the classical 2D Frank-van-der-Merwe growth mode. This also means that, once formed, a Fe@Au NP should be very stable against partial dewetting. (Note that the $\gamma_{\text {surface }}^{A u}$ being underestimated compared to experimental data (Table I), the wetting factor predicted could also be underestimated. However, the full wetting is experimentally attested in the NPs from ref [25] and from our work.)

Above, $\Phi_{A u /(3 D) F e}$ is positive and the growth requires an increasing positive sursaturation.

Here we can make an analogy with the 3D Stranski-Krastanov (SK) growth mode, the third classical growth mode adopted by a film grown on an infinite 2D substrate (in near-equilibrium conditions) [13]. In the SK growth mode, the wetting is complete and the layer first grows 2 D , then after a so-called critical thickness, islands take place on the wetting layer. It has been formally
described by considering that the wetting factor, negative at first at the onset of the growth, increases with the deposited thickness so that it becomes positive [13, 26]. In strained semiconducting heterostructures, where the SK growth mode is often observed, this increase is due to the integration in the wetting factor of the elastic energy, a term proportional to the deposited thickness $[13,26]$. The driving force for a 3D morphology is thus the relaxation of the elastic energy by the island faces [26, 27].

In our model, the increase of the wetting factor $\Phi_{A u /(3 D) F e}$ with the $\mathrm{Au} / \mathrm{Fe}$ volume ratio and the change of its sign is due to the shell surface increase. Note that this phenomenon is expected independently of the core and shell crystallography. However, a 3D SK like growth mode will occur only if a modulation of the shell thickness decreases this wetting factor. As investigated in the next section, this will be possible here thanks to the polycrystalline character of the shell engendered by the accommodation of its crystallographic lattice (fcc) to the core one (bcc).

## IV. EQUILIBRIUM SHAPE OF THE FE@AU NANOPARTICLES AT CONSTANT VOLUMES

In this section, we determine the equilibrium shape (ES) of a NP of given Au and Fe volumes, and we identify the driving forces at the origin of the shape transformations allowed in our geometrical model. For this purpose, we analyze the pathways leading to the ES. We consider as initial configurations the cases where the shell adopts the same external shape as the core i.e. when $f=f_{\max }$ as in section III. Then we assume that a shape transformation can only occur by progressive change of $R_{F e}$ and/or $f$.

A large $\mathrm{Au} / \mathrm{Fe}$ volume ratio of 1.71 is first considered then a small one of 0.57 .

## A. Excess energy of a nanoparticle

The excess energy of the NP compared to the same Au and Fe volumes in their bulk phases is

$$
\begin{equation*}
E_{N P}=E_{\text {surface }}^{A u}+E_{\text {interface }} \tag{26}
\end{equation*}
$$

While $E_{\text {interface }}$ is the same as in section III. B., here the surface contribution includes the pyramid facets:

$$
\begin{equation*}
E_{\text {surface }}^{A u}=12 \gamma_{111}^{A u} S_{111}^{A u}+6\left(\gamma_{001}^{A u} S_{001 p y r}^{A u}+4 \gamma_{111}^{A u} S_{111 p y r}^{A u}\right) \tag{27}
\end{equation*}
$$

$$
\begin{equation*}
E_{\text {interface }}=12 \gamma_{(111) A u /(110) F e}^{\text {interf }} S_{110}^{F e}+6 \gamma_{(001) A u /(001) F e}^{\text {interf }} S_{001}^{F e} \tag{28}
\end{equation*}
$$

The solutions of the energy minimization as a function of the two aspect ratios $R_{F e}$ and $f$ are numerically determined for a Fe volume $V_{F e}=175.6 \mathrm{~nm}^{3}$. The core edge when the shape is a cube is 5.6 nm that corresponds to the average experimental value [7]. The core aspect ratio $R_{F e}$ varies from $1 / \sqrt{2}$ (perfect rhombic dodecahedron with $12\{110\} F e$ faces) to $\sqrt{2}$ (perfect \{001\}Fe oriented cube). As discussed in section II. C., the upper limit of $f$ is $f_{\max }$ from Eq. (15) (the whole Au volume is distributed in the homothetic shell), then $f$ can be lowered by the progressive building of the pyramids up to $f_{\text {min }}$.

## B. Equilibrium shape for a large $\mathrm{Au} / \mathrm{Fe}$ volume ratio

We present in detail the case of $V_{A u} / V_{F e}=1.71$ with $V_{F e}=175.6 \mathrm{~nm}^{3}$. Fig. 6 displays the total excess energy $E_{N P}$ as a function of the two aspect ratios $R_{F e}$ and $f$. The upper limit of $f$, common to all $R_{F e}$ values, is $f_{\max }=1.39$ (from Eq. (15)). For all $R_{F e}$ the $\mathrm{Au} / \mathrm{Fe}$ volume ratio is large enough to build untruncated pyramids $(t=0)$ without completely emptying the homothetic shell ( $f$ remains strictly larger than 1).


FIG. 6. Excess energy of a single NP with $V_{F e}=175.6 \mathrm{~nm}^{3}$ and $V_{A u} / V_{F e}=1.71$, displayed as a function of $f$ and $R_{F e} . f_{\max }=1.39$ corresponds to a homothetic shell without pyramids. The ES $\left(R_{F e}=\right.$ $\sqrt{2}$ and $f=1.073$ or $t=0.535$ ) is shown.

As long as $R_{F e}$ is lower than 1 , the minimum of energy is for $f_{\max }$ (shape without pyramids). For a given core shape $R_{F e}$ larger than 1 , the minimum is for a lower $f$, demonstrating the possibility of a 3D SK like growth mode, where a part of the shell is transferred into pyramids at the detriment of the homothetic shell.

Comparing the different core shapes, it is then seen that the minimum of $E_{N P}$ is for $R_{F e}=\sqrt{2}$ (cube-shaped core) and $f=1.073$. The homothetic shell thickness under the pyramids $h_{001}^{A u}$ and the
pyramid height $h_{p y r}^{A u}$ are then respectively 0.21 nm (about $1\{001\} \mathrm{Au} M \mathrm{ML}$ ) and 1.98 nm (about 10 $\{001\} A u$ MLs) and the truncation $t=0.535$. This ES is displayed in inset.

## C. From initial shapes to optimal shapes for a large $\mathrm{Au} / \mathrm{Fe}$ volume ratio

To analyze the driving forces leading to this ES, we report in Fig. 7 (blue curve) the excess energies of NPs with a homogeneous shell thickness ( $f=f_{\max }$ or $t=1$ ). The four core shapes illustrated in Fig. 7 are $R_{F e}=1 / \sqrt{2}, 1.015,1.2$ and $\sqrt{2}$. The possible pathways reducing $E_{N P}$ at constant volumes from these initial configurations are then decomposed in two distinct shape transformations. The excess energies are taken from Fig. 6.


FIG. 7. Shape transformations reducing the NP energy at constant Au and Fe volumes, for large volume ratios. The illustrations are at the scale for $V_{F e}=175.6 \mathrm{~nm}^{3}$ and $V_{A u} / V_{F e}=1.71$.
(i) 2D-3D Stranski-Krastanov like transition: This transformation consists in building <001>Au oriented pyramids at the detriment of $<111>A u$ oriented platelets, the core shape being unchanged (golden arrows). The driving force to form pyramids is the reduction of surface energy by increasing the part of $\{111\} \mathrm{Au}$ surfaces compared to $\{001\} \mathrm{Au}$ (respectively the lateral and top facets of the pyramids). We call this mechanism a 2D-3D SK-like transition because it reduces the effective wetting factor by the modulation of the shell thickness. This mechanism is active only for $R_{F e}$ larger than 1 .
(ii) Core shape transformation: One possible driving force is the reduction of the interface energy. The core shape minimizing the interface energy $E_{\text {interface }}$ is found by application of the Wulff theorem and is $R_{F e}=\gamma_{(111) A u /(110) F e}^{\text {interf }} / \gamma_{(001) A u /(001) F e}^{\text {interf }}$. As here this ratio is larger than $\sqrt{2}$ (from

Table I), the $\{111\} \mathrm{Au} /\{110\} F e$ interfaces are unstable against $\{001\} \mathrm{Au} /\{001\} \mathrm{Fe}$ interfaces and should disappear at the benefit of the shape of a cube. However this would be true only without surface effects. Indeed the cost due to larger $\{001\} A u$ facets with the shape of a cube could be uncompensated by the gain in interface energy. As a consequence, it is seen in Fig. 7 that as long as the shell thickness is homogeneous ( $t=1$ or $f=f_{\max }$ ), the minimum of energy is for $R_{F e}$ around 1 . Actually, this optimal core shape can be found analytically with the Wulff theorem, by considering that each core face experiences the interface energy plus the surface energy of the Au platelet above, and is:

$$
\begin{equation*}
R_{F e}^{\text {homo }}=\frac{\gamma_{(111) A u /(110) F e}^{\text {interf }}+f_{\max }^{2} \gamma_{111}^{A u}}{\gamma_{(001) A u /(001) F e}^{\text {interf }}+f_{\max ^{2}} \gamma_{001}^{A u}} \tag{29}
\end{equation*}
$$

Here with $V_{A u} / V_{F e}=1.71, R_{F e}^{\text {homo }}$ equals 1.025 . The blue arrows in Fig. 7 indicates the pathway from various initial core shapes toward this minimum, the shell thickness remaining homogeneous.
(iii) Finally, the two mechanisms of transformation can cooperate and the minimum of energy can be reached by simultaneous changes in $R_{F e}$ and $f$ as illustrated by the green arrows. The ES combines the cube-shaped core minimizing the interface energy, and the $\{001\} A u$ pyramids optimizing the ratio between the areas of the two gold facet types.

In our model the bare iron is characterized applying the Wulff theorem by an aspect ratio $R_{F e}^{W u l f f}=\gamma_{110}^{F e} / \gamma_{001}^{F e}$. If, at the onset of the shape transformation, the core is close to this value (0.98 from Table I), it can easily evolve towards $R_{F e}^{\text {homo }}$ (1.015), these two values being by chance very close. As the pyramid formation also begins around $R_{F e}=1$, the two mechanisms cooperate leading to the ES.

## D. Equilibrium shape for a small $\mathrm{Au} / \mathrm{Fe}$ volume ratio

Figure 8 displays the NP excess energy for the small ratio $V_{A u} / V_{F e}=0.57$. Here $f_{\max }=1.162$. As for larger volume, for a given core shape $R_{F e}$ larger than about 1, the minimum is not obtained for the homogenous shell thickness. Allowing change in $R_{F e}$, there exist two minima (the corresponding shapes are shown in inset). The first minimum (M1) presents the same shape (cube) for the iron core as for the large volume ratio. The homothetic shell however is empty $(f=1)$ and $A u$ is entirely distributed in 0.62 nm thick pyramids (about $3\{001\} A u$ MLs). The truncation is $t=0.842$.


FIG. 8. Excess energy of a single NP with $V_{F e}=175.6 \mathrm{~nm}^{3}$ and $V_{A u} / V_{F e}=0.57$ displayed as a function of $f$ and $R_{F e} . f_{\max }=1.162$ corresponds to a homothetic shell without pyramids. The shapes in inset correspond to the M 1 minimum ( $R_{F e}=\sqrt{2}$ and $f=1$ ) and M 2 minimum ( $R_{F e}=1.14$ and $f=1.135$ ). SP is the saddle point ( $R_{F e}=1.35$ ).

The second minimum (M2) is for $R_{F e}=1.14$ and $f=1.135$. The homothetic shell thicknesses are $h_{111}^{A u}=0.46 \mathrm{~nm}$ and $h_{001}^{A u}=0.4 \mathrm{~nm}$, i.e. 2 MLs . The <001> oriented Au platelets are covered by flat pyramids with height $h_{p y r}^{A u}=0.2 \mathrm{~nm}$ (about one \{001\}Au MLs) and truncation $t=0.934$. The existence of this second minimum is related to the increase introduced in surface and interface energies for low $h_{111}^{A u}$ to account for the transition between full wetting and bare iron as described in section II. E. 2 (Fig. 4). Indeed, while the NPs examined in section IV. B. and IV. C. always experience the full wetting for any explored shape (Au thickness always larger than 2 MLs ), here the reduction of the $V_{A u} / V_{F e}$ dramatically reduces the ability to form a full Au wetting layer for any explored shape. In other words, the Au layer is thinner than 2 MLs for some sets of aspect ratios $\left(f, R_{F e}\right)$.

## E. From initial shapes to optimal shapes for a small $\mathrm{Au} / \mathrm{Fe}$ volume ratio

Figure 9 describes the possible pathway to ES from initial shapes with homogeneous shells ( $E_{N P}$ are taken from Fig. 8). The blue curve if for the core transformation remaining a homogeneous shell ( $f=f_{\max }$ ). Its minimum is given by Eq. (29) and is $R_{F e}^{h o m o}=1.086$ for $V_{A u} / V_{F e}=0.57$. As a reduction of $E_{N P}$ by pyramid formation is possible from about $R_{F e}=1$, the minimum M 2 is achieved by the cooperation of core transformation and pyramid development (green arrow). M1 and M2 however are separated by a saddle point (SP) of first order located at $R_{F e}=1.35, f=1.09$ (from Fig. 8). The barrier between M 2 and SP is so high $\left(9 \times 10^{-18} \mathrm{~J}\right.$ or 58 eV ) that it fully prevents the transition from M 2 to M1 through the geometrical transformations included in our model. The minimum M1 could be achieved from the initial shape of a cube by the pyramid formation (golden arrow).


FIG. 9. Shape transformations reducing the NP energy at constant Au and Fe volumes, for small volume ratios. The illustrations are at the scale for $V_{F e}=175.6 \mathrm{~nm}^{3}$ and $V_{A u} / V_{F e}=0.57$.

## V. DISCUSSION AND COMPARISON WITH EXPERIMENTAL NANOPARTICLES

We then explore the ES as a function of the $\mathrm{Au} / \mathrm{Fe}$ volume ratio from 0.47 to 2.5 . Fig. 10 displays the evolution of $R_{F e}$ (a), $t$ (b) and Au thicknesses (c) for the two minima M 1 and M 2 as a function of the volume ratio. Here we chose to represent the shell shape through $t$ rather than $f$, because $t$ provides a direct focus on the pyramid shape. To predict the shape evolution, we describe and discuss separately the evolution of the M 1 and M 2 minima, then we discuss which minimum is likely to form at each volume ratio.
(a)




FIG. 10. Evolution of $R_{F e}(\mathrm{a}), t(\mathrm{~b})$ and Au thicknesses (c) for the two minima M 1 and M 2 as a function of the $\mathrm{Au} / \mathrm{Fe}$ volume ratio (for $V_{F e}=175.6 \mathrm{~nm}^{3}$ ).

## A. Evolution of the optimal shape M1 towards an optimal truncation

The M1 minimum (green curves in Fig. 10) exists for all volume ratios. While the core shape is consistently a cube, there exist two regimes for the shell shape.
(i) For volume ratio below 1.2: The homothetic layer below the pyramids is empty ( $h_{001}^{A u}=0$ in Fig. 10(c), corresponding to $f=1$ ). The Au volume increase entirely contributes to develop the <001> oriented pyramids as shown by the progressive decrease of the truncation (Fig. 10(b)), and the corresponding increase of $h_{p y r}^{A u}$ (Fig. 10(c)).
(ii) For volume ratio above 1.2: When $V_{A u} / V_{F e}$ reaches 1.2 , the truncation reaches 0.534 . Then the shell morphology changes, with the apparition of the homothetic layer. As seen in Fig. 10(c) (green curves), the increase in the volume ratio above 1.2 indeed results in the concomitant thickens of the layers under the pyramids $\left(h_{001}^{A u}\right)$ and increase of the pyramid height $\left(h_{p y r}^{A u}\right)$. It is important to note that the truncation does no more evolve (Fig. 10(b)), meaning that the pyramid shape (above the homothetic shell) is unchanged.

These different steps are illustrated at the same scale in Fig. 11 (green squares) on which one can see that the homothetic layer is absent and all the gold is within the truncated pyramids up to $V_{A u} / V_{F e}=1.2$. Beyond 1.2 we observe the appearance of the homothetic layer and the stabilization of the truncation at $t_{\text {opt }}=0.534$.


FIG. 11. Optimal shapes illustrated for $V_{A u} / V_{F e}=0,0.47,1.2,1.7$ and 2.5 volume ratios. The shapes are at the scale for $V_{F e}=175.6 \mathrm{~nm}^{3}$.

## B. Evolution of the optimal shape M2

For $V_{F e}=175.6 \mathrm{~nm}^{3}$, the M 2 minimum (red curves in Fig. 10) exists from $V_{A u} / V_{F e}=0.47$ which is the minimal ratio ensuring a full wetting with 2 MLs : the pyramids are indeed empty ( $h_{p y r}^{A u}=0$ and $t=1)$. Then the core shape evolves monotonically from $R_{F e}=1.11$ for $V_{A u} / V_{F e}=0.47$ to $R_{F e}=1.39$ for
$V_{A u} / V_{F e}=1.7$ (Fig. 10(a)). At the same time, the thickness of the $<111>$ oriented Au platelets ( $h_{111}^{A u}$ in Fig. 10(c)) is stable at 0.46 nm and the Au volume increase entirely contributes to develop the <001> pyramids. The stabilization at $0.46 \mathrm{~nm}(2 \mathrm{MLs})$ of $h_{111}^{A u}$ is a strong indication that $h_{111}^{A u}$ is driven by the wetting of Au on $\{110\} \mathrm{Fe}$. This is indeed the critical thickness chosen in section II. E. 2 (Fig. 4). To verify this hypothesis, we modified this critical thickness and we found that effectively this value directly monitors $h_{111}^{A u}$ of the M 2 minima.

To summarize, the M 2 minimum progressively evolves from a shape purely driven by the wetting (for $V_{A u} / V_{F e}=0.47$ ) to an intermediate shape driven by the combination of the wetting for \{111\}Au on $\{110\} \mathrm{Fe}$, the interface optimization, and the formation of $<001>$ oriented Au pyramids (the 3D SK-like growth mode), as illustrated in Fig. 11 (red points). Around the volume ratio 1.7, the M2 minimum disappears and the M1 minimum only subsists.

## C. Shape evolution with the volume ratio and comparison with experimental NPs

We now discuss which shape is the most likely to develop as a function of the volume ratio, assuming that the growth conditions allow to reach one of the two optimal shapes (M1 or M2 minimum) at each step, as reported in Fig. 11. For $V_{F e}=175.6 \mathrm{~nm}^{3}, V_{A u} / V_{F e}=0.47$ is the minimal ratio ensuring a full wetting with 2 MLs . The M2 minimum can thus be given by Eq. (29) for $f=1$ and is $R_{F e}^{\text {homo }}=1.11$. As the latter value is by chance close to $R_{F e}^{W u l f f}=0.98$ corresponding to the Wulff shape of bare iron (yellow point in Fig. 11), M2 can be easily achieved. Although M1 has a lower energy than M 2 , the barrier $\Delta E_{M 2 \rightarrow S P}$ from M 2 to the saddle point $S P$ is so high $\left(10^{-17} \mathrm{~J}\right.$ from Fig. 12, i.e. 60 eV$)$ that it prevents the M 2 to M 1 transition. (In addition, the physical meaning of M 1 is questionable for low volume ratios. Indeed, our geometrical model does not correctly ensure the full wetting at the core corners below $V_{F e}=1.2$.)

The barrier $\Delta E_{M 2 \rightarrow S P}$ decreases from $10^{-17} \mathrm{~J}$ (or 60 eV ) for $V_{A u} / V_{F e}=0.47$ to zero for $V_{A u} / V_{F e}$ close to 1.7 (Fig. 12). When it is sufficiently low, a shape transition from M2 to M1 will occur, at the latest for $V_{A u} / V_{F e}=1.7$ (Fig. 11). Once the M1 shape is reached, the NP shell will continue to grow with the optimal truncation $t_{\text {opt }}=0.534$ that reflects an optimal distribution of the areas of the $\{111\} A u$ and $\{001\} A u$ surfaces within our model.


FIG. 12. Energy barrier $\Delta E_{M 2 \rightarrow S P}$ separating the $M 2$ minimum and the saddle point $S P$ (for $V_{F e}=175.6 \mathrm{~nm}^{3}$ ).

Note that the M1 shapes are not sensitive to changes in the interface energy densities, providing that their ratio is larger than $\sqrt{2}$. The details of the M 2 shapes are sensitive to these parameters, and to the critical thicknesses chosen for Au wetting. The relative hierarchy of M1 and M2 and the barrier between them (position of the SP) also depend on these parameters. However, the scenarios described in Fig. 8, Fig. 9 and Fig. 11 are unchanged.

These morphologies are in excellent agreement with those experimentally observed.

In [7] (and in its supplemental material), one collection of bare Fe NPs and two different collections of Fe@Au NPs were achieved with the same size distribution of core and similar relative size dispersions before and after gold deposit. As the Fe volume explored in detail here was chosen to fit the mean experimental core size, a direct comparison with the shapes predicted in Figs. 7, 9 and 11 is possible.

The NP in Fig. 1(b) and those from the same collection in [7] present a polyhedral shape with an octagonal section and a full coverage of about 2 or 3 Au MLs, which is completely accounted for by the M 2 minimum predicted for low volume ratio.

Regarding the NPs from the second collection reported in [7] (and from [9]), most of them present a core shape close to a cube and well-formed pyramids, as displayed in Fig. 1(a), and an experimental $\mathrm{Au} / \mathrm{Fe}$ volume ratio estimated in the range 1.6 to 2.3 . The $\{111\} \mathrm{Au} /\{110\} \mathrm{Fe}$ interfaces barely even exist and the core corners are covered by 2 or more Au MLs. Moreover, homogeneous thickness or octagonal section are not observed when the $\mathrm{Au} / \mathrm{Fe}$ volume ratio is in this range. The observed NPs are thus very well reproduced by the M 2 minimum shortly before the M 2 to M 1 shape transition. In addition, we compare in Fig. 13 the predicted optimal truncation and the experimental
truncations measured in 20 pyramids from 5 NPs (TEM observations in <001>Fe zone axis allow to explore 4 of the 6 pyramids of a NP). Given the error bars, we estimate that:
(i) About $30 \%$ of observed pyramids have reached the calculated optimal truncation $\left(t \sim t_{\text {opt }}\right)$.
(ii) About $60 \%$ are more truncated ( $t>t_{\text {opt }}$ ).
(iii) Only $10 \%$ pyramids are slightly less truncated ( $t<t_{\text {opt }}$ ).

So, the experimental observations are very consistent with our model, where pyramids flatter than the optimal shape can exist due to an insufficient amount of gold, but sharper pyramids cannot form.

To conclude, the comparison of the experimental NPs with the predictions of the model indicate that most of them are close to their individual equilibrium shape, given the core size and the available gold amount.


FIG. 13. (a) TEM images (the scale bars are 5 nm ) and (b) experimental truncations $t$ in five different NPs with well-formed pyramids, measured from the TEM images. The dashed line is the optimal truncation (0.534) predicted by the model for large $\mathrm{Au} / \mathrm{Fe}$ volume ratio.
D. Impact of the iron volume $V_{F e}$ on the optimal shape

Let us briefly discuss the role of the core size.
(i) As long as the surface and interface energy densities do not depend on Au thickness, i.e. as long as the Au thicknesses are larger than the critical thicknesses defined in section II. E. 2. (Fig. 4), the NP energy as displayed in Fig. 6 and Fig. 7 scales with $\left(V_{F e}\right)^{2 / 3}$. A universal function $E_{N P} /\left(V_{F e}\right)^{2 / 3}$ could be used to describe the NP energy, reducing the number of geometrical parameters to 3: $f, R_{F e}$ and $V_{A u} / V_{F e}$. This quantity is homogeneous to a surface energy density. A consequence is that the M 1 minimum shape is regardless the core volume.
(ii) The change due to the thinness of the Au shell below 2 MLs depends also on $\left(V_{F e}\right)^{1 / 3}$, so that the core size impacts the M 2 minimum. Figure 14 displays the core shape of the M 2 minimum as a function of $V_{A u} / V_{F e}$ for the reference volume $V_{F e}=175.6 \mathrm{~nm}^{3}$, a volume 8 times smaller, and a volume 512 times larger (red, blue and yellow curves respectively). The M2 minimum disappears more quickly with $V_{A u} / V_{F e}$ for large $V_{F e}$ than for small $V_{F e}$. For a same $V_{A u} / V_{F e}, R_{F e}$ of the M2 minimum is closer to $\sqrt{2}$ for a large $V_{F e}$ than for small $V_{F e}$. The last remark could explain the observations of almost cube-shaped Fe [10] or FeCo [11] cores for smaller volume ratios than discussed above. In these two references, the core sizes were significantly larger than in our particles.


FIG. 14. Core shape $R_{F e}$ of the optimal shapes as a function of the $\mathrm{Au} / \mathrm{Fe}$ volume ratio for various iron volumes. The red curve is for the reference volume $V_{F e}=175.6 \mathrm{~nm}^{3}$, the blue curve is for a volume 8 times smaller, the yellow curve is for a volume 512 times larger (the core edges when the shape is a cube being respectively $5.6 \mathrm{~nm}, 2.8 \mathrm{~nm}$ and 44.8 nm ). The onset of each M 2 curve is for the full wetting of the core by 2 Au MLs.

## E. Other driving forces (elastic energy and grain boundary energy)

Finally, we briefly discuss the possible roles of elastic energy and Au grain boundary energy, which are not included in our model.

Elastic energy in <001>Au grains is due to the moderate misfit ( $0.6 \%$ ) resulting from the rotation of the Au and Fe lattices at the $\{001\} \mathrm{Au} /\{001\} \mathrm{Fe}$ interfaces. In $<111>\mathrm{Au}$ grains, this $0.6 \%$ misfit exists in the $<110>A u / /<001>F e$ direction and a residual misfit could subsists in the <$112>\mathrm{Au} / /<110>$ Fe direction after relaxation through misfit dislocations. In a (001) 2D Au layer with a $0.6 \%$ misfit, the elastic energy is $2.8 \times 10^{6} \mathrm{~J} / \mathrm{m}^{3}$ [9] and we estimated by the finite element method that it can be reduced by more than $50 \%$ thanks to the pyramidal shape [9, 28]. For the small Au volumes discussed in Fig. 8 and Fig. 9, the gain in elastic energy provided by the pyramids is thus 2 orders of magnitude smaller than $\Delta E_{M 2} \rightarrow s p$. So the relaxation of elastic energy also favors pyramids and slightly reduces the volume ratio for which the barrier separating M 2 and M 1 is null or can be crossed, but it plays a very minor role compared to surface and interface optimization.

Regarding the grain boundaries in Au, their areas and thus the associated crystalline disorder are reduced in our model thanks to the constrain on the shell to adopt the same shape as the core (a shape divergence would increase the grain boundaries areas). The formation of 6 pyramidal grains at the detriment of the homothetic shell also reduces the boundary areas and thus their energetical costs. So, this additional driving force should also contribute to favor the formation of pyramids.

## VI. CONCLUSION

Fe@Au NPs achieved through an ultra-high vacuum growth process present a crystalline Fe core embedded in a polycrystalline Au shell, with core and shell morphologies both depending on the $\mathrm{Au} / \mathrm{Fe}$ volume ratio.

First the growth mode of a shell on a nanocrystal was discussed. If, for given surface and interface energy densities, a layer on a 2D infinite substrate follows the classical 2D Franck-van-derMerve growth mode (full wetting), another growth mode takes place due to the finite size of the core. It presents some analogies with the Stranski-Krastanov growth mode (increase of the wetting factor with the deposited amount).

To elucidate the various morphologies of the observed NPs, the excess energy of the NPs compared to the same Au and Fe amounts in their bulk phases was calculated using a geometrical
model. The energies densities of the surfaces and of the two possible $\mathrm{Au} / \mathrm{Fe}$ interfaces were computed by DFT. The shapes that minimize the excess energy were determined as a function of the $\mathrm{Au} / \mathrm{Fe}$ volume ratio from 0.47 to 2.5 .

On the basis of the observed morphologies, our model provides two shape transformations that can modify the excess energy at constant Au and Fe volumes: (i) the transformation of the core, a regular polyhedron that can adopt any intermediate shape from a \{110\}Fe rhombohedral dodecahedron to a perfect $\{001\}$ Fe cube, (ii) the thickness modulation of the shell, through the formation of Au pyramids with a variable truncation rate.

Three different driving forces were identified: (i) the very strong wetting of Au on Fe that tends to preserve a minimal Au layer of about 2MLs on each Fe core face. (ii) the interface energy minimization, that favors the cube-shaped core. This is due to the ratio
$\gamma_{(111) A u /(110) \mathrm{Fe}}^{\text {interf }} / \gamma_{(001) \mathrm{Au} /(001) \mathrm{Fe}}^{\text {interf }}$ larger than $\sqrt{2}$ (about 1.8 from our DFT calculations). (iii) The minimization of Au surface energy, that can promote a 3D Stranski-Krastanov like growth mode of Au on a Fe nanosubstrate. They can compete or cooperate depending on the $\mathrm{Au} / \mathrm{Fe}$ volume ratio.

For large $\mathrm{Au} / \mathrm{Fe}$ volume ratios, there is no competition between the three driving forces, and the two shape transformations cooperate so that the equilibrium shape cumulates the full wetting, the cube-shaped Fe core $\left(R_{F e}=\sqrt{2}\right)$ that minimizes the interface energy and the Au pyramids that minimize the surface energy. From a volume ratio of 1.2, the pyramid truncation is stabilized at 0.535 . This optimal theoretical truncation is in excellent agreement with the experimental observations.

For small $\mathrm{Au} / \mathrm{Fe}$ volume ratio, another optimal shape with an octagonal section and a quasihomogeneous Au shell thickness is favored. It is mainly driven by the full wetting of the iron core by gold. When the $\mathrm{Au} / \mathrm{Fe}$ volume ratio just allows the coverage of Fe by 2 Au MLs , the shape is calculated by the Wulff theorem, considering on each core face the sum of the interface energy and Au surface energy (Eq. (29)). For the Fe volume of $175.6 \mathrm{~nm}^{3}$ (same order of magnitude as the experimental volumes), this volume ratio is 0.47 and the core aspect ratio calculated is then $R_{F e}=1.11$. This calculated shape is very close to experimental NPs with an octagonal section as in Fig. 1(b) (observed $R_{F e}$ around 1 and a coverage of about 2.5 MLs).

Between these two extreme shapes, the wetting can be fully preserved, and the optimal shape progressively evolves with the $\mathrm{Au} / \mathrm{Fe}$ volume ratio by the concomitant development of the Au pyramids and the transformation of the polyhedral core towards a cube.

The comparison of experimental Fe@Au NPs with the predictions of the model indicate that most of them are close to their individual equilibrium shape, given the core size and the available gold amount.

This analysis can be extended to other systems, including those where the epitaxial accommodation of two different crystalline systems result in NPs with a crystalline core and a polycrystalline shell, where it can provide a rational tool for predicting the NPs equilibrium shapes and thus be helpful in mastering the shape dependent physical properties.

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## DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

## APPENDIX A: CORE GEOMETRY

Figure 15(a) displays the (001)Fe cross-section of the core, built by applying Eq. (1). A \{001\}Fe face is a square with the edge:

$$
\begin{equation*}
b_{001}^{F e}=2\left(\sqrt{2} h_{110}^{F e}-h_{001}^{F e}\right)=2 h_{001}^{F e}\left(\sqrt{2} R_{F e}-1\right) \tag{A1}
\end{equation*}
$$

Figure 15(b) displays a \{110\}Fe face, an hexagon with the width:

$$
\begin{equation*}
b_{110}^{F e}=\sqrt{2}\left(2 h_{001}^{F e}-\sqrt{2} h_{110}^{F e}\right)=2 h_{001}^{F e}\left(\sqrt{2}-R_{F e}\right) \tag{A2}
\end{equation*}
$$

The angle $\alpha$ between the $<011>$ and $<111>$ edges of a $\{110\} F e$ face is $35.26^{\circ}(\cos \alpha=\sqrt{2} / \sqrt{3})$.
(a) (001) cross-section of the Fe core
(b) $\{110\} \mathrm{Fe}$ facet


FIG. 15. Cross-section (a) and $\{110\}$ facet (b) of the core.

## APPENDIX B: GEOMETRICAL LIMITS OF THE SHELL FOR A GIVEN CORE SHAPE $\boldsymbol{R}_{\mathrm{Fe}}$

The maximum values of $f$ and $t, f_{\max }$ and $t_{\max }$, depend only on $V_{A u} / V_{F e}$. They are reached when the pyramids are empty $\left(V_{A u}^{p y r}=0\right.$ and $\left.V_{A u}=V_{A u}^{h o m o}\right)$. From Eq. (8) it comes

$$
\begin{align*}
& f_{\max }=\left(1+V_{A u} / V_{F e}\right)^{1 / 3} ; t_{\max }=1  \tag{B1}\\
& h_{p y r}^{A u}=0 \text { and } h_{111 \_\max }^{A u}=h_{110}^{F e}\left(f_{\max }-1\right) \tag{B2}
\end{align*}
$$

The minimal values $f_{\min }$ and $t_{\min }$ result from the progressive building of the pyramids from the above situation. For a given $R_{F e}$, there exists a threshold volume ratio $\left(V_{A u} / V_{F e}\right)_{t h}$ for which $f=1$ and $t=0$ arise simultaneously so that $V_{A u}=6 V_{A u}^{p y r}$. It comes from Eq. (6), (7) and (12):

$$
\begin{equation*}
\left(\frac{V_{A u}}{V_{F e}}\right)_{t h}=\frac{\sqrt{2}\left(S_{001}^{F e}\right)^{3 / 2}}{V_{F e}}=\frac{2\left(\sqrt{2} R_{F e}-1\right)^{3}}{\sqrt{2}-3 R_{F e}\left(\sqrt{2}-R_{F e}\right)^{2}} \tag{B3}
\end{equation*}
$$

There are three possibilities:
(i) if $\left(V_{A u} / V_{F e}\right)>\left(V_{A u} / V_{F e}\right)_{t h}$ : the homothetic shell subsists when the pyramids are complete so that $t_{\text {min }}=0$. From Eq. (7), (8), (12) and (B3) it comes:

$$
\begin{gather*}
f_{\min }=\left(\frac{V_{A u}+V_{F e}}{V_{F e}+\sqrt{2} S_{001}^{F e}}\right)^{1 / 3}=\left[\frac{1+\left(V_{A u} / V_{F e}\right)}{1+\left(V_{A u} / V_{F e}\right)_{t h}}\right]^{1 / 3}  \tag{B4}\\
h_{111 \_ \text {min }}^{A u}=h_{110}^{F e}\left(f_{\text {min }}-1\right) \tag{B5}
\end{gather*}
$$

(ii) if $\left(V_{A u} / V_{F e}\right)=\left(V_{A u} / V_{F e}\right)_{t h}$ : the homothetic shell is empty and the pyramids are complete simultaneously so that $f_{\text {min }}=1$ and $t_{\text {min }}=0$.
(iii) if $\left(V_{A u} / V_{F e}\right)<\left(V_{A u} / V_{F e}\right)_{t h}$ : then $f_{\min }=1$ because the homothetic shell is empty before the pyramids can be complete. In this situation $h_{111 \_\min }^{A u}=h_{100_{\_} \min }^{A u}=0$ and $V_{A u}^{p y r}=V_{A u} / 6$. From Eq. (12) and (B3) the minimal truncation is

$$
\begin{equation*}
t_{\min }=\left(1-\frac{V_{A u}}{\sqrt{2} S_{001}^{F e^{3 / 2}}}\right)^{1 / 3}=\left[1-\frac{\left(V_{A u} / V_{F e}\right)}{\left(V_{A u} / V_{F e}\right)_{t h}}\right]^{1 / 3} \tag{B6}
\end{equation*}
$$

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(c)

(001)Au/(001)Fe
[1-10]Au
[001]Fe $[-1-12] \mathrm{Au}$
$[1-10] \mathrm{Fe}$
(111)Au/(110)Fe
(a) Fe core faces

Au shell surfaces

$<001>$ oriented
Au grain
$<111>$ oriented
Au grain
$\{001\}$ Au surface
$\{111\}$ Au surface

$---\quad\{001\} \mathrm{Au} /\{001\} \mathrm{Fe}$ interface

-     -         -             - \{111\}Au/\{110\}Fe interface
- . . - homothetic part of the shell


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(b)

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$\begin{array}{ll}\text { (a) }(001) \text { cross-section of the Fe core } & \text { (b) }\{110\} F \mathrm{Facet}\end{array}$

## s <br> ic 10 B

$\frac{2}{4}$



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[^1]:    ${ }^{1}$ Note that Fig. 5 is presented regardless the NP size. However, this is valid only when the Au thickness is larger than the critical thickness defined in section II. E. 2. for the full wetting condition. For instance, for the 175.6 $\mathrm{nm}^{3} V_{F e}$ discussed in the next section, and $R_{F e}$ around 1, Fig. 5 is valid for $V_{A u} / V_{F e}$ larger than 0.47 if the critical thickness is 2 MLs .
    ${ }^{2}$ There however exists a particular volume ratio where the 3D wetting factor $\Phi_{A u /(3 D) F e}$ is identical whatever $R_{F e}$ and its hierarchy as a function of $R_{F e}$ undergoes an inversion. It is given from (15) and (23) by

    $$
    V_{A u} / V_{F e}=\left[1+\left(\Phi_{(001) A u /(001) F e}-\Phi_{(111) A u /(110) F e}\right) /\left(\gamma_{111}^{A u}-\gamma_{001}^{A u}\right)\right]^{3 / 2}-1
    $$

    and is 3.4 here. For lower volume ratios, the larger the predefined $R_{F e}$ of the core (i.e. the larger the (001)Au/(001)Fe interface area), the larger the energetical benefit to form a shell homothetic to the core as shown by the smaller $\Phi_{A u /(3 D) F e}$. This is because of the hierarchy of the 2D wetting factors $\left(\Phi_{(111) A u /(110) F e}>\Phi_{(001) A u /(001) F e}\right)$. As the volume ratio increases, the surface term in the shell formation energy $\Delta F$ becomes dominant. As $\gamma_{111}^{A u}<\gamma_{001}^{A u}$, the cost to form a shell becomes larger ( $\Phi_{A u /(3 D) F e}$ becomes larger) with large $R_{F e}$ than with small $R_{F e}$. If $\left(\Phi_{(001) A u /(001) F e}-\right.$ $\left.\Phi_{(111) A u /(110) F e}\right)$ and $\left(\gamma_{111}^{A u}-\gamma_{001}^{A u}\right)$ had opposite sign, this inversion would not exist.

