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Lie Algebroidal Categories

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Universidade Federal de Minas Gerais

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Plain

1. Categorification
2. Lie Algebroidal Categories
3. Connections with Lie Algebroids
4. Speculations

Categorification

Intuition

- ▶ Roughly speaking:
 - ▶ **categorification** is a process that takes a set-theoretic concept and produces an analogous categorical-theoretic concept;
- ▶ There are at least two of such processes:
 - ▶ vertical categorification and
 - ▶ horizontal categorification

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 - ▶ **horizontal categorification**

Vertical Categorification

- ▶ Consists in two steps:
 1. to describe the set-theoretic concept to be categorified in terms of categorical structures in **Set**;
 2. to internalize the defining categorical structures in **Cat**.

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Vertical Categorification

- ▶ in few words, we must replace:
 1. sets by categories;
 2. functions between sets by functors between categories;
 3. commutative diagrams of functions (describing properties) by natural transformations satisfying some universal condition.

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Examples

- A “monoid” is given by:
1. a set X ;
 2. a function $* : X \times X \rightarrow X$;
 3. a distinguished element $e \in X$;
 4. such that

$$x * (y * z) = (x * y) * z \quad (1)$$

$$x * e = x = e * x \quad (2)$$

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- ▶ Remark: 3. and 4. are not categorical!

Examples

- a “monoid” is given by:
1. a set X ;
 2. a function $*$: $X \times X \rightarrow X$;
 3. a function $e : 1 \rightarrow X$, where 1 is a singleton;
 4. commutative diagrams

$$\begin{array}{ccccc}
 X \times (X \times X) & \xrightarrow{\cong} & (X \times X) \times X & \xrightarrow{* \times id} & X \times X \\
 \downarrow id \times * & & & & \downarrow * \\
 X \times X & \xrightarrow{*} & & & X
 \end{array}$$

$$\begin{array}{ccccc}
 1 \times X & \xrightarrow{1 \times id} & X \times X & \xleftarrow{id \times 1} & X \times 1 \\
 \searrow \cong & & \downarrow * & & \swarrow \cong \\
 & & X & &
 \end{array}$$

Examples

- a categorified monoid (“monoidal category”) is given by:
1. a category \mathbf{C} ;
 2. a functor $*$: $\mathbf{C} \times \mathbf{C} \rightarrow \mathbf{C}$;
 3. a functor $e : 1 \rightarrow \mathbf{C}$, where 1 is the categorical singleton;
 4. analogous commutative diagrams

$$\begin{array}{ccccc}
 \mathbf{C} \times (\mathbf{C} \times \mathbf{C}) & \xrightarrow{\cong} & (\mathbf{C} \times \mathbf{C}) \times \mathbf{C} & \xrightarrow{* \times id} & \mathbf{C} \times \mathbf{C} \\
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Examples

set-theoretic concept	vertical categorification
monoid	monoidal category

Table: Examples of Vertical Categorification

Examples

set-theoretic concept	vertical categorification
monoid	monoidal category
commutative monoid	braided monoidal category symmetric monoidal category
group	monoidal category s.t. $\text{Ob}(\mathbf{C}) \subset \text{Pic}(\mathbf{C})$
free and fin. gen. monoid	rigid monoidal category
⋮	⋮

Table: Examples of Vertical Categorification

Motivation

- ▶ why should an algebraist be interested in vertical categorification?
- ▶ Folklore: Let “P” be some algebraic concept which can be vertically categorified;
- ▶ then its category of representations \mathbf{Rep}_P has the structure of a vertical categorification of “P”;
- ▶ **Tannaka duality.** *Let “P” be associative and suppose that it can be vertically categorified. If \mathbf{C} is a category which realizes the vertical categorification of “P”, then $\mathbf{C} \simeq \mathbf{Rep}_P$.*
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Lie Algebroidal Categories

Idea

set-theoretic concept	vertical categorification
monoid	monoidal category
Lie algebra	??

Table: Vertical Categorification of Lie Algebra

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Table: Vertical Categorification of Lie Algebra

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- A “Lie algebra²” is given by:
1. an abelian group \mathfrak{g} ;
 2. a bilinear map $[\cdot, \cdot] : \mathfrak{g} \times \mathfrak{g} \rightarrow \mathfrak{g}$;
 3. such that

$$[x, y] + [y, x] = 0 \quad (3)$$

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- Remark: 3. is not categorical!

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Lie Algebra

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 1. the polynomial identity

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 - ▶ THUS...

Lie Algebroidal Categories

set-theoretic concept	vertical categorification
monoid	monoidal category
Lie algebra	Lie algebroidal category of Type I
	Lie algebroidal category of Type II
	Lie algebroidal category of Type III

Table: Vertical Categorification of Lie Algebra



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Table: Vertical Categorification of Lie Algebra

- ▶ **Remark:** It seems (at least to me) that the corresponding categories **LieCatI**, **LieCatII** and **LieCatIII** are not equivalent.

Lie Algebroids

Horizontal Categorification

- ▶ Consists in taking a set-theoretic concept “P” and finding for a class of categories \mathbf{Cat}_P such that categories $\mathbf{C} \in \mathbf{Cat}_P$ with a single object are equivalent to “P”.
- ▶ In simple terms, horizontally categorifying “P” is to search for some way of considering a “many objects version of P”

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Examples

- ▶ If “P” is the concept of group, \mathbf{Cat}_P is the category of all categories in which every morphism is an isomorphism.
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Lie algebra	??

Table: Examples of Horizontal categorification

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Table: Examples of Horizontal categorification

Lie Algebroids

- ▶ A *Lie algebroid* over a manifold M is given by:
 1. a vector bundle $E \rightarrow M$;
 2. a Lie algebra structure on the space of global sections $\Gamma(E)$;
 3. a vector bundle morphism $\rho : E \rightarrow TM$ such that:
 - 3.1 it is a derivation relative to the action of the ring $C^\infty(M)$;
 - 3.2 the induced map $\Gamma(\rho) : \Gamma(E) \rightarrow \Gamma(TM)$ is a morphism of Lie algebras.
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Lie Algebroids

set-theoretic concept	vertical categorification
group	groupoid
Lie algebra	Lie algebroid

Table: Horizontal Categorification of Lie Algebra

From Lie Algebroids to Lie Algebras

- ▶ On the other hand...
- ▶ “Lie algebroid” over the point $M \simeq pt$ is equivalent to “Lie algebra”.
- ▶ This motivates us to ask:

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- ▶ This motivates us to ask:
- ▶ Question: *Can we embed $\mathbf{LieAlgd}_M$ in some category of categories $\mathbf{C} \subset \mathbf{Cat}$ such that when regarded as object of this new category “Lie algebroid” becomes a horizontal categorification of “Lie algebra”?*

From Lie Algebroids to Lie Algebras

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From Lie Algebroids to Lie Algebras

- ▶ We propose a solution (at least after restriction to \mathbb{Z}):
- ▶ **Theorem:** *For every manifold M the category $\mathbf{LieAlg}_M^{\mathbb{Z}}$ of \mathbb{Z} -Lie algebroids over M can be fully embedded in $\mathbf{LieCatIII}$. Furthermore, inside $\mathbf{LieCatIII}$ the one-object limit of $\mathbf{LieAlg}_M^{\mathbb{Z}}$ coincides with the one-point limit.*

Speculations

From Lie Algebras to Lie Algebroids

- ▶ The idea is to use the following steps:
 1. to consider a definition/result about Lie algebras;
 2. to redefine/reprove the result using only categorical structure, i.e, to show that it is internal to $\mathbf{LieAlg}_{\mathbb{Z}}$;
 3. to apply vertical categorification in order to get an analogous definition/result internal to $\mathbf{LieCatIII}$;
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From Lie Algebras to Lie Algebroids

- ▶ Example: what is the Lie algebroid version of the classification of complex semisimple Lie algebras?
- ▶ OBS: a “semisimple Lie algebroid” should be a Courant algebroid fulfilling additional conditions...
- ▶ Example: what is representation theory for Lie algebroids??
- ▶ and so on.

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From Lie Algebroidal Categories to Lie Algebras

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- ▶ can we build some version of Tannaka duality for “nonassociative algebraic objects”?
- ▶ in affirmative case, any Lie algebroidal category should be equivalent (under Tannaka duality) to the category of representations of some Lie algebra.
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Thank you.

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