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# The progressive 6-year-old conserver: Numerical saliency and sensitivity as core mechanisms of numerical abstraction in a Piaget-like estimation task

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MECHANISMS OF NUMERICAL ABSTRACTION

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3 The progressive 6-year-old conserver: numerical saliency and sensitivity as core mechanisms  
4 of numerical abstraction in a Piaget-like estimation task

5

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1           **The progressive 6-year-old conserver: numerical saliency and sensitivity as core**  
2           **mechanisms of numerical abstraction in a Piaget-like estimation task**

3

4   **Abstract**

5   In Piaget's theory of number development, children do not possess a true concept of number  
6   until they are able to reason on numerical quantity regardless of changes in other  
7   nonnumerical magnitudes, such as length. Recent studies have echoed this result by arguing  
8   that abstracting number from nonnumerical dimensions of magnitude is a developmental  
9   milestone and a strong predictor of mathematics achievement. However, the mechanisms  
10   supporting such abstraction remain largely underspecified. We aimed to study how  
11   identification of the numerical equivalence in a Piaget-like estimation task by 6-year-old  
12   children is affected by (a) the degree of interference between number and nonnumerical  
13   magnitudes and (b) children's spontaneous orientation to numerosity. Six-year-old children  
14   first performed a card sorting task assessing their spontaneous orientation towards  
15   numerosity, spacing, or item size in a set of dots. Then, they completed a Piaget-like  
16   same/different numerical estimation task using two rows of dots in which the length ratio  
17   between the two rows varied systematically. Children were less likely to accept the numerical  
18   equivalence in the Piaget-like estimation task (a) as the difference in spacing between the dots  
19   increased and (b) as the children were more spontaneously oriented towards spacing over  
20   number in the card sorting task. Our results suggest that abstracting number depends on its  
21   saliency, which varies both as a function of the context (i.e., length ratio between the two  
22   rows) and of individual differences in children's sensitivity to the numerical aspects of their  
23   environment. These factors could be at the root of the observed development of performance  
24   in the seminal number-conservation task, which appears as a progressive abstraction of  
25   number rather than a conceptual shift, as Piaget hypothesized.

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27 **Keywords**

28 Numerical Estimation; Spontaneous Orientation to Magnitudes; Individual differences;

29 Heuristics

30

## 31 1. Introduction

32 In Piaget's theory of numerical development, children do not reach a proper understanding of  
33 the concept of number until they are able to perceive the conservation of the numerical  
34 properties of an array of objects despite visual transformations (Piaget, 1952). Indeed, in the  
35 seminal number-conservation task, until approximately 6 or 7 years of age, children fail to  
36 perceive the numerical equivalence between two rows of tokens after the length of one row is  
37 transformed (i.e., by spreading the tokens apart), although they are able to do so when the two  
38 rows of tokens have the same length (i.e., before the transformation). Several studies provided  
39 converging evidence that success in the number-conservation task in children and adults relies  
40 in part on the executive ability to solve the interference between irrelevant nonnumerical  
41 dimensions of magnitude (the length or density of the rows of tokens) and the pertinent  
42 numerical dimension (Houdé & Guichart, 2001, Houdé et al., 2011). The authors suggest that  
43 the participants do so by inhibiting an automatic but misleading 'length equals number'  
44 strategy.

45 Since Piaget's work, research in the field of numerical cognition has revealed the existence of  
46 numerical abilities long before children's success in Piaget's number-conservation task.

47 Children's development of the concept of number is thought to build, at least partly, upon  
48 preverbal representations of quantities that are present very early in infancy (Dehaene, 2001;  
49 Izard, Sann, Spelke, & Streri, 2009). One of these numerical intuitions consists in the ability  
50 to perceive the approximate quantity represented by a set of objects (e.g., an array of dots).

51 Echoing Piaget's work, many studies investigating these nonsymbolic numerical  
52 representations have reported the impact of visuospatial cues on numerical estimation (Gebuis  
53 & Reynvoet, 2012). For instance, congruency effects have been observed throughout  
54 development in the context of numerical estimation tasks, showing that the participants'  
55 performances are influenced by irrelevant spatial dimensions of the stimuli, such as the size of

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56 the dots, or the extent of the space occupied by the array (i.e., field area, often measured by  
57 the convex hull surrounding the dot array). In particular, field area has been shown to  
58 influence numerosity judgments in preschool and school-aged children (e.g., Gilmore, Cragg,  
59 Hogan, & Inglis, 2016). In numerical comparison tasks, number/field area incongruent trials  
60 (i.e., where the most numerous array of dots is the one occupying less field area) are  
61 performed less accurately and more slowly than congruent trials (i.e., where the most  
62 numerous array is also the one occupying more field area). Recent studies have observed that  
63 the developmental trajectories of these congruency effects can differ depending on the  
64 interfering spatial dimension (Gilmore et al., 2016). In the case of field area, its impact on  
65 numerical representation can be observed all the way into adulthood (Clayton, Gilmore, &  
66 Inglis, 2015; Gilmore et al., 2016). As in the number-conservation task (whereby field area is  
67 equivalent to the length of the rows of tokens), inhibition seems to play a critical role in  
68 blocking the influence of irrelevant spatial information to make judgments based on the  
69 numerical aspects of the stimuli in numerical estimation tasks (Leibovich, Katzin, Harel, &  
70 Henik, 2017). These observations support the idea that, beyond children's precision of their  
71 numerical representations, their ability to abstract number from other nonnumerical  
72 dimensions of magnitude is a strong predictor of math achievement (Gilmore et al., 2013; but  
73 see Keller & Libertus, 2015). This is further supported by studies demonstrating that  
74 conservation abilities in pre- and early-school years are related to children's mathematical  
75 abilities such as math fluency (Cooper & Schleser, 2006; Ramos Christian, Schleser, & Varn,  
76 2008).

77 Children's numerical estimation is not only related to their ability to inhibit irrelevant spatial  
78 information. For instance, individual differences in children's spontaneous orientation to  
79 number are predictive of math achievement (e.g., Hannula-Sormunen, Lehtinen, & Räsänen,  
80 2015). Moreover, children's spontaneous orientation towards numerosity is related to their

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81 ability to perform number/size incongruent trials in a nonsymbolic numerical comparison task  
82 (Viarouge et al., 2018). In the study of Viarouge et al. (2018), the authors assessed children's  
83 relative spontaneous orientation towards three dimensions of magnitude by asking them to  
84 sort cards representing arrays of dots. No further instruction was provided on the sorting  
85 criterion the children should use. The cards could be sorted on the basis of three dimensions  
86 of magnitude: number, size, or spacing. Children who sorted the cards more frequently by  
87 number in the card sorting task (hereafter referred to as the SOMAG task for "Spontaneous  
88 Orientation to MAGnitudes") performed better on number/size incongruent trials in a  
89 numerical comparison task. This indicated that children's spontaneous orientation towards  
90 different dimensions of magnitude is related to their ability to solve interferences due to  
91 irrelevant visual cues in numerical estimation. Consistent with these results, a study  
92 demonstrated that kindergarteners who succeed in the number-conservation task focus more  
93 spontaneously on number than kindergarteners who fail the number-conservation task, and  
94 vice versa for length (Miller & Heller, 1976).

95 Taken together, these findings suggested that the development of children's numerical  
96 representations relies on a variety of mechanisms allowing them to abstract the numerical  
97 aspects of their environment from other nonnumerical dimensions of magnitude. Hence, what  
98 had been described by Piaget as a shift between two stages of numerical development, from a  
99 nonconserving to a conserving response in contexts such as the number conservation task now  
100 appears as a progressive ability to focus on the numerical aspects of one's environment.

101 However, so far, the mechanisms behind this progressive abstraction remain underspecified.  
102 Our study is thus aimed to better characterize the mechanisms involved in numerical  
103 abstraction, which is crucial to understand the predictive value of early numerical  
104 representations for math achievement.

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105 Both the ability to inhibit irrelevant dimensions and the spontaneous orientation to these  
106 dimensions are likely to contribute to this development. From this perspective, children's  
107 performance in the seminal number conservation task should depend both on the strength of  
108 the interference between the numerical and nonnumerical dimensions and on children's  
109 sensitivity to the numerical dimension of the task. In classic nonsymbolic numerical  
110 comparison tasks using arrays of dots, isolating nonnumerical dimensions of magnitude to  
111 study their impact on performance is particularly tedious. Indeed, all the dimensions of  
112 magnitude cannot be controlled for simultaneously. This is why we will use a Piaget-like  
113 number conservation task, whereby a same-different numerical judgement is performed on  
114 two horizontally arranged rows of tokens. This context allows investigating the impact of  
115 length/field area on numerical representation, while keeping other dimensions of magnitude  
116 such as size and total surface area constant. To date, no study has manipulated this level of  
117 saliency in a systematic manner, while conjointly taking into account individual measures of  
118 spontaneous orientation towards magnitudes.

119  
120 In the current study, we asked 6-year-old children to perform the SOMAG task used in  
121 Viarouge et al. (2018) to assess their relative spontaneous orientation towards different  
122 magnitudes (number, size, spacing). The participants were also asked to perform a  
123 computerized "same/different" numerical estimation task with horizontally arranged dots (as  
124 in Piaget's number-conservation task after the length of one row is transformed). The spacing  
125 of the dots in one row (and thus the length ratio between the two rows) was systematically  
126 varied. Using a Piaget-like same-different task will allow isolating for the first time the  
127 selective impact of the field area dimension on numerical equivalence perception by  
128 manipulating the strength of the interference between field area and number in a systematic  
129 manner at a crucial age of numerical development in Piaget's theory. We investigated whether



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130 children's ability to perceive the numerical equivalence between the two rows of dots (a)  
131 varies systematically with the degree of interference between the nonnumerical and numerical  
132 dimensions of magnitude and (b) is related to children's spontaneous orientation towards the  
133 various dimensions of magnitude present in the tasks. We reasoned that, if succeeding in the  
134 same-different numerical estimation task requires inhibition of irrelevant nonnumerical  
135 dimensions, then children should make more errors as the difference in dot spacing increases.  
136 Indeed, as the difference in dot spacing increases, the nonnumerical dimensions of magnitude  
137 will interfere to a greater extent with numerical processing. Additionally, children who  
138 spontaneously focus more on number over spacing in the SOMAG task should make less  
139 errors in the same-different numerical estimation task, because spacing is less salient for these  
140 children and thus interferes to a lesser extent with their numerical processing. Note that, since  
141 item size is maintained constant across the trials of the same-different task, children's  
142 performances should only correlate with their spontaneous orientation towards number over  
143 spacing, but not with their orientation towards number over size.

144

## 145 2. Materials and Methods

### 146 2.1. Participants

147 We collected data on the SOMAG task and a numerical same-different task described below  
148 in ninety-three typically developing children (47 girls, mean age =  $6.1 \pm 0.62$  years) from  
149 kindergarten and first grade classes at three public schools. Sensitivity analyses were  
150 performed using G\*Power 3.1.9.2 (Faul, Erdfelder, Lang, & Buchner, 2007), revealing that,  
151 given our sample size, an alpha-level of .05 and a level of power of .80, minimum effect sizes  
152 that can be significantly detected were  $f = .12$  for the repeated-measures ANOVA and  $r = .23$   
153 for the correlation analyses described below.

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154 The same group of children also performed a nonsymbolic numerical comparison task, and  
155 the results were reported in (Viarouge et al., 2018). Participation was voluntary after  
156 obtaining signed informed consent from the children’s parents. Since the SOMAG task  
157 assesses children’s spontaneous orientation towards either numerical or nonnumerical  
158 dimensions of magnitude, this task was always administered first for all children. The  
159 numerical same-different and comparison tasks were counterbalanced across the participants.  
160 The results described below remained identical when controlling for the order of the two  
161 tasks. Children were tested individually by groups of three or four in a quiet room. Apart from  
162 encouraging comments, no feedback on accuracy in the numerical same-different task or on  
163 the chosen sorting criteria in the SOMAG task was given. Each child received a certificate  
164 with stickers for their participation in the study. The teachers and parents were instructed not  
165 to talk with the children about the specific content and goals of the study. The study was  
166 carried out in accordance with national and international norms that govern the use of human  
167 research participants.

168

## 169 2.2. Material and design

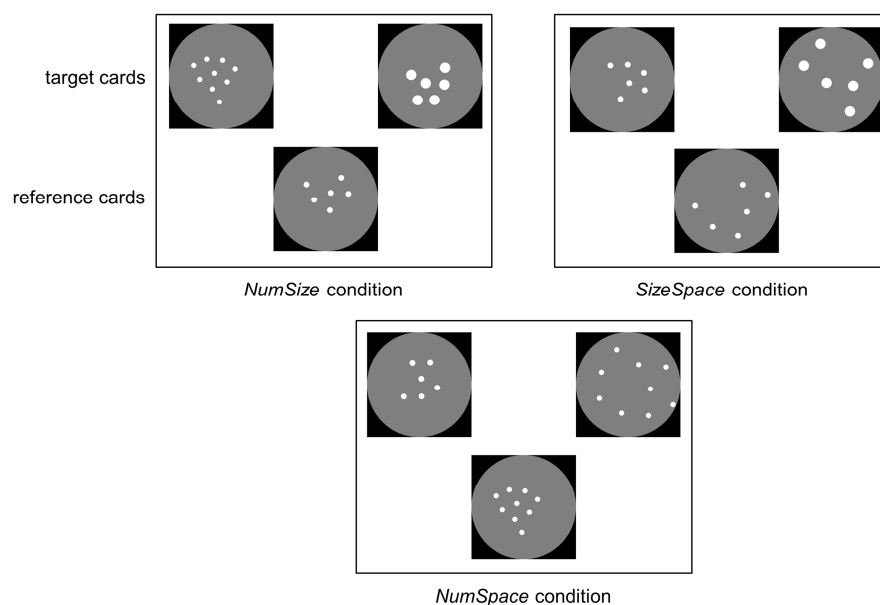
170 **SOMAG task.** In each trial of this computerized task, children were presented with three  
171 images introduced to them as cards showing arrays of dots. Two cards (targets) were  
172 horizontally aligned at the top of the screen, while the third card (reference) was centered at  
173 the bottom of the screen. Children were asked to spontaneously pick one target card to go  
174 with the reference card. The instructions were as follows: “Which card goes best with that one  
175 (pointing at the bottom card)? This one (pointing at the card on the left) or that one (pointing  
176 at the card on the right)? Give me your answer without thinking too much!” The  
177 experimenters were instructed not to mention any of the dimensions of magnitude tested in  
178 this task (number, size and spacing). Children responded using the same keys as in the

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179 numerical same-different task. They were instructed to press the right key (green sticker)  
180 when they wanted to pick the top right card and the left key (red sticker) when they wanted to  
181 pick the top left card.

182 The collections of dots shown on the cards could be sorted according to the number of dots,  
183 aspects of the dot size or of the spacing of the dots (the *Size* and *Spacing* dimensions were  
184 defined following DeWind, Adams, Platt, & Brannon, 2015). The task included three  
185 experimental conditions, depending on the two competing dimensions of magnitude matching  
186 with the bottom card (see Figure 1): in one condition, the top cards matched with the bottom  
187 card in either number or *Size*, while *Spacing* was maintained constant across the three sets of  
188 dots (later referred to as the “NumSize” condition); in one condition, the top cards matched  
189 the bottom one in *Number* or *Spacing*, while *Size* was maintained constant (“NumSpace”  
190 condition); and in one condition, the top cards matched the bottom one in either *Size* or  
191 *Spacing*, while the number of dots was maintained constant (“SizeSpace” condition).

192



**Figure 1:** Examples of the stimuli used in the SOMAG task, for each of the three experimental conditions. In each condition, one dimension (Number, Size, or Spacing) is maintained constant. The bottom reference card can be sorted with one of the two top target cards according to one of the two remaining dimensions (e.g. NumSize condition pits the Number and Size dimensions against each other).

193

194

195 For each dimension, two levels of magnitude were used, which were selected so that they

196 would be easily discriminable by children. A ratio of 1:2 was used for variations in *Size* and197 *Spacing*, while a ratio of 2:3 was used for the number of dots (sets of either six or nine dots).

198 Across all trials, we counterbalanced the location of the higher level of magnitude (the larger

199 *Size*, *Spacing* or number could either be on the two top cards or on the bottom card), and the

200 location of the two competing dimensions of magnitude (either on the top left or top right

201 card). Each trial started with the presentation of a centered fixation cross for 1500 ms,

202 followed by the presentation of the three cards for a maximum duration of 4500 ms. The next

203 trial started when the child gave his/her response, either before or after the end of the

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204 presentation of the cards. There were 16 trials per condition, resulting in 48 trials total, with a  
205 randomized presentation, and for a total duration of the task of approximately 4 minutes.

206

207 **Numerical same-different task.** In each trial of the numerical same-different task, children  
208 were presented with two horizontal rows of evenly spaced blue dots on a white background  
209 and were asked to perform a “same/different” numerical judgment. The two rows were left  
210 aligned with each other and were located respectively on the top and bottom half of the screen  
211 (at a  $1.7^\circ$  visual angle from the center). The spacing between the dots of each row was varied  
212 so that the length of the two rows differed by one of four possible length ratios (1:2, 2:3, 6:7,  
213 and 9:10). The location (top or bottom) of the longer row was counterbalanced across trials. In  
214 24 test trials, both rows had an equal number of dots (6, 7 or 8). The length of the longest row  
215 represented either  $12.4^\circ$ ,  $14.6^\circ$ , or  $16.7^\circ$  of visual angle, depending on the number of dots  
216 presented, and the length of the shortest row varied according to the length ratio tested. To  
217 prevent children from learning that the number of dots was the same in both rows while  
218 maintaining a short duration of the experiment, 16 additional trials had a different number of  
219 dots (6 vs. 7, or 7 vs. 8 dots), leading to a total of 40 trials. These numerical ratios were shown  
220 to be perceivable by 6-year-olds with an accuracy level of approximately 75% (Halberda &  
221 Feigenson, 2008). The task started with four training trials to make sure that the children  
222 understood the instructions correctly. All four training trials showed rows with a length ratio  
223 of 1:2, two “equal number” trials (6 tokens in each row), and two “unequal number” trials (6  
224 vs. 7 tokens), with counterbalanced location of the longest row. Each trial started with the  
225 presentation of a centered fixation cross for a duration of 1500 ms, followed by the  
226 presentation of the two rows of dots for a maximum duration of 1200 ms. This presentation  
227 time was chosen to prevent the use of counting during the task. Children were asked the  
228 following: “Do the two rows have the same number of dots, or a different number of dots?”

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229 Children were instructed to respond using the K and D keys on an AZERTY keyboard. One  
230 green and one red sticker were placed on each of the two keys, and children were instructed to  
231 press the green key (“K” on the right) when they thought the number of dots was the same in  
232 both rows and the red key (“D” on the left) when they thought the number of dots in each row  
233 was different. Due to the tendency of some of the children to spontaneously give their  
234 responses verbally or by pointing at the screen, the experimenters then pressed the  
235 corresponding response key themselves. For this reason, reaction times were not included in  
236 the analyses reported below. When children responded that the number of dots in each row  
237 was different, the experimenter followed by asking, “Which row had more?” The child gave a  
238 verbal response, which was coded by the experimenter by pressing the up or down arrow key,  
239 depending on the location of the chosen row on the screen. The next trial started as soon as a  
240 “same number” response or the location of the row judged as containing more dots was  
241 provided. The 40 trials were presented in a different random order for each child, and the total  
242 duration of the task was approximately 5 minutes.

243

244 **3. Results**

245

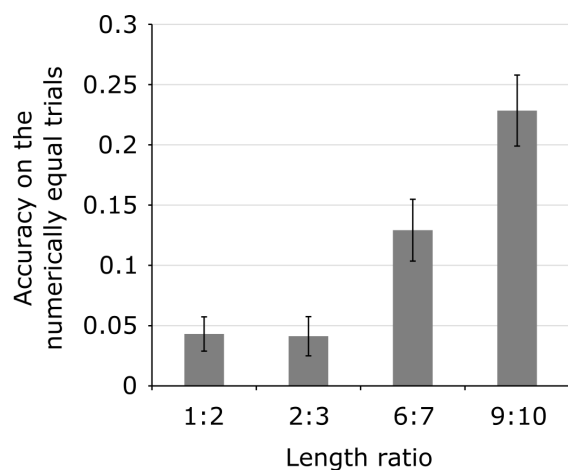
246 **Effect of the strength of the number/length interference**

247 Three children systematically alternated the right and left responses throughout the task  
248 without following the instructions. Their data were removed from the analyses described  
249 below. One child’s performance fell over the average  $\pm 2.5$  standard deviations on both  
250 accuracy levels for numerically equal and different trials and was not further analyzed, which  
251 led to a total of 89 children included in the analyses. All the results of the statistical analyses  
252 reported below were similar when adding the data from this child. For each child, we  
253 computed the average accuracy score on the 24 test trials (same number of dots in each row)

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254 for each length ratio. Children judged the two rows as containing a different number of dots  
 255 89% of the time across all trials. However, a repeated-measures ANOVA using length ratio as  
 256 a within-subject factor showed a significant effect of the ratio on the children's performances,  
 257  $F(3, 264) = 24.36, p < .0001, \eta_p^2 = .22$ , with accuracy scores increasing with the length ratio  
 258 (1:2 = 4.31%; 2:3 = 4.12%; 6:7 = 12.92%; 9:10 = 22.85%). Post hoc Bonferroni tests  
 259 confirmed a significant difference in performance between the 2:3 and 6:7 length ratio ( $p <$   
 260  $.01$ ) and between the 6:7 and 9:10 length ratio ( $p < .001$ ) but not between the first two length  
 261 ratios ( $p = 1$ , Figure 2).

262



**Figure 2:** Performance on the numerically equal trials of the numerical estimation task, for each level of length ratios between the two rows of dots. Error bars represent  $\pm$  one standard error of the mean.

263

264 It is worth noting that 86 of the 89 children who judged two numerically equal rows as  
 265 different subsequently stated that there were more dots in the longer row (significant Binomial  
 266 tests,  $p < .012$ ), confirming the use of a misleading “length equals number” rather than a  
 267 “density equals number” strategy. We computed, for each participant, the slope of the linear  
 268 regression of the accuracy scores against the level of length ratio. This measure allowed us to  
 269 evaluate individual differences in the impact of the length ratio in the numerical same-  
 270 different task. We observed a significant correlation between this slope and the slope obtained

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271 from the accuracy scores on the 16 filler trials (numerically different rows),  $r(87) = -0.86$ ,  $p <$   
272  $.001$ , showing that the ratio of length impacted the children's performances both on the  
273 numerically equal and different trials, and confirming the reliability of our measure.

274

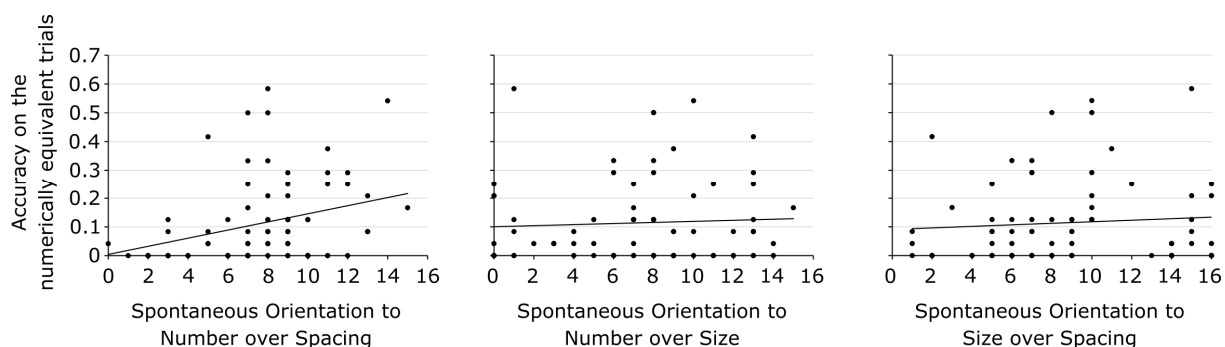
275 **Effect of the spontaneous orientation to numerical and nonnumerical dimensions of**  
276 **magnitude**

277 Despite the same instructions being repeated throughout the SOMAG task, some children had  
278 a hard time choosing sorting criteria. Of the 89 children included in the analyses of the  
279 numerical same-different task, 13 children did not follow the instructions of the SOMAG task  
280 and systematically alternated between the left and right response keys. These children might  
281 have been confused by the absence of a clear rule to follow and decided to follow a reassuring  
282 strategy. Their data were removed from the following analyses, which were run on a total of  
283 76 children. We verified that the effect of length ratio on the performance in the same-  
284 different task was still present in this sample ( $F(3, 225) = 23.6$ ,  $p < .001$ ,  $\eta_p^2 = .22$ ), with  
285 similar increasing accuracy scores (1:2 = 4.39%; 2:3 = 3.73%; 6:7 = 13.16%; 9:10 = 24.34%).  
286 The SOMAG task provided individual scores corresponding to the total number of times each  
287 dimension of magnitude (numerosity, *Size*, *Spacing*) was spontaneously chosen as a sorting  
288 criterion by the child in the three experimental conditions (NumSize, NumSpace, and  
289 SizeSpace). We ran correlational analyses between the SOMAG task scores and children's  
290 average performances in the numerical same-different task (see Table 1 for a full description  
291 of the correlations). The average performance in the test trials of the numerical same-different  
292 task (i.e., trials in which there was a conflict between number and length/field area) was  
293 associated with the score in the NumSpace condition of the SOMAG task,  $r(74) = .30$ ,  $p <$   
294  $.01$ , but not with the NumSize or the SizeSpace condition,  $p > .49$ . This correlation remained  
295 significant after controlling for the exact age of the participants,  $r(74) = .35$ ,  $p < .01$ . This



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296 association showed that children who chose to sort the cards according to their numerosity  
 297 rather than the spacing of the dots in the SOMAG task were also more likely to make correct  
 298 numerical judgments regardless of the difference in the spacing of the dots between the two  
 299 rows in the numerical same-different task (Figure 3).



**Figure 3:** Correlation between the spontaneous orientation to Magnitudes in the three experimental conditions of the SOMAG task and the performance in the numerical estimation task.

300  
 301  
 302 Note that, while there was a significant correlation between the two SOMAG conditions  
 303 involving Size ( $r(74) = .75, p < .001$ ) we found no significant relation between the two  
 304 conditions of the SOMAG involving the Number dimension ( $r(74) = .03, p = .77$ , Table 1).  
 305 Thus, choosing Number as a sorting criterion over Spacing was not systematically related to a  
 306 general tendency of orienting towards numerical over nonnumerical dimensions of magnitude.  
 307 A multiple regression analysis confirmed that adding the NumSize and SizeSpace conditions  
 308 to the model did not significantly change the observed relation between NumSpace and our  
 309 same-different task ( $\Delta R^2 = .01, F(2, 72) < 1$ ).

310

311

312

313

Measures	1	2	3	4
1. Same-Different Task	-	0.3 **	0.06	0.08
2. SOMAG NumSpace (Number)		-	0.03	0.14
3. SOMAG NumSize (Number)			-	- 0.75 ***
4. SOMAG SizeSpace (Size)				-

**Table 1:** Table of correlations between the performance on the test trials of the same-different task and the different SOMAG conditions. For each of the SOMAG conditions, the correlations were computed based on the dimension between parentheses.

314

315

316 

#### 4. Discussion

317 By conjointly taking into account the impact of the strength of the interference between

318 numerical and nonnumerical dimensions of magnitude and individual differences in

319 spontaneous orientation to number, the current study shed new light on the mechanisms

320 underlying the development of children's ability to abstract number. We showed that both

321 factors significantly contribute to children's ability to handle the number/length interference

322 in a Piaget-like same-different numerical task.

323 First, we provided evidence that a systematic manipulation of the length ratio between the two

324 rows of dots in a Piaget-like same/different numerical estimation task affects children's

325 performance. Specifically, we found that children's ability to make a "same" judgment on two

326 rows containing an equal number of dots decreased progressively as the difference in the

327 lengths of the two rows increased. Thus, children's performance on this task decreased as the

328 interference between the nonnumerical and numerical magnitudes increased. This result adds

329 further support to the idea that the performance observed in numerical judgement tasks are

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330 impacted by nonnumerical magnitudes, even in a same-different task thought to tap more  
331 directly into magnitude representation (Van Opstal & Verguts, 2011).

332 One of the key advantages of the Piaget-like same/different numerical estimation task  
333 designed in the present study is it allows systematic manipulation of the strength of the  
334 interference between number and a nonnumerical dimension, with a fixed level of numerical  
335 difficulty. Indeed, in our test trials (where the two rows contained the same number of  
336 tokens), the evolution of the performance cannot be explained in terms of numerical difficulty  
337 and can only be attributed to an influence of the nonnumerical aspects of the task. We argue  
338 that manipulating the length ratio modified the strength of the interference between length and  
339 number, and concurrently the probability to rely on the “length equals number” misleading  
340 heuristic. In turn, this manipulation varied the level of inhibitory control required to block  
341 such a misleading strategy in order to perceive the numerical equivalence in our same-  
342 different task. This is in line with previous findings showing that the inhibition of the ‘length  
343 equals number’ strategy is needed to overcome systematic errors in a Piaget-like number-  
344 conservation task (Houdé & Guichart, 2001, Houdé et al., 2011).

345 Second, we found that children tended to make fewer errors in the numerical same-different  
346 task when they sorted the cards in the SOMAG task more frequently by number than by  
347 spacing. This correlation suggests that the prepotency to rely on the ‘length equals number’  
348 misleading strategy in the number conservation task is also dependent on individual  
349 differences in children’s spontaneous orientation towards relevant or irrelevant dimensions of  
350 the stimuli.

351 Note that the children’s performance in our numerical same-different task was not related to  
352 their spontaneous orientation towards number over size or towards size over spacing, as  
353 assessed by the SOMAG task. This is consistent with the material of our same-different task,  
354 whereby the size of the dots is maintained as a constant and only dimensions related to

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355 density and field area vary. Thus, our results suggest a specificity of spontaneous orientation  
356 towards magnitudes, with different predictive values of orientation towards number over size  
357 v. over spacing, depending on the experimental context and/or possibly age. This hypothesis  
358 is further supported by the lack of correlation between the NumSize and NumSpace  
359 conditions in the SOMAG task.

360 This specificity is interesting to put in perspective with the results reported in the study of  
361 Viarouge et al. (2018) based on data collected for the same group of children. These results  
362 showed that both the NumSize and the NumSpace SOMAG conditions explained significant  
363 and distinct parts of the variance in the performance on Size/Number incongruent trials of a  
364 nonsymbolic numerical comparison task.

365 The different patterns of correlation with the SOMAG reported in the current study might be  
366 explained by differences between the numerical same-different and comparison tasks. As  
367 mentioned above, one important aspect of the current study is that the same-different task  
368 allows separating the Size and Spacing dimensions more clearly than in the numerical  
369 comparison tasks, such as the one used by Viarouge et al. (2018). Indeed, in the latter task,  
370 given that the Spacing dimension is related both to field area and sparsity (DeWind et al.,  
371 2015), variations in these dimensions could have occurred, which in turn might explain why  
372 the NumSpace condition was correlated with performance on Size/number incongruent trials.  
373 Importantly, in the numerically equal trials of our same-different task, both the size and total  
374 surface area of the two rows of dots are maintained as constant, and the impact of different  
375 levels of dots spacing in the context of a fixed numerical difficulty is fully isolated. Another  
376 potential explanation for this different pattern of correlations might have to do with what the  
377 different conditions of the SOMAG are actually capturing. The correlation between the two  
378 conditions involving *Size* suggests that children's choices were strongly influenced by this  
379 dimension. Added to the lack of correlation between the NumSpace and the other two

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380 conditions, this suggests that the NumSpace condition might be capturing children's  
381 spontaneous orientation to Number more specifically. This could then explain why this  
382 condition relates to both tasks involving interferences with number, regardless of the  
383 competing nonnumerical dimension.

384 A limitation of our study is the lack of a separate measure of children's inhibitory control  
385 ability. This would have allowed us to determine the respective contribution of inhibitory  
386 control ability and spontaneous orientation towards number over spacing to the performance  
387 observed in our Piaget-like same-different task. Further studies are needed to better  
388 understand the relation between the ability to process number in contexts in which irrelevant  
389 non-numerical dimensions interfere with it, the spontaneous orientation towards relevant and  
390 irrelevant magnitudes, and the ability to inhibit these irrelevant dimensions.

391 Finally, our study provides new insights into the possible mediating factors of the relation  
392 between early numerical abilities and formal mathematics. In particular, these findings help  
393 specify the mechanisms involved in the abstraction of number by suggesting to take into  
394 account individual differences in spontaneous orientation to numerical and nonnumerical  
395 magnitudes. Indeed, recent studies have questioned the predictive value of numerical  
396 intuitions for math achievement. In particular, some authors have suggested that inhibitory  
397 control acts as a mediating factor of this relation, with mixed results. According to this view,  
398 children's capacity to grasp the invariance of number with regard to other dimensions of  
399 magnitude would be crucial for their true understanding of the concept of number, beyond the  
400 precision of their early numerical representations. This capacity to abstract number from other  
401 quantities might be particularly important in early school years, when the covariance of  
402 number with dimensions such as length might be unintentionally reinforced through  
403 education. For instance, the use of nonsymbolic representations of integers via rows of evenly  
404 spaced images might strengthen the misleading "number equals length" heuristic.

405

406

407 **5. Conclusion**

408 Altogether, these results indicated that the development of numerical representations in  
409 children relies on a variety of factors contributing to children's ability to abstract number  
410 from other irrelevant, nonnumerical dimensions of magnitude. In particular, we showed that  
411 conserving responses in a Piaget-like numerical estimation task appear progressively  
412 dependent on the saliency of the numerical dimension. This saliency depended both on the  
413 degree of interference with nonnumerical magnitudes and on individual differences in  
414 spontaneous orientation towards these dimensions.

415 Our results also provided additional evidence that cognitive development does not rely  
416 exclusively on the acquisition of knowledge of increasing complexity (Piaget, 1952) but also  
417 on children's ability to inhibit misleading strategies and previously acquired knowledge  
418 (Bjorklund & Harnishfeger, 1990; Houdé, 2000) and ability that can be expressed more or  
419 less easily by the children depending on the context (i.e., the level of interference) and their  
420 spontaneous orientation towards different dimensions of the context.

421 In light of current debates regarding the predictive value of numerical intuitions for math  
422 achievement, our study helps specify the factors involved in numerical development and  
423 provides new insights regarding the possible mediating factors of this relation.

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