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# Lyapunov-Based Synchronization of Networked Systems: From Continuous-Time to Hybrid Dynamics

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## Abstract

Synchronization pertains to the property of interconnected systems according to which their dynamic behavior is coordinated in an appropriate sense. That is to say, some of their state variables, or functions of the latter for that matter, converge to each other. Synchronization may occur naturally or may be induced, controlled, and it may be present between two systems or among a large number. In the latter case, it is convenient to speak of a *network* of interconnected systems. Understanding synchronization, and how to control it, is an important paradigm as it is present in a variety of scenarios. These involve, *e.g.*, networks of technological systems (robots and vehicles of different kinds), social networks (by which people exchange opinions and agree, or not), networks of biological systems (uni or pluricellular), *etc.* Owing to the context, the mathematical models to describe such networks and to define synchronization formally, varies dramatically. Ordinary continuous-time or discrete-time models for which modern control theory and Lyapunov stability theory are tailored result inappropriate to incorporate hybrid phenomena that intervene in the network. These may stem from sudden topology changes, the use of digital or intermittent control strategies, the presence of impacts in the intrinsic dynamics of the nodes, *etc.* In this invited paper, we give an overview of synchronization control problems, mostly of cooperative control of networks of autonomous vehicles (based on continuous-time models). For that matter, the first part of the paper focuses on the main contributions of [1]. Then, we give further perspectives on what we consider significant open problems on synchronization of hybrid systems and hybrid networks.

*Keywords:* Control engineering; Synchronization; Networks of dynamical systems; Safety; Hybrid dynamical systems; Lyapunov methods.

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## 1. Introduction

Generally speaking, for a network of dynamical systems, inducing synchronization consists in designing a controller (often one for each node in the network) so that an output or a state variable of each system acquires, asymptotically, a common behavior. For example, the spatial positions of network-interconnected autonomous vehicles (such as drones, mobile robots, platoons of cars, *etc.*) may be controlled to form a specified geometric pattern, a formation of sorts; the currents and voltages in a network of DC-AC inverters may be controlled to reach the same frequency of oscillation.

Depending on the nature of interaction among the nodes in the network, we distinguish between *centralized* and *decentralized* control schemes. In the first case, each node receives global information dictating its desired behavior. This may consist in a desired set-point (a specific position or oscillating frequency), a reference

trajectory, or a desired dynamical behavior in more complex cases. In centralized control approaches, each node has an independent task to ensure, which is defined by a central entity. These approaches are often used when the network is composed of a relatively small number of nodes, but abundant computation and communication capacities may be required for the control entity. Decentralized approaches, as we consider in this article, overcome these limitations. There, the control input for each node depends only on local information provided by a given set of nodes called neighbors. The network topology, which determines how the information among the neighbors flows, is typically defined using a communication graph [1]. The communication graph plays a key role in the analysis and design of networked systems.

Decentralized synchronization in networked systems is an active and challenging research topic that has continuously attracted undivided attention from many researchers over the previous two decades —see, for instance, [1–4] in which both continuous and discrete-time networks are considered. Such is the case in engineering

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disciplines where technological solutions involve groups of systems required to *cooperate* to complete a common task. Some common examples of these are related, but are not limited, to robotic manipulation and the use of autonomous vehicles [5–9], but they are also common in energy networks [10], for instance. Other disciplines include neuroscience, in which networks and sub-networks (clusters) formed due to neuronal population activities and interactions are studied [11], social science, in which a subject of study is that of opinion networks formed due to the interactions among individuals [12], and geography, in which groups of populations that interact when reacting to catastrophes are studied [13]. While the systems models involved in each of these instances are different, a common denominator is the presence of a network composed of nodes having a dynamic behaviors that interact locally with the aim at reaching a common global goal. From a systems-theory viewpoint, at least two primary challenges can be identified when contributing to synchronization problems. The first one arises from recognizing that communication among the nodes is subject to constraints; this includes scenarios where the transfer of information is unreliable, time-dependent, or subjected to delays. The second challenge stems from considering the complexity of the nodes’ dynamics; these may be linear, nonlinear or hybrid in nature and, moreover, may vary throughout the network — such is the case of heterogeneous networks.

Some of these aspects have been addressed in [14], in scenarios pertaining to electromechanical engineering and robotics. The main originality of [14] is to propose advanced Lyapunov-based approaches inspired by techniques for stand-alone nonlinear time-varying systems to analyze the coordination problems after transforming it into global stabilization of a closed, but unbounded, set. Then, Lyapunov-based techniques, via the construction of strict and differentiable Lyapunov functions, are proposed for the network’s model after interconnection. This allows to derive several systematic methods to analyze the coordination task, the network’s performance, and its robustness with respect to perturbations and delays affecting the transfer of information among the nodes. Note that, in related literature, the coordination task is usually analyzed via trajectory-based approaches or using weak Lyapunov functions, in the sense that the Lyapunov function does not systematically allow the verification the coordination task. In that way, for autonomous mobile robots, different coordination tasks are studied in [14], such as consensus and trajectory-tracking formation control or, for networks of oscillators, the frequency synchronization problem has been addressed. In regards to the nature of interaction and transfer of information among the nodes, several scenarios are considered; these include time-varying interconnections as well as communication delays. As far as the intrinsic dynamics of the nodes is considered, only some continuous-time models, such as single and double integrators, time-varying

planar oscillators, and nonholonomic unicycles, have been considered.

The results contained in [14] concern exclusively interconnected systems described by continuous-time models. However, the problems solved therein lead to more general open questions that transcend these systems and are, in our opinion, of sufficient significance to awake the interest of a wide readership. The problems that we describe are related to the study of synchronization problems described previously, but in the context of hybrid networked systems. In particular, it is of major importance to handle the presence of discrete phenomena that are coupled with the continuous-time dynamics of the nodes that compose the network. A prominent example that makes this problem relevant is that of digital or intermittent control strategies that interact with continuous-time models, which captures constrained control actions as in power networks [15, 16], in valve-based control systems [17], or, in networked control systems [18, 19]. It also captures the creation (resp. loss) of links and the addition (resp. removal) of nodes as in social networks [20–22]. Furthermore, in the intrinsic behavior of the nodes, one must also consider systems affected by discontinuities such as impacts and instantaneous jumps like in mechanical systems with impacts [23, 24] or in cyber-physical systems [25–27]. Hence, such research aims at covering unexplored aspects which, nonetheless, are naturally motivated by intrinsic characteristics of modern systems networks.

The rest of this paper is organized as follows. In the next section, we describe the main contributions of [14]. In section 3, we discuss the relevance and challenges related to synchronization in hybrid networks. Finally, in Section 4, we provide some research directions.

## 2. Synchronization in Continuous-Time Networked Systems

### 2.1. Main Idea

In [14], original Lyapunov-based approaches are developed to analyze networks where the nodes are interconnected via decentralized control laws. More precisely, the proposed approaches consist in transforming the coordination task into global stabilization of a closed set. The analysis of the coordination task has been conducted using Lyapunov’s direct method. That is, original constructions of strict and differentiable Lyapunov functions, for classes of nonlinear time-varying systems modeling the network after interconnection, are provided. Compared to related literature, where the coordination task is usually analyzed using trajectory-based approaches or using weak Lyapunov functions, in the sense that the Lyapunov function does not systematically allow the verification the coordination

task, the solutions proposed in [14] allow a systematic analysis of the coordination task as well as the network's performance, e.g., in terms of convergence rates. Moreover, a systematic robustness analysis with respect to perturbations and delays affecting the transfer of information among the nodes is also carried out. Thanks to these approaches, various important coordination problems have been solved in [14]. These problems have been solved for different graph topologies, depending on how the information flows among the nodes, and for diverse target trajectories.

## 2.2. Leader-Follower Formation Control for Nonholonomic Vehicles

In leader-follower formation problems for mobile vehicles, the coordination task consists in making the vehicles maintain a given geometric posture in terms of both, positions and orientations, relative to that of a leader and track its trajectory [28]. For a network of nonholonomic vehicles and due to the nonholonomic restriction, solving this problem under a general leader's trajectory, using a unique continuous feedback control law, is a challenging problem, especially to establish uniform asymptotic stability. Indeed, it is shown in [29] that nonholonomic systems are not asymptotically stabilizable to arbitrary leaders trajectories having piecewise continuous velocities via continuous controllers, not even if the controllers are time-varying (the possibility of using discontinuous controllers is not analyzed in [29] and we are not aware of any other work for that matter). In the literature, for the simpler one-leader-and-one-follower case, the tracking problem has been studied in e.g., [30–34] under particular scenarios of the leader's trajectories. To the best of our knowledge, the only article that allows for general leader's trajectories and for a network of nonholonomic vehicles is [28]. The control design method in the latter reference follows the framework proposed in [35]. However, it is only established that the formation errors converge to an arbitrarily small compact ball centered at the origin.

In [14], see also the associated publications [34, 36–39], decentralized feedback control laws are proposed to solve the leader-follower formation control problem for a network of nonholonomic mobile robots. The interconnections among the nodes is assumed to be a directed spanning-tree topology; namely, each node has only one leader and can have multiple followers. The formation pattern is specified in terms of both the positions and the orientations. The network of vehicles is shown to asymptotically reach the desired geometric pattern around the leader and to track the leader's trajectory. Although the communication graph is simple, the originality of this work consists in providing strong stability guarantees; namely, global uniform asymptotic stability, for general leader's trajectories and using continuous-time feedback laws. That is, the leader's trajectory is allowed to con-

verge to a static location (parking problem), and can also have a general time-varying velocity profile.

## 2.3. Leaderless Formation Control for Nonholonomic Vehicles

In the context of the previously described problems, the network is assumed to contain a leader. However, in some applications, the formation does not have a leader. Hence, the coordination task consists in making the vehicles maintain a given geometric posture, in terms of positions and orientations, around a center point. This problem has been studied for a network of nonholonomic vehicles in [40–44] using decentralized continuous control laws. In the aforementioned references, it is shown that the nodes converge asymptotically to the required formation. However, the stability properties of the formation errors have not been analyzed. While ensuring convergence to the desired formation is important, it is also essential to have guarantees about the convergence times, in particular that two nearby initial conditions have approximately to the same convergence rate, as well as to make sure that when the robots are “close” to form the formation, they preserve this property for all times. One way to capture these features is to ensure that uniform global asymptotic stability properties are satisfied.

The leaderless formation control problem for a network of nonholonomic mobile vehicles is studied in [14], see also [45–47]. The communication graph, in this case, is assumed to be a general bidirectional graph. A continuous and time-varying decentralized interconnection algorithm is proposed and the network after interconnection is showed to converge to the desired formation in terms of both positions and orientations. A significant contribution consists in establishing uniform global asymptotic stability of the formation error for the closed-loop network. Furthermore, in contrast to most of the previous-cited works, where only the kinematics model is considered, we use a model augmented by two integrators, which represent the feedback-linearized velocity dynamics. Furthermore, original tools, via the construction of strict Lyapunov functions, are proposed to analyze stability of the network in closed loop. Using these tools, in [46, 47], we allowed the transfer of information among the nodes to be affected by general time-varying delays. The same approach was used in [44] for a network of Euler-Lagrange systems.

## 2.4. Synchronization in Heterogeneous Networks Using Strong Interconnections

In general, the intrinsic dynamics of the nodes are not necessarily the same. Such is the case, for example, of networks of oscillators, in which the intrinsic frequency of the nodes is not identical [48]. We refer to such networks as *heterogeneous*. For a network of nonlinear systems, in which all the nodes are identical and the interconnection gain is sufficiently high, it is expected that the nodes synchronize;

namely, that the state variables reach a common value. Furthermore, it is intuitively clear that a synchronized behavior of such a network corresponds, roughly speaking, to all the nodes adopting the motion of a single isolated unit [49]. However, in the case of heterogeneous networks, the paradigm of asymptotic synchronization is much more complex: the existence of a synchronization manifold is not guaranteed [50]. Hence, in lieu of adopting the behavior of one of the nodes, some type of internal model is necessary and sufficient for synchronization [50] and, in general, only “practical” synchronization is usually ensured in the literature considering heterogeneous networks [51–56]. In [53, 54] a novel framework of analysis of heterogeneous networks was developed upon the premise that the collective behavior of network-interconnected systems is dichotomic: it consists, on the one hand, in a “weighted averaged” motion determined by the so-called *emergent dynamics* and, on the other, in the dynamics of the *synchronization errors* of each individual unit in the network, relatively to the collective behavior. Then, we say that the network synchronizes or, more precisely, achieves dynamic consensus if all the nodes adopt the emergent behavior in steady state.

This approach was pursued and extended in [57, 58]<sup>1</sup>, where it is assumed that the emergent dynamics admits a limit cycle with certain stability properties. It is shown that each individual interconnected node also admits a limit cycle with the same frequency and the same stability properties as the one in the emergent dynamics. The different nodes are interconnected via a general directed graph. For the first time in the literature of synchronization, the analysis was based on singular-perturbation theory. Indeed, due to the strong interconnection, it is showed that the emergent dynamics corresponds to a slow system while the dynamics of the synchronization errors form a fast one.

### 2.5. Consensus Under Time-Varying Graphs

A last contribution of [14] is dedicated to the consensus problem where the graph is time-varying. Considering time-varying graphs captures different scenarios in which the transfer of information among the nodes is unreliable, the information flow through a restricted amount of channels, or, when the nodes have a limited sensing range. Consensus problem under time-varying and switching graphs has been widely studied in the literature, see e.g., [3, 59–65], mostly for nodes with single or double integrator dynamics, under different graph-connectivity conditions. The approach in the aforementioned references uses non-systematic trajectory-based approaches via non-smooth Lyapunov functions, which makes the analysis of robustness challenging.

<sup>1</sup>The contributions in [57, 58] have not been included in [14] and [58] will be submitted soon.

In [66–68]<sup>2</sup>, a novel analysis approach for time-varying consensus protocols employing the notions of persistency of excitation and strict Lyapunov functions are provided. Those approaches rely on stability theory of so-called gradient-descent and model-reference-adaptive-control systems, known in adaptive control theory [69]. That is, the classical consensus paradigm is recast into a problem of stability analysis for systems with persistency of excitation. The analysis relies on the edge-agreement transformation based on the relationship between node and edge-agreement protocol [70]. The interconnection graph is assumed time-varying while maintaining a mild persistent connectivity among the nodes. The time-varying interconnections, individually, are assumed to be persistently exciting. Furthermore, networks composed of single-integrator nodes, double-integrator nodes, and time-varying planar oscillators are studied. Finally, different convergence rate estimates, for the consensus error coordinates, are provided.

## 3. Towards Hybrid Networked Systems

All the results mentioned in Section 2 concentrate on networked systems with continuous-time dynamics. It appears that in many situations, the overall system also exhibits discontinuous/jump dynamics either at the nodes’ level or at the network’s level. Before describing hybrid networks, in the next section, hybrid dynamical systems are introduced according to the modeling framework of [71].

### 3.1. Hybrid Dynamical Systems

In [71], a hybrid dynamical system is seen as the following combination of a constrained differential equation and a constrained difference equation

$$\begin{cases} (x, u) \in C & \dot{x} = f(x, u) \\ (x, u) \in D & x^+ = g(x, u). \end{cases} \quad (1)$$

The state of the hybrid system, represented by  $x$ , can change according to the differential equation  $\dot{x} = f(x, u)$  while in the flow set  $C$ , and it can change according to a difference equation  $x^+ = g(x, u)$  while in the jump set  $D$ . When  $x \in C \cap D$ , the solution can either jump or flow if flowing keeps the solution in  $C$ . The notation  $\dot{x}$  represents the time derivative of the state  $x$ , while  $x^+$  represents the value of the state after an instantaneous jump.

The strength of this formalism resides on the possibility of capturing, in a single representation, heterogeneous dynamics involving both continuous and discrete evolutions. It allows to cover many real-world applications that cannot be modeled precisely as purely continuous-time systems nor as purely discrete-time systems. Finally, it allows

<sup>2</sup>The works in [67, 68] have not been included [14].

the use and the extension of state-of-the-art tools to analyze continuous and discrete-time systems (stability, robustness, safety, etc). Indeed, a range tools for analysis and control design of systems described by (1) are by now available in the literature [71].

### 3.2. Motivation

Hybrid phenomena at the node's level are common in the case of mechanical systems with impacts, such as walking robots [23, 72], juggling mechanisms [24, 73], and classes of cyber-physical systems [25, 74]. Furthermore, hybrid phenomena at the network's level are common in many applications. For example, when interconnecting nodes under constrained control actions, which is the case in power converters within energy grids [15, 16] and in valve-based control systems [17], where the input switches among a finite set of operation modes. In networks of event-triggered and self-triggered control systems [19, 25, 75–77], where the input is updated only at particular events and remains constant between events. Finally, in network control systems [18, 26, 27], where all the data (inputs and measurements) flow through the same channel; hence, the measurements and the control actions are updated only at particular intervals of time.

While there are now numerous approaches to analyze and design continuous-time and discrete-time interconnected systems, there is a need for methodological tools, which are adapted to hybrid phenomena in the literature.

### 3.3. Handling Hybrid Phenomena in Networked Systems

In the literature, fundamental links between algebraic graph theory and synchronization have been established for different scenarios of continuous and discrete-time networked systems. However, the tools that are available to date to analyze synchronization are not always applicable when both continuous and discrete phenomena coappear. Indeed, assume that the interaction among the hybrid nodes occurs only during their continuous-time, or only during their discrete-time evolution. Such is the case for mechanical nodes with impacts [23] or for networks under any event-based or intermittent interconnections [78]. In this case, one needs to show that the network's attitude to synchronize during one of the two modes (flowing or jumping) is robust with respect to the other mode along which the interconnection is lost. However, most of the existing techniques to show synchronization for purely continuous-time or purely discrete-time networks are trajectory-based approaches [62, 79] and cannot be easily extended in order to analyze robustness. Another example, where classical tools do not apply, is that of hybrid networks operating in clusters. In this case, sub-networks are formed due to their strong interconnections. The nodes within the same sub-network tend to synchronize quickly (fast dynamics) before interacting with the other sub-networks (slow dynamics) [80–83]. The global hybrid network exhibits a multi-time-scale dynamics [84, 85]. As discussed in Section 2.4, singular-perturbation theory is useful to analyze

synchronization in the presence strong interconnections. However, for stand-alone hybrid singularly perturbed systems and even for a one-system case, only few results are available, see e.g., [86–88] for more discussion.

The main originality of [14] consists in providing a range of advanced Lyapunov-based approaches for set-stability, inspired by techniques for nonlinear time-varying systems, for interconnected continuous-time systems. Similar Lyapunov-based tools exist for hybrid systems [71]. In particular, a range of tools are available for the analysis of set stability. Hence, there is hope to follow similar approaches when analyzing hybrid networks. For this purpose, several key questions would need to be addressed.

1. The modeling of the interconnections among the nodes, which is not a trivial task and requires special care to preserve the so-called *basic conditions* for the network in closed loop [89, 90]. Hybrid basic conditions are essential for hybrid systems to ensure some robustness properties, see [71].
2. The current approaches to address hybrid interconnected systems consist in modeling the overall network as a hybrid system and to apply related results to conclude about stability like in [25]. For instance, it would be much more natural to derive global properties based on the local properties of each node and the interconnection graph, as suggested in [91].
3. The persistence of excitation property, which have been extensively used in [14], extends to the notions of persistent flow and persistent jump in the hybrid systems [92], which need to be better understood in terms of their implication on synchronization.
4. The possibility of a mismatch between the times when the hybrid nodes jump would considerably increase the difficulty to analyze synchronization. This fact has been mentioned in the particular context of hybrid observers in [93].

## 4. Challenging Problems

The aforementioned questions are general and fundamental. It therefore seems appropriate to follow a divide-and-conquer strategy by considering, separately, situations where the hybrid phenomena are due to the node dynamics, from those where these are due to the control strategy as discussed in the following.

### 4.1. Synchronizing Intrinsically Hybrid Nodes

Networks of hybrid nodes include networks of switched systems (like power converters), mechanical systems with impacts, or cyber-physical systems. Because such networks are challenging, it would be relevant to first concentrate on situations where the interaction among the systems occurs only during their continuous-time evolution, which is the case in walking robots, or only during

their discrete-time evolution, which is the case of juggling mechanisms. In each of the previously-cited scenarios, we need to provide robustness-analysis tools in order to conclude when the network's attitude to synchronize during one of the two modes of evolution (flowing or jumping) is robust with respect to the other mode, where the interconnections are not present, and also with respect to the mismatch in the jump times as the nodes do not necessarily jump at the same time. The qualitative Lyapunov-based techniques for time-varying systems proposed in [14], as described in Section 2 and [67, 94], is relevant here and would need to be adapted to this context.

#### 4.2. Synchronization Using Hybrid Controllers

A node may also be hybrid because of the used local controller. There, a hybrid controller is a hybrid system admitting the system or part of the system's state variables as inputs and generates, as outputs, the system's control inputs. First, this class of controllers can overcome fundamental limitations of continuous-time controllers e.g. [95–97]. They also offer the possibility to solve the combined stability-safety control problem robustly [98]. More importantly, hybrid controllers offer the flexibility to model different scenarios of constrained interconnection among the nodes [99] and they naturally arise in presence of sampling [25, 27, 78, 89, 100].

A concrete research problem, related to synchronization in networks of non-hybrid nodes using hybrid interconnections, is to analyze networks operating in clusters under event-based interconnections. In this case, the global network may exhibit a multi-time-scale dynamics. Moreover, since the interconnections are intermittent, the resulting closed-loop network exhibits hybrid phenomena. A promising research direction that contribute to this question, consists in extending the singular-perturbation-based framework proposed in [101, 102] as described in Section 2.4. The tools developed in the aforementioned references exploit the continuity of the stability properties of classes of singularly perturbed systems under small perturbations. Showing such a continuity property for classes of hybrid singularly perturbed systems is an open question to the best of our knowledge.

#### 4.3. Safety Specifications in Hybrid Networks

Usually, in many real-world applications, safety requirements need to be guaranteed when solving synchronization problems. Safety is also named *obstacle avoidance* in the particular context of mobile robotics. Safety while synchronizing a network of dynamical systems means that the trajectories of the network in closed loop, when starting from a given set of initial conditions, remain within a safe region of the state space while the variables of interest converge to the agreement. Barrier functions are analogues to Lyapunov functions and have been usually used to guarantee safety in dynamical systems [103, 104]. They have been also used for networked systems in [105–109]. Recently, in [110], barrier functions are used to show

safety for hybrid systems in the form of  $\mathcal{H}$ . However, to the best of our knowledge, establishing distributed safety criteria for hybrid networks has not been done yet.

## 5. Conclusion

This article reviews the main contributions presented in [14] where original Lyapunov-based approaches for set stability have been developed for the synchronization of networked continuous-time systems. The approach of [14] together with the recent advances on the Lyapunov stability theory for hybrid systems allow us to envision a range of exciting perspectives for the synchronization of hybrid interconnected systems. The main motivations of this research direction are discussed and some open problems are proposed.

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