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Distributed Consensus-formation of Force-controlled Nonholonomic Robots with Time-varying Delays

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Abstract

We solve the leaderless consensus-formation control problem for nonholonomic robots under the influence of time-varying communication delays and via smooth feedback. The control objective is to ensure that both the Cartesian positions of the vehicles and their orientations on the plane converge to a given position in a formation pattern, relatively to an *a priori* unknown barycenter and a common orientation, respectively. Our controller, being of proportional-plus-damping type and smooth time-varying, is easy to implement. Furthermore, it relies on δ -persistency of excitation to overcome the well-known obstacle that for nonholonomic systems a set-point is not stabilizable by smooth time-invariant feedback. We establish asymptotic convergence of the tracking errors and we provide some simulation results that support our theoretical findings.

Key words: Nonholonomic vehicles; formation control; consensus control; systems with time-delays.

1 Introduction

For multiagent systems the well-studied consensus control problem consists in making all the systems' states converge to a common equilibrium (Ren and Beard, 2005; Jadbabaie et al., 2003; Olfati-Saber and Murray, 2004). For mobile robots, because several of these can obviously not occupy the same position simultaneously, consensus must be formulated as a problem of *formation* control, that is, in which the robots stabilize near an equilibrium point while forming a geometric pattern (Lin et al., 2005). We may distinguish two consensus-formation problems: the leader-follower, in which the agreement equilibrium point is given as a desired value to a set of follower agents in the network (Maghenem, 2017), and the leaderless, in which such common value results from the network's topology (Cao and Ren, 2011; Wang, 2014; Hatanaka et al., 2015; Dong and Farrell, 2008a). Furthermore, it is important to distinguish works in which both Cartesian coordinates and orientation are controlled (Dimarogonas and Kyriakopoulos, 2007; Maghenem, 2017) from those in which orientation is neglected (Lin et al., 2005; Roza et al., 2018). In this paper we focus on the leaderless full-consensus problem, that is, the set-point is not given a priori and consensus is reached both in Cartesian positions and orientation angles.

Another important distinction lies in the model considered. It can be of first order, in which case either only the kinematics resulting from the nonholonomic constraints is considered (Lin et al., 2005; Roza et al., 2018; Peng et al., 2015; Dong and Farrell, 2008a) or it may be simplified even further to a point-mass on the plane, in which case the consensus problem boils down to that of consensus of simple integrators; a problem completely solved under various scenarios (Ren and Beard, 2005; Ren, 2008; Olfati-Saber and Murray, 2004). A more realistic model, however, is force-controlled; this model is of second order and, besides the nonholonomy and underactuation, it presents the difficulty of controlling a higher relative-degree system. This is the model that we consider in this paper; see also (Do et al., 2004; Bautista-Castillo et al., 2016; Maghenem et al., 2019).

From a control viewpoint, the nonholonomy of the robot imposes significant difficulty to set-point stabilization; it is well known that these systems are not stabilizable via time-invariant smooth feedback (Brockett, 1983); for multiagent systems, necessary conditions for consensus are laid in (Lin et al., 2005). Thus, consensus of multiagent nonholonomic systems is solvable either via discontinuous time-invariant control (Dimarogonas and Kyriakopoulos, 2007) or time-varying smooth feedback (Yang et al., 2016; Jin and Gans, 2017).

From an implementation viewpoint, there are many kinds of controllers that may be designed depending

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on the available sensors and their capabilities. In this article we assume that the robots are equipped with global-measurement sensors and they transmit and receive measurements from “neighbor” robots that are interconnected over a network. It is most sensible to assume that communication flows in both directions (hence the network topology is undirected) and delays are time-varying (Hatanaka et al., 2015; Abdessameud et al., 2017). Such scenario is common, for instance, in the case that a group of robots are initially too far apart for relative-measurement sensors to be of any use. When within range, however, relative measurements may result more practical than global ones in missions of formation-tracking control (after the rendez-vous has been achieved). Relative-measurement based control is considered, for instance, in (Roza et al., 2018), but without communication delays and neglecting to control the orientation. Delays in the communication have been considered, *e.g.*, in (Dong and Farrell, 2008b; Maghenem, 2017). In the former delays are considered to be constant and in the latter they are time-varying, but the control gains, as in (Roza et al., 2018), must satisfy a condition that requires global connectivity information.

Thus, in this paper we consider the full-consensus problem for second-order (force-controlled) nonholonomic systems interconnected over an undirected graph and with time-varying measurement delays. We propose a smooth controller of proportional-plus-damping type that achieves the control goal via a δ -persistently-exciting (δ -PE) term —*cf.* (Loría et al., 1999). Our controller is completely distributed as the control gains depend only on the actual strength of the interconnection for each agent. To the best of our knowledge, this problem, under the conditions considered here, has only been considered in (Maghenem, 2017). Other important aspects, such as collision avoidance (Jin and Gans, 2017) and robustness with respect to external disturbances (Ajlrou et al., 2015; Maghenem et al., 2018) are out of scope in this work.

The rest of the paper is organized as follows. In the next section we present the model and formulate the control problems that we address. In Section 3 we present our main theoretical results; these are illustrated via numerical simulations in Section 4. We conclude with some remarks in Section 5.

2 Model and problem formulation

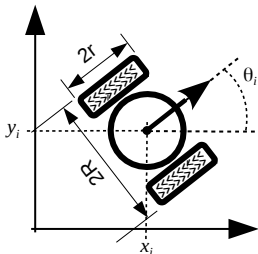


Fig. 1. Schematics of a differential wheeled mobile robot.

Consider a group of N force-controlled nonholonomic vehicles, like the one depicted in Fig. 1, each of which is modeled by the usual unicycle kinematics equations and two added integrators, that is, for each $i \leq N$, we have

$$\dot{z}_i = \varphi_i(\theta_i)v_i, \quad \varphi_i(\theta_i) := [\cos(\theta_i) \quad \sin(\theta_i)]^\top \quad (1a)$$

$$\dot{v}_i = u_{vi}, \quad u_{vi} := \frac{1}{r_i m_i} [\tau_{i1} + \tau_{i2}] \quad (1b)$$

$$\dot{\theta}_i = \omega_i \quad (2a)$$

$$\dot{\omega}_i = u_{\omega i}, \quad u_{\omega i} := \frac{2R_i}{I_i r_i} [\tau_{i1} - \tau_{i2}], \quad (2b)$$

where $z_i := [x_i, y_i]^\top \in \mathbb{R}^2$ are the Cartesian coordinates of the robot on the plane, $\theta_i \in \mathbb{R}$ denotes its orientation, v_i and ω_i are the linear and angular velocities of the center of mass, respectively, m_i is the mass, I_i is the moment of inertia, r_i the radius of the wheels, R_i is the distance between point Q_i and the wheels, u_{vi} and $u_{\omega i}$ are the control inputs, and τ_{ij} are the torques at the wheels, that is, for the purpose of control implementation, we have

$$\begin{bmatrix} \tau_{i1} \\ \tau_{i2} \end{bmatrix} = \frac{r_i}{2} \begin{bmatrix} m_i & I_i/2R_i \\ m_i & -I_i/2R_i \end{bmatrix} \begin{bmatrix} u_{vi} \\ u_{\omega i} \end{bmatrix}. \quad (3)$$

The leaderless consensus-formation control problem, which we solve in this paper, involves making the robots acquire a desired formation pattern relatively to a barycenter with coordinates $z_c := [x_c \ y_c]^\top$ and a common orientation, denoted θ_c . It is assumed that both, z_c and θ_c are unknown; they result from the topology of the interconnection graph and the multiagent system’s initial conditions. The formation is determined by defining, for each robot, a constant vector denoted $\delta_i \in \mathbb{R}^2$, $\delta_i := [\delta_{xi} \ \delta_{yi}]$ and the relative position errors $\bar{z}_i := z_i - \delta_i$. Then, the control goal is to steer $\bar{z}_i \rightarrow z_c$ and $\theta_i \rightarrow \theta_c$ for all $i \leq N$.

We assume that each robot possesses position and velocity sensors and is able to communicate them to a number of neighbors through a relatively reliable network. More precisely, we assume that

(A1) the interconnection graph is undirected, static, and connected.

Hence, the interconnection graph may be modeled by a constant Laplacian matrix, $L := [\ell_{ij}] \in \mathbb{R}^{N \times N}$, where

$$\ell_{ij} = \begin{cases} \sum_{k \in \mathcal{N}_i} a_{ik} & i = k \\ -a_{ik} & i \neq k, \end{cases} \quad (4)$$

$\mathcal{N}_i \subset \mathbb{Z}$ is the set of indexes corresponding to robots transmitting information to the i th robot, $a_{ik} > 0$ if $k \in \mathcal{N}_i$ and $a_{ik} = 0$ otherwise. By construction, $L\mathbf{1}_N = 0$, where $\mathbf{1}_N = [1 \cdots 1]^\top$ and, after Assumption A1, L is symmetric, it has a unique zero-eigenvalue, and the rest of its spectrum is strictly positive. Thus, $\text{rank}(L) = N - 1$.

On the other hand, although the network interconnections are assumed to be static, the network is logically as-

sumed to be wireless. This has as a natural consequence that the communications are affected by time-varying delays (Hatanaka et al., 2015; Abdessameud et al., 2017). Hence, we also assume that

(A2) the communication from the j th to the i th robot is subject to a variable time-delay denoted $T_{ji}(t)$ that is bounded by a known upper-bound $\bar{T}_{ji} \geq 0$ and has bounded time-derivatives.

Then, the control problem consists in designing u_{vi} and u_{ω_i} so that

$$\lim_{t \rightarrow \infty} v_i(t) = 0, \quad \lim_{t \rightarrow \infty} \omega_i(t) = 0, \quad (5)$$

$$\lim_{t \rightarrow \infty} \bar{z}_i(t) = z_c, \quad \lim_{t \rightarrow \infty} \theta_i(t) = \theta_c, \quad (6)$$

for all $i \leq N$.

Remark 1 *Being inherently a set-point stabilization problem, leaderless consensus differs greatly from other formation control problems, notably from moving-leader-follower consensus, which is intrinsically a tracking control problem. Firstly, in clear contrast with the case of holonomic systems, set-point stabilization cannot be considered as a particular case of trajectory tracking (Lizárraga., 2004). Secondly, from a systems theory viewpoint, an important distinction lies in that nonholonomic systems are controllable about a trajectory provided that the leader velocity is nonzero (de Luca et al., 2016). While this is relevant in tracking-control scenarios, in the present setting the main difficulty resides in that a set-point is not asymptotically stabilizable by smooth feedback, unless the latter is time-varying.*

3 Main result

The main contribution in this paper is to provide an original solution to the problem previously stated: full leaderless consensus control of multiple second-order non-holonomic systems.

For the purpose of control design and analysis, we start by reformulating the control goal in (6) as

$$\lim_{t \rightarrow \infty} e_{zi}(t) = 0, \quad \lim_{t \rightarrow \infty} e_{\theta_i}(t) = 0 \quad (7)$$

for all $i \leq N$, where

$$e_{zi} := \sum_{j \in \mathcal{N}_i} a_{ij} [\bar{z}_i - \bar{z}_j(t - T_{ji}(t))] \quad (8)$$

denotes the consensus errors in terms of the Cartesian coordinates of the i th robot and the measurements received from its \mathcal{N}_i neighbours, with different time-varying delays, and

$$e_{\theta_i} := \sum_{j \in \mathcal{N}_i} a_{ij} [\theta_i - \theta_j(t - T_{ji}(t))] \quad (9)$$

denotes the consensus error in terms of the orientation angles, also affected by time-varying delays.

The control approach is based on designing a controller for each part of the system's dynamics separately. One

for the dynamics corresponding to the translational part, Eq. (1), and another one for the orientation-angles dynamics. Since the respective systems (1) and (2) are of second order it is natural to design controllers of proportional-derivative type —*cf.* (Ren and Beard, 2005). Nonetheless, the system being subject to non-holonomic constraints, a set-point is not stabilizable via time-invariant smooth feedback (Brockett, 1983). Furthermore, because it is also desired to reach consensus in the orientation angles, controllers tailored for point-mass models do not apply either. Thus, our controller is of proportional-derivative type and it is smooth time-varying.

First, for the translational dynamics of the i th robot we introduce the nonlinear control law —*cf.* (Nuño et al., 2013),

$$u_{vi} = -p_{vi}\varphi_i(\theta_i)^\top e_{zi} - d_{vi}v_i, \quad (10)$$

where the parameters p_{vi} and d_{vi} are, respectively, the so-called proportional and derivative gains and are positive constants by design. The derivative term is simply proportional to the forward velocities $v_i \in \mathbb{R}$ but, in view of the robot's kinematics, the proportional-feedback term, $-p_{vi}\varphi_i(\theta_i)^\top e_{zi}$, which depends on the consensus errors $e_{zi} \in \mathbb{R}^2$, involves the rank-deficient vector $\varphi_i(\theta_i)$. Some readers may see here an analogy between the proportional-feedback term and the Jacobian-transpose controller used for robot manipulators, in which the Jacobian is not required to have full rank to achieve regulation.

Then, the closed-loop equation, corresponding to the translational dynamics, is

$$\Sigma_v : \begin{cases} \dot{\bar{z}}_i = \varphi_i(\theta_i)v_i \\ \dot{v}_i = -d_{vi}v_i - p_{vi}\varphi_i(\theta_i)^\top e_{zi}. \end{cases} \quad (11)$$

In contrast to the case of an ordinary double integrator the system (11) has multiple equilibria, other than points including $e_{zi} = 0$. These equilibria correspond to points in the set $\mathcal{E}_v := \{\bar{z}_i, \theta_i, v_i, e_{zi} : v_i = 0 \wedge \varphi_i(\theta_i)^\top e_{zi} = 0\}$. This, in addition to the fact that set-points are not stabilizable for nonholonomic systems, via smooth time-invariant feedback, considerably increases the difficulty of the consensus problem, relatively to that for linear systems or even systems with holonomic constraints.

The so-needed external function of time is introduced through the second control law, at the orientation-angle dynamics. Let

$$u_{\omega_i} = -p_{\omega_i}e_{\theta_i} - d_{\omega_i}\omega_i + \alpha_i(t, \theta_i, e_{zi}), \quad (12)$$

where p_{ω_i} and d_{ω_i} are positive gains. Clearly, the first two terms on the right-hand side of (12) are also of the proportional-derivative type. The function α_i , which introduces the needed time-varying terms, is defined as

$$\alpha_i(t, \theta_i, e_{zi}) := k_{\alpha_i}f_i(t)\varphi_i(\theta_i)^\perp e_{zi}, \quad (13)$$

where $k_{\alpha_i} > 0$, $\varphi_i(\theta_i)^\perp = [-\sin(\theta_i) \cos(\theta_i)]^\top$ is the annihilator of φ_i , *i.e.*, $\varphi_i(\theta_i)^\top \varphi_i(\theta_i)^\perp = \varphi_i(\theta_i)^\perp \varphi_i(\theta_i) = 0$,

and $f_i \in \mathcal{C}^2(\mathbb{R}_{\geq 0}, \mathbb{R})$ satisfies, by construction, $f_i, \dot{f}_i, \ddot{f}_i \in \mathcal{L}_\infty$, $\lim_{t \rightarrow \infty} f_i(t) \neq 0$, and $\lim_{t \rightarrow \infty} \dot{f}_i(t) \neq 0$.

The stabilization mechanism of u_{ω_i} may be explained by observing the closed-loop equations relative to the orientation-angle dynamics,

$$\Sigma_\omega : \begin{cases} \dot{\theta}_i = \omega_i \\ \dot{\omega}_i = -d_{\omega_i} \omega_i - p_{\omega_i} e_{\theta_i} + \alpha_i(t, \theta_i, e_{z_i}). \end{cases} \quad (14)$$

With $\alpha_i \equiv 0$, these equations correspond to those of a stable second-order time-invariant system with equilibria in $\mathcal{E}_\omega := \{\theta_i, \omega_i : e_{\theta_i} = 0, \omega_i = 0\}$. Hence, without the term α_i in the controller, complete system's trajectories may belong to $\mathcal{E}_\omega \cap \mathcal{E}_v$ which includes, but is not restricted to, the consensus manifold. On the other hand, because $\varphi_i(\theta_i)$ and $\varphi_i(\theta_i)^\perp$ are orthogonal, we have $\varphi_i(\theta_i)^\perp e_{z_i} \neq 0$ for all points in $\mathcal{E}_v \setminus \{v_i = 0, e_{z_i} = 0\}$. Now, in view of the assumptions imposed on f_i in (13), the function α_i evaluated along complete trajectories belonging to \mathcal{E}_v cannot vanish. Thus, α_i may be seen as a ‘‘controlled perturbation’’ that persistently impedes all the trajectories, except the trivial one, to remain in $\mathcal{E}_\omega \cap \mathcal{E}_v$.

Remark 2 *The excitation property of α_i is reminiscent of δ -persistency of excitation (Lor a et al., 2005). See also (Lor a et al., 1999) where the so-called δ -persistently-exciting controllers were introduced for stabilization of nonholonomic systems.*

Our main result emerges from the previous rationale.

Proposition 1 *Consider the system (1)–(2), under Assumptions A1 and A2, in closed loop with (10), (12)–(13) and with f_i as defined above. Then, the leaderless consensus control goal is achieved, that is (5) and (6) hold, provided that*

$$d_{v_i} > \frac{1}{2} p_{v_i} \sum_{j \in \mathcal{N}_i} a_{ij} \left(\beta_i + \frac{\bar{T}_{ij}^2}{\beta_j} \right) \quad (15)$$

$$d_{\omega_i} > \frac{1}{2} p_{\omega_i} \sum_{j \in \mathcal{N}_i} a_{ij} \left(\varepsilon_i + \frac{\bar{T}_{ij}^2}{\varepsilon_j} \right) \quad (16)$$

for some $\beta_i, \varepsilon_i > 0$, for all $i \leq N$. \square

Proof of Proposition 1

The proof relies on Barbalat’s lemma and standard signal-chasing arguments. We start by showing that $v_i \rightarrow 0$ and that the solutions of Σ_v are bounded. To that end, consider the Lyapunov-Krasovskii functional

$$\mathcal{V} := \sum_{i \leq N} \left[\frac{1}{2 p_{v_i}} v_i^2 + \frac{1}{4} \sum_{j \in \mathcal{N}_i} a_{ij} |\bar{z}_i - \bar{z}_j|^2 + \frac{1}{2 \beta_i} \sum_{j \in \mathcal{N}_i} a_{ij} \bar{T}_{ji} \int_{\bar{T}_{ji}}^0 \int_{t+\sigma}^t v_j(\eta)^2 d\eta d\sigma \right]$$

whose total derivative, along the trajectories of (11),

yields

$$\dot{\mathcal{V}} = - \sum_{i \leq N} \left[\frac{d_{v_i}}{p_{v_i}} v_i^2 + \dot{z}_i^\top \sum_{j \in \mathcal{N}_i} a_{ij} (\bar{z}_i - \bar{z}_j(t - T_{ji}(t))) - \frac{1}{2} \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{z}_i - \dot{z}_j)^\top (\bar{z}_i - \bar{z}_j) - \frac{1}{2 \beta_i} \sum_{j \in \mathcal{N}_i} a_{ij} \left(\bar{T}_{ji}^2 v_j^2 - \bar{T}_{ji} \int_{t-\bar{T}_{ji}}^t v_j^2(\sigma) d\sigma \right) \right].$$

Then, we use

$$\bar{z}_j - \bar{z}_j(t - T_{ji}(t)) = \int_{t-T_{ji}(t)}^t \dot{z}_j(\sigma) d\sigma,$$

and Lemma 6.1 from (Cao and Ren, 2011) to obtain, under Assumption A1,

$$\frac{1}{2} \sum_{i \leq N} \sum_{j \in \mathcal{N}_i} a_{ij} (\dot{z}_i - \dot{z}_j)^\top (\bar{z}_i - \bar{z}_j) = \sum_{i \leq N} \sum_{j \in \mathcal{N}_i} a_{ij} \dot{z}_i^\top (\bar{z}_i - \bar{z}_j).$$

Therefore,

$$\dot{\mathcal{V}} = - \sum_{i \leq N} \left[\frac{d_{v_i}}{p_{v_i}} v_i^2 + \sum_{j \in \mathcal{N}_i} a_{ij} \dot{z}_i^\top \int_{t-T_{ji}(t)}^t \dot{z}_j(\sigma) d\sigma - \frac{1}{2 \beta_i} \sum_{j \in \mathcal{N}_i} a_{ij} \left(\bar{T}_{ji}^2 v_j^2 - \bar{T}_{ji} \int_{t-\bar{T}_{ji}}^t v_j^2(\sigma) d\sigma \right) \right].$$

Next, we apply Young’s and Cauchy-Schwarz’ inequalities on the second right-hand-term to obtain

$$\begin{aligned} -\dot{z}_i^\top \int_{t-T_{ji}}^t \dot{z}_j(\sigma) d\sigma &\leq \frac{\beta_i}{2} |\dot{z}_i|^2 + \frac{1}{2 \beta_i} \left| \int_{t-T_{ji}}^t \dot{z}_j(\sigma) d\sigma \right|^2 \\ &\leq \frac{\beta_i}{2} |\dot{z}_i|^2 + \frac{\bar{T}_{ji}^2}{2 \beta_i} \int_{t-\bar{T}_{ji}}^t |\dot{z}_j(\sigma)|^2 d\sigma, \end{aligned}$$

for any $\beta_i > 0$. Now, since $\varphi_i^\top \varphi_i = 1$, we have $|\dot{z}_i|^2 = v_i^2$ and, in turn,

$$\dot{\mathcal{V}} \leq - \sum_{i \leq N} \left[\left(\frac{d_{v_i}}{p_{v_i}} - \frac{\beta_i}{2} l_{ii} \right) v_i^2 - \sum_{j \in \mathcal{N}_i} a_{ij} \frac{\bar{T}_{ji}^2}{2 \beta_i} v_j^2 \right],$$

where $l_{ii} := \sum_{j \in \mathcal{N}_i} a_{ij}$ is the i th element in the diagonal of the Laplacian matrix. Now, following (Nu o et al., 2013) and defining $s(v_i^2) := [v_1^2 \cdots v_N^2]^\top$ and

$$\Psi = \begin{bmatrix} \frac{d_{v_1}}{p_{v_1}} - \frac{\beta_1}{2} l_{11} & -\frac{\bar{T}_{21}^2}{2 \beta_1} a_{12} & \cdots & -\frac{\bar{T}_{N1}^2}{2 \beta_1} a_{1N} \\ -\frac{\bar{T}_{12}^2}{2 \beta_2} a_{21} & \frac{d_{v_2}}{p_{v_2}} - \frac{\beta_2}{2} l_{22} & \cdots & -\frac{\bar{T}_{N2}^2}{2 \beta_2} a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\bar{T}_{1N}^2}{2 \beta_N} a_{N1} & -\frac{\bar{T}_{2N}^2}{2 \beta_N} a_{N2} & \cdots & \frac{d_{v_N}}{p_{v_N}} - \frac{\beta_N}{2} l_{NN} \end{bmatrix},$$

we obtain $\dot{\mathcal{V}} \leq -1_N^\top \Psi s(v_i^2)$ or, equivalently,

$$\dot{\mathcal{V}} \leq - \sum_{i \leq N} \left[\frac{d_{vi}}{p_{vi}} - \sum_{j \in \mathcal{N}_i} a_{ij} \left(\frac{\beta_i}{2} + \frac{\bar{T}_{ij}^2}{2\beta_j} \right) \right] v_i^2.$$

Thus, after (15), it follows that there exists $\lambda_i > 0$ such that $\dot{\mathcal{V}} \leq - \sum_{i \leq N} \lambda_i v_i^2$. This implies that $v_i \in \mathcal{L}_2$ and, since \mathcal{V} is positive definite and radially unbounded with respect to v_i and $\bar{z}_i - \bar{z}_j$, the latter belong to \mathcal{L}_∞ and, consequently, $e_{zi} \in \mathcal{L}_\infty$. Now, since φ_i is uniformly bounded, it follows from (11) that $\dot{v}_i \in \mathcal{L}_\infty$ and, therefore, $\lim_{t \rightarrow \infty} v_i(t) = 0$.

So far, we have established the first limit in (5). Now, we analyze the behavior of the solutions of (14) with the aim at establishing the second limit. To that end, consider the system Σ_ω with (for the time-being) $\alpha_i \equiv 0$ and the Lyapunov-Krasovskii functional

$$\mathcal{W} := \sum_{i \leq N} \left[\frac{1}{2p_{\omega i}} \omega_i^2 + \frac{1}{4} \sum_{j \in \mathcal{N}_i} a_{ij} (\theta_i - \theta_j)^2 + \frac{1}{2\varepsilon_i} \sum_{j \in \mathcal{N}_i} a_{ij} \bar{T}_{ji} \int_{-\bar{T}_{ji}}^0 \int_{t+\sigma}^t \omega_j^2(\eta) d\eta d\sigma \right],$$

where $\varepsilon_i > 0$. Proceeding as we did previously for the differentiation of \mathcal{V} , we obtain

$$\dot{\mathcal{W}} \leq - \sum_{i \leq N} \left[\left(\frac{d_{\omega i}}{p_{\omega i}} - \frac{\varepsilon_i}{2} l_{ii} \right) \omega_i^2 - \sum_{j \in \mathcal{N}_i} a_{ij} \frac{\bar{T}_{ji}^2}{2\varepsilon_i} \omega_j^2 \right],$$

so

$$\dot{\mathcal{W}} \leq - \sum_{i \leq N} \left(\frac{d_{\omega i}}{p_{\omega i}} - \sum_{j \in \mathcal{N}_i} a_{ij} \left(\frac{\varepsilon_i}{2} + \frac{\bar{T}_{ij}^2}{2\varepsilon_j} \right) \right) \omega_i^2.$$

Therefore, after (16), it follows that there exists $\lambda_i > 0$ such that $\dot{\mathcal{W}} \leq - \sum_{i \leq N} \lambda_i \omega_i^2$. Since \mathcal{W} is positive definite

and radially unbounded with regards to ω_i and $\theta_i - \theta_j$, it follows that $\omega_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\theta_i - \theta_j \in \mathcal{L}_\infty$. This, in turn, implies that $\dot{\omega}_i \in \mathcal{L}_\infty$ so, after Barbalat's Lemma, $\lim_{t \rightarrow \infty} \omega_i(t) = 0$.

We proceed now to establish the limit in (6). To that end, observe that, since

$$\lim_{t \rightarrow \infty} \int_0^t \dot{\omega}_i(\sigma) d\sigma = \lim_{t \rightarrow \infty} \omega_i(t) - \omega_i(0) = \omega_i(0),$$

it follows, from Barbalat's lemma, that $\lim_{t \rightarrow \infty} \dot{\omega}_i(t) = 0$ if $\dot{\omega}_i$ is uniformly continuous. The latter holds since

$$\ddot{\omega}_i = -\frac{1}{I_i} \left[p_{\omega i} \dot{e}_{\theta i} + d_{\omega i} \dot{\omega}_i \right]$$

is bounded; indeed, this follows from Assumption A2 and the fact that $\dot{\omega}_i, \omega_i \in \mathcal{L}_\infty$.

Now, from the second equation in (14), restricted to $\alpha_i \equiv 0$, it follows that $\lim_{t \rightarrow \infty} \dot{\omega}_i(t) = 0$ and $\lim_{t \rightarrow \infty} \omega_i(t) = 0$ imply that $\lim_{t \rightarrow \infty} e_{\theta i}(t) = 0$ or, equivalently, that

$$\lim_{t \rightarrow \infty} \sum_{j \in \mathcal{N}_i} a_{ij} \left(\theta_i(t) - \theta_j(t) + \int_{t-T_{ji}(t)}^t \omega_j(\sigma) d\sigma \right) = 0.$$

Then, since $\omega_i \rightarrow 0$ and $T_{ji}(t)$ is globally bounded (see A2), we have

$$\lim_{t \rightarrow \infty} \sum_{j \in \mathcal{N}_i} a_{ij} [\theta_i(t) - \theta_j(t)] = 0$$

which implies that $\lim_{t \rightarrow \infty} L\theta(t) = 0$, where $\theta = \text{col}(\theta_i) \in \mathbb{R}^N$. Thus,

$$\lim_{t \rightarrow \infty} \theta_i(t) = \theta_c. \quad (17)$$

All the previous arguments hold provided that $\alpha_i \equiv 0$. If this is not the case, we remark that since, f_i, e_{zi} and φ_i^\perp are bounded along trajectories, so is α_i —see Eq. (13). This and the fact that Σ_ω is a marginally stable linear time-varying system with uniformly bounded time-delays ensure that $\dot{\omega}_i, \omega_i \in \mathcal{L}_\infty$, by Proposition 3 in (Wang, 2020) and so is $e_{\theta i}$.

The rest of the proof mostly consists in establishing that $e_{zi} \rightarrow 0$ which, since $v_i \rightarrow 0$, implies that $\bar{z}_i \rightarrow \bar{z}_c$, thereby completing (6). To see the latter, note that

$$e_{zi} = \sum_{j \in \mathcal{N}_i} a_{ij} \left(\bar{z}_i - \bar{z}_j + \int_{t-T_{ji}(t)}^t \dot{\bar{z}}_j(\sigma) d\sigma \right),$$

so, defining $e_z := [e_{z1} \cdots e_{zN}]^\top$ and $\bar{z} := [\bar{z}_1 \cdots \bar{z}_N]$ respectively, and using $\dot{\bar{z}}_j(t) = \varphi_j(t) v_j(t)$, we obtain

$$e_z = (L \otimes I_2) \bar{z} + \text{col} \left(\sum_{j \in \mathcal{N}_i} a_{ij} \int_{t-T_{ji}(t)}^t \varphi_j(\sigma) v_j(\sigma) d\sigma \right).$$

Therefore, $e_{zi} = 0$ and $v_i = 0$ imply that $(L \otimes I_2) \bar{z} = 0$ which, in view of the properties of L , implies the existence of $\bar{z}_c \in \mathbb{R}^2$ such that $\bar{z} = 1_N \otimes \bar{z}_c$, or $\bar{z}_i(t) = \bar{z}_c$ for all $i \leq n$.

We proceed now to show that $e_{zi} \rightarrow 0$. To that end, we stress that, since $\lim_{t \rightarrow \infty} v_i(t) = 0$,

$$\lim_{t \rightarrow \infty} \int_0^t \dot{v}_i(\sigma) d\sigma = \lim_{t \rightarrow \infty} v_i(t) - v_i(0) = v_i(0).$$

Moreover, in view of Assumption A2 and the fact that $\theta_i, \omega_i, \dot{\omega}_i, v_i, \dot{v}_i$, and $e_{zi} \in \mathcal{L}_\infty$, so does

$$\ddot{v}_i = -\frac{p_{vi}}{m_i} \left[\omega_i \varphi_i^{\perp \top} e_{zi} + \varphi_i^\top \dot{e}_{zi} + \frac{d_{vi}}{p_{vi}} \dot{v}_i \right]. \quad (18)$$

After Barbalat's lemma, it follows that $\lim_{t \rightarrow \infty} \dot{v}_i(t) = 0$ and, in turn after (11), also $\lim_{t \rightarrow \infty} \varphi_i(\theta(t))^\top e_{zi}(t) = 0$. Note, however, that the solutions to $\varphi_i(\theta)^\top e_{zi} = 0$ are of the form $e_{zi} = c\varphi_i^\perp$. Notwithstanding, so are the solutions to $\varphi_i(\theta)^\perp e_{zi} = 0$ on the right-hand side of (18). Therefore, the unique solution to both equations is $c = 0$ or, equivalently, $e_{zi} = 0$, since $\varphi_i^\perp \varphi_i^\perp = 1$. Thus, it is left to establish that $\varphi_i(\theta(t))^\perp e_{zi}(t) \rightarrow 0$. To that end, we use once more Barbalat's lemma to show that $\ddot{v}_i \rightarrow 0$. First, note that

$$\lim_{t \rightarrow \infty} \int_0^t \ddot{v}_i(\sigma) d\sigma = \lim_{t \rightarrow \infty} \dot{v}_i(t) - \dot{v}_i(0) = -\dot{v}_i(0).$$

Then, to show that \ddot{v}_i is uniformly continuous we use (18) and $\dot{\varphi}^\perp = -\omega_i \varphi_i$ to obtain

$$\ddot{v}_i = -\frac{p_{vi}}{m_i} \left[\dot{\omega}_i \varphi_i^\perp e_{zi} + 2\omega_i \varphi_i^\perp \dot{e}_{zi} - \omega_i^2 \varphi_i^\perp e_{zi} + \varphi_i^\top \ddot{e}_{zi} + \frac{d_{vi}}{p_{vi}} \ddot{v}_i \right] \quad (19)$$

which is bounded, in view of Assumption A2 and the boundedness of all functions on the right-hand side. This implies that $\lim_{t \rightarrow \infty} \ddot{v}_i(t) = 0$. Similar arguments apply to establish that $\lim_{t \rightarrow \infty} \ddot{v}_i(t) = 0$ and $\lim_{t \rightarrow \infty} v_i^{(4)}(t) = 0$, where

$$v_i^{(4)} = -\frac{p_{vi}}{m_i} \left[\ddot{\omega}_i \varphi_i^\perp e_{zi} + 3\dot{\omega}_i \varphi_i^\perp \dot{e}_{zi} - \omega_i^3 \varphi_i^\perp e_{zi} + \varphi_i^\top \ddot{\ddot{e}}_{zi} + 3\omega_i (\varphi_i^\perp \ddot{e}_{zi} + \dot{\omega}_i \varphi_i^\top e_{zi} + \omega_i \varphi_i^\top \dot{e}_{zi}) + \frac{d_{vi}}{p_{vi}} \ddot{\ddot{v}}_i \right] \quad (20)$$

Hence, since $v_i, \dot{v}_i, \ddot{v}_i, v_i^{(4)}$ and $\varphi_i^\top e_{zi}$ converge to zero, so do $\dot{e}_{zi}, \ddot{e}_{zi}$ and $\ddot{\ddot{e}}_{zi}$. It follows that all the terms in (18), (19), and (20) converge to zero. Thus, from (18) it follows that $\omega_i \varphi_i^\perp e_{zi} \rightarrow 0$, from (19) we have $\dot{\omega}_i \varphi_i^\perp e_{zi} \rightarrow 0$, and (20) implies that $\ddot{\omega}_i \varphi_i^\perp e_{zi} \rightarrow 0$.

Next, we use

$$\ddot{\omega}_i = -\frac{k_{\alpha i}}{I_i} \left[\frac{p_{\omega i}}{k_{\alpha i}} \dot{\theta}_i + \frac{d_{\omega i}}{k_{\alpha i}} \dot{\omega}_i + f_i \omega_i \varphi_i^\top e_{zi} - f_i \varphi_i^\perp \dot{e}_i - f_i \varphi_i^\perp e_{zi} \right]$$

in the first right hand side term of (20) to conclude that the term $\frac{p_{\omega i}}{k_{\alpha i}} \dot{\theta}_i \varphi_i^\perp e_{zi} - f_i (\varphi_i^\perp e_{zi})^2$ converges to zero. Since, on one hand $\omega_i \varphi_i^\perp e_{zi}$ converges to zero and, on the other hand, $e_{zi} = c\varphi_i^\perp$ and $\varphi_i^\perp \varphi_i^\perp = 1$, it follows that ω_i also converges to zero. In turn, we also have $\dot{\theta}_i \varphi_i^\perp e_{zi} \rightarrow 0$. Finally, the design of f_i guarantees that f_i does not vanish, so we conclude that $\lim_{t \rightarrow \infty} \varphi_i^\perp (\theta_i(t)) e_{zi}(t) = 0$. Therefore α_i vanishes and the limits in (5), as well as (17), follow. ■

4 Simulations

We present now some numerical simulation results to illustrate our theoretical findings. The simulations are performed using a network of six differential-drive robots interconnected as illustrated in Fig. 2, below. For simplicity, the physical parameters of all robots are taken to be equal: 10Kg for the mass; 3Kgm² for the moment of inertia; 0.3m of the distance between point Q and the wheels, and 0.05m for the radius of the wheels *cf.* (Shojaei et al., 2011).

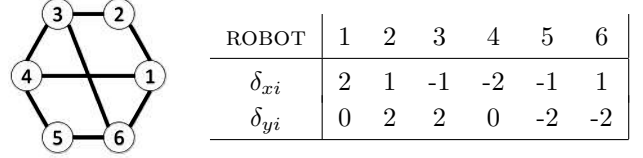


Fig. 2. Communication topology and hexagonal desired formation pattern (left). The latter is defined as an hexagon, determined by the distances δ_{xi} and δ_{yi} as per in the table on the right.

The interconnection time-delays are defined as a normal Gaussian distribution with a mean of 0.2s and a variance of 0.001s hence, they are bounded by $\bar{T}_{ji} = 0.33s$. The functions $t \mapsto f_i$ are designed using a truncated Fourier series of a square wave, that is,

$$f_i(t) = \frac{5}{2} + \sum_{k=1}^5 \frac{4}{(2k-1)\pi} \sin\left(\frac{(2k-1)t}{2}\right) \quad \forall i \leq N.$$

The robots are assumed to be initially standing still at the coordinates

$$x_i(0) := [5 \ 7 \ 7 \ 3 \ 1 \ 1]^\top \quad y_i(0) := [2 \ 5.5 \ 3.5 \ 2 \ 3.5 \ 5.5]^\top$$

and with the following orientations (in rad)

$$\theta(0) = \left[\frac{\pi}{2} \ 0 \ -\frac{\pi}{8} \ \frac{\pi}{8} \ -\frac{\pi}{8} \ -\frac{\pi}{8} \right]^\top.$$

The controller gains are set to $p_{\omega i} = 30$, $d_{\omega i} = 60$, $k_{\alpha i} = 150$, $p_{vi} = 300$ and $d_{vi} = 600$, for all $i \leq N$.

As it may be appreciated from Fig. 4, the robots converge to the desired formation pattern with a barycenter at $z_c = [3.35, 3.96]^\top$; their orientations converge to a common angle $\theta_c = 5.77$ rad, —see Fig. 3 below.

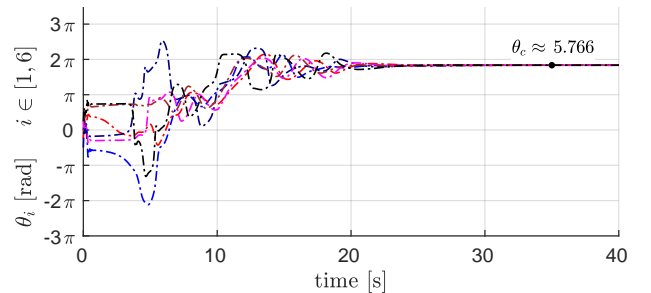


Fig. 3. Orientations of the six robots against time

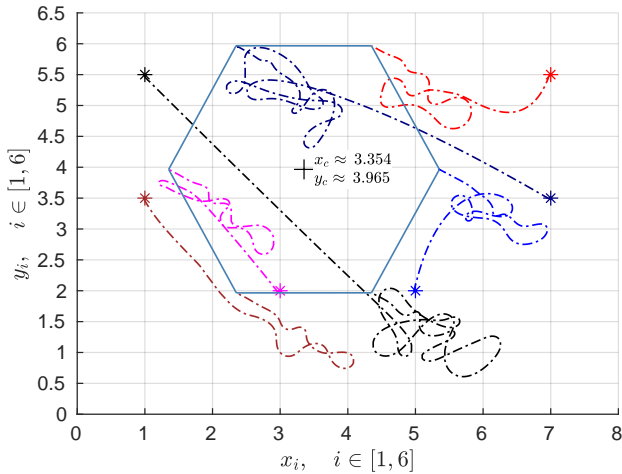


Fig. 4. Position trajectories in the Cartesian xy -plane

5 Conclusions

A solution to the leaderless consensus problem for force-controlled nonholonomic robots, in the presence of time-varying measurement delays, was presented. To cope with the difficulty imposed by the nonholonomic constraints to achieve set-point stabilization via smooth feedback, our controller is time-varying. It relies on a condition of persistency of excitation imposed on the term $\alpha_i(t, \theta_i, e_{zi})$ —see Eq. (13). Such condition is reminiscent of δ -persistency of excitation, which for nonlinear time-varying systems is *necessary* for uniform asymptotic stability (Loría et al., 2005). Even though this property is not established here (it is subject of current research), the previous rationale justifies the little conservatism of our assumptions.

From an implementation viewpoint, it is important to stress that the controllers have a simple proportional-plus-damping structure. Hence, the stability conditions (15) and (16) reflect the obvious fact that if the proportional gains are increased, to obtain a faster response, the dissipation gain must also be increased. Surprisingly, even though the system exhibits nonholonomic restrictions, the resulting conditions are the same as that reported in (Nuño et al., 2013) for the unrestricted case. On the other hand, the choice of f_i also impacts the performance of the controller.

Current research focusses on moving-leader-follower consensus, on establishing stronger stability properties, and on determining a performance-driven tuning approach of the controller parameters.

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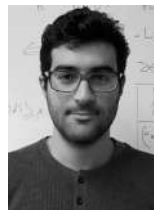
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