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Public Debt, Policy Mix and European Stability

Francois Langot* †

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Abstract

This paper highlights the specifics of a monetary union, such as the Euro area, regarding the possible choices of monetary and fiscal policies. As the dynamics of public debt are specific to the choices made by each government, I show that the dynamic stability of the area requires coordination of fiscal policies, particularly in the case of a liquidity trap situation. My results suggest that a fiscal union, taking the form of a common debt, guarantees the dynamic stability of the area, notwithstanding the monetary policy, chosen or constrained—thus improving institutional robustness of the European Union.

Keywords: Euro area, Taylor rule, Fiscal rule, public debt, NK model, Fiscal theory of price level

JEL Classification: E32, E52, E62, F41, H63

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1 Introduction

When a major crisis occurs, the strategic choice of policy mix for a large stimulus policy is obviously at the center of macroeconomic analysis. These exceptional times are also good tests of our institutions. More precisely, our institutions cannot be founded only on a single conventional view of economic stability based on the effectiveness of the Taylor principle. More robust rules must be found, ensuring equilibrium stability even when the conventional view does not apply. In this context, European Union (EU) requires specific analysis. Indeed, unlike a federal system like the United States (US), the decision-making centers of monetary and fiscal policy are not both centralized and, therefore, do not limit the process of political coordination between two actors. Therefore, the European policy mix also requires coordination of fiscal decisions, involving multiple decision-makers.

In this paper, I determine which set of rules, that is, which monetary-fiscal policies, make it possible to guarantee the stability of a monetary union such as EU. My analysis determines the rules for ensuring the existence of a single path around the steady state (a prerequisite for economic stabilization), leaving aside the analysis of multipliers that are specific to each of the policy mix regimes. Not surprisingly, but without our prior knowledge, I show that this set of rules is not unique, and it strongly depends on the effective margins of monetary policy. More specifically, I show that a fiscal union, that would take the form of a common debt, are necessary for European stability to overcome the limits of monetary policy when interest rates can no longer be lowered. Therefore, the paper results provide support to the recent initiative, called “Next Generation EU”, allowing the European Commission to issue bonds on the financial markets on behalf of the EU.¹

As this has been done since Leeper (1991), our paper considers that both monetary and fiscal authorities can only choose between two types of policies, active or passive. In a closed economy, the traditional policy mix associating an “active” monetary policy (application of the Taylor principle) and a “passive” fiscal policy (fiscal rule with a debt brake) ensures the dynamic stability of the New-Keynesian (hereafter, NK) model. However, another policy mix ensures equilibrium stability: it is based on a combination of a “passive” monetary policy (failure to respect the Taylor principle) and an “active” fiscal policy

¹President von der Leyen announced on May 27, 2020 that the bulk of the recovery measures proposed by EU will be powered by the “Next Generation EU” with financial firepower of €750 billion. The financing will be made possible by borrowing up to €750 billion on behalf of the EU, through the issuance of bonds. See the speech of President von der Leyen, https://ec.europa.eu/commission/presscorner/detail/en/speech_20_941.

(fiscal rule without debt brake). On this last equilibrium, the Fiscal Theory of the Price Level (hereafter, FTPL) applies.² When considering the case of the EU, a cursory analysis could lead to the conclusion that European policy makers have the same arbitrations than the US because in open economies with a monetary union the NK model has the same dynamic characteristics for the inflation and output gap than a closed economy. However, adding the analysis of country-specific public debts of open economies within a monetary union may not lead to differences in the case of a closed economy. In this paper, I show that this comfortable transposition of a policy mix analysis in a closed economy to the European case is not trivial. Indeed, given that each government of the Euro area maintains its fiscal independence, there are multiple public debt dynamics, a priori not controlled by the same debt brake.³ By taking into account these differences in public debt dynamics across the EU members, I show that despite the absence of a commitment on an explicit debt brake, an active monetary policy is a coordination device because it reduces the political decision of each government to a binary choice: to fight or not to fight the explosiveness of its public debt, stability of its inflation, and its output gap being insured by the European Central Bank (ECB). Note, however, that the stability of the area depends on the assumption that all the members have the same perception of monetary policy (active in this case), than making them all opt for controlling the stability of their public debt (voluntary choice of a passive fiscal policy). By contrast, when the ECB policy does not respect the Taylor principle (passive monetary policy), I show that only one country of the Euro area must implement an active fiscal policy. This result is due to the integration of inflation rates as well as output gaps: with unique inflation rate dynamics in the area, only a single public debt is needed to anchor the price level. However, it is not clear which country of the area will be free to escape the Maastricht treaty. This lack of a coordination device advocates a fiscal union for coordinating fiscal policies: this can be done by issue bonds on behalf of the EU, thus adding a new public debt in the Euro area that would be adopt a fiscal policy in accordance with the monetary policy, thereby ensuring the dynamic stability of the Euro area.⁴

²Leeper (1991), Sims (1994), Woodford (1998), and Cochrane (2001) developed the FTPL. See Bassetto (2002) for an analysis of a game between market players and the government, where the equilibrium solution provides the foundations of the FTPL. There are proponents of (e.g., Sims (2013)) and opponents to (e.g., Ljungqvist and Sargent (2004)) the FTPL. However, explaining this debate is beyond the scope of this paper: see, for e.g., Christiano and Fitzgerald (2000), Leeper and Leith (2016), or Cochrane (2019) for surveys on this debate.

³This unique rule is lacking in the current treaties. Indeed, the European treaties provide some “general principles” for the fiscal policy, but not an explicit fiscal rule that is uniform for each state.

⁴I complement the arguments of Farhi and Werning (2017) and Berger, Dell’Ariccia and Obstfeld (2018) showing that a fiscal union has greater ability to stabilize asymmetric shocks. In my paper, I

Indeed, in an open economy, dynamic stability is more demanding than in a closed one. In a system describing the joint dynamics of inflation, that is, output gap and public debt,⁵ the conditions of Blanchard and Kahn (1980) impose that there are two eigenvalues greater than one for the two jump variables, which are inflation and output gap, and one or less than one for the stock of public debt. If there is only one public debt, then dynamic stability is obtained for two types of policy mix: first, monetary policy is active and fiscal policy is passive (conventional view); second, fiscal policy is active and monetary policy is passive (FTPL). Stability is ensured because the central bank's decision constrains a government's choice. However, when there are two public debts, one for each independent state, the monetary union leaves the joint dynamics of inflation and output gap unchanged, but there is an additional eigenvalue that must be controlled to ensure dynamic stability. If monetary policy is active, thus stabilizing inflation and the output gap, then the two governments must implement a fiscal brake to stabilize the dynamics of their public debt. Conversely, if monetary policy is passive (then, the inflation-output system is no longer stable with an eigenvalue greater than one and another less than one), then one of the two countries must have an active fiscal policy, allowing the dynamic system to recover a value greater than one. Thus, it becomes clear that the stability of the area depends on the coordination of the states to implement the right rules at the right time. The fiscal union, that takes the form of a common debt, then makes it possible to provide a simple answer to this problem of coordination.

The impractical implementation of the Taylor principle is a reality in economies today, constrained by nominal interest rates equal to zero. Policy choices for the management of the current COVID-19 crisis, such as the "Next Generation EU" initiative, must, therefore, be implemented in this context where monetary policy is highly constrained. In a liquidity trap, or at the Zero-Lower-Bond (hereafter, ZLB), I show that a fiscal union, or a common debt, is a very useful tool to ensure the dynamic stability of the Euro area. Indeed, at the ZLB, dynamic stability must be ensured without using an active monetary policy. The situation is, therefore, close to an equilibrium where only an active budgetary policy, implemented by a unique European government (the European Council), would ensure

focus on the ability of the fiscal union, that takes the form of a common debt, to guarantee the dynamic stability of the area, whatever may be the nature of the shocks. I also complement the arguments of Aguiar et al. (2015) that show why existence of a fiscal externality can rationalizes the imposition of debt ceilings in a monetary union, but above all, show that a high-debt country is less vulnerable to rollover crises in sovereign debt markets, and have higher welfare, when it belongs to a union with an intermediate mix of high- and low-debt members, than one where all other members are low-debt.

⁵There is not only one inflation dynamics but also one output gap dynamics, due to the real exchange dynamics and aggregate resource constraints.

the dynamic stability of the Euro area. Indeed, the multiplicity of public debts then raises the problem of the designation of only the state which will need to have an active fiscal policy, thus allowing to anchor the price level in the Euro area. Then, this suggests that a fiscal union (issue of a common debt) will solve this coordination problem. Of course, the liquidity trap (or ZLB) can only be perceived as transitory by agents. In this case, I show that what matters for the stability of the equilibrium are the agent's expectations on the policies implemented after this episode of liquidity trap. To this end, I follow Christiano, Eichenbaum, and Rebelo (2011)⁶ by assuming a stochastic duration for this ZLB episode. I show that if agents expect the monetary policy will become active after the ZLB period, then a stable equilibrium of the Euro area will be possible if each European country respects its commitment to the Maastricht Treaty by choosing a passive fiscal policy. Conversely, if the agents expect the monetary policy to remain passive after the ZLB period, then a stable equilibrium of the Euro area will be possible only if one of the European countries does not respect its commitment for the Maastricht Treaty. This last result supports a fiscal union unifying the multiple debt dynamics of the Euro area, and thus making it possible to resolve the problem of the designation of the only state that can avoid the fiscal constraints of the Maastricht Treaty.

The remainder of the paper is structured as follows. In section 2, I present a two-country model of the Euro area in which the countries are asymmetric, and derive the implied terms-of-trade (TOT) dynamics. In section 3, I describe the dynamic properties of inflation, output gap, and public debts in the functioning of the policy choices of the central bank and different governments. Section 4 extends the analysis to episodes when the monetary policy is constrained by a zero lower bound. The link between the results of this paper and previous literature is discussed in section 5. Finally, section 6 concludes.

2 A Two-Country Model in a Monetary Union

The Euro area is modeled as an economy comprising two asymmetric “countries”. The first one is “a real country”, whereas the second one is “the rest of the Euro area”, that is, aggregation of all the other Euro area countries. The Euro area is populated by a continuum of infinitely lived households indexed by $j \in [0, 1]$. The population in segment $j \in [0, m)$ belongs to country H (Home country) and the population in segment $j \in [m, 1]$ belongs to country F (Foreign country). Therefore, m is the relative size of country H

⁶These authors have based their analysis on the previous works of Eggertsson and Woodford (2003), Christiano (2004), and Eggertsson (2004)

in the Euro area. The small Home country case can be derived by taking the limit of the two-country model as $m \rightarrow 0$.

Home bias and country size. The home bias in Home (Foreign) households' preferences is crucial because it is the only source of real exchange rate fluctuation in the model. Therefore, I assume that the share of imported goods denoted α (α^*) should decrease with the relative size of country H (F) and with the degree of home bias in Home (Foreign) households' preferences. A tractable way to formalize these ideas is to define $\alpha = \bar{\alpha}(1 - m)$ ($\alpha^* = \bar{\alpha}^*m$), where the exogenous parameter $\bar{\alpha}$ ($\bar{\alpha}^*$) is inversely related to the degree of home bias in H (F) households' preferences. In the following, I assume symmetric preferences, namely $\bar{\alpha} = \bar{\alpha}^*$.

2.1 Equilibrium

The complete set of equations describing the equilibrium dynamics are the following:⁷

The New Phillips curves (NP curves)

$$\hat{\pi}_{H,t} = \beta \mathbb{E}_t [\hat{\pi}_{H,t+1}] + \kappa \hat{y}_t - \lambda(\alpha m + \omega - 1) \hat{s}_t + \lambda t_t \quad (1)$$

$$\hat{\pi}_{F,t}^* = \beta \mathbb{E}_t [\hat{\pi}_{F,t+1}^*] + \kappa \hat{y}_t^* + \lambda(\omega^* - \alpha m) \hat{s}_t + \lambda t_t^*, \quad (2)$$

The Euler Equations (IS curves)

$$\hat{y}_t = \mathbb{E}_t [\hat{y}_{t+1}] - \frac{1}{\sigma} (\hat{r}_t - \mathbb{E}_t [\hat{r}_{t+1}]) + \frac{1 - \bar{\alpha}m - \omega}{\sigma} \mathbb{E}_t \Delta \hat{s}_{t+1} \quad (3)$$

$$\hat{y}_t^* = \mathbb{E}_t [\hat{y}_{t+1}^*] - \frac{1}{\sigma} (\hat{r}_t - \mathbb{E}_t [\hat{r}_{t+1}^*]) + \frac{\omega^* - \bar{\alpha}m}{\sigma} \mathbb{E}_t \Delta \hat{s}_{t+1}, \quad (4)$$

The Terms-Of-Trade (TOT)

$$\hat{s}_t - \hat{s}_{t-1} = \hat{\pi}_{F,t} - \hat{\pi}_{H,t} \quad (5)$$

$$\hat{s}_t = \frac{\sigma}{\omega + \omega^*} (\hat{y}_t - \hat{y}_t^*), \quad (6)$$

The Taylor Rule

$$\hat{r}_t = \alpha_\pi (m \pi_{H,t} + (1 - m) \pi_{F,t}^*) + \alpha_y (m \hat{y}_t + (1 - m) \hat{y}_t^*) + \varepsilon_t^M \quad (7)$$

$$\varepsilon_t^M = \rho^M \varepsilon_{t-1}^M + e_t, \quad (8)$$

⁷The definitions of all the reduced form parameters as functions of the structural model's parameters are given in Appendix H. The complete description of the model is provided in Appendix G and a summary of the equations leading to the final reduced form is given in Appendix F.

Public Debt and the Fiscal Rules

$$\widehat{b}_t^r = \beta^{-1}(\widehat{b}_{t-1}^r - \widehat{\pi}_{H,t}) + \widehat{r}_t - (\beta^{-1} - 1)\widehat{s}_t^r \quad (9)$$

$$\widehat{s}_t^r = \gamma \frac{1}{\beta^{-1} - 1} (\widehat{b}_{t-1}^r - \widehat{r}_{t-1}) + \varepsilon_t^F \quad (10)$$

$$\varepsilon_t^F = \rho \varepsilon_{t-1}^F + \nu_t \quad (11)$$

$$\widehat{b}_t^{r*} = \beta^{-1}(\widehat{b}_{t-1}^{r*} - \widehat{\pi}_{F,t}) + \widehat{r}_t - (\beta^{-1} - 1)\widehat{s}_t^{r*} \quad (12)$$

$$\widehat{s}_t^{r*} = \gamma^* \frac{1}{\beta^{-1} - 1} (\widehat{b}_{t-1}^{r*} - \widehat{r}_{t-1}) + \varepsilon_t^{F*} \quad (13)$$

$$\varepsilon_t^{F*} = \rho^* \varepsilon_{t-1}^{F*} + \nu_t^*, \quad (14)$$

where $\widehat{\pi}_{H,t}$ and $\widehat{\pi}_{F,t}^*$ are the inflation rates based on Production Price Index (PPI) and \widehat{y}_t and \widehat{y}_t^* are the output gaps in the Home and Foreign countries, respectively.⁸ The deviation of the nominal interest rate from its natural value is denoted by \widehat{r}_t . The government's choices are given by an *ad hoc* rule called a *fiscal rule*, where it is assumed that the real surplus of the Home government s_t^r reacts to the real debt gap b_t^r adjusted by the nominal interest rate that provides the price of the bond on the stock market (see, e.g., Leeper and Leith (2016)). These fiscal rules can be viewed as reduced forms of the European fiscal treaties. By leaving the budget autonomy to each government, the European treaties give each state the opportunity to choose the “strictness” of the fiscal rule: the choices of γ and γ^* summarize these policy strategies. Finally, firms pay tax based on their wage bill in each country, with the tax rates being t_t and t_t^* .

After integrating Equations (10) in (9) and (13) in (12) and the Taylor rule (7) in the IS curves (Equations (3) and (4)) and in the government budget constraints (Equations (9) and (12)), I obtain a system of eight equations, where the seven unknowns are $\{\widehat{\pi}_{H,t}, \widehat{\pi}_{F,t}^*, \widehat{y}_t, \widehat{y}_t^*, \widehat{s}_t, \widehat{b}_{H,t}^r, \widehat{b}_{H,t}^{r*}\}$ and five exogenous variables $\{\varepsilon_t^M, \varepsilon_t^F, \varepsilon_t^{F*}, t_t, t_t^*\}$.⁹ In this system, there are four jump variables $\{\widehat{\pi}_{H,t}, \widehat{\pi}_{F,t}^*, \widehat{y}_t, \widehat{y}_t^*\}$, two predetermined variables $\{\widehat{b}_{H,t}^r, \widehat{b}_{H,t}^{r*}\}$, one static relation between \widehat{s}_t and $\{\widehat{y}_t, \widehat{y}_t^*\}$ and a definition that links \widehat{s}_t with $\{\widehat{s}_t, \widehat{\pi}_{H,t}, \widehat{\pi}_{F,t}^*\}$. The static relationship emphasizes that the two IS curves are not independent equations, whereas the definition of the TOT provides the link between the country specific inflation rates. Therefore, the system dimension must be reduced to four independent equations. In the following, I discuss how to solve this dynamic system and the restrictions on the monetary and fiscal policies necessary to ensure the existence of a unique stable equilibrium.

Note that with this linearized version of the model, the conditions under which a unique

⁸These gaps are expressed in a log-deviation from the steady state.

⁹The dynamics of these exogenous variables are described by Equations (8), (11), and (14).

stable path exists can be determined in the deterministic version of the model. Therefore, for simplicity, I abstract in the following for the stochastic part of the model and consider that the exogenous variables $\{\varepsilon_t^M, \varepsilon_t^F, \varepsilon_t^{F*}, t_t, t_t^*\}$ are a deterministic process.

2.2 The Dynamics of Terms-Of-Trade

In this section, I show that the dynamics of the terms-Of-trade (TOT) can be solved independent of the rest of the model.

Proposition 1. *The TOT dynamics do not depend on the rules of monetary and/or fiscal policies (α_π , α_y , γ , and γ^*) as well as on their shocks (ε_t^M , ε_t^F , and ε_t^{F*}). The TOT dynamics only depend on fiscal gaps in labor costs ($t_t^* - t_t$).*

Proof. The difference between the two Phillips curves (Equations (1) and (2)) leads to:

$$\widehat{\pi}_{F,t}^* - \widehat{\pi}_{H,t} = \beta (\widehat{\pi}_{F,t+1}^* - \widehat{\pi}_{H,t+1}) + \kappa(\widehat{y}_t^* - \widehat{y}_t) + \lambda(\omega^* + \omega - 1)\widehat{s}_t + \lambda(t_t^* - t_t)$$

Given the definitions of ω and ω^* ,¹⁰ we deduce, using Equation (6), the equilibrium dynamics of TOT as follows:

$$0 = \widehat{s}_{t+1} - b\widehat{s}_t + \frac{1}{\beta}\widehat{s}_{t-1} + \frac{\lambda}{\beta}(t_t^* - t_t), \quad (15)$$

where $b = \left(1 + \beta + \frac{\kappa + (\kappa - \sigma)\bar{\alpha}(2 - \bar{\alpha})(\sigma\eta - 1)}{\sigma}\right) / \beta$. The solution is such that

$$(1 - r_1L)(1 - r_2L)\widehat{s}_{t+1} = \frac{\lambda}{\beta}(t_t^* - t_t) \Rightarrow \widehat{s}_{t+1} = r_1\widehat{s}_t - \lambda r_1 \sum_{j=0}^{\infty} \left(\frac{1}{r_2}\right)^j (t_{t+1+j}^* - t_{t+1+j}),$$

where $r_i = \frac{b \pm \sqrt{b^2 - 4/\beta}}{2}$, for $i = 1, 2$ and s.t. $0 < r_1 < 1 < r_2$ (See Appendix A). \square

This result emphasizes the first property of an open economy dynamics: the TOT introduces backward and forward-looking components, independent of the monetary and fiscal policies. The predetermined component of the TOT comes from the sticky prices, coupled with the fixed exchange rate assumption (Currency union). The TOT fluctuations are driven by tax rate shocks (t_t, t_t^*). These results are used by Farhi et al. (2014) to derive the optimal sequence of these taxes that would allow the equilibrium of a fixed exchange rate open economy to reach the equilibrium of an economy with flexible nominal exchange rates, better suited for damping business cycle shocks.

¹⁰See Appendix H for details on the parameters.

Therefore, without any shocks on the marginal costs induced by a fiscal policy implemented in the sticky price economy, the TOT dynamics is deterministic. Thus, $\widehat{s}_t = 0$ if its initial condition is its equilibrium value and $t_t, t_t^* = 0, \forall t$. Using Equation (6), we deduce that $\widehat{y}_t = \widehat{y}_t^*$.

2.3 A Limit Case: A Small Open Economy

The case of a small open economy (SOE) is obtained when $m \rightarrow 0$. Hence, the monetary policy, given by $\widehat{r}_t = \alpha_\pi \pi_{F,t}^* + \alpha_y \widehat{y}_t^* + \varepsilon_t^M$, does not depend on Home aggregates. Then, the Home aggregates have an autonomous dynamic, with the Foreign variables and the interest rate being exogenous. For simplicity, I thus assume $\widehat{\pi}_{F,t}^* = \widehat{y}_t^* = t_t^* 0$. This leads to $\widehat{s}_t = -\widehat{p}_{H,t}$. The equilibrium dynamic of an SOE is thus given by Equation (15), where \widehat{s}_t is simply replaced by $-\widehat{p}_{H,t}$. Given the equilibrium path for $\widehat{p}_{H,t}$, we deduce the output gap using Equation (6), the equation being evaluated for $m \rightarrow 0$. This shows that the dynamics of the Home output gap is given by a Home price dynamic (here, the opposite of the TOT) and an inflation dynamic (the simple difference between the current and the past price levels). As for the TOT, the equilibrium price dynamic is deduced from Equation (15), where $\widehat{s}_t = -\widehat{p}_{H,t}$. Its solution is $\widehat{p}_{H,t+1} = r_1 \widehat{p}_{H,t} - \lambda r_1 \sum_{j=0}^{\infty} \left(\frac{1}{r_2}\right)^j t_{t+j}$, where the roots are $r_i = \frac{b \pm \sqrt{b^2 - 4/\beta}}{2}$, for $i = 1, 2$ and s.t. $0 < r_1 < 1 < r_2$. This is the result presented in Erceg and Linde (2012). It is also a corollary of Proposition 1: in an SOE, the equilibrium dynamics of Home price (opposite of the TOT in an SOE) does not depend on monetary and fiscal policies (rules and shocks).

Proposition 2. *In a small open economy of the Euro area, a debt brake, of a minimal size $\gamma > 1/\beta - 1$, is required to ensure the existence of a saddle path.*

Proof. See Appendix B □

Given that the path of price gives both output and inflation dynamics, the equilibrium of an SOE is defined only by price and public debt dynamics. The fact that the price dynamic of the Home country does not depend on fiscal policy induces that price and public debt are independent processes in an SOE. The stability of the debt can be only ensured by the implementation of a fiscal brake.

From a policy perspective, we deduce that if each SOE of the Euro area adopts a fiscal rule that respect $1/\beta - 1 < \gamma$, then the aggregate public debt of the Euro area will have a sufficient debt break to ensure its stability. Therefore, the individual fiscal policies of each

country imply the stability of the Euro aggregate. This is a credible strategy if all the countries expect that the ECB respects or implements its mandate to stabilize inflation via the implementation of the Taylor principle (active monetary policy). Indeed, given that inflation and output gap abroad are taken as exogenous and stationary at the level of each SOE, the ECB must apply the European treaties by respecting the Taylor principle in order to stabilize the aggregate output-inflation dynamics of the Euro area.¹¹ If each SOE of the Euro area does not respect the fiscal rule, there is no chance for the ECB to correct this instability. This is true if we consider only one SOE for the Euro area, because the dynamic of this SOE does not depend on the monetary policy. However, I will see later that the analysis of the global dynamics of the Euro area provides an equilibrium, where the fiscal theory of price level applies, thereby supporting a stable equilibrium with an active fiscal policy and passive monetary policies.

3 Inflation, Output and Public Debt in the Euro Area

In the first subsection, I analyze a specific case where it is assumed that all the governments of the Euro area adopt the same fiscal rule. This case corresponds to the one where all the governments behave as a representative one, obviously a hypothetical scenario in the current Euro area. I show that two equilibria are possible: first, the ECB has an active monetary policy (Taylor principle) and all governments have a passive fiscal policy (implementation of a fiscal brake); and second, the ECB has a passive monetary policy and all governments have an active fiscal policy (no debt brake). In the second subsection, I analyze the case where governments have different fiscal policies, implying that the equilibrium cannot be summarized by aggregates determined by representative decision makers.

¹¹For analyzing the SOE, it is assumed that $\hat{\pi}_{F,t}^* = \hat{y}_t^* = 0$ for simplicity, but what matters is the stationarity of both $\hat{\pi}_{F,t}^*$ and \hat{y}_t^* . This stationarity is ensured if it exists as a unique equilibrium path for these two variables representing inflation and the output gap of Euro area.

3.1 Stability of the Euro Aggregates When All Governments Adopt the Same Fiscal Rule

Assume that fiscal brakes are identical across countries.¹² Note that only the component of the fiscal policy that accounts for the debt brake (γ and γ^*) must be homogeneous across countries for the dynamic stability of the Euro aggregate.¹³ The component of the fiscal policy that accounts for discretionary decisions (ρ and ρ^*) can be heterogeneous across countries. Thus, we have $\gamma = \gamma^* \equiv \gamma_u$, but $\rho \neq \rho^*$. The Euro aggregate is defined as follows: inflation is $\widehat{\pi}_t^u = m\widehat{\pi}_{H,t} + (1-m)\widehat{\pi}_{F,t}$, output gap is $\widehat{y}_t^u = m\widehat{y}_t + (1-m)\widehat{y}_t^*$, and debt is $\widehat{b}_t^{ru} = m\widehat{b}_t^r + (1-m)\widehat{b}_t^{r*}$. By directly analyzing the aggregates of the Euro area, the monetary policy described by the following Taylor rule (Equation (7))

$$\widehat{r}_t = \alpha_\pi \widehat{\pi}_t^u + \alpha_y \widehat{y}_t^u + \varepsilon_t^M \quad (16)$$

links the interest rate to aggregate inflation and output gap, without distinguishing countries by their weight on these aggregates.

By aggregating the two Phillips curves (Equations (1) and (2))

$$\widehat{\pi}_t^u = \beta \widehat{\pi}_{t+1}^u + \kappa \widehat{y}_t^u$$

the two IS curves (Equations (3) and (4))

$$\widehat{y}_t^u = \widehat{y}_{t+1}^u - \frac{1}{\sigma} (\widehat{r}_t - \widehat{\pi}_{t+1}^u)$$

the two debt dynamics (Equations 9) and (12) as well as the surpluses (Equations (10) and (13))

$$\begin{aligned} \frac{1}{\beta^{-1} - \gamma_u} \widehat{b}_{t+1}^{ru} &= \widehat{b}_t^{ru} - \widehat{\pi}_{t+1}^u - \frac{\beta^{-1} - 1}{\beta^{-1} - \gamma_u} (m\varepsilon_{t+1}^F + (1-m)m\varepsilon_{t+1}^{F*}) \\ &\quad + \frac{1}{\beta^{-1} - \gamma_u} (\alpha_\pi \widehat{\pi}_{t+1}^u + \alpha_y \widehat{y}_{t+1}^u) + \frac{\gamma_u}{\beta^{-1} - \gamma_u} (\alpha_\pi \widehat{\pi}_t^u + \alpha_y \widehat{y}_t^u) \end{aligned}$$

and using the Taylor rule (Equation (16)), the equilibrium paths for $\{\widehat{\pi}_t^u, \widehat{y}_t^u, \text{and } \widehat{b}_t^{ru}\}$ can be solved.¹⁴

¹²In this section, it is assumed for simplicity that $t_t = t_t^* = 0, \forall t$. These exogenous variables simply add a shock to the aggregate Phillips curve of the Euro area, which is not useful at this stage.

¹³Aggregation is also possible under the assumption that the structural parameter that governs nominal rigidities (Phillips curves) is the same across countries (assuming that all Europeans share the same preferences). This assumption is not rejected by the data (see Martin and Philippon (2017)).

¹⁴Another way to obtain this dynamic system is to assume that Europeans have no home bias $\bar{\alpha} = 1$. In this case, the real exchange rate is constant as well as the TOT: the risk-sharing condition leads to $\widehat{c}_t = \widehat{c}_t^*$. Assuming for simplicity that $m = 1/2$, this implies that $\widehat{y}_t = \widehat{y}_t^* = \frac{\widehat{y}_t + \widehat{y}_t^*}{2} \equiv \widehat{y}_t^u$, $\widehat{\pi}_{H,t} = \widehat{\pi}_{F,t}^* = \frac{\widehat{\pi}_{H,t} + \widehat{\pi}_{F,t}^*}{2} \equiv \widehat{\pi}_t^u$, $\widehat{b}_t^r = \widehat{b}_t^{r*} = \frac{\widehat{b}_t^r + \widehat{b}_t^{r*}}{2} \equiv \widehat{b}_t^{ru}$ and $\widehat{d}_t^{r,u} \equiv \frac{\widehat{d}_t^r + \widehat{d}_t^{r*}}{2}$ are solutions of the same dynamic system.

Proposition 3. *When all the governments adopt the same fiscal rules, the stability of the Euro aggregates is ensured if a “passive” fiscal policy is implemented in each European state when the Taylor principle applies.*

However, if the Taylor principle does not apply, it can also exist as a stable equilibrium if fiscal policies are the same everywhere and “active” in each country.

Proof. Using $\varepsilon_{t+1}^F = \rho \varepsilon_t^F$ and $\varepsilon_{t+1}^{F*} = \rho^* \varepsilon_t^{F*}$, the Philips and IS curves as well as the debt dynamics of the Euro area aggregates lead to

$$\begin{bmatrix} \widehat{\pi}_t^u \\ \widehat{y}_t^u \\ \widehat{b}_t^u \end{bmatrix} = \begin{bmatrix} A & 0 \\ a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} \widehat{\pi}_{t+1}^u \\ \widehat{y}_{t+1}^u \\ \widehat{b}_{t+1}^u \end{bmatrix} + \Omega \begin{bmatrix} -\kappa & 0 & 0 \\ -1 & 0 & 0 \\ b_1 & m\rho b_2 & (1-m)\rho^* b_2 \end{bmatrix} \begin{bmatrix} \varepsilon_t^M \\ \varepsilon_t^F \\ \varepsilon_t^{F*} \end{bmatrix}$$

$$\text{where } \begin{cases} \Omega = \frac{1}{\sigma + \alpha_y + \kappa \alpha_\pi} & a_1 = -\frac{\beta}{1 - \gamma_u \beta} (\alpha_y - \beta^{-1} + \gamma_u \sigma (\alpha_\pi \beta - 1) \Omega) \\ a_2 = -\frac{\beta}{1 - \gamma_u \beta} (\alpha_y + \gamma_u \sigma (\alpha_y + \alpha_\pi \kappa) \Omega) & a_3 = \frac{\beta}{1 - \gamma_u \beta} \\ b_1 = -\gamma_u \frac{\sigma \beta}{1 - \gamma_u \beta} & b_2 = (1 - \beta) \frac{\sigma + \alpha_\pi \kappa + \alpha_y}{1 - \gamma_u \beta} \end{cases}$$

This system is block-diagonal with an upper block defined by

$$\begin{bmatrix} \widehat{y}_t^u \\ \widehat{\pi}_t^u \end{bmatrix} = A \begin{bmatrix} \widehat{y}_{t+1}^u \\ \widehat{\pi}_{t+1}^u \end{bmatrix} + B \varepsilon_t^M$$

$$\text{where } A = \Omega \begin{bmatrix} \kappa + \beta(\sigma + \alpha_y) & \kappa \sigma \\ 1 - \beta \alpha_\pi & \sigma \end{bmatrix}, \quad B = -\Omega \begin{bmatrix} \kappa \\ 1 \end{bmatrix}$$

can be solved first. The two eigenvalues of A are positive, and the characteristic polynomial is such that $\mathbf{P}_A(1) = 1 - \text{tr}(A) + \det(A) = \frac{\kappa(\alpha_\pi - 1) + (1 - \beta)\alpha_y}{\sigma + \alpha_y + \kappa \alpha_\pi}$, which indicates that both roots are lower than 1 if $\kappa(\alpha_\pi - 1) + (1 - \beta)\alpha_y > 0$. This is the Taylor principle.

This strategy for monetary policy enforces *all* the governments of the Euro area to implement a strict fiscal rule, that is, to choose a parameter $\gamma = \gamma^* \equiv \gamma^u$ such that public debt does not explode. A simple restriction is $\gamma^u > \beta^{-1} - 1$, which represents the “strictness” of the fiscal rule. \square

This analysis reveals that the stability of the Euro aggregates does not depend on the nature of the shocks, but on the coordination of the policy rules.¹⁵ This framework,

¹⁵Note that this analysis, based on the aggregates of the Euro area, does not imply that the output gap and inflation of each country are the same and equal to the ones of the Euro area. This will be the case only if no other country-specific shocks exist: that is, in this particular case, the TOT does not exist and the analysis will be reduced to one of a closed economy.

where all the countries have exactly the same fiscal policy, can be interpreted as **Federal state** where the central government coordinates the fiscal policy which is thus unique. Therefore, we return to the same economy studied by Lepeér, Traum, and Walker (2017) in the case of the US modeled as a closed economy. In this case, it exists as a menu for the policy mix ensuring economic stability: an active monetary policy accompanied by a passive fiscal policy, or a passive monetary policy accompanied by an active fiscal policy. For the Euro area, this simplified model with a representative country is close to the one used by Woodford (1996) in discussing the impact of the Maastricht treaty on maintaining price stability in Europe. **If** the Taylor principle is satisfied, namely $\kappa(\alpha_\pi - 1) + (1 - \beta)\alpha_y > 0$, **then** a passive fiscal rule **applied in each country** is necessary to ensure that public debt remains non-explosive. Indeed, with $\{\widehat{\pi}_t^u, \widehat{y}_t^u\}$ (the stationary solutions of the dynamic system for the aggregate economy when the Taylor principle is respected), debt is sustainable if $\gamma > \beta^{-1} - 1$. This fiscal break ($\gamma > \beta^{-1} - 1$) represents the “strictness” of the fiscal rule. The commitment of each government of the Euro area is crucial to ensure economic stability: this was the aim of the Maastricht treaty. **By contrast**, if the ECB does not respect the Taylor principle, then all the countries of the Euro area (in fact, the representative government of the Euro area) must be coordinate in order to simultaneously abandon fiscal rules incorporating a fiscal brake. This is an easy strategy for a federal state, but less easy for the Euro area with its Maastricht treaty obligation to maintain independence for fiscal policies. Note that this problem is not a theoretical curiosity: it can be concrete when an economy enters a liquidity trap, such as the period when the ZLB constraint is perceived as very persistent.

3.2 Stability of the Euro Area when Fiscal Rules Are Different Across Countries

We now break the simplistic view of a unique IS curve and a unique Phillips curve in Europe. Hence, with heterogeneous paths of output gaps and inflation, the choice of fiscal policy can/must differ from those that a fictional European government would take. Using Proposition 1, all the variables can be expressed as a function of the solution for the TOT dynamics. Given this property, it is obvious that the Taylor principle can ensure the stability of the Euro area if we abstract from the public debt dynamics. I introduce the public debt dynamics specific to each country and show that instability can emerge.

The complete dynamics of the two-country model, where each country has a specific fiscal policy (rule and shock), is the solution for the following system (using Equations (1), (3),

(9), and (12)):

$$\begin{aligned}
\widehat{\pi}_{H,t} &= \beta\widehat{\pi}_{H,t+1} + \kappa\widehat{y}_t - \lambda(\alpha m + \omega - 1)\widehat{s}_t + \lambda t_t \\
\widehat{y}_t &= \widehat{y}_{t+1} - \frac{1}{\sigma}(\widehat{r}_t - \widehat{\pi}_{H,t+1}) + \frac{1 - \bar{\alpha}m - \omega}{\sigma}(\widehat{s}_{t+1} - \widehat{s}_t) \\
\frac{1}{\beta^{-1} - \gamma}\widehat{b}_{t+1}^r &= \widehat{b}_t^r - \widehat{\pi}_{H,t+1} - \frac{\beta^{-1} - 1}{\beta^{-1} - \gamma}\varepsilon_{t+1}^F + \frac{1}{\beta^{-1} - \gamma}\widehat{r}_{t+1} + \frac{\gamma}{\beta^{-1} - \gamma}\widehat{r}_t \\
\frac{1}{\beta^{-1} - \gamma^*}\widehat{b}_{t+1}^{r*} &= \widehat{b}_t^{r*} - (\widehat{s}_{t+1} - \widehat{s}_t) - \widehat{\pi}_{F,t+1} - \frac{\beta^{-1} - 1}{\beta^{-1} - \gamma^*}\varepsilon_{t+1}^{F*} + \frac{1}{\beta^{-1} - \gamma^*}\widehat{r}_{t+1} + \frac{\gamma}{\beta^{-1} - \gamma^*}\widehat{r}_t,
\end{aligned}$$

where the Taylor rule, given by

$$\widehat{r}_t = \alpha_\pi\widehat{\pi}_{H,t} + \alpha_y\widehat{y}_t + (1 - m)\left(\alpha_\pi - \alpha_y\frac{\omega + \omega^*}{\sigma}\right)\widehat{s}_t - \alpha_\pi(1 - m)\widehat{s}_{t-1} + \varepsilon_t^M \quad (17)$$

shows that despite the asymmetric weight of each country, the sensitivity of the Taylor rule to inflation and output gap does not depend on the weight of each country. This result is a direct implication of the TOT equation (6).

After integrating Equation (17) in the IS and public debt equations, this system can be rewritten as follow $Z_t = \widehat{M}Z_{t+1} + \widehat{D}S_t + \widehat{P}\varepsilon_t$, where $Z_t = [\widehat{\pi}_t^H, \widehat{y}_t, \widehat{b}_t^r, \widehat{b}_t^{r*}]'$, $S_{t+1} = [\widehat{s}_{t+1}, \widehat{s}_t, \widehat{s}_{t-1}]'$, and $\varepsilon_t = [\varepsilon_t^M, \varepsilon_t^F, \varepsilon_t^{F*}]'$. This system has two jump variables $\{\widehat{\pi}_t^H, \widehat{y}_t\}$ and two predetermined variables $\{\widehat{b}_t^r, \widehat{b}_t^{r*}\}$. Its stability is then ensured if \widehat{M} has two eigenvalues inside and two outside the unit circle. The solution for \widehat{s}_t is given in Proposition 1.¹⁶ Using this solution, \widehat{y}_t^* is deduced from $\widehat{y}_t^* = \widehat{y}_t - \frac{\omega + \omega^*}{\sigma}\widehat{s}_t$, and $\widehat{\pi}_{F,t}$ from $\widehat{\pi}_{F,t} \equiv \widehat{s}_t - \widehat{s}_{t-1} + \widehat{\pi}_{H,t}$.

Proposition 4. *If the Taylor principle is respected, the stability of the Euro area is ensured if all the countries have a passive fiscal policy, that is, if all the countries implement a fiscal brake satisfying $\gamma, \gamma^* > \beta^{-1} - 1$.*

If the Taylor principle is not respected, the stability of the Euro area is ensured if only one country implement an active fiscal policy, all others must respect their commitment to implement a fiscal brake: $\gamma > \beta^{-1} - 1$ & $\gamma^ < \beta^{-1} - 1$, or $\gamma < \beta^{-1} - 1$ & $\gamma^* > \beta^{-1} - 1$.*

Proof. Given that \widehat{M} is triangular (See appendix C), its eigenvalues are those of the matrix A and $\left\{\frac{1}{\beta^{-1} - \gamma}, \frac{1}{\beta^{-1} - \gamma^*}\right\}$. In Proposition 3, it is shown that the two eigenvalues of A are inside the unit circle if $\kappa(\alpha_\pi - 1) + (1 - \beta)\alpha_y > 0$ (Taylor principle).

If $\kappa(\alpha_\pi - 1) + (1 - \beta)\alpha_y > 0$, then the two eigenvalues of A are inside the unit circle. Therefore, the two last eigenvalues of \widehat{M} must be outside the unit circle: $\frac{1}{\beta^{-1} - \gamma} > 1$ and $\frac{1}{\beta^{-1} - \gamma^*} > 1$, that is, $\gamma, \gamma^* > \beta^{-1} - 1$.

¹⁶See appendix C for the complete description of this system.

If $\kappa(\alpha_\pi - 1) + (1 - \beta)\alpha_y < 0$, then one eigenvalue of A is inside the unit circle, the other being outside the unit circle. Therefore, only one of the two last eigenvalues of \widehat{M} must be outside the unit circle. This implies that one set of the following restrictions must be satisfied: *i)* $\frac{1}{\beta^{-1}-\gamma} > 1$ and $\frac{1}{\beta^{-1}-\gamma^*} < 1$, that is, $\gamma > \beta^{-1} - 1$ and $\gamma^* < \beta^{-1} - 1$, or *ii)* $\frac{1}{\beta^{-1}-\gamma} < 1$ and $\frac{1}{\beta^{-1}-\gamma^*} > 1$, that is, $\gamma < \beta^{-1} - 1$ and $\gamma^* > \beta^{-1} - 1$. \square

If the Taylor principle is respected, that is, when the ECB implements an “active” monetary policy rule, **all** countries of the Euro area must implement a fiscal brake. This shows that these two policies are complementary to ensure the stability of the area. The active monetary policy implemented by the ECB can be viewed as a coordination device guiding all governments to respect the European treaties.

If the Taylor principle is not respected, only one country is constrained to adopt a passive fiscal policy. The other country must have an active fiscal policy. Therefore, when the monetary policy of the ECB does not respect the Taylor principle, **one** of the two countries of the Euro area must adopt an active fiscal policy, that is, they must not implement the fiscal brake of the Fiscal Stability Treaty. In this case, the fiscal theory of price level applies, showing that a non-unified fiscal policy provides the freedom to choose which country can renounce its Maastricht treaty commitments. This degree of freedom can also lead to a coordination problem, unforeseen by the European treaties.

Instability of the Euro area. The two regimes of Proposition 4 suggests that a *fiscal union*, that takes the form of a common debt, would be the best solution to avoid any destabilization of the Euro area linked to the non-compliance of one of the member states to the fiscal stability treaties.

The bonds issue of common debt leads “simply” to an additional public debt dynamics, managed by the European Council. Therefore, the dynamic system of the Euro area would be composed by five equations: the NP and the IS curves, the public debts of each of the two countries and the common public debt of the EU. In this context, even if the monetary policy is not “active”, the stability of the Euro area can be ensured by an “active” fiscal policy for the common EU budget, the stability of the two other public debts being ensured by the implementation of debt brakes (“passive” fiscal policy inside each country). By contrast, if monetary policy can be “active”, then the three fiscal policies must be “passive”. However, the risk of instability for the Euro area seems lower when the monetary policy is active: if one country deviates, it knows that it could lead to global instability in the area, its gains then being able to exist only in the very short term. Moreover, if this country is small, Proposition 2 suggests that it is not rational

for an SOE to find grounds for non-compliance with treaties. Obviously, this reasoning also applies to the governance of the European common debt. Hence, when the monetary policy is active, nobody seems to be tempted to give up its commitment to the European treaties. An active monetary policy also acts as a coordination device.

This optimism about the Euro system is undermined in situations where the ECB would choose not to have an active monetary policy. In this case, one of the members must renounce these European commitments, but no one knows which among the states will be able to have an active fiscal policy. This indeterminacy of equilibrium presents a clear problem of stability in the Euro area. The critical situation in the Euro area would then be the moment when the ECB would be forced not to have an active monetary policy and nobody would take the decision to issue bonds behalf of the EU managed by a fiscal policy rectifying the shortcoming of the monetary policy.

4 Zero-Lower-Bound: A Risk Arguing in Favor of a Fiscal Union

When the Euro area is caught in a liquidity trap, for instance, when the ECB's monetary policy constrained by the ZLB, it could be difficult for the active monetary policy to coordinate with the governments of the Euro area on an equilibrium where they all choose a passive fiscal policy (Proposition 4). This section presents the options for a policy mix in this context by highlighting that these strategies mainly depend on agent expectations.

4.1 A highly persistent Zero-Lower-Bound

If the ZLB is expected to be highly persistent and can be approximated by a permanent state, a passive monetary policy will be favored in the agent's expectations. Therefore, Proposition 4 shows that one country of the Euro area must leave the Fiscal Stability Treaty in order to stabilize the Euro area. However, which country will do this? If each country considers itself as one among the many SOEs in the Euro area, then none of them will be interested in choosing an active fiscal policy (abandonment of the fiscal brake of the European treaties) because this strategy destabilizes an SOE (see proposition 2). However, in the case of larger economies of the Euro area, such as Germany, France, Italy or Spain, it is possible that these governments do not consider their countries as an SOE, and therefore rely on their partners to ensure the stability of the Euro area,

while “enjoying” an active fiscal policy. However, without a coordination system, each country can consider itself as one that can have an active fiscal policy, thereby leading to instability in the Euro area.

Proposition 5. *When ZLB is perceived to be permanent, a fiscal union stabilizes the Euro area by selecting a uniquely active fiscal policy at the federal state level.*

Proof. Using proposition 4, we know that if the monetary policy is passive (the Taylor principle cannot be respected), an active fiscal policy must be implemented because one eigenvalue of A is outside the unit circle.

Given that the monetary policy is constrained to be passive at the ZLB, a fiscal union that would take the form of a unification of all public debts, can ensure that $\gamma = \gamma^* \equiv \gamma^u$ with $\gamma^u < \beta^{-1} - 1$. With only one real-debt dynamics (fiscal union), the dynamics would have two eigenvalues inside the unit circle and one outside.

Another way, consist to issue bonds behalf the EU, that will manage them by implementing an active fiscal policy. At the same time, all the states must respect their European commitments by implementing a passive fiscal policy, as in the case where monetary policy is active. \square

Proposition 5 shows that fiscal union is a solution to ensure stability in the Euro area when deprived of an active monetary policy. The policy of this fiscal union would then be to select the equilibrium where the Fiscal Theory of Price Level (FTPL) would apply. The non-Ricardian properties of this equilibrium would, therefore, allow fiscal transfer policies to revive the Euro area economy. The budgetary tool for regulating economic activities would, therefore, be greatly appreciated during periods when the interest rate is constant, that is, locked in at zero. It seems easier to issue new public debt on behalf of the EU than to unify the debts of the member states. This is the Franco-German proposal (Macron-Merkel), presented by the President of the European Commission on May 27, 2020.

Note that even in an extreme context of permanence of the ZLB constraint, policies other than those leading to an equilibrium where the FTPL applies are possible. Specifically, it is possible to envisage rules inducing fiscal devaluations in response to the business cycle and then allowing to stabilize the Euro area around a “Ricardian” equilibrium. On this equilibrium, the fiscal policy of each State must remain passive (a fiscal brake exists) as each of them is committed to it by signing the European treaties. At this equilibrium, employment subsidies in each State depend on economic activities (inflation and output

gaps), and then induce fluctuations in the real exchange rate. This policy rule replaces a stabilization by the nominal interest rate.¹⁷ This selection of a stable equilibrium via adjustments to the TOT is a specificity of open economies that opted for a monetary union.

4.2 Transitory Zero-Lower-Bound

The assumption of permanence of the ZLB constraint is certainly too extreme. Nonetheless, it allows us to see how essential anticipation is for the design of a policy mix: by considering that the ZLB constraint will be permanent, then the stability of the economy can only be assessed around this stationary state where the monetary policy is doomed to be silent. In a more realistic case where the ZLB period is finite in expectation, the dynamics of the Euro area depend on agents' expectations of the monetary policy that the ECB will likely lead, once the economy has left the ZLB. To model a ZLB constraint that is only transitory, a stochastic ending of this regime à la Christiano, Eichenbaum, and Rebelo (2011) is considered.

Proposition 6. *If the ZLB is only a transitory period, what matters is the expected monetary policy at the end of the liquidity trap:*

- *If the monetary policy is expected to be active after the period when ZLB is binding, then **all** governments must choose a passive fiscal policy. The fiscal union is not necessary to coordinate fiscal policies to achieve stability.*
- *If the monetary policy is expected to be passive after the period when ZLB is binding, then one of the government must choose an active fiscal policy, or a common public debt must ensure this stabilizing task.*

Proof. See Appendix D. □

A fiscal union that take the form of bond issues behalf the EU makes it possible to guarantee the stability of the Euro area whatever maybe the expectations of agents. This will be necessary only if the agents expect that an active monetary policy will no longer be possible in the future, even when the economy will leave the ZLB regime. When the

¹⁷Appendix E creates restrictions on the rules such that $t_t = \nu_y \hat{y}_t + \nu_\pi \hat{\pi}_{H,t}$, allowing to have a stable dynamics of inflation and the output gap in the absence of the Taylor rule (passive monetary policy or ZLB). Given this stability in the inflation-output gap dynamics, the fiscal policy must then be passive to ensure the dynamic stability of the Euro area.

agents expect that an active monetary policy will revive, a fiscal union is not necessary; however, cannot destabilize the dynamics if European council adopts a passive fiscal policy, as all the member states. Therefore, a fiscal union can be viewed as a guarantee, ensuring, whatever maybe agents' expectations, the constraints on monetary policy, and the non-explosiveness of the Euro area dynamics.

5 Discussion

My paper relates to several strands of literature.

First, it offers complementary arguments in favor of fiscal union to those presented in Farhi and Werning (2017) and Berger, Dell’Ariccia, and Obstfeld (2018). These authors show that the loss of adjustment margin induced by the fixed exchange rate of a currency union, can be compensated by transfers between countries, or organized by a fiscal union. They also show that a fiscal union can discipline moral hazard problems induced by this more efficient risk sharing. Hence, such studies focus on the higher ability of a fiscal union to stabilize the asymmetric shocks. I complement these arguments by highlighting the ability of a fiscal union to guarantee the dynamic stability of the area, irrespective of the nature of the shocks. This is shown by considering explicitly the public debt dynamics of each country of the area, a dimension too often overlooked in business cycle analysis.

Second, my analysis provides answers to the stability problems of the Euro area raised by Woodford (1996) and Sims (1999):

“Even if one government is fiscally responsible and keeps its real primary deficit at some sustainable constant level, variations in the budget deficit of the other government will result in price level instability for that government as well. Thus, there is a clear reason for a government concerned to maintain stable prices to care about fiscal policies of the other governments with which it shares a common currency.” Woodford (1996), p. 30

“What is the weakness of the EMU system in other circumstances—the fiscal free rider problem—would work toward resolving the difficulty. Even one country that is sufficiently fiscally expansive, despite the Maastricht rules, could undo the liquidity trap, with the resultant reversal of deflation benefiting all members of the EMU. On the other hand, if the logical foundations of the need for fiscal coordination are not understood, the need to break the Maastricht

rules in this situation could undermine adherence to them more generally."
Sims, (1999), p. 425

Following these questions, I show that the construction of European institutions cannot be founded on a single vision of price stability, based on the effectiveness of Taylor's principle. If, the policy mix of the Euro area is open to a wider range of practices than the conventional ones, exposed in Gali's (2008) textbook, the Fiscal Theory of Price Level (FTPL), presented by Leeper (1991), Sims (1994), Woodford (1998), and Cochrane (2001), can help to design new and more robust rules.¹⁸ In this paper, I show that a fiscal union is not necessary when the monetary policy can be active, no country has the incentive to deviate from the implementation of the fiscal brake suggested by the Treaties, but it can be a necessity when the monetary policy cannot be active. In this case where the FTPL applies, the Euro area's inflation is determined by the nominal public debt dynamics of one country of area that must implement an active fiscal policy, whereas the others must respect their fiscal commitments (passive fiscal policy by implementing a fiscal brake). The degree of freedom concerning the choice of the country that must have an active fiscal policy, advocate in favor of a fiscal union that trivially solves this coordination problem. Therefore, my paper complements Bergin (2000) and Dupor (2000), who analyzed the implications of FTPL in the framework of open economy. If the analysis in flexible exchange rates proposed by Dupor (2000) does not apply directly to the current European experience, that of Bergin (2000), in a monetary union, shows the integration of the securities of various public debts of the European countries through the balance sheet of an unique central bank unifies the dynamics of these public assets, the redistribution of the dividends taking then the form of lump sum transfers. Bergan's solution is based on a unifying central bank that controls the issue of new money through open market bond purchases, and returns its interest income back to the national governments via lump sum transfers.

Third, since the 2008 global financial crisis and for a period that will last certainly for another few years after the COVID-19 crisis, I also contribute to the analysis of the Euro area's policy mix under the ZLB constraints, as it has already been investigated by Farhi

¹⁸As emphasized by Woodford (1998), the study by D'Autume and Michel (1987) is important in the literature on the fiscal theory of the price level: "*It thus appears to be usually an admissible policy for the government to distribute to the public subsidies financed by an ever-increasing monetary debt. The reason, of course, is that the government does not pay any interest on this special kind of debt*" (p. 1351). "*If the government freely chooses the paths of its instruments without caring about its intertemporal budget constraint, the choice of the initial price level must ensure that [...] the real value of initial debt are such that the government budget constraint is satisfied ex post*" (p. 1363).

and Werning (2016). These authors focus on the fiscal multiplier size during a liquidity trap period and highlight the limits of conventional stabilizers in this context.¹⁹ My study suggests that other fiscal multipliers can be evaluated in the context where individuals expect that monetary policy will never be active in the future. But, beyond the size of the multipliers, my results suggest that issue bonds on the financial markets on behalf of the EU could also reinforce the dynamics stability of the Euro area, and thus ensure the robustness of the recovery measures.

6 Conclusion

In this paper, I show that, in a monetary union, the dynamic stability of open economies, with each having a public debt, depends on the combination of the choice of fiscal and monetary policy rules. If an active monetary policy can be implemented, or agents expect its implementation after experiencing a period of a liquidity trap situation, then I show that each country will choose to have a passive fiscal policy. Indeed, if the central bank is able to ensure the dynamic stability of the inflation and output gap by applying the Taylor principle, then each government will voluntarily choose to implement a fiscal brake so that its debt does not explode. The fiscal union and even the fiscal stability treaties are not rationally necessary in this environment, with all governments choosing the same option, which is more stringent than the implicit fiscal rules of the European treaties.

Conversely, when monetary policy is constrained, as has been the case for a few years now and seems inevitable for many years to come due to the recent COVID-19 crisis, it becomes necessary to adapt our current institutions. Thus, I show that a fiscal union, that would take the form of bond issues on behalf of the EU, and thus define a public debt common to the countries of the Euro area, makes it possible to overcome the coordination problem induced by the need to manage differently a single public debt among those of the area. In fact, if the monetary union and risk sharing in the financial markets induce a single European inflation dynamic, then it becomes necessary for an actor to have an active budgetary policy to guarantee the stability of inflation on a balance where the fiscal theory of the price level applies. This actor could be the European Council, as it will be case for the management of the “Next Generation EU” initiative financing by a common debt.

¹⁹Indeed, exchange rate adjustments greatly reduce the impact of conventional stimuli, as the fiscal multipliers are less than one in an open economy in a currency union and greater than one in the context of a closed economy with a liquidity trap.

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Appendix

A Roots of the Terms-Of Trade Dynamic Equation

The following equation gives the deterministic part of the TOT dynamic:²⁰

$$0 = \widehat{s}_{t+1} - \underbrace{\left(\frac{1 + \beta + \frac{\kappa + (\kappa - \sigma)\bar{\alpha}(2 - \bar{\alpha})(\sigma\eta - 1)}{\sigma}}{\beta} \right)}_{=b} \widehat{s}_t + \frac{1}{\beta} \widehat{s}_{t-1}$$

The solution is such that $(1 - r_1L)(1 - r_2L)\widehat{s}_t = 0$, where the roots are

$$r_i = \frac{b \pm \sqrt{b^2 - 4/\beta}}{2} \quad i = 1, 2 \quad \text{where } 0 < r_1 < 1 < r_2$$

First, remark that

$$b = \frac{1 + \beta + \frac{\kappa + (\kappa - \sigma)\alpha(2 - \alpha)(\sigma\eta - 1)}{\sigma}}{\beta} = \frac{1 + \beta}{\beta} + \frac{\lambda(\varphi + \sigma) + \lambda\varphi\alpha(2 - \alpha)(\sigma\eta - 1)}{\beta\sigma} \equiv \frac{1 + \beta}{\beta} + \Gamma,$$

where $\Gamma > 0$ when $\eta\sigma > 1$. Therefore, assuming $\eta\sigma > 1$, we have $|r_1| < 1$ and $|r_2| > 1$ iff:

$$\begin{aligned} |r_1| < 1 & & |r_2| > 1 \\ \frac{b - \sqrt{b^2 - 4/\beta}}{2} < 1 & & \frac{b - \sqrt{b^2 + 4/\beta}}{2} > 1 \\ \frac{1 + \beta}{\beta} < \frac{1 + \beta}{\beta} + \Gamma & & \frac{1 + \beta}{\beta} < \frac{1 + \beta}{\beta} + \Gamma \end{aligned}$$

These two inequalities lead to the same restriction, which is always satisfied. Thus, given that $0 < r_1 < 1 < r_2$ when $\eta\sigma > 1$, the TOT equation has a unique saddle path.

B Equilibrium Dynamics of an SOE

The system describing the equilibrium is²¹

$$\begin{aligned} \widehat{p}_{H,t+1} &= b\widehat{p}_{H,t} - \frac{1}{\beta}\widehat{p}_{H,t-1} \\ \widehat{b}_{t-1}^r &= \frac{\beta}{1 - \beta\gamma}\widehat{b}_t^r + \frac{1}{1 - \beta\gamma}(\widehat{p}_{H,t} - \widehat{p}_{H,t-1}) + \frac{\beta^{-1} - 1}{\beta^{-1} - \gamma}\varepsilon_t^F \end{aligned}$$

²⁰See Appendix H for a description of the model parameters.

²¹See Appendix H for a description of the model parameters.

This system can be rewritten as follows:

$$\begin{aligned} \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{\beta} & b & 0 \\ \frac{1}{1-\beta\gamma} & -\frac{1}{1-\beta\gamma} & 1 \end{bmatrix} \begin{bmatrix} \widehat{p}_{H,t-1} \\ \widehat{p}_{H,t} \\ \widehat{b}_{t-1} \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{\beta}{1-\beta\gamma} \end{bmatrix} \begin{bmatrix} \widehat{p}_{H,t} \\ \widehat{p}_{H,t+1} \\ \widehat{b}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\beta^{-1}-1}{\beta^{-1}-\gamma} \end{bmatrix} \varepsilon_t^F \\ \Rightarrow \begin{bmatrix} \widehat{p}_{H,t-1} \\ \widehat{p}_{H,t} \\ \widehat{b}_{t-1} \end{bmatrix} &= \begin{bmatrix} b\beta & -\beta & 0 \\ 1 & 0 & 0 \\ \frac{1-b\beta}{1-\beta\gamma} & \frac{\beta}{1-\beta\gamma} & \frac{\beta}{1-\beta\gamma} \end{bmatrix} \begin{bmatrix} \widehat{p}_{H,t} \\ \widehat{p}_{H,t+1} \\ \widehat{b}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\beta^{-1}-1}{\beta^{-1}-\gamma} \end{bmatrix} \varepsilon_t^F \end{aligned}$$

The eigenvalues of its deterministic dynamics are

$$\lambda_{1,2} = \beta \frac{b \pm \sqrt{b^2 - 4/\beta}}{2} < 1, \quad \lambda_3 = \beta \frac{1}{1-\beta\gamma} \leq 1$$

We have $\lambda_3 > 1$ if $1/\beta - 1 < \gamma$. This restriction gives the minimum size of the debt brake.

C Dynamic System for the Two-Country Model

The dynamic system for the two-country model when each country has a specific fiscal policy is:²²

$$\begin{aligned} \widehat{\pi}_{H,t} - \kappa \widehat{y}_t &= \beta \widehat{\pi}_{H,t+1} + \lambda(1-m)\nu \widehat{s}_t \\ \widehat{y}_t \left(1 + \frac{\alpha_y}{\sigma}\right) + \frac{\alpha_\pi}{\sigma} \pi_{H,t} &= \widehat{y}_{t+1} + \frac{1}{\sigma} \widehat{\pi}_{H,t+1} \\ &\quad + \frac{1-m}{\sigma} \nu \widehat{s}_{t+1} - \frac{(1-m)}{\sigma} (\nu + \zeta) \widehat{s}_t + \frac{(1-m)}{\sigma} \alpha_\pi \widehat{s}_{t-1} - \frac{1}{\sigma} \varepsilon_t^M \\ \frac{1}{\beta^{-1}-\gamma} \widehat{b}_t^r &= \widehat{b}_t^r - \widehat{\pi}_{H,t+1} - \frac{\beta^{-1}-1}{\beta^{-1}-\gamma} \varepsilon_{t+1}^F + \frac{1}{\beta^{-1}-\gamma} \varepsilon_{t+1}^M + \frac{\gamma}{\beta^{-1}-\gamma} \varepsilon_t^M \\ &\quad + \frac{1}{\beta^{-1}-\gamma} (\alpha_\pi \widehat{\pi}_{H,t+1} + \alpha_y \widehat{y}_{t+1}) + \frac{\gamma}{\beta^{-1}-\gamma} (\alpha_\pi \widehat{\pi}_{H,t} + \alpha_y \widehat{y}_t) \\ &\quad + \frac{(1-m)\zeta}{\beta^{-1}-\gamma} \widehat{s}_{t+1} - \frac{(1-m)\gamma}{\beta^{-1}-\gamma} \widehat{s}_{t-1} - \frac{(1-m)(\alpha_\pi - \gamma\zeta)}{\beta^{-1}-\gamma} \widehat{s}_t \\ \frac{1}{\beta^{-1}-\gamma^*} \widehat{b}_t^{r*} &= \widehat{b}_t^{r*} - \widehat{\pi}_{H,t+1} - \frac{\beta^{-1}-1}{\beta^{-1}-\gamma^*} \varepsilon_{t+1}^{F*} + \frac{1}{\beta^{-1}-\gamma^*} \varepsilon_{t+1}^M + \frac{\gamma}{\beta^{-1}-\gamma^*} \varepsilon_t^M \\ &\quad + \frac{1}{\beta^{-1}-\gamma^*} (\alpha_\pi \widehat{\pi}_{H,t+1} + \alpha_y \widehat{y}_{t+1}) + \frac{\gamma^*}{\beta^{-1}-\gamma^*} (\alpha_\pi \widehat{\pi}_{H,t} + \alpha_y \widehat{y}_t) \\ &\quad - \left[1 - \frac{(1-m)\zeta}{\beta^{-1}-\gamma^*}\right] \widehat{s}_{t+1} - \frac{(1-m)\gamma^*}{\beta^{-1}-\gamma^*} \widehat{s}_{t-1} + \left[1 - \frac{(1-m)(\alpha_\pi - \gamma^*\zeta)}{\beta^{-1}-\gamma^*}\right] \widehat{s}_t \end{aligned}$$

²²See Appendix H for a description of the model parameters.

given that

$$\begin{aligned}\widehat{\pi}_{F,t+1}^* &= \widehat{s}_{t+1} - \widehat{s}_t + \widehat{\pi}_{H,t+1} \\ \widehat{r}_t &= \alpha_\pi \widehat{\pi}_{H,t} + \alpha_y \widehat{y}_t + (1-m) \left(\alpha_\pi - \alpha_y \frac{\omega + \omega^*}{\sigma} \right) \widehat{s}_t - \alpha_\pi (1-m) \widehat{s}_{t-1} + \varepsilon_t^M\end{aligned}$$

We deduce that

$$\begin{bmatrix} \widehat{\pi}_{H,t} \\ \widehat{y}_t \\ \widehat{b}_t^r \\ \widehat{b}_t^{r*} \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ m_1 & m_2 & m_3 & 0 \\ m_1^* & m_2^* & 0 & m_3^* \end{bmatrix} \begin{bmatrix} \widehat{\pi}_{H,t+1} \\ \widehat{y}_{t+1} \\ \widehat{b}_{t+1}^r \\ \widehat{b}_{t+1}^{r*} \end{bmatrix} + N \begin{bmatrix} \widehat{s}_{t+1} \\ \widehat{s}_t \\ \widehat{s}_{t-1} \end{bmatrix} + P \begin{bmatrix} \varepsilon_{t+1}^M \\ \varepsilon_t^M \\ \varepsilon_{t+1}^F \\ \varepsilon_t^F \\ \varepsilon_{t+1}^{F*} \\ \varepsilon_t^{F*} \end{bmatrix}$$

where, given that $m_3 = \frac{1}{\beta^{-1}-\gamma}$ and $m_3^* = \frac{1}{\beta^{-1}-\gamma^*}$,

$$\begin{aligned}m_1 &= 1 - \alpha_\pi m_3 - \gamma \Omega m_3 [\alpha_y + \alpha_\pi (\kappa + \beta \sigma)], & m_2 &= -m_3 (\alpha_y + \sigma \gamma (\alpha_y + \alpha_\pi \kappa) \Omega) \\ m_1^* &= 1 - \alpha_\pi m_3^* - \gamma \Omega m_3^* [\alpha_y + \alpha_\pi (\kappa + \beta \sigma)], & m_2^* &= -m_3^* (\alpha_y + \sigma \gamma (\alpha_y + \alpha_\pi \kappa) \Omega)\end{aligned}$$

D Model solution at the ZLB

As in Christiano, Eichenbaum and Rebelo (2011)²³, I assume that the economy hits the ZLB after a “shock” on the discount factor. The discount factor follows a two-state Markov process, where the low value is an absorbing state. At the time of shock, the discount factor is at its high value and is sufficient to ensure that the zero bound binds. Thereafter, the discount factor can return to its low value with a probability of $1 - p$.

- A)** If the monetary policy is expected to be active after the ZLB, the model is solved by the agents by taking $\{\widehat{\pi}_{H,t}, \widehat{y}_t\}$ as the two jump variables of the economy. This is the solution provided by Christiano, Eichenbaum and Rebelo (2011). Given the paths of $\{\widehat{\pi}_{H,t}, \widehat{y}_t\}$ that solve this system with stochastic regime switches, it is then necessary to impose stability conditions on the real debt dynamics.
- B)** If the monetary policy is expected to be passive after the ZLB, the model is solved by the agents by taking $\{\widehat{\pi}_{H,t}, \widehat{b}_t^r\}$ as the two jump variables of the economy. After solving for $\{\widehat{\pi}_{H,t}, \widehat{b}_t^r\}$, we deduce the dynamics of the Home output gap followed by the Foreign variables.

²³These authors have based their analysis on the previous works of Eggertsson and Woodford (2003), Christiano (2004), and Eggertsson (2004)

First, I show how to solve the model in this case when the ZLB is not binding.²⁴ Introducing the Taylor rule in the debt dynamics, we obtain

$$\begin{aligned}\widehat{b}_{t-1}^r &= \beta \widehat{b}_t^r + (1 - \beta \alpha_\pi) \widehat{\pi}_{H,t} + (1 - \beta) \widehat{s}_t^r - \beta \varepsilon_t^M \\ \sum_{i=0}^{\infty} \beta^i \widehat{\pi}_{H,t+i} &= \frac{1}{1 - \beta \alpha_\pi} \left[\widehat{b}_{t-1}^r - (1 - \beta) \sum_{i=0}^{\infty} \beta^i \widehat{s}_{t+i}^r + \beta \sum_{i=0}^{\infty} \beta^i \varepsilon_{t+i}^M \right],\end{aligned}\quad (18)$$

where the left-hand side provides the expected value of government revenues, while the right-hand side is the expected value of inflation. The expected value of inflation (left-hand side) is the solution of the inflation dynamic, which is given by:

$$\widehat{\pi}_{H,t+1} = \lambda_1 \widehat{\pi}_{H,t} + \frac{\sigma^{-1} \kappa}{\beta \lambda_2} \varepsilon_t^M \sum_{j=0}^{\infty} \left(\frac{\rho_M}{\lambda_2} \right)^j = \lambda_1 \widehat{\pi}_{H,t} + \frac{\sigma^{-1} \kappa}{\beta (\lambda_2 - \rho_M)} \varepsilon_t^M,$$

where $|\lambda_1| < 1 < |\lambda_2|$ are the eigenvalues of A and using $\varepsilon_{t+1} = \rho_M \varepsilon_t$. The present value of the expected inflation is then given by

$$L^{-j} \widehat{\pi}_{H,t} = \lambda_1^j \widehat{\pi}_{H,t} + (\lambda_1^{j-1} + \lambda_1^{j-2} L^{-1} + \dots + \lambda_1 L^{-(j-2)} + L^{-(j-1)}) \frac{\sigma^{-1} \kappa}{\beta (\lambda_2 - \rho_M)} \varepsilon_t^M$$

Given that $L^{-i} \varepsilon_t = \rho_M^i \varepsilon_t$, the expected value of inflation is thus:

$$\sum_{i=0}^{\infty} \beta^i \widehat{\pi}_{H,t+i} = \sum_{i=0}^{\infty} \beta^i L^{-i} \widehat{\pi}_{H,t} = \frac{1}{1 - \beta \lambda_1} \widehat{\pi}_{H,t} + \frac{\sigma^{-1} \kappa}{\beta (\lambda_2 - \rho_M)} \frac{1}{1 - \beta \lambda_1 \rho_M} \varepsilon_t^M$$

Integrating this result in Equation (18), the equilibrium is defined by

$$\begin{aligned}& \frac{1}{1 - \beta \lambda_1} \widehat{\pi}_{H,t} + \frac{\sigma^{-1} \kappa}{\beta (\lambda_2 - \rho_M)} \frac{1}{1 - \beta \lambda_1 \rho_M} \varepsilon_t^M \\ &= \frac{1}{1 - \beta \alpha_\pi} \left[\widehat{b}_{t-1}^r - (1 - \beta) \sum_{i=0}^{\infty} \beta^i \widehat{s}_{t+i}^r + \beta \sum_{i=0}^{\infty} \beta^i \varepsilon_{t+i}^M \right] \\ &= \frac{1}{1 - \beta \alpha_\pi} \left[\widehat{b}_{t-1}^r - \frac{1 - \beta}{1 - \beta L^{-1}} \widehat{s}_t^r + \frac{\beta}{1 - \beta \rho_M} \varepsilon_t^M \right] \\ \widehat{\pi}_{H,t} &= \frac{1 - \beta \lambda_1}{1 - \beta \alpha_\pi} \left[\widehat{b}_{t-1}^r - \frac{1 - \beta}{1 - \beta L^{-1}} \widehat{s}_t^r \right] \\ &+ (1 - \beta \lambda_1) \left(\frac{\beta}{(1 - \beta \alpha_\pi)(1 - \beta \rho_M)} - \frac{\sigma^{-1} \kappa}{\beta (1 - \beta \lambda_1 \rho_M)(\lambda_2 - \rho_M)} \right) \varepsilon_t^M\end{aligned}$$

If the fiscal policy is active, as for instance, $\gamma = 0$, then $\widehat{s}_t^r = \varepsilon_t^F$ and thus

$$\begin{aligned}\widehat{\pi}_{H,t} &= \frac{1 - \beta \lambda_1}{1 - \beta \alpha_\pi} \left[\widehat{b}_{t-1}^r - \frac{1 - \beta}{1 - \beta \rho_F} \varepsilon_t^F \right] \\ &+ (1 - \beta \lambda_1) \left(\frac{\beta}{(1 - \beta \alpha_\pi)(1 - \beta \rho_M)} - \frac{\sigma^{-1} \kappa}{\beta (1 - \beta \lambda_1 \rho_M)(\lambda_2 - \rho_M)} \right) \varepsilon_t^M\end{aligned}$$

²⁴In the following, I always assume that $\alpha_y = 0$ without loss of generality, but with significant gains for the exposal simplicity.

We deduce the equilibrium path of the output gap using the Phillips curve

$$\widehat{y}_t = \frac{1}{\kappa}(1 - \beta L^{-1})\widehat{\pi}_{H,t},$$

where $t_t = t_t^* = 0$ leading to $\widehat{s}_t = 0, \forall t$, for simplicity. The dynamics of Foreign aggregates y_t^* and $\widehat{\pi}_{F,t}$ are deduced from Equations (6) and $\widehat{\pi}_{F,t} \equiv \widehat{\pi}_{H,t}$, given that $t_t = t_t^* = 0$ implying $\widehat{s}_t = 0, \forall t$. Therefore, γ^* must be larger than $\beta^{-1} - 1$ (fiscal brake) in order to ensure the stability of the Foreign real debt, given the stationarity of the others aggregates $\{\widehat{y}, \widehat{y}_t^*, \widehat{\pi}_{H,t}, \widehat{\pi}_{F,t}, \widehat{b}_t^r\}$.

Now, assuming that the ZLB binds at the initial period. The debt dynamic is then

$$\begin{aligned} \widehat{b}_{t-1}^r &= \beta \widetilde{b}_t^r + \widetilde{\pi}_{H,t} + (1 - \beta)\widetilde{s}_t^r \\ \widehat{b}_{t-1}^r &= \beta \left[\begin{aligned} &\beta p \widetilde{b}_{t+1}^r + \beta(1 - p)\widehat{b}_{t+1}^r + p\widetilde{\pi}_{H,t+1} + (1 - \beta\alpha_\pi)(1 - p)\widehat{\pi}_{H,t+1} \\ &+ (1 - \beta)p\widetilde{s}_{t+1}^r + (1 - \beta)(1 - p)\widetilde{s}_{t+1}^r - (1 - p)\beta\varepsilon_{t+1}^M \end{aligned} \right] \\ &\quad + \widetilde{\pi}_{H,t} + (1 - \beta)\widetilde{s}_t^r \\ &= \beta^2 p \widetilde{b}_{t+1}^r + \beta^2(1 - p)\widehat{b}_{t+1}^r + \sum_{i=0}^1 (\beta p)^i \widetilde{\pi}_{H,t+i} + (1 - \beta) \sum_{i=0}^1 (1 - p)^i \widetilde{s}_{t+i}^r \\ &\quad + (1 - \beta)p\widetilde{s}_{t+1}^r + (1 - p) \left[(1 - \beta\alpha_\pi)\widehat{\pi}_{H,t+1} - \beta\varepsilon_{t+1}^M \right] \\ &= \beta^2 p \left[\begin{aligned} &\beta p \widetilde{b}_{t+2}^r + \beta(1 - p)\widehat{b}_{t+2}^r + p\widetilde{\pi}_{H,t+2} + (1 - \beta\alpha_\pi)(1 - p)\widehat{\pi}_{H,t+2} \\ &+ (1 - \beta)p\widetilde{s}_{t+2}^r + (1 - \beta)(1 - p)\widetilde{s}_{t+2}^r - (1 - p)\beta\varepsilon_{t+2}^M \end{aligned} \right] \\ &\quad + \beta^2(1 - p) \left[\beta \widehat{b}_{t+2}^r + (1 - \beta\alpha_\pi)\widehat{\pi}_{H,t+2} + (1 - \beta)\widetilde{s}_{t+2}^r - \beta\varepsilon_{t+2}^M \right] \\ &\quad + \sum_{i=0}^1 (\beta p)^i \widetilde{\pi}_{H,t+i} + (1 - \beta) \sum_{i=0}^1 (1 - p)^i \widetilde{s}_{t+i}^r \\ &\quad + (1 - \beta)p\widetilde{s}_{t+1}^r + (1 - p) \left[(1 - \beta\alpha_\pi)\widehat{\pi}_{H,t+1} - \beta\varepsilon_{t+1}^M \right] \\ &= \sum_{i=0}^{\infty} (\beta p)^i \widetilde{\pi}_{H,t+i} + (1 - \beta)\beta p \sum_{i=0}^{\infty} (\beta p)^i \widetilde{s}_{t+1+i}^r + (1 - \beta)\widehat{s}_{t+i} \\ &\quad + (1 - p)\beta^2 \sum_{i=0}^{\infty} (\beta p)^i \widehat{b}_{t+1+i}^r \end{aligned}$$

This gives the value of expected inflation:

$$\sum_{i=0}^{\infty} (\beta p)^i \widetilde{\pi}_{H,t+i} = \widehat{b}_{t-1}^r - \frac{(1 - \beta)\beta p}{1 - \beta p L^{-1}} \widetilde{s}_{t+1}^r + (1 - \beta)\widetilde{s}_t^r - \frac{(1 - p)\beta^2}{1 - \beta p L^{-1}} \widehat{b}_{t+1}^r$$

After the shock, the dynamics of inflation must also consider the stochastic end of

the ZLB:

$$\begin{aligned}
\text{No ZLB:} \quad & \widehat{\pi}_{H,t+2} - \frac{1 + \beta + \sigma^{-1}\kappa}{\beta} \widehat{\pi}_{H,t+1} + \frac{1 + \alpha_\pi \sigma^{-1}\kappa}{\beta} \widehat{\pi}_{H,t} = -\frac{\sigma^{-1}\kappa}{\beta} \varepsilon_t^M \\
\text{with ZLB:} \quad & p^2 \widetilde{\pi}_{H,t+2} + (p(1-p) + (1-p)^2) \widehat{\pi}_{H,t+2} \\
& - \frac{1 + \beta + \sigma^{-1}\kappa}{\beta} p \widetilde{\pi}_{H,t+1} - \frac{1 + \beta + \sigma^{-1}\kappa}{\beta} (1-p) \widehat{\pi}_{H,t+1} + \frac{1}{\beta} \widetilde{\pi}_{H,t} = -\frac{\sigma^{-1}\kappa}{\beta} \varepsilon_t^M \\
\Leftrightarrow \quad & \widetilde{\pi}_{H,t+2} - \frac{1 + \beta + \sigma^{-1}\kappa}{\beta p} \widetilde{\pi}_{H,t+1} + \frac{1}{\beta p^2} \widetilde{\pi}_{H,t} \\
& = \frac{1-p}{p^2} \underbrace{\left[\frac{1 + \alpha_\pi \sigma^{-1}\kappa}{\beta} \widehat{\pi}_{H,t} + \frac{\sigma^{-1}\kappa}{\beta} \varepsilon_t^M \right]}_{x_t}
\end{aligned}$$

The two roots of the second-order differential equation for inflation are $\lambda_i = \frac{1}{2} \frac{1}{p\beta} (a \pm \sqrt{a^2 - 4\beta})$ with $a = 1 + \beta + \sigma^{-1}$, with $|\lambda_1| < 1 < |\lambda_2|$, for $p \in]0.5; 1[$. Using $(L^{-1} - \lambda_1)(L^{-1} - \lambda_2) \widetilde{\pi}_t = x_t$, we deduce the solution of the inflation dynamic when the ZLB binds:

$$\widetilde{\pi}_{H,t+1} = \lambda_1 \widetilde{\pi}_{H,t} - \lambda_1 \beta p \sum_{i=0}^{\infty} \lambda_2^{-i} x_{t+i}$$

The expected value of inflation is then

$$\sum_{i=0}^{\infty} (\beta p)^i \widetilde{\pi}_{H,t+i} = \frac{1}{1 - \beta p \lambda_1} \widetilde{\pi}_{H,t} - \frac{\lambda_1 \beta p}{1 - \beta p \lambda_1 L^{-1}} x_t$$

After equalizing the two expressions of the expected inflation, I obtain:

$$\frac{1}{1 - \beta p \lambda_1} \widetilde{\pi}_{H,t} - \frac{\lambda_1 \beta p}{1 - \beta p \lambda_1 L^{-1}} x_t = \widehat{b}_{t-1}^r - \frac{(1-\beta)\beta p}{1 - \beta p L^{-1}} \widehat{s}_{t+1}^r + (1-\beta) \widehat{s}_t^r - \frac{(1-p)\beta^2}{1 - \beta p L^{-1}} \widehat{b}_{t+1}^r$$

When the fiscal policy is active, for instance, $\gamma = 0$, the equilibrium inflation is

$$\widetilde{\pi}_{H,t} = (1 - \beta p \lambda_1) \left[\widehat{b}_{t-1}^r - \frac{(1-\beta)(2\beta p \rho - 1)}{1 - \beta p \rho} \varepsilon_t^F - \frac{(1-p)\beta^2}{1 - \beta p L^{-1}} \widehat{b}_{t+1}^r + \frac{\lambda_1 \beta p}{1 - \beta p \lambda_1 L^{-1}} x_t \right],$$

where the sign of the impact of a redistributive shock ε_t^F depends on the inequality $\beta p \rho \lesseqgtr 1/2$. As for the previous case, the equilibrium path of the output gap is deduced from the Phillips curve

$$\widehat{y}_t = \frac{1}{\kappa} (1 - \beta L^{-1}) \widehat{\pi}_{H,t},$$

where $t_t = t_t^* = 0$ leading to $\widehat{s}_t = 0, \forall t$, for simplicity. The dynamics of Foreign aggregates y_t^* and $\widehat{\pi}_{F,t}$ are deduced from Equations (6) and $\widehat{\pi}_{F,t} \equiv \widehat{\pi}_{H,t}$, given that $t_t = t_t^* = 0$ implying $\widehat{s}_t = 0, \forall t$.

E Fiscal Devaluation and Stability of the Euro Area

When the monetary policy is constrained to be passive ($\widehat{r}_t = 0$), I show in this appendix that it is possible to stabilize inflation and output via the introduction of policy rules on payroll tax. This policy allows governments to respect their European commitments by implementing a budgetary brake.

I assume the rates of the payroll tax change as follows:

$$\begin{aligned} t_t &= \nu_y \widehat{y}_t + \nu_\pi \widehat{\pi}_{H,t} \\ t_t^* &= \nu_y^* \widehat{y}_t^* + \nu_\pi^* \widehat{\pi}_{F,t} \end{aligned}$$

Therefore, the dynamics of the inflation and output gap are given by, knowing the TOT dynamics:

$$\begin{aligned} (1 - \lambda \nu_\pi) \widehat{\pi}_{H,t} &= \beta \widehat{\pi}_{H,t+1} + (\kappa + \lambda \nu_y) \widehat{y}_t - \lambda (\alpha m + \omega - 1) \widehat{s}_t \\ \left(1 + \frac{\alpha_y}{\sigma}\right) \widehat{y}_t &= \widehat{y}_{t+1} + \frac{1}{\sigma} \widehat{\pi}_{H,t+1} - \frac{\alpha_\pi}{\sigma} \widehat{\pi}_{H,t} + \frac{1 - \bar{\alpha} m - \omega}{\sigma} \widehat{s}_{t+1} + \frac{\alpha_\pi (1 - m)}{\sigma} \widehat{s}_{t-1} \\ &\quad - \frac{1}{\sigma} \left[1 - \bar{\alpha} m - \omega + (1 - m) \left(\alpha_\pi - \alpha_y \frac{\omega + \omega^*}{\sigma} \right) \right] \widehat{s}_t - \frac{1}{\sigma} \varepsilon_t^M \\ \widehat{s}_{t+1} &= \left(b - \frac{\lambda}{\beta} \left(\nu_\pi^* - \nu_y^* \frac{\omega + \omega^*}{\sigma} \right) \right) \widehat{s}_t - \frac{1}{\beta} (1 - \nu_\pi^*) \widehat{s}_{t-1} \\ &\quad - \frac{\lambda}{\beta} \left[(\nu_\pi^* - \nu_\pi) \widehat{\pi}_{H,t} + (\nu_y^* - \nu_y) \widehat{y}_t \right] \end{aligned}$$

This system can be rewritten as $Z_t = MZ_{t+1} + R\varepsilon_t^M$, where $Z_t = [\widehat{\pi}_{H,t}, \widehat{y}_t, \widehat{s}_t, \widehat{s}_{t-1}]$. This dynamic system will be stable if and only if three of the four eigenvalues of M are inside the unit circle. These three dimensions correspond to the two jump variables that are inflation and the output gap and to the “forward” component of the real exchange rate. The last eigenvalue must be outside the unit circle because it corresponds to the “backward” component of the real exchange rate.

The eigenvalues of the matrix M , for different values of the parameters $\{\nu_y, \nu_\pi, \nu_y^*, \text{ and } \nu_\pi^*\}$, are presented in Figure 1.²⁵ This figure only reports cases where the tax rates are countercyclical, that is, cases where governments implement a fiscal devaluation when the country’s economic activity is poor. This figure shows two results. The first is that it is more likely to obtain a stable dynamic system when the tax rates depend simultaneously on inflation and the output gap. Indeed, panel (a) of Figure 1 shows that two eigenvalues

²⁵For this numerical illustration, the model parameters are the following: $\beta = 0.98$, $m = 0.5$, $\sigma = 2$, $\alpha = 0.5$, $\eta = 2$, $\theta = 0.8$, $\varphi = 3$.

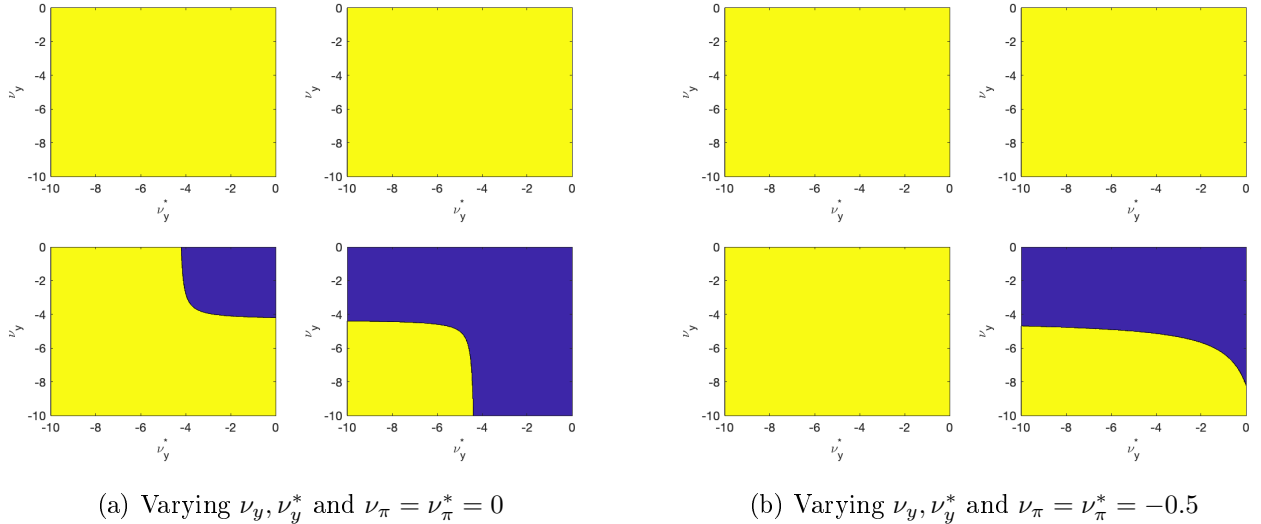


Figure 1: **Stability of the Euro Area with Fiscal Devaluation Policies.** The four graphics on each panel ((a) and (b)) represent the regions, where an eigenvalue is larger (purple) or lower (yellow) than one. In panel (a), $\nu_\pi = \nu_\pi^* = 0$, whereas in panel (b) $\nu_\pi = \nu_\pi^* = 1.5$. In these two cases, we have $\nu_y \in [-10, 0]$ and $\nu_y^* \in [-10, 0]$, respectively.

can be simultaneously greater than one, if $\nu_\pi = \nu_\pi^* = 0$, while in panel (b), where both $\nu_\pi < 0$ and $\nu_\pi^* < 0$, only one eigenvalue can be greater than unity. However, even in this case, the uniqueness of the dynamic path is not always guaranteed. The stable path is unique only when ν_y is not less than a threshold value, here around -4 (see panel (b) of the Figure 1). Pour l'ensemble des paramètres permettant à cette politique de dévaluation fiscale "automatique" de garantir la stabilité dynamique, il serait alors intéressant d'évaluer son impact sur le welfare, comparativement au cas où la stabilisation est obtenue via une politique budgétaire active implémentée sur l'une des dette de la zone Euro. This study is left to future research.

F Summary of the Model's Equations

This appendix provides the equations used to define the model equilibrium. The individual behaviors allowing us to derive these reduced forms are explained in Appendix G. Appendix H provides a complete description of the model parameters.

F.1 Households

Consumption demand

$$\widehat{c}_t = \mathbb{E}_t[\widehat{c}_{t+1}] - \frac{1}{\sigma} (\widehat{r}_t - \mathbb{E}_t[\widehat{\pi}_{t+1}]) \quad (19)$$

$$\widehat{c}_t^* = \mathbb{E}_t[\widehat{c}_{t+1}^*] - \frac{1}{\sigma} (\widehat{r}_t - \mathbb{E}_t[\widehat{\pi}_{t+1}^*]) \quad (20)$$

Labor supply

$$\sigma \widehat{c}_t + \varphi \widehat{n}_t = \widehat{w}_t - \widehat{p}_t \quad (21)$$

$$\sigma \widehat{c}_t^* + \varphi \widehat{n}_t^* = \widehat{w}_t^* - \widehat{p}_t^* \quad (22)$$

F.2 Prices, exchange rates, and international risk sharing

Terms-of-Trade.

$$\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}} \Rightarrow \widehat{s}_t = \widehat{p}_{F,t} - \widehat{p}_{H,t}$$

Price indexes.

$$P_t = ((1 - \alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta})^{\frac{1}{1-\eta}} \Rightarrow \frac{P_t}{P_{H,t}} = ((1 - \alpha) + \alpha \mathcal{S}_t^{1-\eta})^{\frac{1}{1-\eta}}$$

$$P_t^* = ((1 - \alpha^*)(P_{F,t}^*)^{1-\eta} + \alpha^*(P_{H,t}^*)^{1-\eta})^{\frac{1}{1-\eta}} \Rightarrow \frac{P_t^*}{P_{H,t}^*} = ((1 - \alpha^*)\mathcal{S}_t^{1-\eta} + \alpha^*)^{\frac{1}{1-\eta}}$$

implying that

$$\widehat{p}_t - \widehat{p}_{H,t} = \alpha \widehat{s}_t = \bar{\alpha}(1 - m)\widehat{s}_t$$

$$\widehat{p}_t^* - \widehat{p}_{H,t}^* = (1 - \alpha^*)\widehat{s}_t = (1 - \bar{\alpha}m)\widehat{s}_t$$

Nominal exchange rate.

$$P_{F,t} = \mathcal{E}_t P_{F,t}^* \Rightarrow \widehat{p}_{F,t} = \widehat{e}_t + \widehat{p}_{F,t}^*$$

Real exchange rate.

$$Q_t = \frac{\mathcal{E}_t P_t^*}{P_t} \Rightarrow \widehat{q}_t = \widehat{e}_t + \widehat{p}_t^* - \widehat{p}_t$$

$$\widehat{q}_t = \widehat{e}_t + \widehat{p}_{H,t}^* + (1 - \alpha^*)\widehat{s}_t - \widehat{p}_{H,t} - \alpha \widehat{s}_t$$

$$\widehat{q}_t = \widehat{e}_t + (1 - \alpha^* - \alpha)\widehat{s}_t + \widehat{p}_{H,t}^* - \widehat{p}_{H,t}$$

In the Euro Area, we have $\widehat{e}_t = 0, \forall t$ and thus

$$\widehat{q}_t = (1 - \alpha^* - \alpha) \widehat{s}_t = (1 - \bar{\alpha}m - \bar{\alpha}(1 - m)) \widehat{s}_t = (1 - \bar{\alpha}) \widehat{s}_t$$

CPI Inflation .

$$\begin{aligned} \widehat{\pi}_t &= \widehat{\pi}_{H,t} + \bar{\alpha}(1 - m)(\widehat{s}_t - \widehat{s}_{t-1}) \\ \widehat{\pi}_t^* &= \widehat{\pi}_{F,t}^* - \bar{\alpha}m(\widehat{s}_t - \widehat{s}_{t-1}) \end{aligned}$$

Financial markets.

$$\left. \begin{aligned} \beta \frac{(C_{t+1}^j)^\sigma}{(C_t^j)^\sigma} &= \frac{P_{t+1}}{P_t} Q(z_{t+1}|z_t) \\ \beta \frac{(C_{t+1}^{j*})^\sigma}{(C_t^{j*})^\sigma} &= \frac{Q_{t+1}}{Q_t} \frac{P_{t+1}}{P_t} Q(z_{t+1}|z_t) \end{aligned} \right\} \Rightarrow C_t = \frac{m}{1 - m} \vartheta Q_t^{\frac{1}{\sigma}} C_t^*, \quad (23)$$

where $\vartheta = Q_t^{-\frac{1}{\sigma}} \frac{C_0}{C_0^*}$ and with $C_t = \int_0^m C_t^j dj = m C_t^j$ and $C_t^* = \int_m^1 C_t^{j*} dj = (1 - m) C_t^{j*}$. This leads to

$$\widehat{c}_t = \frac{1}{\sigma} \widehat{q}_t + \widehat{c}_t^* \quad (24)$$

F.3 Price-setting rule

The Phillips curves are

$$\widehat{\pi}_{H,t} = \beta \mathbb{E}_t [\widehat{\pi}_{H,t+1}] + \kappa \widehat{y}_t - \lambda(\alpha m + \omega - 1) \widehat{s}_t \quad (25)$$

$$\widehat{\pi}_{F,t}^* = \beta \mathbb{E}_t [\widehat{\pi}_{F,t+1}^*] + \kappa \widehat{y}_t^* + \lambda(\omega^* - \alpha m) \widehat{s}_t, \quad (26)$$

which are the Home and Foreign New Phillips curves, where $\kappa = \lambda(\varphi + \sigma)$.

F.4 Resource constraints

$$\begin{aligned} Y_t &= C_{H,t} + C_{H,t}^* = (1 - \alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha^* \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^* \\ Y_t^* &= C_{F,t} + C_{F,t}^* = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t + (1 - \alpha^*) \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\eta} C_t^* \end{aligned}$$

Using (59) $\Leftrightarrow \frac{1-m}{m} \frac{1}{\vartheta} Q_t^{-\frac{1}{\sigma}} C_t = C_t^*$, we have

$$Y_t = \left[(1 - \alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} + \alpha^* \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} \frac{1 - m}{m} \frac{1}{\vartheta} Q_t^{-\frac{1}{\sigma}} \right] C_t,$$

where

$$\begin{aligned}\mathcal{Q}_t &= \frac{\mathcal{E}_t P_t^*}{P_t} = \frac{((1 - \alpha^*)\mathcal{S}_t^{1-\eta} + \alpha^*)^{\frac{1}{1-\eta}}}{((1 - \alpha) + \alpha\mathcal{S}_t^{1-\eta})^{\frac{1}{1-\eta}}} \\ \frac{P_{H,t}}{P_t} &= ((1 - \alpha) + \alpha\mathcal{S}_t^{1-\eta})^{-\frac{1}{1-\eta}} \\ \frac{P_{H,t}^*}{P_t^*} &= ((1 - \alpha^*)\mathcal{S}_t^{1-\eta} + \alpha^*)^{-\frac{1}{1-\eta}}\end{aligned}$$

showing that $Y_t = \mathcal{Y}(\mathcal{S}_t, C_t)$. Log-linearizing this equation leads to

$$\hat{y}_t = \hat{c}_t + \frac{\omega + \bar{\alpha} - 1}{\sigma} \hat{s}_t \quad (27)$$

$$\hat{y}_t^* = \hat{c}_t^* - \frac{\omega^*}{\sigma} \hat{s}_t, \quad (28)$$

where $\omega = 1 - \bar{\alpha}m + (1 - m)\bar{\alpha}(2 - \bar{\alpha})(\sigma\eta - 1) > 0$ and $\omega^* = \bar{\alpha}m + m\bar{\alpha}(2 - \bar{\alpha})(\sigma\eta - 1) > 0$. Remark that (72) and (73) can be rewritten as follows:

$$\hat{y}_t = \hat{c}_t + \frac{\omega + \bar{\alpha} - 1}{\sigma} (\hat{p}_{F,t} - \hat{p}_{H,t}) \quad (29)$$

$$\hat{y}_t^* = \hat{c}_t^* - \frac{\omega^*}{\sigma} (\hat{p}_{F,t} - \hat{p}_{H,t}) \quad (30)$$

Therefore, the IRS condition (60) can be rewritten as follows:

$$\hat{y}_t - \hat{y}_t^* = \frac{\omega + \omega^*}{\sigma} (\hat{p}_{F,t} - \hat{p}_{H,t}) = \frac{\omega + \omega^*}{\sigma} \hat{s}_t$$

G Individual Behaviors

G.1 Households

The quantities and prices in country F are denoted by an asterisk and those in country H are without asterisks. The representative households' preferences are given by

$$\mathcal{U}^j = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\frac{1}{1 - \sigma} (C_t^j)^{1-\sigma} - \frac{1}{1 + \varphi} (N_t^j)^{1+\varphi} \right) \right] \quad (31)$$

$$\mathcal{U}^{j*} = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \left(\frac{1}{1 - \sigma} (C_t^{j*})^{1-\sigma} - \frac{1}{1 + \varphi} (N_t^{j*})^{1+\varphi} \right) \right], \quad (32)$$

where $\beta \in (0, 1)$ is the subjective discount factor. $N_t = \int_0^m N_t^j dj = mN_t^j$ and $N_t^* = \int_m^1 N_t^{j*}(i) di = (1 - m)N_t^{j*}$ are the total hours worked in countries H and F , respectively.

C_t^j and C_t^{j*} denote the consumption index of a household in countries H and F , and are defined as follows:

$$C_t^j = \left((1 - \alpha)^{-\frac{1}{\eta}} (C_{H,t}^j)^{\frac{\eta-1}{\eta}} + \alpha^{-\frac{1}{\eta}} (C_{F,t}^j)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \quad (33)$$

$$C_t^{j*} = \left((1 - \alpha^*)^{-\frac{1}{\eta}} (C_{F,t}^{j*})^{\frac{\eta-1}{\eta}} + (\alpha^*)^{-\frac{1}{\eta}} (C_{H,t}^{j*})^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \quad (34)$$

where the index of the consumption of goods produced in country H , and the index of the consumption of goods produced in country F are

$$C_{H,t}^j = \left(\left(\frac{1}{m} \right)^{\frac{1}{\varepsilon}} \int_0^m C_t^j(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (35)$$

$$C_{F,t}^{j*} = \left(\left(\frac{1}{1-m} \right)^{\frac{1}{\varepsilon}} \int_m^1 C_t^{j*}(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (36)$$

For simplicity, assume that the number of goods produced in each “country” is equal to the number of people living in each “country” (one skill-one good). The elasticity of substitution between the differentiated goods produced in each country satisfies $\varepsilon > 1$. The trade openness of each “country” is measured by $\alpha, \alpha^* \in [0, 1]$. The elasticity of substitution between domestic and foreign goods from the viewpoint of the domestic consumer is given by $\eta \in (0, 1)$.

The maximizations of (31) and (32) are subject to sequences of budget constraints of the following form:

$$\begin{aligned} & \int_0^m P_t(h) C_t^j(h) dh + \int_0^m P_t(f) C_t^j(f) df + \mathbb{E}_t[Q_{t,t+1} A_t^j] + \frac{1}{R_t} B_t^j \\ &= A_{t-1}^j + B_{t-1}^j + W_t N_t^j + T_t^j \end{aligned} \quad (37)$$

$$\begin{aligned} & \int_m^1 P_t^*(h) C_t^{j*}(h) dh + \int_m^1 P_t^*(f) C_t^{j*}(f) df + \mathbb{E}_t[Q_{t,t+1} A_t^{j*}] + \frac{1}{R_t} B_t^{j*} \\ &= A_{t-1}^{j*} + B_{t-1}^{j*} + W_t^* N_t^* + T_t^{j*}, \end{aligned} \quad (38)$$

where $P_t(h)$ and $P_t(f)$ are the prices of goods produced in country H and country F , respectively. A_t denotes the nominal payoff during period t of the portfolio (Arrow securities) bought during period $t - 1$ and $Q_{t,t+1}$ the prices of these contingent assets (i.e., the price in t to have 1 € in period $t + 1$). B_t denotes the nominal payoff during period t of the government bonds bought during period $t - 1$ and $1/R_t$ the nominal price at t of an asset that pays 1 € during period $t + 1$. The asset price ($Q_{t,t+1}$) and bond price $1/R_t$ are the same in both countries because all exchanges take place in the Euro area. W_t and T_t are the nominal wage and lump-sum transfers (taxes), respectively. The transfers include

the sum of dividends redistributed by the firm to households and the lump-sum tax net of government transfers.

Within each category of goods, the optimal choices are given by

$$C_t^j(h) = \frac{1}{m} \left(\frac{P_t(h)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}^j, \quad C_t^j(f) = \frac{1}{1-m} \left(\frac{P_t(f)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t}^j \quad (39)$$

$$C_t^{j*}(h) = \frac{1}{m} \left(\frac{P_t^*(h)}{P_{H,t}^*} \right)^{-\varepsilon} C_{H,t}^{j*}, \quad C_t^{j*}(f) = \frac{1}{1-m} \left(\frac{P_t^*(f)}{P_{F,t}^*} \right)^{-\varepsilon} C_{F,t}^{j*}, \quad (40)$$

where $P_{H,t} = \left(\frac{1}{m} \int_0^m P_t(h)^{1-\varepsilon} dh \right)^{\frac{1}{1-\varepsilon}}$ and $P_{F,t} = \left(\frac{1}{1-m} \int_m^1 P_t(f)^{1-\varepsilon} df \right)^{\frac{1}{1-\varepsilon}}$ denote the PPI. The price indices of imported goods, $P_{H,t}^*$ and $P_{F,t}^*$, are defined analogously to $P_{H,t}$ and $P_{F,t}$. The optimal allocations of expenditure between domestic and imported goods are

$$C_{H,t}^j = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t^j, \quad C_{F,t}^j = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t^j \quad (41)$$

$$C_{H,t}^{j*} = \alpha^* \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\eta} C_t^{j*}, \quad C_{F,t}^{j*} = (1-\alpha^*) \left(\frac{P_{F,t}^*}{P_t^*} \right)^{-\eta} C_t^{j*}, \quad (42)$$

where the CPIs are given by

$$P_t = \left((1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right)^{\frac{1}{1-\eta}} \quad (43)$$

$$P_t^* = \left((1-\alpha^*)P_{H,t}^{*\,1-\eta} + \alpha^* P_{F,t}^{*\,1-\eta} \right)^{\frac{1}{1-\eta}} \quad (44)$$

Equations (39) and (40) imply that the total expenditure in country H is the sum of $\int_0^m P_t(h)C_t^j(h)dh = P_{H,t}C_{H,t}^j$ and $\int_m^1 P_t(f)C_t^j(f)df = P_{F,t}C_{F,t}^j$. Similarly, the total expenditure in country F is the sum of $\int_m^1 P_t^*(h)C_t^{j*}(h)dh = P_{H,t}^*C_{H,t}^{j*}$ and $\int_0^m P_t^*(f)C_t^{j*}(f)df = P_{F,t}^*C_{F,t}^{j*}$. Then, from Equations (41) and (42), the following relations hold: $P_{H,t}C_{H,t}^j + P_{F,t}C_{F,t}^j = P_tC_t^j$ and $P_{H,t}^*C_{H,t}^{j*} + P_{F,t}^*C_{F,t}^{j*} = P_t^*C_t^{j*}$. These expressions can be used to rewrite Equations (37) and (38) as

$$P_tC_t^j + \mathbb{E}_t[Q_{t,t+1}A_t^j] + \frac{1}{R_t}B_t^j = A_{t-1}^j + B_{t-1}^j + W_tN_t^j + TR_t \quad (45)$$

$$P_t^*C_t^{j*} + \mathbb{E}_t[Q_{t,t+1}A_t^{j*}] + \frac{1}{R_t}B_t^{j*} = A_{t-1}^{j*} + B_{t-1}^{j*} + W_t^*N_t^{j*} + TR_t^{j*} \quad (46)$$

Using the definition of the aggregates $C_t = mC_{H,t}^j$, $C_t^* = (1-m)C_{H,t}^{j*}$, $N_t = mN_t^j$, and $N_t^* = (1-m)N_t^{j*}$, the first-order conditions of the maximizations of (31) subject to either

(45) or (32) and subject to (46) are

$$\beta \frac{C_{t+1}^\sigma P_t}{C_t^\sigma P_{t+1}} = Q(z_{t+1}|z_t) \quad \forall z \quad (47)$$

$$\beta \mathbb{E}_t \left[\frac{C_{t+1}^\sigma P_t}{C_t^\sigma P_{t+1}} \right] = \frac{1}{R_t} \quad (48)$$

$$m^{-(\sigma+\varphi)} C_t^\sigma N_t^\varphi = \frac{W_t}{P_t} \quad (49)$$

$$\beta \frac{(C_{t+1}^*)^\sigma P_t^*}{(C_t^*)^\sigma P_{t+1}^*} = Q(z_{t+1}|z_t) \quad \forall z \quad (50)$$

$$\beta \mathbb{E}_t \left[\frac{(C_{t+1}^*)^\sigma P_t^*}{(C_t^*)^\sigma P_{t+1}^*} \right] = \frac{1}{R_t} \quad (51)$$

$$(1-m)^{-(\sigma+\varphi)} (C_t^*)^\sigma (N_t^*)^\varphi = \frac{W_t^*}{P_t^*}, \quad (52)$$

where $Q_{t,t+1}$ is the vector of $Q(z_{t+1}|z_t)$. Equations (47) and (50) imply that

$$\beta \mathbb{E}_t \left[\frac{C_{t+1}^\sigma P_t}{C_t^\sigma P_{t+1}} \right] = \mathbb{E}_t [Q(z_{t+1}|z_t)] \equiv Q_t \quad (53)$$

$$\beta \mathbb{E}_t \left[\frac{(C_{t+1}^*)^\sigma P_t^*}{(C_t^*)^\sigma P_{t+1}^*} \right] = \mathbb{E}_t [Q(z_{t+1}|z_t)] \equiv Q_t \quad (54)$$

The short-term nominal interest rate, R_t , which is also the ECB's policy instrument, is linked to the nominal bond price, namely $1/R_t$. The no-arbitrage condition leads to $1/R_t = Q_t$.

Log-linearizing around the steady state for the symmetric case (i.e., where $\bar{\alpha} = \bar{\alpha}^*$), Equations (53) and (54), and using the expression for R_t , I find that Home aggregate consumption is

$$\widehat{c}_t = \mathbb{E}_t[\widehat{c}_{t+1}] - \frac{1}{\sigma} (r_t - \mathbb{E}_t[\pi_{t+1}] + \log(\beta)) \quad (55)$$

$$\widehat{c}_t^* = \mathbb{E}_t[\widehat{c}_{t+1}^*] - \frac{1}{\sigma} (r_t - \mathbb{E}_t[\pi_{t+1}^*] + \log(\beta)), \quad (56)$$

where $\widehat{x}_t = \log(X_t/X)$, $\forall x_t$ with X the steady-state value of X_t and $x_t = \log(X_t)$. Therefore, $\pi_t = p_t - p_{t-1}$ ($\pi_t^* = p_t^* - p_{t-1}^*$) is the Home (Foreign) CPI inflation.

G.2 PPI and CPI inflation rates, real exchange rate, TOT, and international risk sharing

To determine the link between CPI and PPI inflation rates, I consider the price of goods produced in country F in terms of the price of goods produced in country H . The TOT

are defined by $\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}}$. The log-linearized CPI inflation rate is

$$\widehat{\pi}_t = \widehat{\pi}_{H,t} + \bar{\alpha}(1-m)(\widehat{s}_t - \widehat{s}_{t-1}) \quad ; \quad \widehat{\pi}_t^* = \widehat{\pi}_{F,t}^* - \bar{\alpha}m(\widehat{s}_t - \widehat{s}_{t-1}), \quad (57)$$

where $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ and $\pi_{F,t}^* = p_{F,t}^* - p_{F,t-1}^*$ denote the PPI inflation in countries H and F , respectively. Equation (57) shows that CPI inflation depends on PPI inflation and changes in the TOT. The effect of a change in the Home country's TOT on the gap between the Home CPI and PPI increases with the weight of the imported (Foreign) goods in Home households' preferences, given by $\bar{\alpha}(1-m) = \alpha$, which decreases with the relative size (m) of country H and with the degree of home bias, inversely related to $\bar{\alpha}$. Therefore, in the specific case of a small country H , when m is close to 0, $p_t^* = p_{F,t}^*$ ($\pi_t^* = \pi_{F,t}^*$); **however**, I maintain a gap between the CPI and PPI in country H ($\pi_t = \pi_{H,t} + \bar{\alpha}(s_t - s_{t-1})$) because $\alpha = \bar{\alpha}(1-m)$ and $m \rightarrow 0$.

I assume that the law of one price (LOP) always holds for individual goods, both for import and export prices. This implies that $P_t(f) = \mathcal{E}_t P_t^*(f)$, $\forall f$ and $P_t(h) = \mathcal{E}_t P_t^*(h)$, $\forall h$, where \mathcal{E}_t denotes the price of country F 's currency in terms of country H 's currency. Substituting these conditions into the price indices for the two countries yields $P_{F,t} = \mathcal{E}_t P_{F,t}^*$ and $P_{H,t} = \mathcal{E}_t P_{H,t}^*$. In the Euro area, I have $\mathcal{E}_t = 1$, therefore, $P_{F,t} = P_{F,t}^*$ and $P_{H,t} = P_{H,t}^*$. However, due to the home bias introduced in the preferences over consumption bundles ($\alpha \neq \frac{1}{2}$), purchasing power parity (PPP) does not hold in general. Hence, $P_t \neq \mathcal{E}_t P_t^*$.

The real exchange rate, denoted by $\mathcal{Q}_t = \frac{\mathcal{E}_t P_t^*}{P_t}$, reduces to $\mathcal{Q}_t = \frac{P_t^*}{P_t}$ in the Euro area, where $\mathcal{E}_t = 1$. Hence, I have²⁶

$$\mathcal{Q}_t = \frac{\left((1-\alpha^*)\mathcal{S}_t^{1-\eta} + \alpha^*\right)^{\frac{1}{1-\eta}}}{\left((1-\alpha) + \alpha\mathcal{S}_t^{1-\eta}\right)^{\frac{1}{1-\eta}}} \Rightarrow \widehat{q}_t = \widehat{s}_t(1-\bar{\alpha}), \quad (58)$$

where I use the properties that for any m , the steady-state solution is such that $\mathcal{S} = 1$, when preferences are identical across countries $\bar{\alpha} = \bar{\alpha}^*$. Hence, home bias is the only

²⁶I use the definitions of price indexes:

$$\begin{aligned} P_t &= \left((1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right)^{\frac{1}{1-\eta}} \Rightarrow \frac{P_t}{P_{H,t}} = \left((1-\alpha) + \alpha\mathcal{S}_t^{1-\eta}\right)^{\frac{1}{1-\eta}} \\ P_t^* &= \left((1-\alpha^*)P_{F,t}^{1-\eta} + \alpha^* P_{H,t}^{1-\eta}\right)^{\frac{1}{1-\eta}} \Rightarrow \frac{P_t^*}{P_{H,t}} = \left((1-\alpha^*)\mathcal{S}_t^{1-\eta} + \alpha^*\right)^{\frac{1}{1-\eta}} \end{aligned}$$

With $\mathcal{Q}_t = \frac{P_t^*}{P_t}$, this leads to

$$\widehat{q}_t = (1-\alpha^* - \alpha)\widehat{s}_t = (1-\bar{\alpha}m - \bar{\alpha}(1-m))\widehat{s}_t = (1-\bar{\alpha})\widehat{s}_t$$

source of the violation of PPP.²⁷ The real exchange rate's volatility increases with the degree of Home bias and volatility in the TOT. Although the LOP holds for all goods individually, the real exchange rate \widehat{q}_t is directly related to the TOT \widehat{s}_t , which fluctuates over time in response to shocks in both countries, because the Foreign (Home) preference places a higher weight on Foreign (Home) goods than the Home (Foreign) preference does.

Turning to the financial market and uncovered interest parity (UIP) conditions, I have in a monetary union (the nominal exchange rate is equal to one, $\mathcal{E}_t = 1$):

$$\left. \begin{aligned} \beta \frac{(C_{t+1}^j)^\sigma}{(C_t^j)^\sigma} &= \frac{P_{t+1}}{P_t} Q(z_{t+1}|z_t) \\ \beta \frac{(C_{t+1}^{j*})^\sigma}{(C_t^{j*})^\sigma} &= \frac{Q_{t+1}}{Q_t} \frac{P_{t+1}}{P_t} Q(z_{t+1}|z_t) \end{aligned} \right\} \Rightarrow C_t = \frac{m}{1-m} \vartheta Q_t^{\frac{1}{\sigma}} C_t^*, \quad (59)$$

where $\vartheta = Q_t^{-\frac{1}{\sigma}} \frac{C_0}{C_0^*}$ and with $C_t = \int_0^m C_t^j dj = m C_t$ and $C_t^{j*} = \int_m^1 C_t^{j*} dj = (1-m) C_t^*$. Hence, home bias allows for a variable gap between Home and Foreign households' consumption growth rates, even if the international financial market structure is complete. The ratio between the Home and Foreign aggregate consumption levels collapses to zero as the Home country becomes a small economy.

Following a general procedure in the literature, I assume the same initial conditions for Home and Foreign households, so that $\vartheta = 1$. Log-linearizing (59) around the steady state for the symmetric case ($\bar{\alpha} = \bar{\alpha}^*$), I obtain

$$\widehat{c}_t = \frac{1}{\sigma} \widehat{q}_t + \widehat{c}_t^* \quad (60)$$

showing that risk sharing implies a perfect international correlation between consumption levels.

G.3 Firms

There is a continuum of firms indexed by $i \in [0, 1]$. Firms on the interval $[0, m)$ are located in country H , while firms on the interval $[m, 1]$ are located in country F . Each monopolistic competitive firm sets the relative price of its differentiated goods, faced with an isoelastic and downward-sloping demand curve and subject to a technological constraint. Firms use only a homogeneous type of labor for production and there is no investment. The labor market is competitive.

²⁷ $\widehat{q}_t = 0$ in every period when there is no home bias (i.e., when $\bar{\alpha} = 1$).

G.3.1 Cost minimization

All Home firms operate an identical constant returns-to-scale technology:

$$Y_t(i) = A_t L_t(i),$$

where $Y_t(i)$ is the Home firm i 's output and $L_t(i)$ is the Home firm i 's labor demand. A_t is the Home's total factor productivity shifter. Its log-linearized counterpart follows an $AR(1)$ process:

$$\hat{a}_t = \log(A_t/A) = \rho \hat{a}_{t-1} + \epsilon_{a,t},$$

where $0 < \rho < 1$ and $\epsilon_{a,t}$ are i.i.d Gaussian shocks. All Foreign firms operate a similar technology, except for the fact that $\rho \neq \rho^*$ is possible. The shocks $\epsilon_{a,t}$ and $\epsilon_{a,t}^*$ may be correlated. Technology constraints imply that the Home and Foreign i th firms' labor demands are given respectively by

$$\begin{aligned} L_t(i) &= \frac{Y_t(i)}{A_t} \\ L_t^*(i) &= \frac{Y_t^*(i)}{A_t^*} \end{aligned}$$

so that the Home and Foreign nominal marginal costs are given by $MC_t^n = (1 + t_t)W_t/A_t$ and $MC_t^{n*} = (1 + t_t^*)W_t^*/A_t^*$, while the Home and Foreign real marginal costs are defined as

$$MC_t = \frac{MC_t^n}{P_{H,t}} = \frac{(1 + t_t)W_t}{P_{H,t}A_t} \quad (61)$$

$$MC_t^* = \frac{MC_t^{n*}}{P_{F,t}^*} = \frac{(1 + t_t^*)W_t^*}{P_{F,t}^*A_t^*}, \quad (62)$$

where t_t and t_t^* are payroll tax rates in Home and Foreign economies.

G.3.2 Price setting

Firms set prices in a staggered way, as in Calvo (1983): every period, a measure of $1 - \theta$ randomly selected firms set a new price, with an individual firm's probability of readjusting at each period being independent of the time elapsed since it last reset its price. A Home firm i adjusting its price in period t sets a new price $\bar{P}_{H,t}$ to maximize the present value of its stream of expected future profits:

$$\max_{\bar{P}_{H,t}(i)} \sum_{\tau=0}^{\infty} \theta^\tau \mathbb{E}_t [Q_{t,t+\tau} (\bar{P}_{H,t}(i) - MC_{t+s}^n) Y_{t+s}(i)]$$

subject to the demand constraint

$$Y_{t+s}(i) = \left(\frac{\bar{P}_{H,t}}{P_{H,t+s}} \right)^{-\varepsilon} C_{t+s}$$

A firm cannot decide on its optimal price in each period. To simplify the notation, I omit the index i for the firm. Following Calvo (1983), on each date, the firm receives a signal advising it whether it can revise its price $P_{H,t}$ in an optimal manner. There is a probability θ that the firm cannot revise its price in a given period. When a firm receives a positive signal (with probability $1 - \theta$), it chooses price $P_{H,t}$ that maximizes its profit discerning that during the next periods it may be unable to choose its price optimally. Let $\tilde{\mathcal{V}}_t$ be the value of a firm that receives a positive signal in period t and $\mathcal{V}_t(P_{H,t-1})$ the value of a firm that receives a negative signal. As a firm that receives a negative signal simply follows the *ad hoc* pricing rule $P_{H,t} = P_{H,t-1}$, its value at time t depends only on $P_{H,t-1}$. Denoting $\Pi(\bar{P}_{H,t}) = (\bar{P}_{H,t} - MC_t^n)Y_t$, the value of a firm that receives a positive signal in period t is

$$\tilde{\mathcal{V}}_t = \max_{\frac{\bar{P}_{H,t}}{P_{H,t}}} \left\{ \Pi(\bar{P}_{H,t}) + \beta \mathbb{E}_t \left[Q_{t+1} \left((1 - \theta) \tilde{\mathcal{V}}_{t+1} + \theta \mathcal{V}_{t+1}(\bar{P}_{H,t}) \right) \right] \right\} \quad (63)$$

The value of a firm that cannot re-optimize is

$$\mathcal{V}_t(P_{H,t-1}) = \Pi(P_{H,t-1}) + \beta \mathbb{E}_t \left[Q_{t+1} \left((1 - \theta) \tilde{\mathcal{V}}_{t+1} + \theta \mathcal{V}_{t+1}(P_{H,t-1}) \right) \right] \quad (64)$$

The first-order and envelope conditions associated with (63)–(64) determine the dynamics of inflation. These are given by

$$\Pi'_t(\bar{P}_{H,t}) + \beta \theta \mathbb{E}_t \left[Q_{t+1} \mathcal{V}'_{t+1}(\bar{P}_{H,t}) \right] = 0 \quad (65a)$$

$$\mathcal{V}'_t(P_{H,t-1}) = \Pi'_t(P_{H,t-1}) + \beta \theta \mathbb{E}_t \left[Q_{t+1} \mathcal{V}'_{t+1}(P_{H,t-1}) \right] \quad (65b)$$

Iterating forward on these first-order conditions, I can obtain

$$\begin{aligned} \mathbb{E}_t \left[\sum_{j=0}^{\infty} \theta^j Q_{t+j} Y_{t+j} \left(\bar{P}_{H,t} - \frac{\varepsilon}{\varepsilon - 1} MC_{t+j}^n \right) \right] &= 0 \\ \mathbb{E}_t \left[\sum_{j=0}^{\infty} \theta^j Q_{t+j} Y_{t+j} \left(\frac{\bar{P}_{H,t}}{P_{H,t-1}} - \frac{\varepsilon}{\varepsilon - 1} MC_{t+j} \frac{P_{H,t+j}}{P_{H,t-1}} \right) \right] &= 0 \end{aligned} \quad (66)$$

Log-linearizing this equation around the zero-inflation steady state and using $Q_{t+j} = \beta^j$, I obtain

$$\widehat{p}_{H,t} - \widehat{p}_{t-1} = (1 - \beta\theta) \sum_{j=0}^{\infty} (\beta\theta)^j \mathbb{E}_t [\widehat{m}c_{t+j} + (\widehat{p}_{t+j} - \widehat{p}_{t-1})],$$

where \widetilde{mc}_t is the deviation of the real marginal cost from its steady-state level. It is equal to $\widetilde{mc}_t = t_t + \widehat{w}_t - \widehat{p}_{H,t} - \widehat{a}_t$.

$$\widehat{\pi}_{H,t} = \beta \mathbb{E}_t [\widehat{\pi}_{H,t+1}] + \lambda \widetilde{mc}_t,$$

where $\lambda = \frac{1-\theta}{\theta}(1 - \theta\beta)$.

Proceeding in the same way with the Foreign country, I obtain

$$\widehat{\pi}_{F,t}^* = \beta \mathbb{E}_t [\widehat{\pi}_{F,t+1}^*] + \lambda \widetilde{mc}_t^*,$$

where \widetilde{mc}_t^* is the deviation of the real marginal cost from its steady-state level. The Home and Foreign real marginal costs under flexible prices are given by $mc_t = mc_t^* = -\log\left(\frac{\varepsilon}{\varepsilon-1}\right)$, where $\log\left(\frac{\varepsilon}{\varepsilon-1}\right)$ is the gross markup (in log).

G.4 Government budget constraints

Each agent j in country H (or F) receives Home and Foreign public debts, $B_t^j = B_{H,t}^j + B_{F,t}^j$ and $B_t^{j*} = B_{H,t}^{j*} + B_{F,t}^{j*}$. At the equilibrium, I have $B_{H,t} = \int_0^n B_{H,t}^j dj + \int_n^1 B_{H,t}^{j*} dj = nB_{H,t}^j + (1-n)B_{H,t}^{j*}$ and $B_{F,t} = \int_0^n B_{F,t}^j dj + \int_n^1 B_{F,t}^{j*} dj = nB_{F,t}^j + (1-n)B_{F,t}^{j*}$.

I assume for simplicity that a lump sum transfer (Tr_t and Tr^*) allows each government to redistribute the revenues of the payroll taxes paid only in sticky price economies: this implies that $Tr_t = t_t W_t N_t$ and $Tr^* = t_t^* W_t^* N_t^*$.

The budget constraint (after receiving and redistributing the payroll taxes) of the state in the Home country is given by

$$\frac{1}{R_t} \frac{B_{H,t}}{P_{H,t}} + D_{H,t}^r = \frac{B_{H,t-1}}{P_{H,t}} \Leftrightarrow \frac{1}{R_t} B_{H,t}^r + D_{H,t}^r = \frac{1}{\pi_{H,t}} B_{H,t-1}^r,$$

where $D_{H,t}^r$ are the real net transfers from the government to households, namely, $D_t^r = Tax_t - Tr_t$, with Tax_t a real lump-sum tax and Tr_t a real lump-sum transfer. I deflate the nominal debt ($B_{H,t}$) by the PPI to define the real debt $B_{H,t}^r$. By the forward iteration of this budgetary constraint, the price level $P_{H,t}$ depends on the sequence of the choice of the state ($\{D_{t+i}^r\}_{i=0}^\infty$), via the valuation of the debt:

$$\frac{B_{t-1}}{P_t} = \mathbb{E}_t \sum_{i=0}^{\infty} \left(\prod_{j=1}^i \frac{\pi_{H,t+j}}{R_{t+j-1}} \right) D_{t+i}^r$$

Given an initial condition for the nominal debt B_{t-1} , this equation shows that the general price level depends on the sequence of net spending by the state of the Home country ($\{D_{t+i}^r\}_{i=0}^\infty$): this is one of the basis of the price tax theory.

The linearized versions of the budget constraints of each state (Home and Foreign) are given by

$$\widehat{b}_t^r = \beta^{-1}(\widehat{b}_{t-1}^r - \widehat{\pi}_{H,t}) + \widehat{r}_t + (1 - \beta^{-1})\widehat{d}_t^r \quad (67)$$

$$\widehat{b}_t^{r*} = \beta^{-1}(\widehat{b}_{t-1}^{r*} - \widehat{\pi}_{F,t}^*) + \widehat{r}_t + (1 - \beta^{-1})\widehat{d}_t^{r*} \quad (68)$$

As for the Taylor rule, one can model the government's choices using a fiscal rule. For example, I can assume that the surplus of the Home government \widehat{d}_t follows the following rule:

$$\begin{aligned} D_t^r &= D^r + \gamma \left(\frac{1}{R_{t-1}} \frac{B_{t-1}}{P_{H,t-1}} - \frac{B^r}{R} \right) + \varepsilon_t^F \\ \Rightarrow \widehat{d}_t^r &= \gamma \frac{1}{\beta^{-1} - 1} (\widehat{b}_{t-1}^r - \widehat{r}_{t-1}) + \varepsilon_t^F \end{aligned} \quad (69)$$

This last equation represents the fiscal rule. The same rule exists in the Foreign country. Introducing (69) into the budget constraints of the government in the Home (Foreign) country (Equations (67) and (68)), I obtain

$$\widehat{b}_t^r = (\beta^{-1} - \gamma)\widehat{b}_{t-1}^r - \beta^{-1}\widehat{\pi}_{H,t} + \widehat{r}_t + \gamma\widehat{r}_{t-1} + (1 - \beta^{-1})\varepsilon_t^F \quad (70)$$

$$\widehat{b}_t^{r*} = (\beta^{-1} - \gamma^*)\widehat{b}_{t-1}^{r*} - \beta^{-1}\widehat{\pi}_{F,t}^* + \widehat{r}_t + \gamma\widehat{r}_{t-1} + (1 - \beta^{-1})\varepsilon_t^{F*} \quad (71)$$

These budgetary constraints depend on the ECB's policy, that is, directly through the interest rate and indirectly through inflation.

G.5 Equilibrium

The log-linearization of the equilibrium equations of the goods market around the steady state symmetric ($\bar{\alpha} = \bar{\alpha}^*$), using the UIP condition (Equation 60), leads to

$$\widehat{y}_t = \widehat{c}_t + \frac{\omega + \bar{\alpha} - 1}{\sigma} \widehat{s}_t \quad (72)$$

$$\widehat{y}_t^* = \widehat{c}_t^* - \frac{\omega^*}{\sigma} \widehat{s}_t, \quad (73)$$

where $\omega = 1 - \bar{\alpha}m + (1 - m)\bar{\alpha}(2 - \bar{\alpha})(\sigma\eta - 1) > 0$ and $\omega^* = \bar{\alpha}m + m\bar{\alpha}(2 - \bar{\alpha})(\sigma\eta - 1) > 0$.

Using Equations (57), (72), and (73), Equations (55) and (56) become

$$\begin{aligned} \widehat{y}_t &= \mathbb{E}_t[\widehat{y}_{t+1}] - \frac{1}{\sigma} (r_t - \mathbb{E}_t[\widehat{\pi}_{H,t+1}] + \log(\beta)) + \frac{1 - \bar{\alpha}m - \omega}{\sigma} \Delta \mathbb{E}_t \widehat{s}_{t+1} \\ \widehat{y}_t^* &= \mathbb{E}_t[\widehat{y}_{t+1}^*] - \frac{1}{\sigma} (r_t - \mathbb{E}_t[\widehat{\pi}_{F,t+1}^*] + \log(\beta)) - \frac{\bar{\alpha}m - \omega^*}{\sigma} \mathbb{E}_t \Delta \widehat{s}_{t+1} \end{aligned}$$

Using Equations (60) and (58), I obtain

$$\widehat{s}_t = \frac{\sigma}{\omega + \omega^*} (\widehat{y}_t - \widehat{y}_t^*) \quad (74)$$

G.5.1 Flexible price equilibrium

In the flexible price economy, it is assumed that $t_t = t_t^* = 0$. Home and Foreign firms optimize their prices in each period. Hence, these prices are equal to a mark-up over the marginal cost. I find that the Home and Foreign real marginal costs are given by

$$mc_t = mc_t^* = -\log\left(\frac{\varepsilon}{\varepsilon - 1}\right),$$

where $mc_t = mc_t^n - p_{H,t}$ and $mc_t^* = mc_t^n - p_{F,t}^*$. Hence, I deduce

$$\bar{s}_t = \frac{1 + \varphi}{1 + \frac{\varphi}{\sigma}(\omega + \omega^*)}(\hat{a}_t - \hat{a}_t^*)$$

and the natural level of the Home and Foreign outputs:

$$\begin{aligned}\bar{y}_t &= \log m - \frac{1}{\varphi + \sigma} \log\left(\frac{\varepsilon}{\varepsilon - 1}\right) + (1 - \Theta) \frac{1 + \varphi}{\varphi + \sigma} \hat{a}_t + \Theta \frac{1 + \varphi}{\varphi + \sigma} \hat{a}_t^* \\ \bar{y}_t^* &= \log(1 - m) - \frac{1}{\varphi + \sigma} \log\left(\frac{\varepsilon}{\varepsilon - 1}\right) + \Theta^* \frac{1 + \varphi}{\varphi + \sigma} \hat{a}_t + (1 - \Theta^*) \frac{1 + \varphi}{\varphi + \sigma} \hat{a}_t^*\end{aligned}$$

with $\Theta = \frac{\sigma(1 - \bar{\alpha}m - \omega)}{\sigma + \varphi(\omega + \omega^*)}$ and $\Theta^* = \frac{\sigma(\bar{\alpha}m - \omega^*)}{\sigma + \varphi(\omega + \omega^*)}$. Using the $AR(1)$ processes for \hat{a} and \hat{a}^* , I obtain

$$\mathbb{E}_t \Delta \bar{s}_{t+1} = \frac{1 + \varphi}{1 + \frac{\varphi}{\sigma}(\omega + \omega^*)} [(\rho - 1)\hat{a}_t - (\rho^* - 1)\hat{a}_t^*]$$

This leads me to deduce the natural expected interest rate ($r_t^e \equiv \bar{r}_t - \mathbb{E}_t \bar{\pi}_{H,t}$ and $r_t^{e*} \equiv \bar{r}_t - \mathbb{E}_t \bar{\pi}_{F,t}^*$):

$$r_t^e = \frac{(1 - \Gamma)\sigma(1 + \varphi)(\rho - 1)\hat{a}_t + \Gamma\sigma(1 + \varphi)(\rho^* - 1)\hat{a}_t^*}{\varphi + \sigma} - \log \beta \quad (75)$$

$$r_t^{e*} = \frac{\Gamma^*\sigma(1 + \varphi)(\rho - 1)\hat{a}_t + (1 - \Gamma^*)\sigma(1 + \varphi)(\rho^* - 1)\hat{a}_t^*}{\varphi + \sigma} - \log \beta, \quad (76)$$

where $\Gamma = \frac{\varphi(\bar{\alpha}m + \omega - 1)}{\sigma + \varphi(\omega + \omega^*)}$ and $\Gamma^* = \frac{\varphi(\omega - \bar{\alpha}m)}{\sigma + \varphi(\omega + \omega^*)}$.

G.5.2 Equilibrium with nominal rigidities

As in Gali and Monacelli (2005), I assume that the deviations of the dispersion of the Home and Foreign firms' output induced by price rigidities around the steady state symmetric are of second order, so that up to a first-order approximation, I can set them to zero. Therefore, using the labor market equilibrium, I have

$$\begin{aligned}mc_t &= (\sigma + \varphi)(y_t - \log m) - (\bar{\alpha}m + \omega - 1)s_t - (1 + \varphi)a_t + t_t \\ mc_t^* &= (\sigma + \varphi)(y_t^* - \log(1 - m)) - (\bar{\alpha}m - \omega^*)s_t - (1 + \varphi)a_t^* + t_t^*\end{aligned}$$

Assuming that t_t and t_t^* are different from zero only in the sticky price economies, I obtain

$$\widehat{\pi}_{H,t} = \beta \mathbb{E}_t [\widehat{\pi}_{H,t+1}] + \kappa \widehat{y}_t - \lambda(\alpha m + \omega - 1) \widehat{s}_t + \lambda t_t \quad (77)$$

$$\widehat{\pi}_{F,t}^* = \beta \mathbb{E}_t [\widehat{\pi}_{F,t+1}^*] + \kappa \widehat{y}_t^* + \lambda(\omega^* - \alpha m) \widehat{s}_t + \lambda t_t^*, \quad (78)$$

which are the Home and Foreign New Phillips curves, where $\kappa = \lambda(\varphi + \sigma)$. The IS curves are given by

$$\widehat{y}_t = \mathbb{E}_t[\widehat{y}_{t+1}] - \frac{1}{\sigma} (r_t - \mathbb{E}_t[\widehat{\pi}_{H,t+1}] - \bar{r}_t^e) + \frac{1 - \bar{\alpha}m - \omega}{\sigma} \mathbb{E}_t \Delta \widehat{s}_{t+1} \quad (79)$$

$$\widehat{y}_t^* = \mathbb{E}_t[\widehat{y}_{t+1}^*] - \frac{1}{\sigma} (r_t - \mathbb{E}_t[\widehat{\pi}_{F,t+1}^*] - \bar{r}_t^{e*}) + \frac{\omega^* - \bar{\alpha}m}{\sigma} \mathbb{E}_t \Delta \widehat{s}_{t+1} \quad (80)$$

Therefore, the equilibrium dynamics of this two-country NK model with asymmetric countries is defined by

- the New Phillips curves (Equations (77) and (78)),
- the IS curves (Equations (79) and (80)), and
- the natural expected interest rates (Equations (75) and (76)),
- The Taylor rule of the ECB.

$$\widehat{r}_t = \alpha_\pi \widehat{\pi}_t^u + \alpha_y \widehat{y}_t^u + \epsilon_{r,t}, \quad (81)$$

where $\widehat{\pi}_t^u = m \widehat{\pi}_{H,t} + (1 - m) \widehat{\pi}_{F,t}^*$ and $\widehat{y}_t^u = m \widehat{y}_t + (1 - m) \widehat{y}_t^*$. The distinction between CPI inflation and PPI inflation, while meaningful at the level of each country, disappears for the ECB. Formally, given that $p_t = p_{H,t} + \bar{\alpha}(1 - n)s_t$ and $p_t^* = p_{F,t}^* - \bar{\alpha}n s_t$, I have $p_t^u = n p_t + (1 - n) p_t^* = n p_{H,t} + (1 - n) p_{F,t}^*$ (see Equation (57)).

- Gap in the TOT is given by:

$$\widehat{s}_t = \frac{\sigma}{\omega + \omega^*} (\widehat{y}_t - \widehat{y}_t^*) \quad (82)$$

- Public debt of each country given by Equations (67) and (68) and the fiscal rules in Equation (69).

For simplicity, I assume in the following that $a_t = a_t^*$, $\forall t$, leading to $r^e = r^{e*} = -\log(\beta)$. This does not change the properties of the dynamic system. The dynamic system is

$$Z_t = A^{-1} B \mathbb{E}_t Z_{t+1} + A^{-1} C e_t$$

H From Structural to Reduced Form Parameters

Parameter		Interpretation
β		Subjective discount factor
σ		Intertemporal elasticity of substitution for consumption
φ		Intertemporal elasticity of substitution for employment
$\bar{\alpha}$		Home Bias
m		Size of the Home country
α	$\bar{\alpha}(1 - m)$	Trade openness in Home country
α^*	$\bar{\alpha}m$	Trade openness in the Foreign country
η		Elasticity of substitution between Home and Foreign baskets
ε		Elasticity of substitution between Home (Foreign) goods
ρ_a		Persistence of technological shocks
θ		Probability to not reset price for a firm
γ, γ^*		Fiscal brake in Home (Foreign) country
ρ, ρ^*		Persistence of fiscal transfer in Home (Foreign) country
α_π		Elasticity of the interest rate to inflation (Taylor rule)
α_y		Elasticity of the interest rate to output gap (Taylor rule)
Parameters for the reduced form		
λ	$\frac{1-\theta}{\theta}(1 - \theta\beta)$	Elasticity of the inflation to marginal costs
κ	$\lambda(\sigma + \varphi)$	Elasticity of the inflation to output gap in a closed economy
ω	$1 - \alpha^*$ $+\alpha(2 - \bar{\alpha})(\sigma\eta - 1)$	Weight of the Home country
ω^*	α^* $+\alpha^*(2 - \bar{\alpha})(\sigma\eta - 1)$	Weight of the Foreign country

Table 1: Model parameters