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Pipe two-phase flow non-invasive imaging using Ultrasound Computed Tomography: a two-dimensional numerical and experimental performance assessment

William Cailly^{a,b,*}, Henri Walaszek^a, Sébastien Brzuchacz^a, Fan Zhang^a, Philippe Lasaygues^b

^aTechnical Centre for Mechanical Industry, CETIM, France

^bAix-Marseille Univ, CNRS, Centrale Marseille, LMA, Marseille, France

Abstract

Pipe two-phase flow non-invasive imaging is of great interest in the field of industry. In particular, small bubble flow imaging through opaque pipes is challenging. Ultrasound computed tomography can be a relevant technique for this purpose. However, perturbation phenomena that are inherent to the configuration (acoustic impedance mismatching, circumferential propagation, reverberation) limit two aspects: the performance of the technique and the use of conventional inversion algorithms. The objectives of the presented work are: (i) to predict the effects of the pipe wall on ultrasonic waves for both metallic and plastic pipe, (ii) to define a consistent inversion algorithm taking into account those effects, (iii) to validate and to assess the limitations of the designed imaging numerical tool using an experimental setup. The benchmark configuration consists of 150 mm diameter 3 mm thick pipes containing 6 mm diameter rods acting as reference scatterers. Two materials of very different acoustical properties were tested: aluminum and PMMA. The results highlighted that the quality of the reconstructed image is very dependent on the pipe material. The results showed that, using an adapted inversion model, consistent target reconstruction is obtained. Based on numerical predictions, performance limitations are reached for metallic pipes.

Keywords: Ultrasound computed tomography, two-phase flow, adjoint tomography

1. Introduction

1.1. Context and purpose

Process monitoring is of great importance in the field of chemical, oil and gas industries. Process tomography regroups imaging techniques that permit the monitoring of industrial processes. Such techniques give information about a state of a process by means of a real-time visualization. In particular, for multiphase flow, the purpose is to

*Corresponding author

Email address: cailly@lma.cnrs-mrs.fr (William Cailly)

7 know the spatial distribution of the various phases flowing through a pipe from non-
8 invasive measurements. The two-phase flow regime is a steady state depending on the
9 gas concentration and on the relative velocity between the two phases. In particular, the
10 bubbly flow occurs when the flow rate of the liquid is significantly higher than the flow
11 rate of the gas. This bubbly flow regime occurs not only during controlled flow in reac-
12 tors but also during heterogeneous advected cavitation phenomena. Small voids that are
13 formed on the inner surface of a pipe are advected when there is a liquid flowing through
14 this pipe. Bubbles are moving and different dynamical effects occur : Archimedes' accel-
15 eration, expansion and break. Heterogeneous cavitation, when it is strong, implies local
16 structural damage such as erosion. Detecting bubbles in order to prevent damage due to
17 cavitation is of great significance for systems like hydraulic circuits, turbines or pumps.

18 1.2. Background

19 Different imaging techniques are currently available for process monitoring : opti-
20 cal imaging, electric tomography, MRI, X-ray tomography, ultrasonic imaging [1]. The
21 design of process tomography systems is subject to the trade-off between : (i) image
22 reconstruction time - (ii) cost of instrumentation - (iii) spatial resolution and (iv) tem-
23 poral resolution. Another restraining factor to non-invasive process tomography is the
24 penetrating power of waves. Electromagnetic waves are limited to relatively thin and low
25 density walls. Moreover, optical imaging is only applicable to transparent walls and liq-
26 uids. Ultrasound imaging techniques seem to be a good solution where electromagnetic
27 wave techniques are not applicable. Ultrasound techniques can be used on materials for
28 which X-ray techniques have little penetrating power, typically thick steel walls. So,
29 non-invasive imaging of small bubble flow is challenging. It is known that Electrical Ca-
30 pacitance Tomography is not suitable for detecting small bubbles because of its lack of
31 spatial resolution performance. Ultrasound techniques should be more affordable for in-
32 dustry because of fewer technical constraints, lower cost, and absence of safety restriction
33 compared with X-ray or MRI techniques[2].

34 It is possible to monitor two-phase flow in a pipe by means of fan-beam ultrasound
35 tomography. The 2D fan-beam tomography can be implemented by electronic scan-
36 ning, that is, an acquisition sequence of signals emitted by a set of N transducers placed
37 around a section of a pipe. In the case of a two-phase flow, the impedance discontinu-
38 ity at a gas-liquid interface makes it possible to reconstruct the position of interfaces,
39 whose curvature radius is greater than the wavelength, from ultrasonic echoes, using
40 transmission and/or reflection tomography. In [3, 4, 5, 6, 7, 8, 9], Hoyle's research group
41 and other researchers from the University of Leeds showed the feasibility of real-time
42 fan-beam ultrasound tomography for bubbly flow imaging. In [10, 11, 12, 13, 14, 15],
43 Ruzairi Abdul Rahim's group achieved good results for the application of transmission
44 ultrasound tomography to two-phase flow in bubble column reactors. Recent results
45 have been presented by Langener *et al.* [16]. It is possible to apply time-of-flight dif-
46 ference tomography to reconstruct a quantitative map of the plane components of the
47 velocity field of a liquid flow [17, 18, 19]. A similar tomographic method has been used
48 to the whole 3-component velocity field [20]. Overall, this background shows that the
49 advantages of ultrasound tomography are its low cost, high contrast and good spatial
50 resolution. However, the major limitations of ultrasound techniques are (i) low temporal
51 resolution, (ii) signal complexity due to refraction and multiple diffraction phenomena
52 (iii) significant mismatching at solid-liquid interfaces. Limitation (i) is related to the

53 speed of sound. The electronic scanning sequence implies that the reconstructed image
 54 is computed from signals emitted at different instants. Limitation (ii) has to be taken
 55 into account especially for the case of small bubbles. Indeed, the diffraction pattern
 56 of a spherical obstacle whose diameter is nearly the wavelength of ultrasound includes
 57 side lobes resulting in a complex directivity. Limitation (iii) depends on the material the
 58 pipe is made of. Acoustic impedance matching systems involve artificially minimizing the
 59 effects of refraction and reflection at the interface between the wall and the liquid to max-
 60 imize the energy transmitted between these two media. The reflection coefficient at the
 61 steel–water interface with normal incidence is around 94 %. The impedance mismatch-
 62 ing at the curved interface of the pipe wall causes refraction of an incident plane wave
 63 with a convergent effect. Most researchers in the field of process ultrasound tomography
 64 conclude that non-invasive imaging of the inside of a steel pipe is made very difficult
 65 because of internal specular reflections [21]. Several solutions to minimize these effects
 66 have been studied [22, 23] : (i) focused probes (expensive solution); (ii) inserts machined
 67 through the wall (invasive); (iii) multi-element probes with different orientations. Cowell
 68 showed that solution (iii) can be a solution to minimize the effect of refraction which
 69 will be discussed in this paper. Moreover, the Lamb waves generated in the wall disturb
 70 the signals transmitted inside the pipe. This disturbance is significant in the case of
 71 steel since most of the energy remains trapped in the wall. An increase in the frequency
 72 causes a concentration of the ultrasound intensity within the wall. The idea developed
 73 by Abbaszadeh is to minimize the disturbance due to the wall waves by selecting the
 74 excitation frequency according to the propagation time of the Lamb waves [21].

75 *1.3. Objectives and content*

76 Within this industrial context and background, the objectives of the presented work
 77 are: (i) to predict the effects of the pipe wall on ultrasonic waves for both metallic and
 78 plastic pipe, (ii) to define a consistent inversion algorithm taking into account those
 79 effects, (iii) to validate and to assess the limitations of the designed imaging numerical
 80 tool using an experimental setup.

81 First, the paper deals with the direct problem. The geometric and acoustic aspects
 82 of the problem are exposed. Then, the 2D numerical model is presented. Two inversion
 83 methods are defined according to the direct model: an adapted filtered back propaga-
 84 tion method (AFBP) and an adjoint field tomography method (AFT). The benchmark
 85 configuration focuses on a single geometry: 150 mm diameter and 3 mm thickness for
 86 aluminum pipes and PMMA pipes, assuming small bubble concentration and small flow
 87 velocity. Results are obtained for a set of both numerical and experimental data from
 88 this configuration. The presented results analysis consists in measuring the detection,
 89 localization and spatial undersampling performance, for 6 rods mm acting as scatterers.
 90 The results are discussed in the last section.

91 **2. Methods**

92 *2.1. Geometry of the configuration*

93 Let us consider a 2D cross section of a pipe containing liquid and bubbles considered
 94 as perfect impenetrable obstacles. The geometry of the configuration showing the chosen
 95 parameters is given in figure 1. The pipe wall Ω_S is a homogeneous elastic isotropic

109 These transfer functions, as first approximation neglecting mode conversion and shear
 110 waves, are valid for normal incidence and large excitation area compared to the thickness.
 111 One can derive the following energy conservation relation

$$T_r T_r^* + T_t T_t^* = 1 \quad (3)$$

112 The maximum of transmission, corresponding to resonance, occurs at $|T_t| = 1$ when

$$\omega_n = \frac{\pi c_L n}{h} \quad (4)$$

113 This determines the center frequency for excitation for maximizing the acoustic wave
 114 amplitude in the pipe. In this paper, we studied two pipe materials with different acous-
 115 tical properties: PMMA and aluminum. The transmission transfer function for these
 116 materials for $h = 3$ mm are plotted in figure 2. The chosen material constant values are
 117 given in table 1.

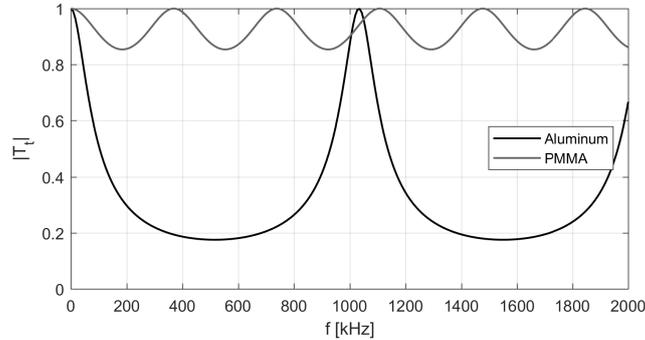


Figure 2: Transmission transfer function of an infinite 3 mm thick layer for PMMA and aluminum.

118 2.3. Adapted filtered backpropagation ray method

119 It is possible to use a geometrical optics analogy principles to describe the phenomena
 120 caused by the acoustic discontinuity: (i) refraction, (ii) in wall reflections, (iii) interfe-
 121 rence of leaky waves. Analytical tomography is based on geometric formulation. Under
 122 this approximation, a spherical wave can be described as ray paths starting from a point
 123 and pointing in all directions. This optics analogy allows to describe reflection, refrac-
 124 tion and diffraction according to Snell's law and Huygens' principle. Refraction is only
 125 negligible for a small range around the center O . For scatterers close to the wall it is nec-
 126 essary to take into account refraction *i.e.*, ray deviation, due to angular incidence. This
 127 approach is valid for small wavelengths compared with the wall thickness, $\omega \gg c_L/h$.

128 The ultrasonic energy contained in the wall is described as multiple reflected rays.
 129 The reverberation and propagation phenomena affect all the sinogram, *i.e.* the measure-
 130 ment at each position around the pipe. The calculated paths are transformed to *phase*
 131 *shift* in order to define a consistent analytical tomography formulation. This formulation
 132 describes the phase and amplitude of a wave propagating from a point source within the
 133 pipe to the outer surface of the wall. The formulation is equivalent to the diffraction
 134 tomography formulation in a homogeneous medium when $c_L \rightarrow c_0$ or $h \rightarrow 0$. The formu-
 135 lation presented below concerns the passive case (inversion source problem) and can be

136 easily extended to the active scattering case (emitter–receiver) applying reciprocity and
 137 Born approximation. The direct source problem is defined by the following mathematical
 138 operation.

$$\mathcal{R} : L^1(\mathbb{R}^2) \rightarrow L^1(S^1 \times \mathbb{R}),$$

$$\mathcal{R}f(\theta, \omega) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} t(\gamma) e^{ik_L l(\gamma)} w(\gamma) \sum_{n=0}^{\infty} Q_n(\gamma) d\gamma \quad (5)$$

139

$$Q_n(\gamma) = |r|^n(\gamma) e^{ink_L 2l(\gamma)} \int_{S_n(\theta, \gamma)} f(\mathbf{x}) H_0^1(k_0 |\mathbf{x} - \mathbf{x}_n|) d\mathbf{x} \quad (6)$$

140 where f is the contrast function to be reconstructed, H_0^1 is the Hankel function known
 141 to be the Green function of the 2D Helmholtz equation in homogeneous medium with
 142 $k_0 := \omega/c_0$, S^1 is the unit circle, r and t are the reflection and transmission coefficients
 143 oriented from liquid to solid, l is the reflection path. The rays $S_n(\theta, \gamma)$ are defined by a set
 144 of geometrical parameters and relations given in appendix. The reconstruction algorithm
 145 is the adjoint of the operator including high pass filtering (Filtered Back Propagation,
 146 FBP). The far field approximation can be used as in classical diffraction tomography.
 147 This approximation reduces the reconstruction computational cost by approximating the
 148 curved arrival time locus to tangent lines Δ perpendicular to the direction of observation
 149 θ (see gray dotted line in figure 1). This algorithm is denoted by adapted FBP (AFBP)
 150 in the paper.

151 2.4. The beamforming and circumferential propagation modes

152 Abbaszadeh *et al.* observed experimentally the effect of circumferential guided waves
 153 propagating in a thick steel pipe wall on the signals transmitted to the liquid [21]. The
 154 authors observed that a decrease in frequency has the effect of reducing the perturbation
 155 caused by circumferential waves. However, to detect scatterers, the decrease in frequency
 156 results in a decrease in resolution. Consequently, there exists a trade-off between reso-
 157 lution and transmission of ultrasonic waves in the liquid. The main parameters involved
 158 are: (i) tube geometry and material, (ii) excitation frequency, (iii) scatterers global posi-
 159 tion and size. To deal with the case where the wavelength is close to or greater than the
 160 thickness of the wall, it is necessary to go through such wave frequency domain approach.
 161 We are particularly interested in the plane-strain circumferential modes with significant
 162 radial vibration, which are the equivalents of antisymmetric Lamb plate modes. Indeed,
 163 the radial displacement has the effect of transmitting leaky waves in the liquid medium
 164 contained in the pipe to be inspected. The analytical approach is described by Gazis,
 165 who showed that the modal behavior is determined by the aspect ratio h/R and the Pois-
 166 son coefficient ν [24, 25]. For the geometry under study $h/R \ll 1$, so the circumferential
 167 wave can be approximated to Lamb plate mode.

168 The ultrasonic beam form, that is, the spatial distribution of transmitted energy, is of
 169 great importance for the inspection performance because it determines the resolution and
 170 detection power. The shape of the beam inside the pipe is determined by the diffraction
 171 occurring at the inner interface. The effect of the wall on the ultrasound beam can be
 172 described using the presented geometrical optics formulation. Each reflected wave in
 173 the wall transmits a wave whose amplitude depends on the transmission coefficient and

174 the phase depending on the multiple reflection path. We are interested in the near-
175 field perturbations. It is possible to apply the Huygens' principle to describe the field
176 diffracted by the inner interface.

177 2.5. Adjoint field computation using a finite element model

178 Finite difference method (FDM) or finite element method (FEM) can be used to
179 predict complex propagation phenomena. Such methods make it possible to solve direct
180 and inverse problems involving heterogeneous media, in our case liquid-solid media. One
181 of the advantages of FEM compared with FDM is that the positions of the nodes are
182 consistent with the domain boundary geometry, in our case a circle. It also makes the
183 implementation of the boundary conditions easier.

184 For the FEM model, we supposed that the in-plane components of the liquid flow
185 velocity are negligible. We supposed that the liquid is a homogeneous acoustic medium
186 with bulk modulus κ and density ρ_0 . We supposed that the speed of a bubble is much
187 lower than the speed of sound $c_0 = \sqrt{\kappa/\rho_0}$, so that a bubble can be considered as a
188 static obstacle during the ultrasound scanning. The 2D approximation implies that the
189 obstacle is a cylinder. A 3D analysis is required to take into account 3D diffraction due
190 to the spherical shape of a bubble. We considered the following formulation: velocity
191 potential ϕ for liquid, plane strain displacement \mathbf{u} for solid and perfect particular velocity
192 and pressure-traction continuity assumption at the solid-liquid interface Γ_i . The bubbles
193 were considered as Dirichlet obstacles ($\phi = 0$ on Γ_D). The reference phantom consisted
194 in three different radial rod positions: $r_0 = 0$ mm, $r_1 = 24.5$ mm, $r_2 = 48.5$ mm. We used
195 the central difference integration scheme to compute the transient result of the elastic-
196 acoustic dynamic system. We checked the results against those obtained by Komatitsch
197 *et al.* for a flat horizontal interface [26]. A spectral element implementation was used in
198 the cited paper. In our case we chose linear quadrangle and triangle P1 elements. The
199 mass matrix was a lumped-mass matrix, *i.e.*, a diagonal mass matrix approximation.
200 A view of the mesh at the pipe interface is given in figure 3. The average size of the
201 elements was 0.3 mm. The integration step was $\Delta t = 0.02\mu\text{s}$.

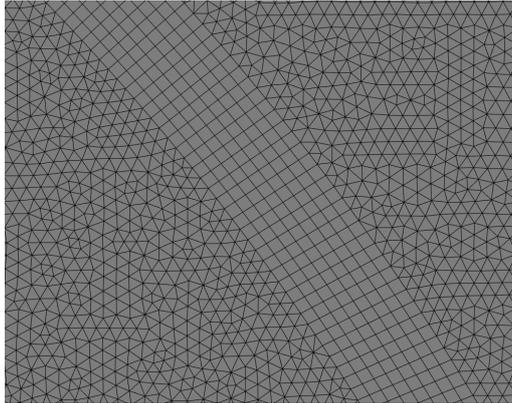


Figure 3: View of the mesh of a 3mm pipe wall with quadrangle elements surrounded by triangle elements.

202 The problem of recovering consistent scattering information coming from a source
 203 perturbed by an aberrating layer is known in acoustics. In the case of a relatively
 204 weak impedance discontinuity, the perturbation can be compensated by a phase-shift
 205 operation. In our case, severe perturbations, *i.e.*, leaky guided waves and reverberation,
 206 due to high impedance contrast, make it difficult to detect the scattering information
 207 coming from a point source. Dorne and Fink showed that a physical time-reversal
 208 operation on the perturbed signals (having integrated the perturbing layer) makes it
 209 possible to focus energy on a point source, and to reconstruct a consistent image [27].
 210 Adjoint field tomography (AFT) consists in computing a time-reversed field to recover
 211 the characteristic function of the scatterer. The AFT image is a representation in the
 212 space of parameters of the gradient of a functional measuring the error between observed
 213 data \mathbf{d}_{obs} and predicted synthetic data \mathbf{d}_{syn} , where $\mathbf{d} := [\phi, \mathbf{u}]$ stands for the degrees of
 214 freedom vector. The functional χ measuring the error is defined by the following equation

$$\chi(\mathbf{m}) := \frac{1}{2} \int_0^T \int_{\Gamma'_f} (\mathbf{d}_{obs}(\mathbf{x}, t) - \mathbf{d}_{syn}(\mathbf{x}, t, \mathbf{m}))^2 d\mathbf{x}dt \quad (7)$$

215 where Γ'_f is a part of the outer surface Γ_f (or a sphere surrounding the object) where
 216 the field is measured. The vector \mathbf{m} contains the parameters of the model at each node.
 217 The gradient of χ can be expressed in terms of the adjoint field \mathbf{d}_{adj} which is defined by
 218 the following equation.

$$L^* \mathbf{d}_{adj} = \mathbf{d}_{obs}(\mathbf{x}', t) - \mathbf{d}_{syn}(\mathbf{x}', t), \mathbf{x}' \in \Gamma_f \quad (8)$$

219

$$\nabla_{\mathbf{m}} \chi \delta \mathbf{m} = \int_0^T \int_{\Omega} \mathbf{d}_{adj}(\mathbf{x}, T-t) \nabla_{\mathbf{m}} L \delta \mathbf{m} d\mathbf{x}dt \quad (9)$$

220 with $\mathbf{x}' \in \Gamma'_f$, and L^* denotes the time-reversal ($T-t$) of the linear differential operator
 221 L associated with the model. The gradient of the functional can be decomposed into
 222 sensitivity kernels [28]. The sensitivity kernels take the following time-domain general
 223 form.

$$K_{\mathbf{m}}(\mathbf{x}) = \int_0^T \mathbf{d}_{adj}(\mathbf{x}, T-t) \nabla_{\mathbf{m}} L \delta \mathbf{m} dt \quad (10)$$

224 The domain of interest here is the liquid with parameters (ρ_0, κ) . Supposing that the
 225 perturbation given by the presence of bubbles is a density perturbation $\delta\rho_0$ (for quali-
 226 tative representation of the gradient). The associated density kernel for a shot index i
 227 takes the following form.

$$K_{\rho_0}^i(\mathbf{x}) = - \int_0^T \dot{\phi}_{adj}^i(\mathbf{x}, T-t) \dot{\phi}_{syn}^i(\mathbf{x}, t) \delta\rho_0 dt \quad (11)$$

228 where $\dot{\phi}$ denotes the time derivative of the velocity potential. A synthetic qualitative
 229 image I_{ρ_0} , can be obtained by summing the kernels on shots i , for N shots around the
 230 object to inspect.

$$I_{\rho_0}(\mathbf{x}) = \sum_{i=1}^N K_{\rho_0}^i(\mathbf{x}) \quad (12)$$

Parameter	Symbol	Value	unit
Pipe outer diameter	$2R$	150	mm
Pipe thickness	h	3	mm
Speed of sound in liquid	c_0	1480	m/s
Scatterer (rod) diameter	d	6	mm
Reference position of scatterer 1	r_0	0	mm
Reference position of scatterer 2	r_1	24.5	mm
Reference position of scatterer 3	r_2	48.5	mm
Aluminum			
Young modulus	E	70	GPa
Density	ρ	2700	kg/m ³
Poisson ratio	ν	0.33	
Plastic			
Young modulus	E	3.3	GPa
Density	ρ	1190	kg/m ³
Poisson ratio	ν	0.37	

Table 1: Numerical values.

231 *2.6. Experimental setup*

232 The experiments consisted in immersion scanning of pipes. Because beamforming is
233 determined mainly by the diffraction at the inner interface, immersion configuration only
234 affects the way the outer interface is excited. Steel rods 6 mm in diameter acting as pure
235 reflectors were placed in the pipe at the chosen positions r_0, r_1, r_2 . Each rod position was
236 measured individually. The diffraction-mode acquisition process was performed using
237 a laboratory ultrasonic immersion system with mechanical scanning. The acquisition
238 consisted in scanning the target in the diffraction tomography mode with 45 shots around
239 the target, giving an emission angular step of 8 degrees. The receiver range was a
240 limited backscattering aperture of 180 degrees divided into 11 positions, giving a reception
241 angular step of 16 degrees. The transducers used were IMASONICTM 500 kHz with an
242 active diameter of 42 mm and IMASONICTM 250 kHz with an active diameter of 44
243 mm. The electric excitation pulse was generated by an Olympus-PanametricsTM pulser-
244 receiver. The signals were digitized and recorded using a LecroyTM oscilloscope. The
245 sampling frequency was set at 50 MHz. The transducers were located at 225mm far from
246 the center.



Figure 4: Photograph of the immersion experimental setup.

247 *2.7. Methodology for experiments*

248 For both numerical and experimental testings, three configurations were selected:

- 249 • PMMA 3 mm 500 kHz, 3 rod positions
- 250 • aluminum 3 mm 500 kHz, 3 rod positions
- 251 • aluminum 3 mm 250 kHz, 3 rod positions

252 The direct ultrasonic field calculations were done according to the experimental setup
 253 configuration. The direct ultrasonic beam was computed considering the measured trans-
 254 ducer impulse response as the source in the model. For the simulations, adjoint fields
 255 were computed for 8 angular positions equally spaced around the pipe. The adjoint
 256 source locus consisted of the full aperture divided into 1024 points. For the experimental
 257 reconstruction, 9 angular positions equally spaced over 45 were computed. For both ad-
 258 joint field and adapted FBP, the source location corresponded to the transducer center
 259 of aperture. The inversion algorithm were applied to the subtracted signal to the empty
 260 pipe (no scatterers) measurements. The inversions were performed on CPU desktop
 261 computer.

262 The image analysis consisted in measuring (i) the consistency of the location of the
 263 reconstructed scatterers and (ii) the amplitude of the reconstructed scatterers. The
 264 amplitude of each scatterer position allowed to measure the radial amplitude decrease
 265 due to undersampling effect near the pipe wall. The rate of decrease was computed using
 266 a normalized (value of 1 at $r = 0$) exponential fit f :

$$f(r) = e^{-r/r_c} \quad (13)$$

267 where r is the radial position and r_c is the parameter of the fit, called the critical radius.
 268 The signal-to-noise ratio (SNR) was measured comparing the average amplitude of the
 269 scatterers A_s to the average amplitude of inconsistent information on the reconstructed
 270 images A_n that included: measurement apparatus electronic noise, circumferential echo
 271 perturbation, and reconstruction artifacts. The SNR is given in dB: $20 \log_{10}(A_s/A_n)$.
 272 The synthetic accuracy criterion for a given reconstruction was defined by the average
 273 absolute error measured taking the difference between reconstructed peak positions and
 274 real scatterer positions.

275 **3. Results**

276 The ultrasonic field computations for the three selected configurations are given in
 277 figure 5. The fields are given in linear normalized scale relative to the maximum ampli-
 278 tude of excitation from outside the pipe. The AFT simulation results are given in figures
 279 6, 7 and 8. The AFPB experiments results are given in figures 9, 10 and 11. The figures
 280 10a and 10b compare the effect of subtracting the empty pipe acquisition to the measured
 281 signals for the same acquisition. The radial cross sections of AFPB for the 3 rod positions
 282 for each configuration are given in figures 12, 13 and 14. Cross section normalization is
 283 done relatively to the centered scatterer amplitude. On the cross section figures, the real
 284 positions of the scatterers are given by vertical dotted lines. AFT experiments results
 285 are given in figures 15. Similarly radial cross sections of AFT for the 3 rod positions for
 286 the two tested configurations are given in figures 16 and 17. The tomography images are
 287 qualitative and are given in linear normalized grayscale. The measurements of critical
 288 radius and SNR (defined in the previous section) for all image reconstruction are given
 289 in table 2.

290 *3.1. Ultrasonic field: numerical results*

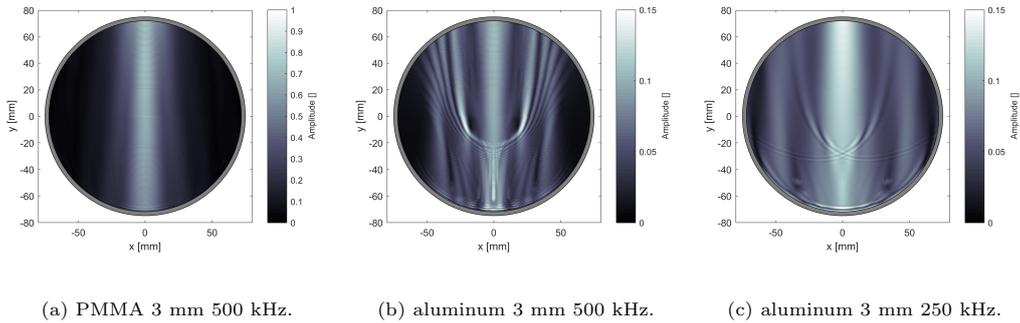
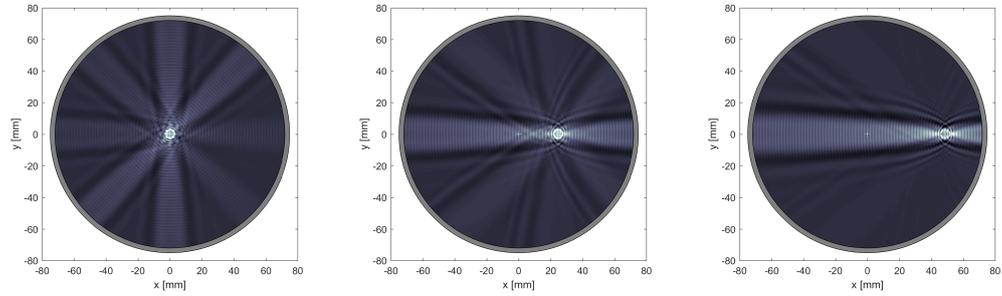
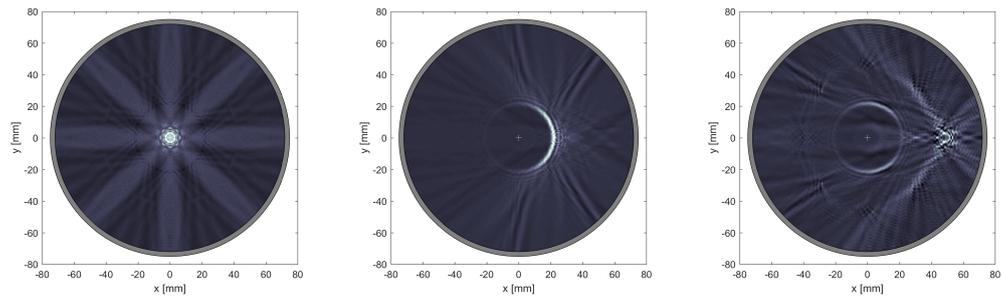


Figure 5: Numerical computations of ultrasonic field in the pipe, given in linear normalized scale relative to the maximum amplitude.



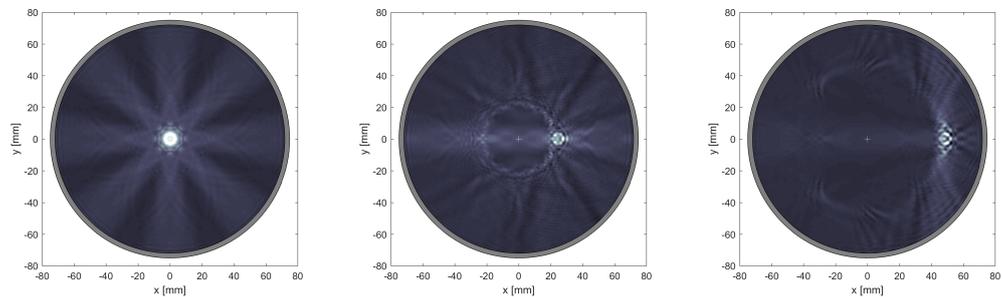
(a) 6 mm diameter rod at position r_0 . (b) 6 mm diameter rod at position r_1 . (c) 6 mm diameter rod at position r_2 .

Figure 6: 8 shots adjoint field tomography from simulated data for the PMMA pipe at 500kHz.



(a) 6 mm diameter rod at position r_0 . (b) 6 mm diameter rod at position r_1 . (c) 6 mm diameter rod at position r_2 .

Figure 7: 8 shots adjoint field tomography from simulated data for the aluminum pipe at 500kHz.

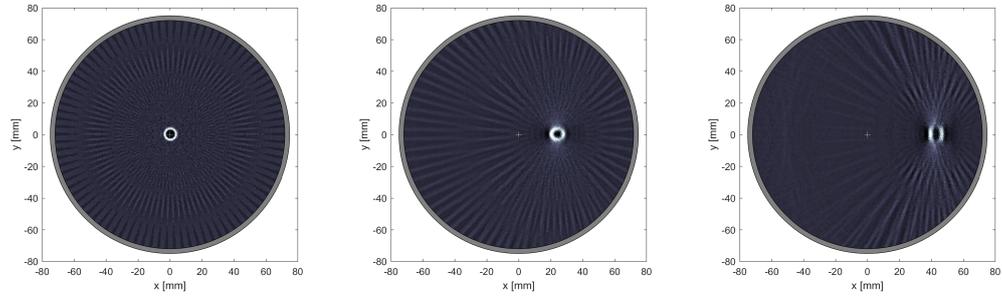


(a) 6 mm diameter rod at position r_0 . (b) 6 mm diameter rod at position r_1 . (c) 6 mm diameter rod at position r_2 .

Figure 8: 8 shots adjoint field tomography from simulated data for the aluminum pipe at 250kHz.

292 3.3. Experimental results

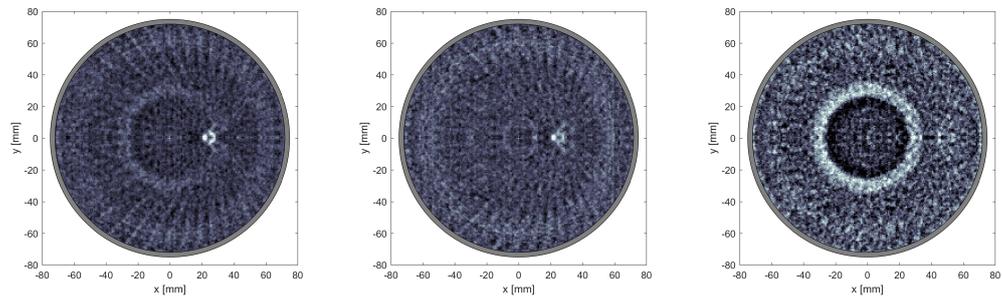
293 3.3.1. PMMA 3mm 500 kHz



(a) 6 mm diameter rod at position r0. (b) 6 mm diameter rod at position r1. (c) 6 mm diameter rod at position r2.

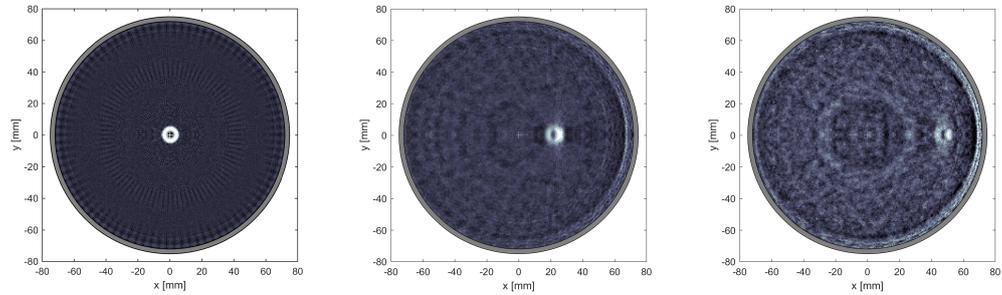
Figure 9: Tomography of PMMA pipe 3 mm 500 kHz, with adapted FPB.

294 3.3.2. ALU 3 mm 500 kHz



(a) 6 mm diameter rod at position r2 without subtraction of empty pipe reference signals. (b) 6 mm diameter rod at position r2 with subtraction of empty pipe reference signals. (c) 6 mm diameter rod at position r3 without subtraction of empty pipe reference signals.

Figure 10: Tomography of aluminum 3 mm pipe, 500 kHz, with adapted FPB.



(a) 6 mm diameter rod at position r_0 . (b) 6 mm diameter rod at position r_1 . (c) 6 mm diameter rod at position r_2 .

Figure 11: Tomography of aluminum pipe 3 mm 250 kHz, with adapted FPB.

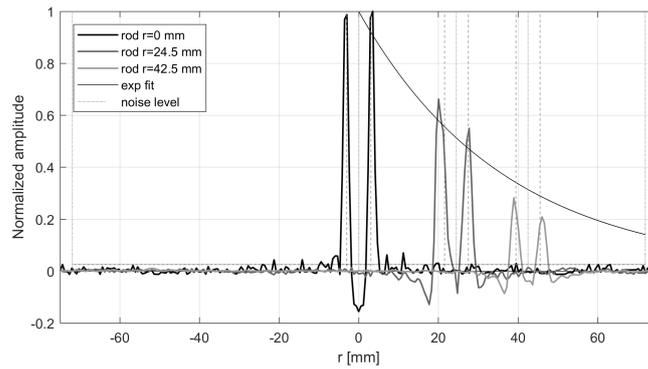


Figure 12: Radial cross section of PMMA 3 mm 500 kHz tomography for the 3 rod positions.

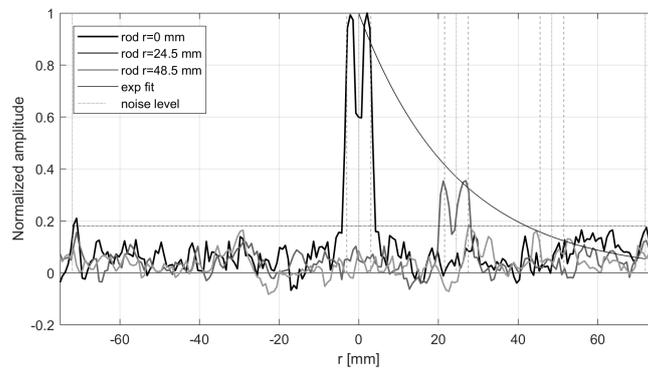


Figure 13: Radial cross section of aluminum 3 mm 500 kHz tomography for the 3 rod positions.

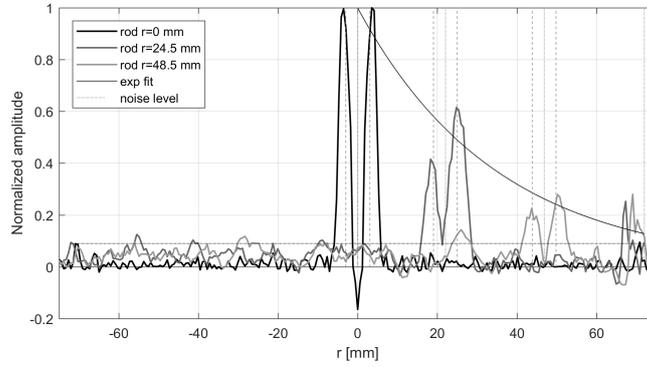
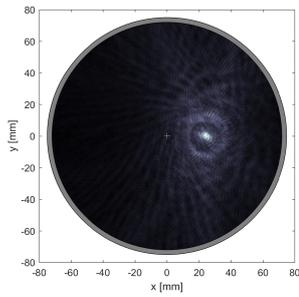
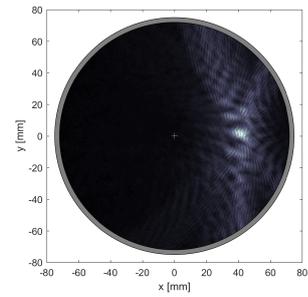


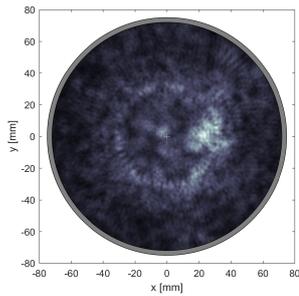
Figure 14: Radial cross section of aluminum 3 mm 250 kHz tomography for the 3 rod positions.



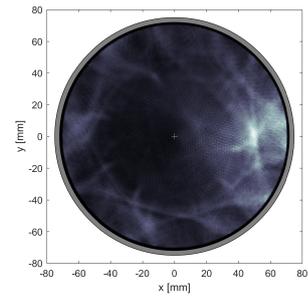
(a) PMMA 3 mm 500 kHz, 6 mm diameter rod at position r1.



(b) PMMA 3 mm 500 kHz, 6 mm diameter rod at position r2.



(c) aluminum 3 mm 250 kHz, 6 mm diameter rod at position r1.



(d) aluminum 3 mm 250 kHz, 6 mm diameter rod at position r2.

Figure 15: Adjoint field tomography results for different configurations, 9 shots.

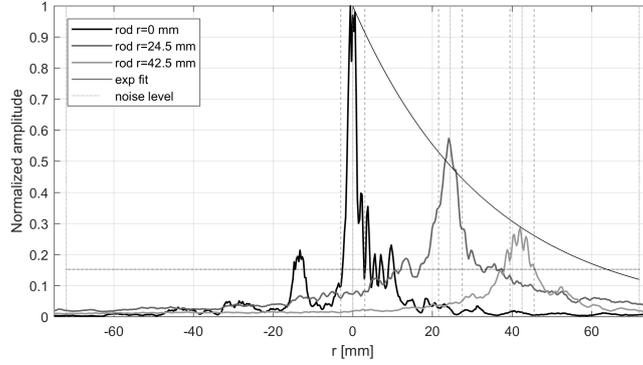


Figure 16: Radial cross section of AFT PMMA 3 mm 500 kHz tomography for the 3 rod positions.

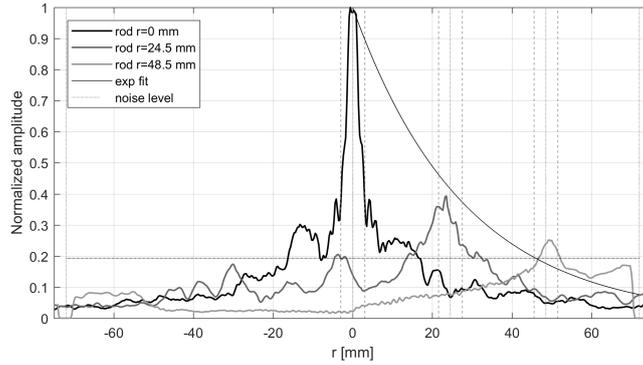


Figure 17: Radial cross section of AFT aluminum 3 mm 250 kHz tomography for the 3 rod positions.

296 3.3.4. Summary of results

Configuration	r_c (mm)	SNR	avg abs err (mm)
Simulation PMMA 3 mm 500 kHz	40.4	24.2 dB	0.21
Simulation aluminum 3 mm 500 kHz	25.7	17.3 dB	0.27
Simulation aluminum 3 mm 250 kHz	39.5	19.7 dB	0.23
Experiment PMMA 3 mm 500 kHz	36.7	32.0 dB	0.56
Experiment aluminum 3 mm 500 kHz	24.6	14.9 dB	0.58
Experiment aluminum 3 mm 250 kHz	36.0	20.9 dB	0.58
Experiment AFT PMMA 3 mm 500 kHz	33.9	16.4 dB	0.27
Experiment AFT aluminum 3 mm 250 kHz	28.0	14.3 dB	0.63

Table 2: Summary of results for all the configurations.

297 **4. Discussion**

298 The results of selected experiments allow to study the influence of (i) the impedance
299 mismatch due to the pipe wall and (ii) the radial position of scatterers on the detection
300 and on the resolution. The configurations are supposed to be valid for relatively small
301 scatterer concentration. As expected, good results are obtained for the PMMA pipe wall.
302 Using the adapted FBP algorithm, it possible to localize the scatterers with good accu-
303 racy, see figure 9. On the contrary, artifacts and low detection due to beam perturbation
304 is observed for the aluminum pipe wall. The direct ultrasonic field computations highlight
305 the beam perturbation. The beam remains unperturbed in the case of plastic, contrary
306 to the aluminum case. As can be seen in figures 5b and 5c, the perturbation is not only
307 an amplitude decrease, but also a narrowing effect and interferences. The adjoint field
308 method makes it possible to take into account the actual beam in the liquid. The simu-
309 lation results make it possible to compare ideal reconstruction to assess the limitations
310 of the method. Concerning the aluminum case, For both simulations and experiments,
311 the results show a ring artifact on the reconstruction that is due to the retropropagation
312 of a circumferential echo. The reconstructed images after subtraction of reference signals
313 (see 10b) show that it is possible to filter this echo. However, it increases the noise level.
314 Due to blinding effect, a dead zone in which the echoes amplitude coming from scatterers
315 are too low, the farthest scatterer from the center, for the aluminum 500 kHz case, is not
316 detected, see figure 10c. Indeed, as shown in the cross section in figure 13 the farthest
317 scatterer amplitude is below the noise level. As predicted by simulation results, better
318 detection is obtained using 250 kHz transducers, see figure 11. For the 250 kHz case,
319 the wavelength is about 6 mm, which is the diameter of the targets. The adjoint field
320 tomography results allow to compare the previous results with an other reconstruction
321 method. The goal is to minimize the number of adjoint field computation. Simulation
322 results with full aperture show that using only 8 shots, consistent scatterer localization
323 is obtained. However, severe artifacts are obtained in the case of aluminum pipe. Re-
324 constructions from experimental data, with limited aperture, show a good localization
325 of the scatterers computing only 9 adjoint fields, see figure 15. Increasing the number of
326 adjoint field computation will increase the contrast, but the computational cost will be
327 higher. The detection and contrast performance can be assessed using the two chosen
328 synthetic quantity: critical radius and SNR. The performance of experimental results are
329 consistent with the simulation results.

330 **5. Conclusion**

331 In this paper we showed that, even for ideal cases the ultrasound computed tomogra-
332 phy for bubble flow imaging has some limitations: spatial undersampling, reconstruction
333 artifacts, blinding effect, and lack of information. The quality of non-invasive imaging
334 through a pipe is strongly dependent on the material the pipe is made of. The ultra-
335 sonic beam is perturbed by the inherent acoustical impedance mismatch at the inner
336 pipe interface. This limits the performance of the reconstruction. Consequently, images
337 suffer presence of artifact, noise, low SNR. The 500 kHz configuration on the aluminum
338 pipe demonstrates the limitations of the method to detect bubble near the pipe wall.
339 However, in the case of aluminum, we showed that selecting a lower frequency increases
340 the detection and the contrast. The finite-element adjoint approach can be applied with

341 reasonable computational cost. An adapted FBP algorithm makes it possible to per-
342 form consistent and accurate reconstruction of scatterers acting as bubbles through a
343 pipe. Using this reconstruction technique, relatively high-quality images, with consistent
344 target localization and contour reconstruction, can be achieved for plastic pipes.

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425 Appendix A. Geometrical definitions and relations

426 The matrices \mathbf{R} are rotation matrices associated to a subscripted angle. The geomet-
427 ric locus $S_n(\theta, \gamma)$ is a ray belonging to the medium Ω_L and obeying the Snell's law for
428 an oblique incident ray making an angle γ with the normal of the circle of center O at
429 the position $a\theta$.

$$430 \quad \mathbf{x}_n = \mathbf{R}_{2\eta}^n \mathbf{R}_\eta a\theta \quad (\text{A.1})$$

$$431 \quad \theta := [\cos \theta, \sin \theta]^T \quad (\text{A.2})$$

431 The Snell's law at the interface $r = a$ is expressed by

$$432 \quad \frac{\sin \alpha}{\sin \vartheta} = \frac{c_0}{c_L} \quad (\text{A.3})$$

432 The ray unit direction vector director is

$$433 \quad \mathbf{v} = -\mathbf{R}_\alpha \frac{\mathbf{x}_n}{a} \quad (\text{A.4})$$

433 The acoustic Fresnel coefficients are

$$434 \quad r_{12} = \frac{z_2 \cos \vartheta - z_1 \cos \alpha}{z_2 \cos \vartheta + z_1 \cos \alpha} \quad (\text{A.5})$$

434

$$t_{12} = \frac{2z_1 \cos \alpha}{z_2 \cos \vartheta + z_1 \cos \alpha} \quad (\text{A.6})$$

435

$$z_i := \rho_i c_i \quad (\text{A.7})$$

436 The in wall propagation length l corresponds to the length of PM and is expressed by

$$l(\gamma) = \sqrt{h(h+2a) + a^2 \cos^2 \vartheta} - a \cos \vartheta \quad (\text{A.8})$$

437 Geometric relations are given by the following equations.

$$\sin \alpha(\gamma) = \frac{R+d}{R} \frac{c_0}{c_1} \sin \gamma \quad (\text{A.9})$$

438

$$\cos \vartheta = \sqrt{1 - \frac{\sin^2 \gamma (a+h)^2}{a^2}} \quad (\text{A.10})$$

439

$$\sin \eta = \frac{l(\gamma)}{a} \sin \gamma \quad (\text{A.11})$$

440 The distance of a refracted ray S_n to the center O is $a \sin \alpha$. The 1D filter along this
 441 variable distance is obtained by a ramp filter of the projections weighted by the factor
 442 $\frac{a+h}{a} \frac{c_0}{c_L}$, which tends to 1 when $c_L \rightarrow c_0, h \rightarrow 0$.