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# Sensor fault detection for switched systems using interval observer with $L_\infty$ performance

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## Abstract

This paper deals with sensor fault detection (FD) for discrete-time switched systems subject to bounded disturbances. A novel approach is investigated to construct residual framers using interval observer with  $L_\infty$  performance. The proposed technique is used to provide more degrees of design freedom and to obtain accurate FD results. The design conditions of the FD observer are given in terms of Linear Matrix Inequalities (LMIs) adopting firstly a common quadratic Lyapunov function, under an arbitrary switching signal and secondly multiple quadratic Lyapunov functions, under an Average Dwell Time (ADT) switching signal. Furthermore, the FD decision is based on residual intervals generated by the proposed interval observer. The effectiveness of the proposed approach is highlighted through simulation results of an academic example.

*Keywords:* Fault detection, switched systems, interval observer,  $L_\infty$  performance

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## 1. Introduction

Due to the increasing demand for higher performance and safety, fault diagnosis for dynamic systems is becoming a major technological challenging issue in

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many engineering fields, such as automotive [1] and aerospace [2, 3]. Faults are  
5 generally unavoidable especially for complex systems and may lead to significant  
performance degradation. Therefore, early alarms of faults in complex systems  
is a crucial task to prevent serious system damage and to improve safety and  
reliability. In presence of uncertainties, robust fault detection is a necessity to  
avoid false alarms in model-based fault diagnosis.

10 In the past decades, several techniques have been introduced for various  
systems in the field of fault diagnosis such as [4, 5, 6, 7, 8]. For instance, in  
[5], a robust FD observer is investigated for a Takagi Sugeno fuzzy system with  
sensor faults and unknown bounded disturbances. Using an augmented model  
combined with an interval observer, the robust actuator fault estimation for a  
15 discrete-time descriptor system with unknown disturbances and measurement  
noises is developed in [9]. In [10], actuator FD and isolation are investigated  
for discrete-time linear descriptor systems. In [11], the authors propose a new  
approach of fault estimation for a class of uncertain nonlinear systems with  
simultaneous unknown input and actuator faults.

20 Switched system is one of the most important complex systems [12] and  
plays an essential role in many engineering applications. They are composed of  
continuous dynamical subsystems combined with a discrete rule that operates  
switching between these subsystems. Many efforts have been devoted to FD  
techniques for switched systems in recent years [13, 14, 15]. This motivation  
25 drives from the fact that the class of switched systems can model and control  
a wide range of physical and engineering systems. In [15], the FD issue is  
investigated for discrete-time switched systems in the finite-frequency domain.  
The finite frequency  $L_2$ -gain is established for characterizing the disturbance  
attenuation performance. The work in [13] is concerned with the  $H_\infty$ -filtering  
30 formulation of FD problems for a class of discrete-time switched systems with  
time-varying state delays. Without using decoupling transformations, a high-  
order sliding mode observer is designed in [16] for the detection of faults that  
occur in the continuous part of the switched system.

In the presence of uncertainties coming either from external disturbances

35 or from the mismatch between the model and the real system, interval ob-  
 servers have been proven to be effective in determining whether the system is  
 faulty or not, since they have been successfully applied to several practical prob-  
 lems and can deal with large uncertainties [17, 18, 19, 20]. In [17], an interval  
 observer-based FD method is investigated for discrete-time linear parameter-  
 40 varying systems. The FD decision is made by determining whether the zero  
 signal is excluded from the residual interval when the faults occur. The pro-  
 posed technique avoids the design of residual evaluation functions and threshold  
 generators. In [18], interval observers are designed for a class of switched sys-  
 tems with unstable modes. A fault detection approach is introduced where  
 45 fault sensitivity as well as disturbance robustness performances are treated. In  
 [19], fault estimation is performed for high-speed railway traction motor in the  
 presence of sensor fault and disturbances using interval and unknown input ob-  
 servers. In [20], a robust fault detection and isolation technique is proposed. By  
 combining the set-invariance approach with a zonotopic interval observer, the  
 50 minimum detectable and isolable fault is characterized. In [21], residual inter-  
 vals are generated based on a switched Takagi-Sugeno interval observer. Such  
 observers provide at each time instant adaptive thresholds adjusted according  
 to the real-time measurements. More recently, the authors in [22] propose a  
 FD observer for continuous-time switched linear systems subject to bounded  
 55 disturbances. Based on the concept of interval observers, the generated residual  
 intervals can be used directly to make the FD decision. In practice, the perfor-  
 mance of model-based fault detection may be affected by modelling uncertain-  
 ties, measured noise and unknown input disturbances. However, the interval  
 observer in [22] cannot provide accurate FD results since the effects of uncer-  
 60 tainties on the residual framers are not minimized. To handle this problem, a  
 robust fault detection technique is required. Many researches are intensively  
 investigated in the field of robust diagnosis using  $H_\infty$  performances to analyze  
 the robustness against uncertainties [23] and [24]. Note that the  $H_\infty$  norm is  
 a measurement of energy-to-energy gain which may not be very convenient for  
 65 residual evaluation. In fact, it requires that the disturbances should be energy-

bounded signals. However, from a practical point of view, some signals cannot be considered energy-bounded but have bounded peak values. Therefore, it is more reasonable to use the  $L_\infty$  analysis which describes the peak-to-peak performance index. It can be considered as an alternative solution to enhance fault  
70 detection performances in many applications such as mechanical systems control, communication networks, embedded control models, power systems, traffic control and some other areas. For instance, in [25], the  $L_\infty$  technique is used for discrete-time Takagi-Sugeno fuzzy systems to measure the worst effect of disturbances on residuals instead of the  $H_\infty$  norm. To the best of the authors' knowledge,  $L_\infty$  interval FD observers have not been fully investigated in the  
75 case of switched systems.

Interval observer design techniques are usually based on the theory of positive systems, which require the non-negativity of the error dynamics. Unfortunately, this assumption is still restrictive and may lead to high computational  
80 complexity for a gain matrix. Some methods propose a coordinate transformation to relax the design conditions of interval observers. However, as mentioned in [26], it is hard to merge the coordinate transformation approach with some additional constraints related to control such as disturbance attenuation performance. In view of this major drawback, the authors in [27] propose a novel  
85 interval observer structure (TNL structure) that can give more degrees of design freedom. The basic idea consists of introducing weighted matrices in addition to the gain matrix. First, it offers a solution when a gain matrix cannot be founded. Second, better performances may be achieved. Inspired by the structure of the interval observer given in [27], this paper presents a new interval  
90 observer-based FD method for a class of discrete-time switched systems.

Motivated by the above discussion, the aim of this paper is to achieve a set-membership technique to detect faults for discrete-time switched systems based on the  $L_\infty$  criterion. Common and multiple Lyapunov functions under an ADT are used to study the stability analysis. The main contributions of this paper  
95 lie in the following aspects:

- Compared with [17, 28], a new observer structure is investigated for switched systems. The proposed approach allows reducing the conservatism of gain matrices and offers more degrees of design freedom by integrating weighted matrices in the structure of the FD observer design.
- 100 • Compared with the traditional methods of designing constant or time-varying thresholds, the present method provides a systematic way for residual evaluation based on a belonging test of the zero signal to the residual framers generated by the proposed FD observers.
- Using common and multiple quadratic Lyapunov functions under an ADT control scheme, novel solvable conditions are derived in terms of LMIs.
- 105

The reminder of this paper is structured as follows. Notations and some preliminaries are introduced in Section 2. In Section 3, the problem statement is presented. Main results are described in Section 4. In Section 5, simulation results are shown to illustrate the effectiveness of the proposed methods. Finally, 110 the paper is concluded in Section 6.

## 2. Preliminaries

**Notation.** In the sequel, the following notations are adopted.  $\mathbb{R}$ ,  $\mathbb{R}^n$  and  $\mathbb{R}^{m \times n}$  denote respectively the sets of real numbers,  $n$  dimensional and  $m \times n$  dimensional Euclidean space. For a signal  $x \in \mathbb{R}^n$ ,  $\|x\|$  represents the Euclidean 115 norm of  $x$  and the  $L_\infty$  norm of a signal  $x$  is defined as  $\|x\|_\infty = \sup_{k \geq 0} \|x(k)\|$ . We denote by  $\mathcal{S} = \overline{1, N}$ ,  $N \in \mathbb{Z}_+$  the set of non-negative integers  $\{1, \dots, N\}$ .  $\mathbb{Z}$  is used to denote the set of all integers. In symmetric block matrices,  $(*)$  denotes the terms introduced by symmetry.  $I_n$  represents the identity matrix and  $O_n$  denotes a  $n \times n$  null matrix. For a matrix  $P$ ,  $P \succ 0$  means that  $P$  120 is positive definite,  $P^\dagger$  represents its pseudo-inverse. For a matrix  $A \in \mathbb{R}^{m \times n}$ , let us define  $A^+ = \max\{0, A\}$ ,  $A^- = A^+ - A$  and  $|A| = A^+ + A^-$ . A class of  $\mathcal{K}$ -functions consists of all functions  $\alpha : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which are continuous, strictly increasing, and satisfying  $\alpha(0) = 0$ .  $\alpha$  is a  $\mathcal{K}_\infty$ -function if it is a  $\mathcal{K}$ -function and

$\alpha(s) \rightarrow \infty$  as  $s \rightarrow \infty$ . Throughout this paper, the following inequalities  $\leq, \geq,$   
125  $<$  and  $>$  should be interpreted elementwise.

The following definitions and lemmas are used in this paper.

**Definition 1.** Consider a linear discrete-time switched system:

$$\begin{cases} x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k) + w(k) \\ y(k) = Cx(k) + v(k) \end{cases}, \sigma(k) \in \mathcal{I} \quad (1)$$

where  $x \in \mathbb{R}^{n_x}$ ,  $u \in \mathbb{R}^{n_u}$ ,  $y \in \mathbb{R}^{n_y}$ ,  $w \in \mathbb{R}^{n_w}$  and  $v \in \mathbb{R}^{n_v}$  are respectively the  
state vector, the input, the output, the state disturbances and the measurement  
130 noises. The switchings between the subsystems are ensured via a switching signal  
represented as a piecewise constant function,  $\sigma(k) : \mathbb{Z}_+ \rightarrow \mathcal{I}$ .

**Lemma 1.** [29] Let  $A \in \mathbb{R}^{m \times n}$  be a constant matrix and  $x \in \mathbb{R}^n$  be a vector  
such that  $\underline{x} \leq x \leq \bar{x}$ , thus

$$A^+\underline{x} - A^-\bar{x} \leq Ax \leq A^+\bar{x} - A^-\underline{x}.$$

**Definition 2.** A matrix  $A \in \mathbb{R}^{n \times n}$  is called non-negative if all its elements are  
non-negative.

**Lemma 2.** [30] Consider the system described by:

$$x(k+1) = Ax(k) + u(k) \quad , \quad u : \mathbb{Z}_+ \rightarrow \mathbb{R}_+^n, \quad k \in \mathbb{Z}_+ \quad (2)$$

with  $x \in \mathbb{R}^n$ . The system (2) is said cooperative or non-negative if and only if  
135  $u(k) \geq 0$  for all  $k \geq 0$ ,  $x(0) \geq 0$  and  $A$  is a non-negative matrix.

**Definition 3.** [31] We denote by  $N_\sigma(k_a, k_b)$  the number of discontinuities of a  
switching signal  $\sigma$  on an interval  $(k_a, k_b)$ .  $\sigma(k)$  has an ADT  $\tau_a$  if there exist  
two positive numbers  $N_0$  and  $\tau_a$  such that:

$$N_\sigma(k_a, k_b) \leq N_0 + \frac{k_b - k_a}{\tau_a}, \quad \forall k_b \geq k_a \geq 0$$

**Lemma 3.** [32] Consider the switched system (1), and let  $0 < \lambda < 1$ ,  $\rho > 1$ . Suppose that there exist  $V_{\sigma(k)} : \mathbb{R}^n \rightarrow \mathbb{R}$  and two  $\mathcal{K}_\infty$  functions  $a_1$  and  $a_2$  such that for each  $\sigma(k) = q, q \neq l$ , the following conditions hold:

$$a_1(\|x(k)\|) \leq V_q(x(k)) \leq a_2(\|x(k)\|) \quad (3)$$

$$\Delta V_q(x(k)) \leq -\lambda V_q(x(k)) \quad (4)$$

$$V_q(x(k)) \leq \rho V_l(x(k)) \quad (5)$$

then, the system (1) is Input to State Stable (ISS) for any switching signal with an ADT  $\tau_a$ .

$$\tau_a \geq \tau_a^* = -\frac{\ln(\rho)}{\ln(1-\lambda)} \quad (6)$$

where  $\tau_a^*$  is the lower bound of  $\tau_a$  determined by both parameters  $\rho$  and  $\lambda$ .

The previous Lemma is common in the literature when the ADT switching signal is considered.

**Lemma 4.** [33] Given matrices  $A \in \mathbb{R}^{a \times b}$ ,  $B \in \mathbb{R}^{b \times c}$  and  $C \in \mathbb{R}^{a \times c}$ , if  $\text{rank}(B) = c$ , then the general solution of the following equation  $AB = C$  is given by

$$A = CB^\dagger + S(I - BB^\dagger)$$

where  $S \in \mathbb{R}^{a \times b}$  is an arbitrary matrix.

### 140 3. Problem statement

Consider the following discrete-time switched system:

$$\begin{cases} x(k+1) = A_q x(k) + B_q u(k) + D_q w(k) \\ y(k) = C x(k) + D_v v(k) + F f(k), \end{cases} \quad (7)$$

where  $x \in \mathbb{R}^{n_x}$ ,  $u \in \mathbb{R}^{n_u}$ ,  $y \in \mathbb{R}^{n_y}$ ,  $f \in \mathbb{R}^{n_f}$ ,  $w \in \mathbb{R}^{n_w}$  and  $v \in \mathbb{R}^{n_v}$  are respectively the state vector, the input, the output, the sensor fault, the state



disturbances and the measurement noises. The known matrices  $A_q, B_q, C, D_q, D_v$  and  $F$  are given with appropriate dimensions. The index  $q$  specifies, at each discrete instant  $k$ , the subsystem that is currently followed.  $q \in \mathcal{S} = \overline{1, N}, N \in \mathbb{Z}_+, N$  is the number of linear subsystems. The switching signal is assumed to be known. Some assumptions are introduced for the rest of the paper.

**Assumption 1.** *Assume that the state disturbances and the measurement noises are unknown but bounded with a priori known bounds such that*

$$\underline{w} \leq w \leq \bar{w}, \quad \underline{v} \leq v \leq \bar{v}$$

where  $\underline{w}, \bar{w} \in \mathbb{R}^{n_w}$  and  $\underline{v}, \bar{v} \in \mathbb{R}^{n_v}$ .

**Assumption 2.** *The pairs  $(A_q, C)$  are detectable,  $\forall q = 1, \dots, N$ .*

In the following, the goal is to design residual framers based on robust FD interval observers for discrete-time linear switched systems subject to sensor faults. An  $L_\infty$  criterion is developed in this paper in order to compute the observer gains and to take into account the presence of state disturbances and measurement noises in the design of the robust FD procedure.

## 4. Main results

In this section, a new technique is investigated to design residual framers based on the TNL structure interval observer. LMIs conditions are derived using firstly the common quadratic Lyapunov function under an arbitrary switching signal and secondly by adopting multiple quadratic Lyapunov functions under an ADT switching signal. The proposed method allows reducing the conservatism of gain matrices by providing more degrees of freedom and obtaining accurate fault detection results.

### 4.1. A common Lyapunov function based approach

In this part, the aim is to design a FD approach for discrete-time switched systems defined by (7). The proposed method is robust against state disturbances and measurement noises. The stability analysis is analyzed by applying the common Lyapunov function.

We introduce the following assumption needed in the remainder of this paper.

**Assumption 3.** *The upper and lower bounds of the initial state,  $\bar{x}(0)$  and  $\underline{x}(0)$  are chosen such that  $\underline{x}(0) \leq x(0) \leq \bar{x}(0)$ .*

Inspired by the structure of the observer proposed in [27], a FD interval observer is introduced. The proposed observer can reduce the conservatism of gain matrices and provide more degree of freedom by introducing weighted matrices  $\bar{T}_q$ ,  $\underline{T}_q$ ,  $\bar{N}_q$  and  $\underline{N}_q$ . The FD interval observer is given as follow:  
 $\forall q = 1, \dots, N$

$$\left\{ \begin{array}{l} \bar{\xi}(k+1) = \bar{T}_q A_q \bar{x}(k) + \bar{T}_q B_q u(k) + \bar{L}_q (y(k) - C \bar{x}(k)) + \bar{\Delta} \\ \bar{x}(k) = \bar{\xi}(k) + \bar{N}_q y(k) \\ \underline{\xi}(k+1) = \underline{T}_q A_q \underline{x}(k) + \underline{T}_q B_q u(k) + \underline{L}_q (y(k) - C \underline{x}(k)) + \underline{\Delta} \\ \underline{x}(k) = \underline{\xi}(k) + \underline{N}_q y(k) \\ \bar{y}(k) = C^+ \bar{x}(k) - C^- \underline{x}(k) + D_v^+ \bar{v} - D_v^- \underline{v} \\ \underline{y}(k) = C^+ \underline{x}(k) - C^- \bar{x}(k) + D_v^+ \underline{v} - D_v^- \bar{v} \\ \bar{r}(k) = \bar{y}(k) - y(k) \\ \underline{r}(k) = \underline{y}(k) - y(k) \end{array} \right. \quad (8)$$

where  $\bar{\xi}(k)$ ,  $\underline{\xi}(k) \in \mathbb{R}^{n_x}$  denote intermediate variables,  $\bar{x}(k)$ ,  $\underline{x}(k) \in \mathbb{R}^{n_x}$  are the upper and lower bounds of  $x(k)$  respectively.  $\bar{\Delta}$  and  $\underline{\Delta}$  are given by:

$$\left\{ \begin{array}{l} \bar{\Delta} = (\bar{T}_q D_q)^+ \bar{w} - (\bar{T}_q D_q)^- \underline{w} + (\bar{L}_q D_v)^+ \bar{v} - (\bar{L}_q D_v)^- \underline{v} \\ \quad + (\bar{N}_q D_v)^+ \bar{v} - (\bar{N}_q D_v)^- \underline{v} \\ \underline{\Delta} = (\underline{T}_q D_q)^+ \underline{w} - (\underline{T}_q D_q)^- \bar{w} + (\underline{L}_q D_v)^+ \underline{v} - (\underline{L}_q D_v)^- \bar{v} \\ \quad + (\underline{N}_q D_v)^+ \underline{v} - (\underline{N}_q D_v)^- \bar{v} \end{array} \right.$$

In (8),  $\bar{L}_q \in \mathbb{R}^{n_x \times n_y}$  and  $\underline{L}_q \in \mathbb{R}^{n_x \times n_y}$  are the observer gains.  $\bar{T}_q \in \mathbb{R}^{n_x \times n_x}$ ,  $\underline{T}_q \in \mathbb{R}^{n_x \times n_x}$ ,  $\bar{N}_q \in \mathbb{R}^{n_x \times n_y}$  and  $\underline{N}_q \in \mathbb{R}^{n_x \times n_y}$  are constant matrices that should be designed to satisfy

$$\bar{T}_q + \bar{N}_q C = I_{n_x} \quad (9)$$

$$\underline{T}_q + \underline{N}_q C = I_{n_x} \quad (10)$$

Based on Lemma 4, the general solutions of (9) and (10) are given by

$$\begin{bmatrix} \bar{T}_q & \bar{N}_q \end{bmatrix} = \begin{bmatrix} I_{n_x} \\ C \end{bmatrix}^\dagger + \bar{S}_q \left( I_{n_x+n_y} - \begin{bmatrix} I_{n_x} \\ C \end{bmatrix} \begin{bmatrix} I_{n_x} \\ C \end{bmatrix}^\dagger \right) \quad (11)$$

$$\begin{bmatrix} \underline{T}_q & \underline{N}_q \end{bmatrix} = \begin{bmatrix} I_{n_x} \\ C \end{bmatrix}^\dagger + \underline{S}_q \left( I_{n_x+n_y} - \begin{bmatrix} I_{n_x} \\ C \end{bmatrix} \begin{bmatrix} I_{n_x} \\ C \end{bmatrix}^\dagger \right) \quad (12)$$

where  $\bar{S}_q, \underline{S}_q \in \mathbb{R}^{n_x \times (n_x+n_y)}$  for  $q = 1, \dots, N$  are arbitrary matrices which will  
175 be designed such that all matrices  $\bar{T}_q, \underline{T}_q$  are of full rank.

Under the designed FD interval observer (8), the objective is to compute the observer gains  $\bar{L}_q$  and  $\underline{L}_q$  that minimize the effect of state disturbances and measurement noises on the upper and lower bounds of the residual vectors  $\bar{r}(k)$ ,  $\underline{r}(k)$ , respectively. Let  $\bar{e}(k) = \bar{x}(k) - x(k)$  and  $\underline{e}(k) = \underline{x}(k) - x(k)$  be the upper and the lower observation errors. By combining (7), (9) and (10),  $x(k+1)$  can be written in two different ways:

$$\begin{aligned} x(k+1) &= (\bar{T}_q + \bar{N}_q C)x(k+1) \\ &= \bar{T}_q x(k+1) + \bar{N}_q (y(k+1) - D_v v(k+1) - Ff(k+1)) \\ &= \bar{T}_q A_q x(k) + \bar{T}_q B_q u(k) + \bar{T}_q D_q w(k) + \bar{N}_q y(k+1) - \bar{N}_q D_v v(k+1) \\ &\quad - \bar{N}_q Ff(k+1) \end{aligned} \quad (13)$$

$$\begin{aligned} x(k+1) &= (\underline{T}_q + \underline{N}_q C)x(k+1) \\ &= \underline{T}_q x(k+1) + \underline{N}_q (y(k+1) - D_v v(k+1) - Ff(k+1)) \\ &= \underline{T}_q A_q x(k) + \underline{T}_q B_q u(k) + \underline{T}_q D_q w(k) + \underline{N}_q y(k+1) - \underline{N}_q D_v v(k+1) \\ &\quad - \underline{N}_q Ff(k+1) \end{aligned} \quad (14)$$

Then, the dynamics of the upper and lower errors are given by:

$$\begin{cases} \bar{e}(k+1) = (\bar{T}_q A_q - \bar{L}_q C) \bar{e}(k) + \bar{\Delta} + \bar{L}_q D_v v(k) + \bar{N}_q D_v v(k+1) \\ \quad - \bar{T}_q D_q w(k) + \bar{L}_q F f(k) + \bar{N}_q F f(k+1) \\ \underline{e}(k+1) = (\underline{T}_q A_q - \underline{L}_q C) \underline{e}(k) + \underline{\Delta} + \underline{L}_q D_v v(k) + \underline{N}_q D_v v(k+1) \\ \quad - \underline{T}_q D_q w(k) + \underline{L}_q F f(k) + \underline{N}_q F f(k+1) \end{cases} \quad (15)$$

We introduce

$$\bar{d}(k) = \begin{bmatrix} \bar{\Delta} - \bar{T}_q D_q w(k) \\ D_v v(k) \\ D_v v(k+1) \end{bmatrix}, \quad \underline{d}(k) = \begin{bmatrix} \underline{\Delta} - \underline{T}_q D_q w(k) \\ D_v v(k) \\ D_v v(k+1) \end{bmatrix}, \quad \tilde{f}(k) = \begin{bmatrix} f(k) \\ f(k+1) \end{bmatrix}$$

Accordingly, the error dynamics in (15) can be rewritten as

$$\begin{cases} \bar{e}(k+1) = (\bar{T}_q A_q - \bar{L}_q C) \bar{e}(k) + \bar{H}_q \bar{d}(k) + \bar{F}_q \tilde{f}(k) \\ \underline{e}(k+1) = (\underline{T}_q A_q - \underline{L}_q C) \underline{e}(k) + \underline{H}_q \underline{d}(k) + \underline{F}_q \tilde{f}(k) \end{cases} \quad (16)$$

where

$$\bar{H}_q = \begin{bmatrix} I_n \\ \bar{L}_q^T \\ \bar{N}_q^T \end{bmatrix}^T, \quad \underline{H}_q = \begin{bmatrix} I_n \\ \underline{L}_q^T \\ \underline{N}_q^T \end{bmatrix}^T, \quad \bar{F}_q = \begin{bmatrix} (\bar{L}_q F)^T \\ (\bar{N}_q F)^T \end{bmatrix}^T, \quad \underline{F}_q = \begin{bmatrix} (\underline{L}_q F)^T \\ (\underline{N}_q F)^T \end{bmatrix}^T$$

**Remark 1.** One can notice that when  $\bar{T}_q = I_{n_x}$ ,  $\underline{T}_q = I_{n_x}$ ,  $\bar{N}_q = O_{n_x}$  and  $\underline{N}_q = O_{n_x}$ , the proposed interval observer in (8) can be written as follows  $\forall q = 1, \dots, N$ :

$$\begin{cases} \bar{x}(k+1) = A_q \bar{x}(k) + B_q u(k) + \bar{L}_q (y(k) - C \bar{x}(k)) + \bar{\Delta} \\ \underline{x}(k+1) = A_q \underline{x}(k) + B_q u(k) + \underline{L}_q (y(k) - C \underline{x}(k)) + \underline{\Delta} \end{cases} \quad (17)$$

where  $\bar{\Delta}$  and  $\underline{\Delta}$  are given by:

$$\begin{cases} \bar{\Delta} = D_q^+ \bar{w} - D_q^- \underline{w} + (\bar{L}_q D_v)^+ \bar{v} - (\bar{L}_q D_v)^- \underline{v} \\ \underline{\Delta} = D_q^+ \underline{w} - D_q^- \bar{w} + (\underline{L}_q D_v)^+ \underline{v} - (\underline{L}_q D_v)^- \bar{v} \end{cases}$$

The error dynamics can be deduced from (17) and given by:

$$\begin{cases} \bar{e}(k+1) = (A_q - \bar{L}_q C)\bar{e}(k) + \bar{\Delta} + \bar{L}_q D_v v(k) - D_q w(k) + \bar{L}_q F f(k) \\ \underline{e}(k+1) = (A_q - \underline{L}_q C)\underline{e}(k) + \underline{\Delta} + \underline{L}_q D_v v(k) - D_q w(k) + \underline{L}_q F f(k) \end{cases} \quad (18)$$

where

$$\bar{d}(k) = \begin{bmatrix} \bar{\Delta} - D_q w(k) \\ D_v v(k) \end{bmatrix}, \quad \underline{d}(k) = \begin{bmatrix} \underline{\Delta} - D_q w(k) \\ D_v v(k) \end{bmatrix}$$

$$\bar{H}_q = \begin{bmatrix} I_n & \bar{L}_q \end{bmatrix}, \quad \underline{H}_q = \begin{bmatrix} I_n & \underline{L}_q \end{bmatrix}$$

In this case, the gain matrices  $\bar{L}_q \in \mathbb{R}^{n_x \times n_y}$  and  $\underline{L}_q \in \mathbb{R}^{n_x \times n_y}$  should be designed such that  $A_q - \bar{L}_q C$  and  $A_q - \underline{L}_q C$  are non-negative  $\forall q = 1, \dots, N$ . This  
180 Assumption is still restrictive and conservative. Consequently, by introducing weighted matrices  $\bar{T}_q$ ,  $\underline{T}_q$ ,  $\bar{N}_q$  and  $\underline{N}_q$ , the proposed FD observer in (8) can reduce the conservatism of gain matrices and provide more degree of freedom.

On the basis of the error system of (16), the non-negativity, the stability and the robustness of the proposed interval observer are studied in the following  
185 theorems.

**Theorem 1.** For system (7), let Assumption 1 hold,  $\bar{x}(k)$  and  $\underline{x}(k)$  in (8) satisfy in the fault free case ( $f = 0$ ) the inclusion

$$\underline{x}(k) \leq x(k) \leq \bar{x}(k)$$

if  $\bar{T}_q A_q - \bar{L}_q C$  and  $\underline{T}_q A_q - \underline{L}_q C$  are non-negative for all  $k \geq 0$  and  $\bar{x}(0)$ ,  $\underline{x}(0)$  are chosen such that  $\underline{x}(0) \leq x(0) \leq \bar{x}(0)$ .

*Proof.* In the fault free case ( $f = 0$ ), according to Assumption 1, we have

$$\begin{aligned} \bar{\Delta} - \bar{T}_q D_q w(k) + \bar{L}_q D_v v(k) + \bar{N}_q D_v v(k+1) &\geq 0 \\ \underline{\Delta} - \underline{T}_q D_q w(k) + \underline{L}_q D_v v(k) + \underline{N}_q D_v v(k+1) &\leq 0 \end{aligned}$$

In addition, let Assumption 3 be satisfied. Then,  $\bar{e}(0) \geq 0$  and  $\underline{e}(0) \leq 0$ . Applying Lemma 2 to (15), the inclusion

$$\underline{x}(k) \leq x(k) \leq \bar{x}(k)$$

holds for all  $k \geq 0$  if  $\bar{T}_q A_q - \bar{L}_q C$  and  $\underline{T}_q A_q - \underline{L}_q C$  are non-negative.  $\square$

In order to study the stability of the proposed residual framers, we propose an augmented state defined by  $\mathcal{E}(k) = [\bar{e}(k)^T \quad \underline{e}(k)^T]^T$  and  $\mathcal{R}(k) = [\bar{r}(k)^T \quad \underline{r}(k)^T]^T$ . The corresponding augmented system can be deduced:

$$\begin{cases} \mathcal{E}(k+1) = \mathcal{A}_q \mathcal{E}(k) + \mathcal{H}_q d(k) + \tilde{\mathcal{F}}_q \tilde{f}(k) \\ \mathcal{R}(k) = \mathcal{C} \mathcal{E}(k) + \mathcal{V} \tilde{v}(k) + \mathcal{F} f(k) \end{cases} \quad (19)$$

where

$$\mathcal{A}_q = \begin{bmatrix} \bar{T}_q A_q - \bar{L}_q C & 0 \\ 0 & \underline{T}_q A_q - \underline{L}_q C \end{bmatrix}, \quad \mathcal{H}_q = \begin{bmatrix} \bar{H}_q & 0 \\ 0 & \underline{H}_q \end{bmatrix}, \quad \tilde{\mathcal{F}}_q = \begin{bmatrix} \bar{F}_q \\ \underline{F}_q \end{bmatrix}$$

$$\mathcal{F} = \begin{bmatrix} -F \\ -F \end{bmatrix}, \quad \mathcal{C} = \begin{bmatrix} C^+ & -C^- \\ -C^- & C^+ \end{bmatrix}, \quad \mathcal{V} = \begin{bmatrix} -D_v & D_v^+ & -D_v^- \\ -D_v & -D_v^- & D_v^+ \end{bmatrix}$$

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$$\tilde{v}(k) = \begin{bmatrix} v(k) \\ \bar{v} \\ \underline{v} \end{bmatrix}, \quad d(k) = \begin{bmatrix} \bar{d}(k) \\ \underline{d}(k) \end{bmatrix}$$

$$d \in \mathbb{R}^{n_d}, \quad n_d = 2n_x + 2n_y.$$

The error dynamics in (19) can be split into two subsystems where the subsystem (20) is decoupled from the effects of  $f(k)$  and the subsystem (21) is only affected by the sensor fault.

$$\begin{cases} \mathcal{E}_d(k+1) = \mathcal{A}_q \mathcal{E}_d(k) + \mathcal{H}_q d(k) \\ \mathcal{R}_d(k) = \mathcal{C} \mathcal{E}_d(k) + \mathcal{V} \tilde{v}(k) \end{cases} \quad (20)$$

$$\begin{cases} \mathcal{E}_f(k+1) = \mathcal{A}_q \mathcal{E}_f(k) + \tilde{\mathcal{F}}_q \tilde{f}(k) \\ \mathcal{R}_f(k) = \mathcal{C} \mathcal{E}_f(k) + \mathcal{F} f(k) \end{cases} \quad (21)$$

where  $\mathcal{E}(k) = \mathcal{E}_f(k) + \mathcal{E}_d(k)$ .

Inspired by [34], and to facilitate the analysis of the error dynamics and the design of the proposed observer, the error dynamics system in (19) are split in both subsystems (20) and (21). The objective in the sequel is to design a

FD observer (8) such that the error system in (20) is stable and the effect of disturbances is minimized and thus the FD accuracy is improved. For this end, inspired by [35], the  $L_\infty$  technique is used such that for given scalars  $\gamma > 0$ ,  $\gamma_1 > 0$ ,  $\gamma_2 > 0$  and  $0 < \lambda < 1$ , the residual signal  $\mathcal{R}_d$  should satisfy the following inequality

$$\|\mathcal{R}_d\| < \sqrt{\gamma_1^2(\gamma(\lambda(1-\lambda)^k V(0) + \gamma\theta_d^2)) + \gamma_2^2\theta_v^2} \quad (22)$$

where  $V(0) = \mathcal{E}_d(0)^T P \mathcal{E}_d(0)$ ,  $P \succ 0 \in \mathbb{R}^{2n_x \times 2n_x}$ ,  $\theta_d$  and  $\theta_v$  are known constants such that  $\theta_d = \|d\|_\infty$  and  $\theta_v = \|\tilde{v}\|_\infty$ . 195

**Remark 2.** *The sensibility analysis of the proposed observer is not addressed in this paper. Based on interval observers, some authors characterized the minimum detectable fault or others  $H_-$  approaches are used to improve the sensibility of the observer [20, 36]. This study should be developed in a future work. Herein, we focus on TNL interval observer with  $L_\infty$  performance.* 200

**Theorem 2.** *Let Assumptions 2 and 3 hold. Given scalars  $\gamma > 0$ ,  $\gamma_1 > 0$ ,  $\gamma_2 > 0$  and  $0 < \lambda < 1$ , the error dynamics system in (20) is stable and  $\mathcal{R}_d$  satisfies the  $L_\infty$  performance, if there exist a constant  $\mu > 0$ , a diagonal matrix*

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \succ 0, P = P^T \succ 0, P \in \mathbb{R}^{2n_x \times 2n_x} \text{ and constant matrices } W_{q1},$$

$W_{q2} \in \mathbb{R}^{n_x \times n_y}$  and  $Y_{q1}, Y_{q2} \in \mathbb{R}^{n_x \times (n_x + n_y)}$  for  $q = 1, 2, \dots, N$  such that:

$$\begin{bmatrix} P_1 \Theta^\dagger \alpha_1 A_q + Y_{q1} \Psi \alpha_1 A_q - W_{q1} C & 0 \\ * & P_2 \Theta^\dagger \alpha_1 A_q + Y_{q2} \Psi \alpha_1 A_q - W_{q2} C \end{bmatrix} \geq 0. \quad (23)$$

$$\begin{bmatrix} \Upsilon_{11} & 0 & \Upsilon_{13} \\ * & -\mu I_{n_d} & \Upsilon_{23} \\ * & * & \Upsilon_{33} \end{bmatrix} \prec 0. \quad (24)$$

$$\begin{bmatrix} \lambda P & 0 & I_{2n_x} \\ * & (\gamma - \mu) I_{n_d} & 0 \\ * & * & \gamma I_{2n_x} \end{bmatrix} \succ 0. \quad (25)$$

$$\begin{bmatrix} \mathcal{C}^T \mathcal{C} - \gamma_1^2 I_{2n_x} & \mathcal{C}^T \mathcal{V} \\ * & -\gamma_2^2 I_{2n_x} + \mathcal{V}^T \mathcal{V} \end{bmatrix} \prec 0. \quad (26)$$

where

$$\Upsilon_{11} = \begin{bmatrix} (\lambda - 1)P_1 & 0 \\ * & (\lambda - 1)P_2 \end{bmatrix}, \quad \Upsilon_{33} = \begin{bmatrix} -P_1 & 0 \\ * & -P_2 \end{bmatrix}$$

$$\Upsilon_{13} = \begin{bmatrix} P_1 \Theta^\dagger \alpha_1 A_q + Y_{q1} \Psi \alpha_1 A_q - W_{q1} C & 0 \\ * & P_2 \Theta^\dagger \alpha_1 A_q + Y_{q2} \Psi \alpha_1 A_q - W_{q2} C \end{bmatrix}^T$$

$$\Upsilon_{23} = \begin{bmatrix} P_1 & W_{q1} & P_1 \Theta^\dagger \alpha_2 + Y_{q1} \Psi \alpha_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_2 & W_{q2} & P_2 \Theta^\dagger \alpha_2 + Y_{q2} \Psi \alpha_2 \end{bmatrix}^T$$

and

$$\alpha_1 = \begin{bmatrix} I_{n_x} \\ 0 \end{bmatrix}, \quad \alpha_2 = \begin{bmatrix} 0 \\ I_{n_y} \end{bmatrix}, \quad \Theta = \begin{bmatrix} I_{n_x} \\ C \end{bmatrix}, \quad \Psi = I_{n_x + n_y} - \Theta \Theta^\dagger$$

Moreover, the observer gains  $\bar{L}_q$ ,  $\underline{L}_q$ ,  $\bar{T}_q$ ,  $\underline{T}_q$ ,  $\bar{N}_q$  and  $\underline{N}_q$  are given by:

$$\begin{cases} \bar{L}_q = P_1^{-1} W_{q1} \\ \underline{L}_q = P_2^{-1} W_{q2} \\ \bar{T}_q = \Theta^\dagger \alpha_1 + P_1^{-1} Y_{q1} \Psi \alpha_1 \\ \underline{T}_q = \Theta^\dagger \alpha_1 + P_2^{-1} Y_{q2} \Psi \alpha_1 \\ \bar{N}_q = \Theta^\dagger \alpha_2 + P_1^{-1} Y_{q1} \Psi \alpha_2 \\ \underline{N}_q = \Theta^\dagger \alpha_2 + P_2^{-1} Y_{q2} \Psi \alpha_2 \end{cases} \quad (27)$$

*Proof.* To prove the stability of (20) and the  $L_\infty$  performance (22), sufficient conditions are given in terms of LMIs based on the common quadratic Lyapunov function given by  $V(k) = \mathcal{E}_d(k)^T P \mathcal{E}_d(k)$ ,  $P^T = P \succ 0$ ,  $P \in \mathbb{R}^{2n_x \times 2n_x}$ . Note that  $P \succ 0$  since it is a diagonal matrix. Thus, according to Theorem 1,  $P \mathcal{A}_q$  is also non-negative for all  $q = 1, \dots, N$ . By replacing  $\bar{T}_q$  and  $\underline{T}_q$  by their expressions in (27), the inequality (23) is satisfied.



The time difference of  $V(k)$  is given by

$$\Delta V(k) = \begin{bmatrix} \mathcal{E}_d(k) \\ d(k) \end{bmatrix}^T \begin{bmatrix} \mathcal{A}_q^T P \mathcal{A}_q - P & \mathcal{A}_q^T P \mathcal{H}_q \\ * & \mathcal{H}_q^T P \mathcal{H}_q \end{bmatrix} \begin{bmatrix} \mathcal{E}_d(k) \\ d(k) \end{bmatrix}$$

If inequality (24) holds, then it can be rewritten as

$$\begin{bmatrix} (\lambda - 1)P & 0 & (P \mathcal{A}_q)^T \\ * & -\mu I_{n_d} & (P \mathcal{H}_q)^T \\ * & * & -P \end{bmatrix} \prec 0 \quad (28)$$

It can also be easily shown by pre- and post- multiplying (28) with  $\begin{bmatrix} I_{2n_x} & 0 & \mathcal{A}_q^T \\ 0 & I_{n_d} & \mathcal{H}_q^T \end{bmatrix}$

and its transpose, respectively, that (29) yields

$$\begin{bmatrix} \mathcal{A}_q^T P \mathcal{A}_q - P & \mathcal{A}_q^T P \mathcal{H}_q \\ * & \mathcal{H}_q^T P \mathcal{H}_q \end{bmatrix} + \begin{bmatrix} \lambda P & 0 \\ * & -\mu I_{n_d} \end{bmatrix} \prec 0 \quad (29)$$

In addition, by pre- and post-multiplying (29) with  $[\mathcal{E}_d(k)^T \quad d(k)^T]$  and its transpose, it follows that

$$\Delta V(k) < -\lambda V(k) + \mu d(k)^T d(k) \quad (30)$$

When  $w(k) = 0$  and  $v(k) = 0$ , it implies that  $d(k) = 0$  and (30) is deduced:

$$\Delta V(k) = V(k+1) - V(k) < -\lambda V(k) < 0$$

Accordingly, the error system in (20) is stable. Moreover, inequality (30) can be rewritten as

$$V(k+1) < (1 - \lambda)V(k) + \mu \theta_d^2 \quad (31)$$

From (31), one can obtain

$$\begin{aligned} V(k) &\leq (1 - \lambda)^k V(0) + \mu \sum_{\tau=0}^{k-1} (1 - \lambda)^\tau \theta_d^2 \\ &\leq (1 - \lambda)^k V(0) + \mu \frac{(1 - \lambda^k)}{\lambda} \theta_d^2 \\ &\leq (1 - \lambda)^k V(0) + \frac{\mu \theta_d^2}{\lambda} \end{aligned} \quad (32)$$

Based on the Schur complement lemma, (25) is equivalent to

$$\begin{bmatrix} \lambda P & 0 \\ * & (\gamma - \mu)I_{n_d} \end{bmatrix} - \frac{1}{\gamma} \begin{bmatrix} I_{2n_x} \\ 0 \end{bmatrix} \begin{bmatrix} I_{2n_x} & 0 \end{bmatrix} \succ 0 \quad (33)$$

Then, pre-multiplying and post-multiplying (33) with  $[\mathcal{E}_d(k)^T \quad d(k)^T]$  and its transpose, one can get

$$\mathcal{E}_d(k)^T \mathcal{E}_d(k) \leq \gamma (\lambda V(k) + (\gamma - \mu)\theta_d^2) \quad (34)$$

According to (32), inequality in (34) gives

$$\begin{aligned} \mathcal{E}_d(k)^T \mathcal{E}_d(k) &\leq \gamma \left( \lambda \left( (1 - \lambda)^k V(0) + \frac{\mu \theta_d^2}{\lambda} \right) + (\gamma - \mu) \theta_d^2 \right) \\ &\leq \gamma \left( \lambda (1 - \lambda)^k V(0) + \gamma \theta_d^2 \right) \end{aligned}$$

In addition, the matrix inequality in (26) implies that

$$\begin{bmatrix} \mathcal{E}_d(k) \\ \tilde{v}(k) \end{bmatrix}^T \begin{bmatrix} \mathcal{C}^T \mathcal{C} - \gamma_1^2 I_{2n_x} & \mathcal{C}^T \mathcal{V} \\ * & -\gamma_2^2 I_{2n_x} + \mathcal{V}^T \mathcal{V} \end{bmatrix} \begin{bmatrix} \mathcal{E}_d(k) \\ \tilde{v}(k) \end{bmatrix} < 0$$

which follows

$$\begin{aligned} \mathcal{R}_d(k)^T \mathcal{R}_d(k) &\leq \gamma_1^2 \mathcal{E}_d(k)^T \mathcal{E}_d(k) + \gamma_2^2 \tilde{v}(k)^T \tilde{v}(k) \\ &\leq \gamma_1^2 \gamma \left( \lambda (1 - \lambda)^k V(0) + \gamma \theta_d^2 \right) + \gamma_2^2 \theta_v^2 \end{aligned}$$

Therefore, the  $L_\infty$  criterion (22) is satisfied.  $\square$

**Remark 3.** *The designed LMIs conditions are relaxed by introducing weighted matrices  $\bar{T}_q$ ,  $\underline{T}_q$ ,  $\bar{N}_q$  and  $\underline{N}_q$  using a common quadratic Lyapunov function under an arbitrary switching signal. However, the existence of such a function is not always guaranteed. Therefore, multiple Lyapunov functions with an ADT switching signal are introduced in the following to reduce the conservatism.*

#### 4.2. Residual framers design based on multiple Lyapunov functions

In this part, the stability analysis is analysed by applying multiple Lyapunov functions under an ADT switching signal. The results given by Theorem 2 are extended and new LMIs conditions are developed hereafter in the following theorem.

**Theorem 3.** Consider the switched system given by (7), let Assumptions 2 and 3 hold and suppose that there exists a piecewise Lyapunov function defined by  $V_q(k) = \mathcal{E}_d(k)^T P_q \mathcal{E}_d(k)$ . If there exist positive definite diagonal matrices  $P_q$ ,  $P_q \in \mathbb{R}^{2n_x \times 2n_x}$ , constant matrices  $M_l \in \mathbb{R}^{2n_x \times 2n_x}$ ,  $W_{q1}, W_{q2} \in \mathbb{R}^{n_x \times n_y}$  and  $Y_{q1}, Y_{q2} \in \mathbb{R}^{n_x \times (n_x + n_y)}$ ,  $a_2 > a_1 > 0$ ,  $\mu > 0$  for given scalars  $\gamma > 0$ ,  $\gamma_1 > 0$ ,  $\gamma_2 > 0$ ,  $0 < \lambda < 1$  and  $0 < \beta < 1$ , then the error dynamics system in (20) are stable,  $\mathcal{R}_d$  satisfies the  $L_\infty$  performance and the following conditions

$$\min_{P_q, M_l} \beta \rho + (1 - \beta) \mu \quad (35)$$

$$a_1 I_{2n_x} \leq P_q \leq a_2 I_{2n_x} \quad (36)$$

$$\begin{bmatrix} P_{q1} \Theta^\dagger \alpha_1 A_q + Y_{q1} \Psi \alpha_1 A_q - W_{q1} C & 0 \\ * & P_{q2} \Theta^\dagger \alpha_1 A_q + Y_{q2} \Psi \alpha_1 A_q - W_{q2} C \end{bmatrix} \geq 0. \quad (37)$$

$$\begin{bmatrix} \Upsilon_{q11} & 0 & \Upsilon_{q13} \\ * & -\mu I_{n_d} & \Upsilon_{q23} \\ * & * & \Upsilon_{q33} \end{bmatrix} \prec 0. \quad (38)$$

$$\begin{bmatrix} \lambda P_q & 0 & I_{2n_x} \\ * & (\gamma - \mu) I_{n_d} & 0 \\ * & * & \gamma I_{2n_x} \end{bmatrix} \succ 0. \quad (39)$$

$$\begin{bmatrix} \mathcal{C}^T \mathcal{C} - \gamma_1^2 I_{2n_x} & \mathcal{C}^T \mathcal{V} \\ * & -\gamma_2^2 I_{2n_x} + \mathcal{V}^T \mathcal{V} \end{bmatrix} \prec 0. \quad (40)$$

$$\begin{bmatrix} M_l & P_q \\ P_q & P_q \end{bmatrix} \succeq 0 \quad (41)$$

hold for all  $q, l \in \mathcal{S}, q \neq l$  where

$$\Upsilon_{q11} = \begin{bmatrix} (\lambda - 1) P_{q1} & 0 \\ * & (\lambda - 1) P_{q2} \end{bmatrix}, \quad \Upsilon_{q33} = \begin{bmatrix} -P_{q1} & 0 \\ * & -P_{q2} \end{bmatrix}$$

$$\Upsilon_{q13} = \begin{bmatrix} P_{q1}\Theta^\dagger\alpha_1A_q + Y_{q1}\Psi\alpha_1A_q - W_{q1}C & 0 \\ * & P_{q2}\Theta^\dagger\alpha_1A_q + Y_{q2}\Psi\alpha_1A_q - W_{q2}C \end{bmatrix}^T$$

$$\Upsilon_{q23} = \begin{bmatrix} P_{q1} & W_{q1} & P_{q1}\Theta^\dagger\alpha_2 + Y_{q1}\Psi\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{q2} & W_{q2} & P_{q2}\Theta^\dagger\alpha_2 + Y_{q2}\Psi\alpha_2 \end{bmatrix}^T$$

and

$$\alpha_1 = \begin{bmatrix} I_{n_x} \\ 0 \end{bmatrix}, \alpha_2 = \begin{bmatrix} 0 \\ I_{n_y} \end{bmatrix}, \Theta = \begin{bmatrix} I_{n_x} \\ C \end{bmatrix}, \Psi = I_{n_x+n_y} - \Theta\Theta^\dagger$$

In addition, if the state  $x$  is bounded, then  $\bar{x}$  and  $\underline{x}$  are also bounded under a switching signal with an ADT  $\tau_a$  satisfying:

$$\tau_a > \tau_a^* = -\frac{\ln(\rho)}{\ln(1-\lambda)} \quad (42)$$

where  $\rho = \frac{\alpha_2}{\alpha_1}$  and the observer gains  $\bar{L}_q$ ,  $\underline{L}_q$ ,  $\bar{T}_q$ ,  $\underline{T}_q$ ,  $\bar{N}_q$  and  $\underline{N}_q$  are given by:

$$\begin{cases} \bar{L}_q = P_{q1}^{-1}W_{q1} \\ \underline{L}_q = P_{q2}^{-1}W_{q2} \\ \bar{T}_q = \Theta^\dagger\alpha_1 + P_{q1}^{-1}Y_{q1}\Psi\alpha_1 \\ \underline{T}_q = \Theta^\dagger\alpha_1 + P_{q2}^{-1}Y_{q2}\Psi\alpha_1 \\ \bar{N}_q = \Theta^\dagger\alpha_2 + P_{q1}^{-1}Y_{q1}\Psi\alpha_2 \\ \underline{N}_q = \Theta^\dagger\alpha_2 + P_{q2}^{-1}Y_{q2}\Psi\alpha_2 \end{cases} \quad (43)$$

Moreover, the interval error (20) satisfies:

$$\lim_{k \rightarrow \infty} \|\mathcal{E}_d(k)\| < \frac{\mu}{a_1\lambda}\theta_d^2$$

*Proof.* First of all, the requirement on the non-negativity property of the matrices  $\bar{T}_qA_q - \bar{T}_qC$  and  $\underline{T}_qA_q - \underline{T}_qC$  is achieved via the inequality (37). We are interested in checking the stability of the observation errors via multiple Lyapunov functions for the proposed interval observer in (8). According to (32), we have

$$V(k) \leq (1-\lambda)^k V(0) + \frac{\mu\theta_d^2}{\lambda}$$

Based on equation (3) in Lemma 3, the following inequality

$$a_1 \|\mathcal{E}_d(k)\| \leq V_q(\mathcal{E}_d(k))$$

allows deducing that

$$\|\mathcal{E}_d(k)\| \leq \frac{1}{a_1} ((1-\lambda)^k V(0) + \frac{\mu \theta_d^2}{\lambda})$$

When  $k \rightarrow \infty$ , inequality in (44)

$$\lim_{k \rightarrow \infty} \|\mathcal{E}_d(k)\| < \frac{\mu}{a_1 \lambda} \theta_d^2 \quad (44)$$

yields. Therefore, it has been shown that the interval error width is asymptotically upper bounded by  $\frac{\mu}{a_1 \lambda} \theta_d^2$  which should be made as small as possible to enhance the accuracy of FD. One can notice that this bound depends on  $\mu$ , for a given  $a_1$  and  $\lambda$ . Consequently, the minimization of  $\mu$  allows one reducing the interval width of the estimation error and thus the interval width of the residual signal. We aim also to minimize  $\rho$  to look for optimum dwell time. The resolution of such a problem leads to solving a problem of linear optimization which consists of seeking a minimization function. Then, the objective function can be added to the LMIs conditions and expressed as:

$$\beta \rho + (1 - \beta) \mu$$

where the weight  $\beta$  is in the range  $[0, 1]$ .

Let us now deal with the stabilization of subsystems at the switching instants. The inequality given in (5) can be written as follows:

$$\rho P_l - P_q \succeq 0 \quad (45)$$

where  $q, l \in \mathcal{I}, q \neq l$ ,  $q$  is the current mode. Then applying the Schur complement lemma, (45) implies:

$$\begin{bmatrix} \rho P_l & I_{2n_x} \\ I_{2n_x} & P_q^{-1} \end{bmatrix} \succeq 0 \quad (46)$$

The multiplication both side of (46) by  $\begin{bmatrix} I_{2n_x} & O_{2n_x} \\ O_{2n_x} & P_q \end{bmatrix}$  yields to  $\begin{bmatrix} M_l & P_q \\ P_q & P_q \end{bmatrix} \succeq 0$  where  $M_l = \rho P_l$ . Therefore, (41) is verified. LMIs conditions in (38), (39) and (40) have been proven in Theorem 2.  $\square$

Under the proposed interval observer (8), the residual evaluation is based on determining whether the zero signal is excluded from the residual interval or not. The corresponding FD decision scheme is made as follows:

$$\begin{cases} 0 \in [\underline{r}(k) & \bar{r}(k)] & \text{Fault-free} \\ 0 \notin [\underline{r}(k) & \bar{r}(k)] & \text{Faulty} \end{cases} \quad (47)$$

The FD evaluation in (47) is deduced from the following relation

$$y(k) \notin [\underline{y}(k) \quad \bar{y}(k)] \quad (48)$$

In fact, in the fault free case, the output signal is consistent with the estimation of the proposed interval observer, i.e.  $y \in [\underline{y} \quad \bar{y}]$ . In contrary case, an inconsistency on the output signal is detected and it indicates the existence of a fault. Based on (48), the consistency test can be written as follow

$$0 \notin [\underline{y}(k) \quad \bar{y}(k)] - y(k) \Rightarrow 0 \notin [\underline{y}(k) - y(k) \quad \bar{y}(k) - y(k)] \quad (49)$$

If zero is contained in the estimated framers, the system is assumed fault free. Otherwise an alarm is triggered.

In the next section, an illustrative example is introduced to show the efficiency of the developed results using both common and multiple Lyapunov functions.

## 5. Numerical example

The numerical example is considered for a discrete-time switched system (7) defined with three subsystems,  $N = 3$ , with:

$$A_1 = \begin{bmatrix} 0.6 & 0.5 & 1 \\ 0.2 & 0 & 1 \\ 0 & 0.2 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.9 & -0.8 & 0 \\ 0.1 & 0 & 1 \\ 0 & 0.5 & 0.1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0.5 & 0.5 & 0.1 \\ 0.4 & 0 & 0.5 \\ 0.1 & 0.2 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 \\ 0.1 \\ 1.3 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 \\ 1 \\ 1 \end{bmatrix}, B_3 = \begin{bmatrix} 1.2 \\ 1 \\ 0.5 \end{bmatrix}, E_1 = \begin{bmatrix} 0.02 \\ 0.01 \\ 0.1 \end{bmatrix}, E_2 = \begin{bmatrix} 0.05 \\ 0.1 \\ 0 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} 0 \\ 0.1 \\ 0.1 \end{bmatrix}, D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, F_1 = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}, C = \begin{bmatrix} 1.2 & 0.01 & 0 \\ 0.1 & 1.1 & 0.1 \end{bmatrix}$$

**Remark 4.** *It is pointed out that the matrices  $C$  and  $F$  are considered to be constant and common for all modes in this paper. However, there is no theoretical difficulty with allowing them to be switched.*

225 In this example,  $w(k) \in \mathbb{R}$  and  $v(k) \in \mathbb{R}^2$  are uniformly distributed bounded signals such that  $|w(k)| \leq 1$  and  $|v(k)| \leq [0.1 \ 0.1]$ . The state initial conditions are set as  $x(0) = [0 \ 0 \ 0]^T$ ,  $\underline{x}(0) = [-0.1 \ -0.1 \ -0.1]^T$  and  $\bar{x}(0) = [0.1 \ 0.1 \ 0.1]^T$  such that  $\underline{x}(0) \leq x(0) \leq \bar{x}(0)$ . Figure 1 shows the evolution of the switching signal. It indicates the active mode of the discrete-time switched system.

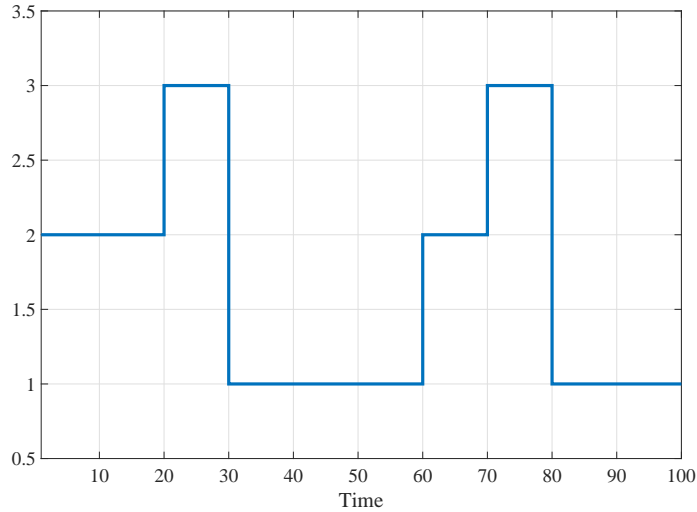


Figure 1: Evolution of the switching signal

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FD results are given in the sequel adopting firstly the common quadratic Lyapunov function and then multiple Lyapunov functions under an ADT switching signal. The numerical simulation was carried out by using Matlab optimization tools (Yalmip/Sedumi).

235 *5.1. Fault detection results based on the common Lyapunov function*

Based on the TNL structure used in (8), the following Lyapunov matrix is given by:

$$P_1 = \begin{bmatrix} 1.28 & 0 & 0 \\ 0 & 1.23 & 0 \\ 0 & 0 & 1.67 \end{bmatrix}, P_2 = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1.92 \end{bmatrix}$$

By solving LMIs in Theorem 2, the matrices  $\bar{L}_q$ ,  $\underline{L}_q$ ,  $\bar{N}_q$ ,  $\underline{N}_q$ ,  $\bar{T}_q$  and  $\underline{T}_q$  can be obtained as follows:

$$\begin{aligned} \bar{L}_1 &= \begin{bmatrix} 0.1 & 0.12 \\ -0.14 & -0.25 \\ -0.05 & 0.09 \end{bmatrix}, \underline{L}_1 = \begin{bmatrix} 0.16 & 0.21 \\ -0.08 & -0.17 \\ -0.06 & 0.03 \end{bmatrix}, \bar{L}_2 = \begin{bmatrix} 0.19 & -0.38 \\ -0.13 & -0.08 \\ 0.01 & 0.17 \end{bmatrix} \\ \underline{L}_2 &= \begin{bmatrix} 0.17 & -0.35 \\ -0.05 & 0 \\ 0.05 & 0.18 \end{bmatrix}, \bar{L}_3 = \begin{bmatrix} 0.2 & 0.18 \\ 0.02 & -0.16 \\ 0.02 & 0.05 \end{bmatrix}, \underline{L}_3 = \begin{bmatrix} 0.19 & 0.2 \\ 0.05 & -0.13 \\ 0.027 & 0.06 \end{bmatrix} \\ \bar{N}_1 &= \begin{bmatrix} 0.36 & 0.26 \\ 0.22 & 0.41 \\ -0.06 & -0.09 \end{bmatrix}, \underline{N}_1 = \begin{bmatrix} 0.40 & 0.25 \\ 0.26 & 0.40 \\ -0.05 & -0.05 \end{bmatrix}, \bar{N}_2 = \begin{bmatrix} 0.51 & -0.12 \\ 0 & 0.63 \\ -0.14 & -0.08 \end{bmatrix} \\ \underline{N}_2 &= \begin{bmatrix} 0.47 & -0.08 \\ 0.01 & 0.60 \\ -0.11 & -0.06 \end{bmatrix}, \bar{N}_3 = \begin{bmatrix} 0.17 & -0.13 \\ -0.02 & 0.39 \\ 0.01 & -0.18 \end{bmatrix}, \underline{N}_3 = \begin{bmatrix} 0.18 & -0.08 \\ 0 & 0.38 \\ 0.02 & -0.14 \end{bmatrix} \\ \bar{T}_1 &= \begin{bmatrix} 0.53 & -0.29 & -0.02 \\ -0.31 & 0.53 & -0.04 \\ 0.08 & 0.10 & 1 \end{bmatrix}, \underline{T}_1 = \begin{bmatrix} 0.48 & -0.28 & -0.02 \\ -0.36 & 0.55 & -0.04 \\ 0.07 & 0.06 & 1 \end{bmatrix} \end{aligned}$$



$$\bar{T}_2 = \begin{bmatrix} 0.39 & 0.13 & 0.01 \\ -0.06 & 0.30 & -0.06 \\ 0.18 & 0.09 & 1 \end{bmatrix}, \underline{T}_2 = \begin{bmatrix} 0.43 & 0.08 & 0 \\ -0.07 & 0.33 & -0.06 \\ 0.14 & 0.073 & 1 \end{bmatrix}$$

$$\bar{T}_3 = \begin{bmatrix} 0.80 & 0.14 & 0.01 \\ 0 & 0.56 & -0.03 \\ 0 & 0.20 & 1.01 \end{bmatrix}, \underline{T}_3 = \begin{bmatrix} 0.78 & 0.09 & 0 \\ -0.03 & 0.58 & -0.03 \\ -0.02 & 0.15 & 1.01 \end{bmatrix}$$

Note that  $\bar{N}_q$ ,  $\underline{N}_q$ ,  $\bar{T}_q$  and  $\underline{T}_q$  are designed to satisfy (9) and (10). In the simulation, an abrupt sensor fault  $f(k)$  is carried out and represented as follows:

$$f(k) = \begin{cases} 1 & 35 \leq k \leq 55 \\ 0 & \text{otherwise} \end{cases}$$

Under the switching sequence shown in Fig. 1, simulation results of the FD interval observer are depicted in Fig. 2 which illustrates the evolution of the residual signals. In the fault free case, the cooperativity property is ensured and one can remark that  $0 \in [r(k) \quad \bar{r}(k)]$ . When a fault occurs ( $k = 35$ ), the additive sensor fault is detected at the time instant  $k = 36$  under the proposed FD TNL approach and  $0 \notin [r(k) \quad \bar{r}(k)]$ .

In the simulation study, the proposed TNL method is compared with the traditional interval approach in [28] based on the common Lyapunov function. The simulation results are shown in Fig. 3. The two proposed methods show their effectiveness for the detection of the sensor fault after its occurrence. Meanwhile, the results show that the FD obtained by the TNL method is more accurate than by the commonly used interval observer.

The case of a small abrupt sensor fault is also considered and simulation results in Fig. 4 show that only by using the TNL technique, the fault can be detected. In fact, based on the traditional interval technique, the cooperativity property is still guaranteed even after the occurrence of the fault and some false alarms can be generated.

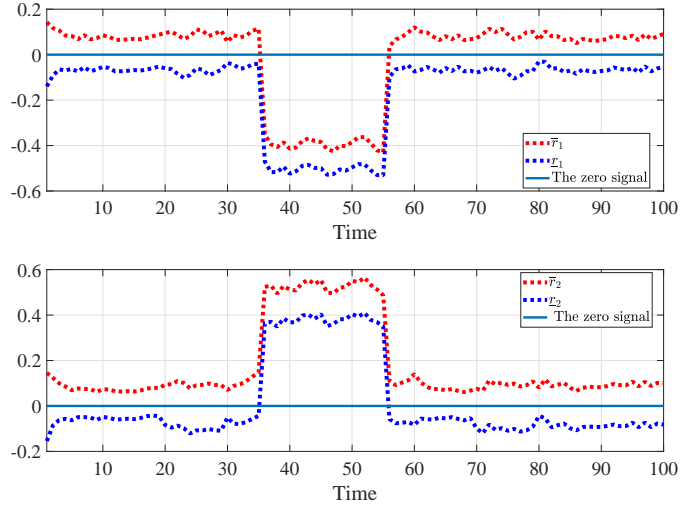


Figure 2: Residual framers using fault detection TNL interval observer

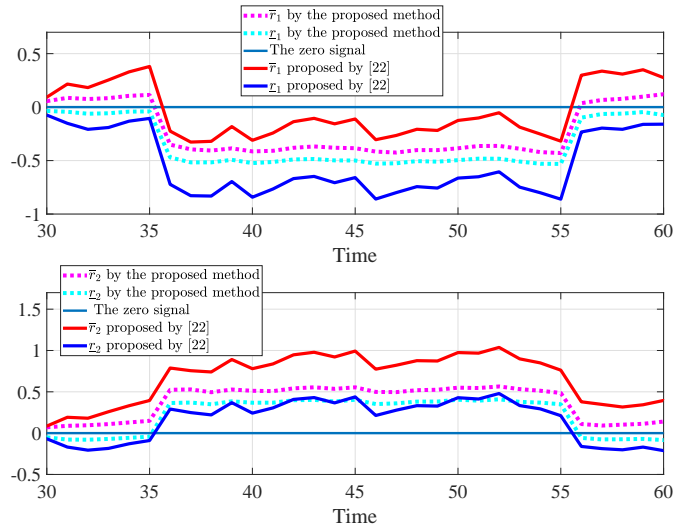


Figure 3: Fault detection performance comparison between the proposed TNL method and the traditional interval approach in [28]

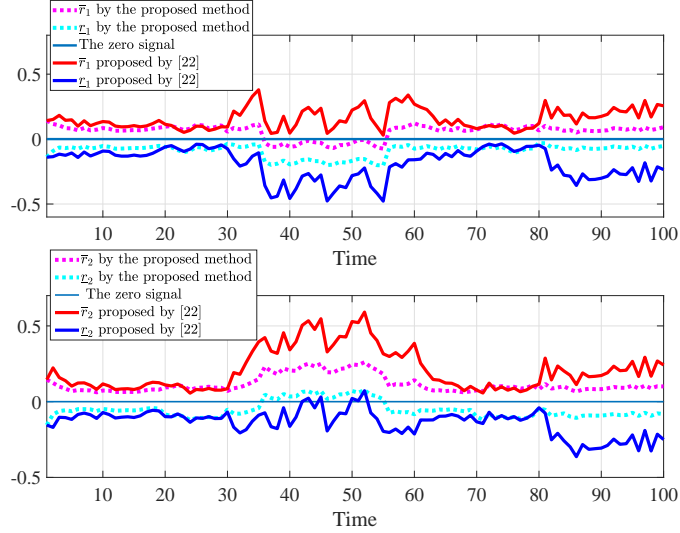


Figure 4: Fault detection performance comparison between the proposed TNL method and the traditional interval approach in [28](Small fault)

255 *5.2. Fault detection results based on multiple Lyapunov functions*

Under the same switched system used in the first example, the optimization problem in (35) can be solved using optimization software Yalmip/Sedumi in MATLAB. The existence of a solution for LMIs in Theorem 3 allows improving the accuracy of FD and to obtain an optimum dwell time. The resolution of the optimization problem in (35) can be then solved and the Lyapunov matrices are obtained:

$$\begin{aligned}
 P_{11} &= \begin{bmatrix} 1.29 & 0 & 0 \\ 0 & 1.26 & 0 \\ 0 & 0 & 1.67 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 1.92 \end{bmatrix}, \quad P_{21} = \begin{bmatrix} 1.35 & 0 & 0 \\ 0 & 1.23 & 0 \\ 0 & 0 & 1.63 \end{bmatrix} \\
 P_{22} &= \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1.49 & 0 \\ 0 & 0 & 1.84 \end{bmatrix}, \quad P_{31} = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1.39 & 0 \\ 0 & 0 & 1.62 \end{bmatrix}, \quad P_{32} = \begin{bmatrix} 1.49 & 0 & 0 \\ 0 & 1.58 & 0 \\ 0 & 0 & 1.77 \end{bmatrix}
 \end{aligned}$$

In the simulation study,  $\mu = 2$  which leads to an ADT  $\tau_a > 1$ . The set of observer gains  $\bar{L}_q, \underline{L}_q$  and the weighted matrices  $\bar{N}_q, \underline{N}_q, \bar{T}_q$  and  $\underline{T}_q$  are computed

according to (43):

$$\begin{aligned}
\bar{L}_1 &= \begin{bmatrix} 0.10 & 0.13 \\ -0.15 & -0.24 \\ -0.05 & 0.11 \end{bmatrix}, \underline{L}_1 = \begin{bmatrix} 0.13 & 0.18 \\ -0.11 & -0.20 \\ -0.04 & 0.11 \end{bmatrix}, \bar{L}_2 = \begin{bmatrix} 0.21 & -0.35 \\ -0.11 & -0.05 \\ 0.03 & 0.20 \end{bmatrix} \\
\bar{L}_2 &= \begin{bmatrix} 0.18 & -0.36 \\ -0.05 & 0.01 \\ 0.08 & 0.25 \end{bmatrix}, \bar{L}_3 = \begin{bmatrix} 0.21 & 0.23 \\ 0.02 & -0.12 \\ 0.03 & 0.06 \end{bmatrix}, \underline{L}_3 = \begin{bmatrix} 0.22 & 0.23 \\ 0.05 & -0.10 \\ 0.04 & 0.07 \end{bmatrix} \\
\bar{N}_1 &= \begin{bmatrix} 0.39 & 0.29 \\ 0.26 & 0.45 \\ -0.04 & -0.08 \end{bmatrix}, \underline{N}_1 = \begin{bmatrix} 0.42 & 0.29 \\ 0.29 & 0.45 \\ -0.03 & -0.08 \end{bmatrix}, \bar{N}_2 = \begin{bmatrix} 0.51 & -0.09 \\ 0 & 0.73 \\ -0.15 & -0.05 \end{bmatrix} \\
\underline{N}_2 &= \begin{bmatrix} 0.48 & -0.06 \\ 0 & 0.71 \\ -0.14 & -0.08 \end{bmatrix}, \bar{N}_3 = \begin{bmatrix} 0.21 & -0.08 \\ 0 & 0.51 \\ 0.02 & -0.18 \end{bmatrix}, \underline{N}_3 = \begin{bmatrix} 0.21 & -0.08 \\ 0.02 & 0.5 \\ 0.03 & -0.16 \end{bmatrix} \\
\bar{T}_1 &= \begin{bmatrix} 0.49 & -0.32 & -0.02 \\ -0.36 & 0.49 & -0.04 \\ 0.06 & 0.08 & 1 \end{bmatrix}, \underline{T}_1 = \begin{bmatrix} 0.45 & -0.32 & -0.02 \\ -0.39 & 0.49 & -0.04 \\ 0.054 & 0.09 & 1 \end{bmatrix} \\
\bar{T}_2 &= \begin{bmatrix} 0.38 & 0.09 & 0 \\ -0.07 & 0.19 & -0.07 \\ 0.19 & 0.06 & 1 \end{bmatrix}, \underline{T}_2 = \begin{bmatrix} 0.42 & 0.06 & 0 \\ -0.07 & 0.21 & -0.07 \\ 0.18 & 0.09 & 1 \end{bmatrix} \\
\bar{T}_3 &= \begin{bmatrix} 0.75 & 0.08 & 0 \\ -0.06 & 0.42 & -0.05 \\ -0.01 & 0.20 & 1.01 \end{bmatrix}, \underline{T}_3 = \begin{bmatrix} 0.75 & 0.09 & 0 \\ -0.08 & 0.44 & -0.05 \\ -0.01 & 0.18 & 1.01 \end{bmatrix}
\end{aligned}$$

A comparison has been performed between FD performances based on the common Lyapunov and multiple Lyapunov functions. The results of the simulation are depicted in Fig. 5 where solid lines present the residual signals obtained by the common Lyapunov function and dashed lines present the residual framers obtained by multiple Lyapunov functions. The simulation results show that the interval width obtained by the construction of multiple Lyapunov functions is

tighter than that given by the common Lyapunov function. With the same initial conditions introduced before, the fault  $f(k)$  can be detected more quickly using Multiple Lyapunov functions.

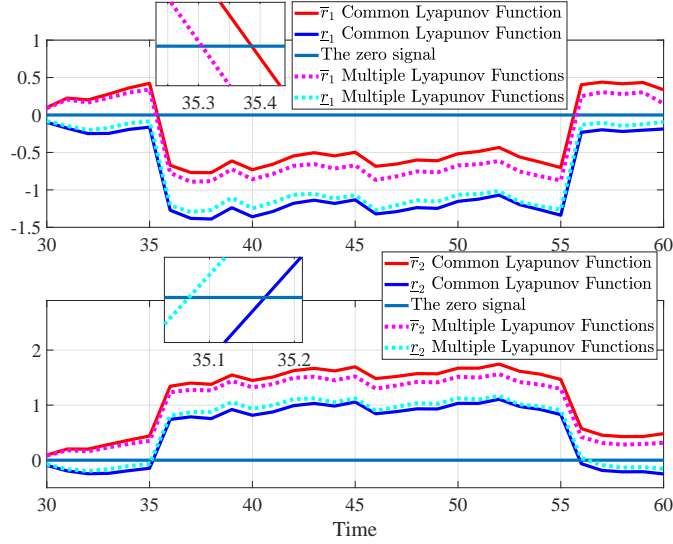


Figure 5: Residual framers using fault detection interval observer

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**Remark 5.** *The proposed new observer structure reduces the conservatism regarding uncertainties in fault detection. Compared to traditional interval observers, the TNL observer combined with the  $L_\infty$  criterion provides a better estimation of residuals. The proposed method can detect small faults without missing alarm.*

270

## 6. Conclusion

This paper presents a new LMIs formulation to design a robust observer-based FD scheme for a discrete-time switched system with sensor faults. An  $L_\infty$  performance is introduced to attenuate the effect of unknown but bounded disturbances and measurement noises. For improvement of performance for detection,

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an FD interval observer based on the TNL structure is investigated. The proposed observer can reduce the conservatism of gain matrices and provide more degree of freedom by introducing weighted matrices. The design conditions of the proposed observer are expressed in terms of LMIs based on common and multiple Lyapunov functions. Finally, simulation results are provided to illustrate the efficiency of the proposed results. For future works, extensions of these results with LPV switched systems will be considered.

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