HAL
open science

Distributed event-triggered formation control for multi-agent systems in presence of packet losses

Christophe Viel, Michel Kieffer, Hélène Piet-Lahanier, Sylvain Bertrand

To cite this version:

Christophe Viel, Michel Kieffer, Hélène Piet-Lahanier, Sylvain Bertrand. Distributed event-triggered formation control for multi-agent systems in presence of packet losses. 2021. hal-02892207v3

HAL Id: hal-02892207

https://hal.science/hal-02892207v3

Preprint submitted on 27 Apr 2021 (v3), last revised 3 Nov 2021 (v5)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Distributed event-triggered formation control for multi-Agent systems in presence of packet losses

Christophe Viel* Michel Kieffer*** Hélène Piet-Lahanier** Sylvain Bertrand**

*CNRS, Lab-STICC, F-29806, Brest, France (e-mail: firstname.lastname@ensta-bretagne.fr).

**Univ Paris-Saclay, ONERA, TIS, F-91123 Palaiseau (e-mail: firstname.lastname@onera.fr).

***Univ Paris-Saclay, CNRS, CentraleSupelec L2S, F-91192 Gif-sur-Yvette (e-mail: firstname.lastname@l2s.centralesupelec.fr)

Abstract

This paper considers an event-triggered approach for the distributed formation control problem of an Euler-Lagrange multi-Agent system with state perturbations, when transmissions are prone to losses. To evaluate its control input, each Agent maintains estimators of its own state and of the states of its neighbors accounting for multiple packet-loss hypotheses. Each Agent is then able to compute the expected estimation error of its own state as evaluated by its neighbors. The communication triggering condition (CTC) exploiting this expected error is then proposed. An analysis of the behavior of the system with such CTC is performed using stochastic Lyapunov functions. Simulations confirm the effectiveness of the proposed approach.

Key words: Communication constraints, event-triggered control, packet losses, formation stabilization, multi-Agent system.

1 Introduction

Distributed control with event-triggered communication is an efficient method to coordinate Multi-Agent Systems (MAS) with a reduced amount of communications between agents. The Communication Triggering Condition (CTC) is instrumental in these approaches to limit communications, while allowing enough information to be exchanged between agents to complete the task assigned to the MAS, [15,18,26,22,14]. Designing a suitable CTC when communications are prone to packet losses is challenging. With event-triggered control, a message is transmitted only when required. A loss of information may thus have a critical impact on the performance and even stability of the MAS.

Packet losses may result from collisions between packets simultaneously transmitted from different agents, from obstacles, or from interference with other communications systems. Considering two packet-loss models, [6] has shown that event-triggered control schemes are more vulnerable to packet losses than time-triggered control strategies. Acknowledgment mechanisms are helpful to detect lost messages, which may then be retransmitted. Nevertheless, acknowledgments or re-transmitted messages may also be lost, which increases communication delays, risk of packet collision, and may lead to desynchronization between agents. In [2,5,6,25,23] packet losses are addressed by combining an H_∞ control and event-triggered communications. For agents with linear dynamics, sufficient conditions are established to ensure the global exponential stability of the system. In [5], communication delays and packet losses are considered simultaneously. In [2], the focus is on a MAS where agents follow several leaders. Each Agent maintains observers of the state of other agents. These observers account for the last received message from the other agents, their dynamics, and perturbations are considered in all these works.

Nonlinear dynamics are studied in [3,4]. In [3], packet losses are taken into account in the estimator models but not in the CTC: New distributed estimators are designed to guarantee the exponential stability of the estimation errors. To update the estimate of the state of other agents, each agent uses its own innovation and the innovation of its neighbors obtained from received packets. This improves the accuracy of the estimates at the cost of an increased sensitivity to losses. The control of a *single* Agent in presence of measurements losses is considered in [4]. An event-triggered strategy is proposed along with two communication protocols, with and without acknowledgments. With acknowledgments, the most recently received measurement can be identified. Without acknowledgment, this information is no longer available and a set of estimators is

used to estimate the measurement lost during transmission, each one using a different hypothesis of the last packet received. As previously stated, the case of a MAS is not considered.

This paper addresses the distributed formation control of a MAS with nonlinear Euler-Lagrange dynamics, state perturbations, and communication with losses. An event-triggered control strategy is proposed to extend the one presented in [19] and address the presence of packet losses. Each Agent maintains several estimators of its *own* state to mimic the estimates of its state maintained by its neighbors, considering different hypotheses of packet reception by these neighbors. This extends the idea of [4], where only two estimators are maintained. In opposite with most of packet losses studies, no explicit feedback mechanisms is considered. Nevertheless, packets received from neighbors provide some implicit feedback which is exploited to reduce the number of considered loss hypotheses, without requiring additional communications. This reduces the amount of estimators of its own state maintained by each Agent. The CTC proposed in [19] is then updated to explicitly account for the potential loss of transmitted packets. The asymptotic convergence of the MAS to the target formation, as well as the absence of Zeno behavior have been proved.

Assumptions and the formation parameterization are introduced in Section 2 and 3. The distributed control law is described in Section 3. State estimators to replace missing information in control law and evaluate the CTC are proposed in Section 4.1. Influence of packet losses on estimator is presented in Section 4.2, to evaluate an expected value of the estimation error. Knowledge of this error is improved using a feedback information from other agents, as described in Section 4.4. The distributed CTC is presented in Section 5. A simulation example is presented in Section 6 to illustrate the reduction of the number of communications obtained by the proposed approach. Finally, Section 7 presents conclusions and perspectives for future work.

2 Notations and hypotheses

Consider a vector $x = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$. The notation $x \geqslant 0$ indicates that each component x_i of x is non-negative, *i.e.*, $x_i \geqslant 0$, $\forall i \in \{1, \ldots, n\}$. The absolute value of the i-th component of x is $|x_i|$ and $|x| = (|x_1|, \ldots, |x_n|)^T$.

2.1 Multi-Agent system

Consider a MAS consisting of N communicating agents with indexes in the set $\mathcal{N} = \{1, \ldots, N\}$. In a global fixed reference frame $\mathcal{R}$, let $q_i \in \mathbb{R}^n$ be the vector of *coordinates* of Agent i and $q = \left[q_1^T, \ldots, q_N^T\right]^T \in \mathbb{R}^{Nn}$ be the *configuration* of the MAS. The relative coordinate vector between two Agents i and j is $r_{ij} = q_i - q_j$.

The evolution of the state $x_i = \left[q_i^T, \dot{q}_i^T\right]^T$ of Agent i is assumed to be described by the Euler-Lagrange model

$$M_i(q_i)\,\ddot{q}_i + C_i(q_i,\, \dot{q}_i)\,\dot{q}_i + G = u_i + d_i, \tag{1}$$

where $u_i \in \mathbb{R}^n$ is some control input, $M_i(q_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix of Agent i, $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n\times n}$ is the matrix of the Coriolis and centripetal terms for Agent i, G accounts for gravitational acceleration supposed to be known and constant, and d_i is a time-varying state perturbation satisfying $\|d_i(t)\| \leqslant D_{\max}$.

One assume that the MAS is such that for each Agent i,

A1) $M_i(q_i)$ is symmetric positive and there exists $k_M > 0$ satisfying $\forall x$, $x^T M_i x \leq k_M x^T x$.

A2) $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric or negative definite and there exists $k_C > 0$ satisfying $\forall x$, $x^T C_i(q_i, \dot{q}_i)\, x \leq k_C \|\dot{q}_i\|\, x^T x$.

A3) the left side of (1) can be linearly parametrized as

$$M_i(q_i)\,\xi_1 + C_i(q_i,\, \dot{q}_i)\,\xi_2 = Y_i(q_i,\, \dot{q}_i,\, \xi_1,\, \xi_2)\,\theta_i \tag{2}$$

for all vectors ξ_1, $\xi_2 \in \mathbb{R}^n$, where $Y_i(q_i,\, \dot{q}_i,\, \xi_1,\, \xi_2)$ is a regressor matrix with known structure identical for all agents, and $\theta_i \in \mathbb{R}^p$ is a vector of constant parameters known by Agent i.

A4) x_i can be measured without error.

A5) an estimate $\hat{x}_i^j(0)$ of the state $x_i(0)$ is known by all its neighbors $j \in \mathcal{N}_i$ and the square norm of the errors $||q_i(0) - \hat{q}_i^j(0)||^2$ and $||\dot{q}_i(0) - \dot{\hat{q}}_i^j(0)||^2$ are bounded with bounds provided in Proposition 6, see Section 5.

N	number of agents
q_i	coordinates of Agent i
q	configuration vector, $q = [q_1, q_2, ..., q_N]$
$\dot{q}_i^*$	target reference velocity of Agent i
r_{ij}	relative coordinate vector between Agents i and j, with $r_{ij} = q_i - q_j$
r_{ij}^*	target relative coordinate vector between Agents i and j
r^*	target relative configuration vector with $r^* = [r_{11}^*, r_{12}^*, ..., r_{1N}^*]$
x_i	state of Agent i with $x_i^T = \left[q_i^T, \dot{q}_i^T\right]$
$\hat{x}_i^j$	estimate of x_i by Agent j with $\left(\hat{x}_i^j\right)^T = \left[\left(\hat{q}_i^j\right)^T, \left(\dot{\hat{q}}_i^j\right)^T\right]$
$\hat{q}_i^{i,\ell}$	estimate of q_i performed by Agent i using the information in its ℓ-th transmitted message and not in the following one
e_i^j	error between q_i and $\hat{q}_i^j$
$\dot{e}_i^j$	error between $\dot{q}_i$ and $\dot{\hat{q}}_i^j$
$\bar{r}_{ij}$	estimated relative coordinate vector between Agents i and j as evaluated by Agent i with $\bar{r}_{ij} = q_i - \hat{q}_j^i$
k_i	index of k_i-th message sent by Agent i
t_{i,k_i}	transmission time of the k_i-th message sent by Agent i
π	packet losses probability
κ	maximum number of consecutive packet losses
δ_{i,k_i}^j	variable indicating whether the k_i-th message sent by Agent i has been received by Agent j ($\delta_{i,k_i}^j = 1$) or lost ($\delta_{i,k_i}^j = 0$)
k_i^j	index of the last message received by Agent j among those sent by Agent i
$k_i^{j,i}$	index known by Agent i of the last message received by Agent j among those sent by Agent i
$p^j_{k_i,\ell\|k_i^{j,i},\rho_i^j}$	probability that the ℓ-th message sent by Agent i (with $k_i^{j,i} \leqslant \ell \leqslant k_i$) has been received by Agent j and that all following messages, including the k_i-th have been lost, knowing that the last packet sent by Agent j has been received by Agent i in $\left[t_{i,\rho_i^j}, t_{i,\rho_i^j}\right[$
m_{ij}	potential energy coefficient between Agent i and j
α_i	sum of coefficients m_{ij} for $j \in \mathcal{N}_i$

Table 1
Main notations

A6) its velocity is bounded,

$$\|\dot{q}_i(t)\| \leq \dot{q}_{\max} \tag{3}$$

and Lipschitz, *i.e*, there exist $K_d > 0$ such that $\forall t, \Delta t$

$$\|\dot{q}_i(t + \Delta t) - \dot{q}_i(t)\| \leq K_d|\Delta t| \tag{4}$$

Assumptions A1, A2, and A3 have been previously considered, *e.g.*, in [10–12,17]. In what follows, the notations M_i and C_i are used in place of $M_i(q_i)$ and $C_i(q_i, \dot{q}_i)$.

2.2 Communication model

The communication topology of the MAS is described by a fixed undirected graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ is the set of edges of the graph. Agent i can communicate with its N_i one-hop neighbors with indexes in $\mathcal{N}_i = \{j \in \mathcal{N} | (i,j) \in \mathcal{E}, i \neq j\}$. One neglects communication delays between agents. The communication link between Agents i and j is unreliable and messages may be lost. Usually, packet losses are due *i*) to collisions (packets are transmitted at the same time instants by different agents), *ii*) to occlusions by obstacles (two agents are not in line of sight), *iii*) to a signal-to-noise ratio below a certain

threshold (agents are too far away). When Agent i broadcasts its k_i-th message at time t_{i,k_i}, Agent $j \in \mathcal{N}_i$ either receives this message without error at time t_{i,k_i} or does not receive it. To limit the amount of communications, there is no acknowledgment mechanism and thus no possible retransmission in case of losses. Let $\{\delta^j_{i,k_i}\}_{k_i \geqslant 1}$ be a sequence of binary variables such that $\delta^j_{i,k_i} = 1$ if the k_i-th message sent by Agent i has been received by Agent j and $\delta^j_{i,k_i} = 0$ else.

Inspired by [7], here, the δ^j_{i,k_i}s are modeled as realizations of time-invariant Markov processes with characteristics identical for all agents, as described in Assumption A7.

A7) *There exists $\kappa > 0$ such that for all pairs of neighbouring Agents (i, j) where $j \in \mathcal{N}_i$, one has*

$$\Pr\left(\delta^j_{i,k_i} = 1 | \sum_{\ell=1}^{\kappa} \delta^j_{i,k_i-\ell} > 0\right) = 1 - \pi \tag{5}$$

$$\Pr\left(\delta^j_{i,k_i} = 0 | \sum_{\ell=1}^{\kappa} \delta^j_{i,k_i-\ell} > 0\right) = \pi \tag{6}$$

and

$$\Pr\left(\delta^j_{i,k_i} = 1 | \sum_{\ell=1}^{\kappa} \delta^j_{i,k_i-\ell} = 0\right) = 1 \tag{7}$$

$$\Pr\left(\delta^j_{i,k_i} = 0 | \sum_{\ell=1}^{\kappa} \delta^j_{i,k_i-\ell} = 0\right) = 0 \tag{8}$$

with $0 \leqslant \pi < 1$.

Assumption A7 implies that at least one of the last κ messages broadcast by Agent i has been received by each of its neighour Agent j.

The packet loss model (5)-(7) is clearly a coarse approximation of reality. This model captures relatively accurately situation i). Packet loss events due to collisions are most often independent from one communication trial to the next one, provided that there is no synchronization between agents (see ALOHA protocol [1]). The considered packet loss model can also represent situation ii) provided that obstacles are small or agents move fast enough to experience only very short occlusions. Situation iii) is more difficult to represent even with the Bernoulli model considered, *e.g.* in [2,5,6,25,23]. Adjusting the transmission power periodically, so as to reach farther agents (even less frequently), may partly address the problem. Nevertheless, this would lead to a time-varying probability π of packet loss. For situations ii) and iii), one may alternatively consider a modification of the agent communication topology, which is out of the scope of this paper. Some of these works use feedback to partially solve the problem, but this method requires extra communications and so increases the risk of collision between packets, as described in situation i). This is why, here, the only feedback information considered is that received from packets sent by other agents considering they have to transmit information.

One cannot ensure (7) rigorously, but for given values of π and ϵ, κ can be chosen sufficiently large to satisfy

$$\Pr\left(\delta^j_{i,k_i} = 1 | \sum_{\ell=1}^{\kappa} \delta^j_{i,k_i-\ell} = 0\right) \geq 1 - \epsilon. \tag{9}$$

for some $\epsilon \ll 1$. For example, considering a loss probability as large as $\pi = 0.5$ and $\epsilon = 0.01$, one can choose $\kappa = 6$, and for $\epsilon = 0.001$, one can choose $\kappa = 9$.

Let $k^j_i \leqslant k_i$ be the index of the last message Agent j has received from its neighbor i. When a communication is triggered at time t_{i,k_i}, Agent i broadcasts a message containing k_i, t_{i,k_i}, $q_i(t_{i,k_i})$, $\dot{q}_i(t_{i,k_i})$, θ_i, and $\{k^i_j\}, j \in \mathcal{N}_i$. By sending $k^i_j \leq k_j$ for all $j \in \mathcal{N}_i$, Agent i indicates the index of the last message received from each of its neighbors.

When Agent j receives a message from Agent i, it updates k^j_i to k_i. Moreover, $q_i(t_{i,k_i})$, $\dot{q}_i(t_{i,k_i})$, and θ_i are used to update its estimator of the state of Agent i, as detailed in Section 4.1. Finally, Agent j keeps track in the variables $k^{i,j}_j$ of the value of k^i_j which represents the index of the last message sent by Agent j and which has been actually received by Agent i. The indice $k^{i,j}_j$ is used by Agent j to evaluate the knowledge Agent i has about x_j (see Figure 1).

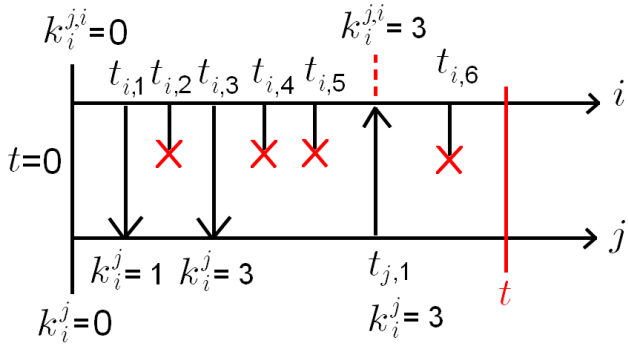


Fig. 1. Communication instants between Agents i and j and evolution of the indexes k_i^j and $k_i^{j,i}$ of last message received; from the packet received at time $t_{j,1}$, Agent i can deduce that Agent j has received the packet sent at time $t_{i,3}$

2.3 Target formation

A potentially time-varying target formation is defined by the set $\mathcal{R} = \left\{ r_{ij}^*(t), (i,j) \in \mathcal{N} \times \mathcal{N} \right\}$, where $r_{ij}^*(t)$ is the target relative coordinate vector between Agents i and j. Without loss of generality, the first agent is considered as the reference agent. Any target relative coordinate vector r_{ij}^* can be expressed as $r_{ij}^*(t) = r_{i1}^*(t) - r_{j1}^*(t)$. The target relative *configuration* vector is $r^*(t) = [\, r_{11}^{*T}(t) \; \ldots \; r_{1N}^{*T}(t) \,]^T$. Each Agent i is assumed to only know the relative coordinate vector with its own neighbors $r_{ij}^*(t)$, $j \in \mathcal{N}_i$. Additionally, a constant target reference velocity $\dot{q}_1^*$ known by all agents is imposed to the MAS. The reference velocities $\dot{q}_i^*$ are expressed as $\dot{q}_i^* = \dot{q}_1^* + \dot{r}_{i1}^*$ and are assumed to satisfy the following assumption.

A8) *For all Agents i, the target velocity $\dot{q}_i^*$ is bounded such that*

$$\|\dot{q}_i^*(t)\| < \dot{q}_{\max}$$

and Lipschitz with constant $K_d^* \leq K_d$, i.e. $\forall t \; \Delta t$,

$$\|\dot{q}_i^*(t + \Delta t) - \dot{q}_i^*(t)\| \leq K_d^* |\Delta t|. \tag{10}$$

Our aim is to evaluate, in a distributed way, the control input for each Agent so that the MAS converges to $\mathcal{R}$, while limiting the number of communications between agents and accounting for losses. For that purpose, the control input of each Agent will have to provide an asymptotic convergence of the MAS to the target configuration vector with a bounded mean-square error. Due to the packet losses, this convergence will only be achievable in the mean-square sense (MSE).

Definition 1 *The MAS asymptotically mean-square converges to the target formation with a bounded MSE (bounded average asymptotic convergence) iff there exists some $\varepsilon_1 > 0$ such that*

$$\forall (i,j) \in \mathcal{N}^2, \; \lim_{t \to \infty} \mathbb{E}\left(\left\| r_{ij}(t) - r_{ij}^*(t) \right\|^2 \right) \leqslant \varepsilon_1, \tag{11}$$

where the expectation is evaluated considering the packet loss events.

2.4 Overview of the proposed approach

A distributed control law is introduced in Section 3 to drive the MAS to its target formation and reference speed. This requires the knowledge by each Agent of the state vector of its neighbors. Since the state vector of a neighbor j is only available at Agent i when Agent j broadcasts its state, Agent i has to maintain an estimator of the state of each of its neighbors. This estimator is described in Section 4.

Moreover, to determine the quality of the estimate of x_i evaluated by its neighbors, Agent i has also to estimate its own state x_i with the information it has transmitted to these neighbors. As soon as a function of the error between this estimate and x_i reaches some threshold, Agent i triggers a communication to allow its neighbors to refresh their estimate of x_i. The main difficulty, compared to [17,19], lies in the fact that estimators have to account for packet losses. In the solution proposed here, each Agent maintains several estimates of its own state accounting for different packet loss hypotheses, and an estimate of

the state of its neighbors with the last information received. As will be seen in Section 4.4, the number of hypotheses can be limited to a manageable amount determined by the last received packet from Agent i.

The CTC relies on the error between the values of the states of agents and of the estimates made by neighboring agents, see Section 5. Since this error cannot be exactly evaluated due to packet losses, only its expected value is used in the CTC. This paper proposes different methods to evaluate or upper-bound this expected error, which is then used to analyze the convergence and the stability of the MAS.

3 Distributed control inputs

Section 3.1 introduces the potential energy $P(q,\, t)$ of the MAS to quantify the discrepancy between the current and target formations. A control input accounting for Agent state estimators is defined in Section 3.2.

3.1 Potential energy of the formation

In [13,24], the *potential energy* of the formation

$$P(q,\, t) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} m_{ij}\left\|r_{ij} - r_{ij}^{*}\right\|^{2} \tag{12}$$

is introduced, where $m_{ij} = m_{ji}$ are some positive or null coefficients. $P(q,t)$ quantifies the discrepancy between the actual and target relative coordinate vectors. We take $m_{ii} = 0$, $m_{ij} = 0$ if $(i,j) \notin \mathcal{E}$, and $m_{ij} > 0$ if $(i,j) \in \mathcal{E}$. Since $\mathcal{G}$ is connected, the minimum number of non-zero coefficients m_{ij} to properly define a target formation is $N-1$.

Proposition 2 *The MAS asymptotically converges to the target formation with a bounded MSE iff there exists some $\varepsilon_2 > 0$ such that*

$$\lim_{t\to\infty} \mathbb{E}\left(P(q,\, t)\right) \leqslant \varepsilon_2, \tag{13}$$

where the expectation is evaluated considering the packet loss events.

The proof of Proposition 2 is provided in Appendix A.1.

3.2 Control input with Agent state estimators

In what follows, a distributed control law is designed so that the MAS asymptotically converges with a bounded MSE. The control law has to make $P(q,\, t)$ decrease. One introduces, as in [24],

$$g_i = \frac{\partial P(q,\, t)}{\partial q_i} = \sum_{j\in\mathcal{N}_i} m_{ij}\left(r_{ij} - r_{ij}^{*}\right), \tag{14}$$

$$\dot{g}_i = \sum_{j\in\mathcal{N}_i} m_{ij}\left(\dot{r}_{ij} - \dot{r}_{ij}^{*}\right), \tag{15}$$

$$s_i = \dot{q}_i - \dot{q}_i^{*} + k_p g_i, \tag{16}$$

where $\dot{q}_i^{*} = \dot{q}_1^{*} - \dot{r}_{1i}^{*}$ is the reference velocity of Agent i. The vectors g_i and $\dot{g}_i$ characterize the evolution with q_i and $\dot{q}_i$ of the discrepancy between the actual and target relative coordinate vectors. In (16), $k_p > 0$ is a scalar design parameter.

According to (14), to make $P(q,\, t)$ decrease, the control input of Agent i requires r_{ij}, and thus q_j, $j \in \mathcal{N}_i$. Nevertheless, q_j is only available to Agent i when it receives a packet from Agent j containing q_j, see Section 2.2. Between the reception of two packets from Agent j, an estimates $\hat{q}_j^i$ of $q_j, j \in \mathcal{N}_i$ needs to be evaluated. This estimate has to account for potentially lost packets, see Section 4.1. Thus, using estimates $\hat{q}_j^i$ and $\dot{\hat{q}}_j^i$ of q_j and $\dot{q}_j$ for all $j \in \mathcal{N}_i$, Agent i is able to evaluate the discrepancies $\bar{r}_{ij} = q_i - \hat{q}_j^i$, $\dot{\bar{r}}_{ij} = \dot{q}_i - \dot{\hat{q}}_j^i$, as well as

$$\bar{g}_i = \sum_{j\in\mathcal{N}_i} m_{ij}\left(\bar{r}_{ij} - r_{ij}^{*}\right) \tag{17}$$

$$\bar{s}_i = \dot{q}_i - \dot{q}_i^{*} + k_p \bar{g}_i. \tag{18}$$

Then, the following control input can be evaluated in a distributed way by Agent i and used in (1)

$$u_i = -k_s \bar{s}_i - k_g \bar{g}_i + G - Y_i \left(q_i,\, \dot{q}_i,\, \bar{p}_i,\, \dot{\bar{p}}_i\right) \theta_i, \tag{19}$$

where $\bar{p}_i = k_p \bar{g}_i - \dot{q}_i^*$ and $\dot{\bar{p}}_i = k_p \dot{\bar{g}}_i - \ddot{q}_i^*$ with the additional design parameters $k_g > 0$ and $k_s \geq 1 + k_p\,(k_M + 1)$.

The convergence properties of the MAS when each Agent i applies the control input (19) is analyzed and ensured in Section 5.

4 State estimators and packet losses

This section describes the estimators involved in the control input (19) of each Agent. These estimators are introduced in Section 4.1. Section 4.2 describes the way Agent i estimates its own state x_i, with the information transmitted to its neighbors, to determine the quality of their estimates of x_i. In Section 4.3, the expected value of the estimation error between the current and the estimated state $\hat{x}_i^j, j \in \mathcal{N}_i$ is evaluated. This estimator accounts for packet losses. In Section 4.4, an implicit feedback, based on packets received from other agents, is described and exploited to improve the evaluation of the state estimation error.

4.1 Estimation of the state of other agents

To evaluate (19), Agent i has to maintain an estimate $\hat{x}_j^i$ of the state x_j of all its neighbors $j \in \mathcal{N}_i$. Assume that Agent j broadcasts its k-th message at time $t_{j,k}$. Then, since communication delays are neglected, depending on whether this message has been received by Agent i, $\hat{x}_j^i$ is updated as follows, see [3]

$$\hat{x}_j^i \left(t_{j,k}^+\right) = \delta_{j,k}^i x_j \left(t_{j,k}\right) + \left(1 - \delta_{j,k}^i\right) \hat{x}_j^i \left(t_{j,k}\right), \tag{20}$$

where $x_j(t_{j,k})$ is obtained from the received packet. For all $t \geqslant t_{j,k}$ and up to the time instant of reception of the next packet sent by Agent j, the components $\hat{q}_j^i$ and $\dot{\hat{q}}_j^i$ of $\hat{x}_j^i$ evolve as

$$M_j \left(\hat{q}_j^i\right) \ddot{\hat{q}}_j^i + C_j \left(\hat{q}_j^i,\, \dot{\hat{q}}_j^i\right) \dot{\hat{q}}_j^i + G = \hat{u}_j^i. \tag{21}$$

where M_j and C_j are evaluated using (2) with Y_j and $\hat{\theta}_j^i = \theta_j$, where the structure of Y_j and θ_j are initially known by Agent i or have been transmitted by Agent j at time $t = 0$. The estimator (21) maintained by Agent i requires itself an estimate $\hat{u}_j^i$ of the control input u_j evaluated by Agent j. This estimate $\hat{u}_j^i$, used by Agent i, is

$$\hat{u}_j^i = -k_s \dot{\hat{\varepsilon}}_j^i + G - Y_j \left(\hat{q}_j^i,\, \dot{\hat{q}}_j^i,\, -\ddot{q}_j^*,\, -\dot{q}_j^*\right) \hat{\theta}_j^i, \tag{22}$$

with $\dot{\hat{\varepsilon}}_j^i = \dot{\hat{q}}_j^i - \dot{q}_j^*$. The control input (22) thus only depends on information available to Agent i.

We consider the following assumption on the components of $\hat{x}_j^i$:

A9) The velocity $\dot{\hat{q}}_j^i$ is bounded such

$$\left\|\dot{\hat{q}}_j^i\right\| \leq \dot{q}_{\max}. \tag{23}$$

and $\dot{\hat{q}}_j^i$ is Lipschitz on all intervals $[t_{j,k}, t_{j,k+1}[$, *i.e* $\forall t \in [t_{j,k}, t_{j,k+1}[$ and $(t + \Delta t) \in [t_{j,k}, t_{j,k+1}[$ such that

$$\left\|\dot{\hat{q}}_j^i \left(t + \Delta t\right) - \dot{\hat{q}}_j^i \left(t\right)\right\| \leq \hat{K}_d |\Delta t|. \tag{24}$$

with $\hat{K}_d > 0$.

This assumtion is consistent with that consider for $\dot{q}_j$, *i.e* A6, since between two communication time instants, (21) is similar to (1).

4.2 Multi hypothesis state estimates

The estimate $\hat{q}_i^j$ of the state of Agent i, evaluated by Agent j, only depends on the information provided by Agent i. The estimate $\hat{q}_i^j$ is reset to q_i as soon as a message sent by Agent i is received by Agent j, see (20). Consequently, when Agent i has sent k_i messages, and wants to evaluate an image of its own state as computed by one of its neighbors, $\kappa+1$ different hypotheses have to be considered, each of which is associated to a different estimator of q_i at time $t \in [t_{i,k_i}, t_{i,k_i+1}[$

- A first estimator considers the k_i-th packet as received,
- A second estimator considers the k_i-th packet as lost, but the k_i-1-th packet as received,
- ...
- If $k_i \geq \kappa$, the last estimator assumes that all packets have been lost, except the $k_i-\kappa+1$-th if $k_i \geq \kappa$,
- If $k_i \leq \kappa$, the last estimator assumes that no packet has been received, but considers the initial state $\hat{x}_i^j(0)$.

At time $t \in [t_{i,k_i},\, t_{i,k_i+1}[$, the state estimates corresponding to these hypotheses are denoted as

$$\hat{x}_i^{i,\ell}(t) = \left[\hat{q}_i^{i,\ell}(t), \dot{\hat{q}}_i^{i,\ell}(t)\right], \tag{25}$$

with $\ell = \max\{k_i-\kappa+1, 0\}, \ldots, k_i$ and $\hat{x}_i^{i,k_i} = \hat{x}_i^i$, introduced in Section 4.1.

Since there are at most $\kappa-1$ consecutive losses, Agent i has only to maintain κ estimates of x_i, denoted $\hat{x}_i^{(1)}(t), ..., \hat{x}_i^{(\kappa)}(t)$. For all $t \in [t_{i,k_i}, t_{i,k_i+1}[$, one has $\hat{x}_i^{(1)}(t) = \hat{x}_i^{i,k_i}(t), \ldots, \hat{x}_i^{(\kappa)}(t) = \hat{x}_i^{i,k_i-\kappa+1}(t)$. These estimates evolve according to the dynamic (21)-(22) introduced in Section 4.1. When a new packet is sent at time t_{i,k_i+1} by Agent i, the estimates are updated as

$$\hat{x}_i^{(1)}(t_{i,k_i+1}) = x_i(t_{i,k_i+1}). \tag{26}$$

$$\hat{x}_i^{(\ell+1)}(t_{i,k_i+1}) = \hat{x}_i^{(\ell)}\left(t_{i,k_i+1}^-\right), \ \ell = 1, \ldots, \kappa-1. \tag{27}$$

4.3 Expected value of the estimation error of $x_i(t)$

At time $t \in [t_{i,k_i},\, t_{i,k_i+1}[$, Agent i has sent k_i packets. Let

$$p_{k_i,\ell}^j = \Pr\left(\delta_{i,\ell}^j = 1, \delta_{i,\ell+1}^j = 0, \ldots, \delta_{i,k_i}^j = 0\right) \tag{28}$$

with $\max\{k_i-\kappa+1, 0\} \leq \ell \leq k_i$, be the probability that the ℓ-th packet has been received by a given neighbor j and that all packets from the $\ell+1$-th to the k_i-th have been lost. By convention,

$$p_{k_i,0}^j = \Pr\left(\delta_{i,1}^j = 0, \ldots, \delta_{i,k_i}^j = 0\right)$$

and

$$p_{k_i,k_i}^j = \Pr\left(\delta_{i,k_i}^j = 1\right). \tag{29}$$

Note that $p_{k_i,\ell}^j$ only depends on the packet loss model (5), and does not depend on the neighbor index j, which is omitted in what follows.

Proposition 3 *For all $k_i > 0$ and $\ell \leq k_i$ one has*

$$p_{1,1}^j = 1-\pi \tag{30}$$

$$p_{1,0}^j = \pi, \tag{31}$$

and

$$\begin{aligned} p_{k_i,\ell}^j &= \pi p_{k_i-1,\ell}^j \quad \textit{if } k_i-\ell \leq \kappa \\ &= 0 \quad \textit{else.} \end{aligned} \tag{32}$$

$$p_{k_i,k_i}^j = 1 - \sum_{\ell=\max\{k_i-\kappa,0\}}^{k_i-1} p_{k_i,\ell}^j. \tag{33}$$

		k_i				
		1	2	3	4	5
ℓ	0	π	π^2	0	0	0
	1	$1-\pi$	$(1-\pi)\pi$	$(1-\pi)\pi^2$	0	0
	2	$*$	$1-\pi$	$(1-\pi)\pi$	$(1-\pi)\pi^2$	0
	3	$*$	$*$	$1-\pi+\pi^3$	$\pi-\pi^2+\pi^4$	$\pi^2-\pi^3+\pi^5$
	4	$*$	$*$	$*$	$1-\pi+\pi^3-\pi^4$	$\pi-\pi^2+\pi^4-\pi^5$
	5	$*$	$*$	$*$	$*$	$1-\pi+\pi^3-\pi^4$

Table 2
Probabilities $p^j_{k_i,\ell}$ that the ℓ-th message sent by Agent i has been received by Agent j and all following messages including the k_i-th one have been lost, here $\kappa = 2$; $*$ represents probabilities not defined.

The proof of Proposition 3 is in Appendix A.2.

At time $t \in [t_{i,k_i},\, t_{i,k_i+1}[$, from $\hat{x}^j_i(t)$ the estimation error of the coordinates of Agent i, as evaluated by Agent j, is

$$e^j_i\,(t) = \hat{q}^j_i\,(t) - q_i\,(t) \tag{34}$$

and its mean-square value is

$$\mathbb{E}\left(||e^j_i\,(t)\,||^2\right) = \sum_{\ell=\max\{k_i-\kappa+1,0\}}^{k_i} p^j_{k_i,\ell}||\hat{q}^{i,\ell}_i\,(t) - q_i\,(t)\,||^2. \tag{35}$$

$\mathbb{E}\left(||e^j_i\,(t)\,||^2\right)$ can be determined by Agent i using $\hat{q}^{i,\ell}_i\,(t)$ and $p_{k_i,\ell}$, $\ell = \max\{k_i-\kappa+1, 0\}, \ldots, k_i$. Consequently, from (35), Agent i is able to determine the quality of the estimate of q_i evaluated by its neighbors. Agent i has thus to maintain κ estimates $\hat{q}^{i,\ell}_i\,(t)$ of $q_i\,(t)$. Note that $\mathbb{E}\left(||\dot{e}^j_i\,(t)\,||^2\right)$ can be obtained in the same way.

4.4 Estimates accounting for received packets

Consider Agent i, the time interval $[t_{i,k_i}, t_{i,k_i+1}[$, and assume that $\left[t_{i,\rho^j_i}, t_{i,\rho^j_i+1}\right[$ is time interval during which the last packet has been received from Agent j. This message contains the index k^j_i of the last message received by Agent j and sent by Agent i, see Figure 1. This index is kept by Agent i in $k^{j,i}_i$, see Section 2.2. This implicit feedback information can significantly improve the evaluation of the mean-square values of $e^j_i\,(t)$ and $\dot{e}^j_i\,(t)$. From this message, Agent i knows that all packets sent in the time interval $\left[t_{i,k^j_i+1}, t_{i,\rho^j_i+1}\right[$ have not been received by Agent j. For example, in Figure 1, the packet received in $[t_{i,k_i}, t_{i,k_i+1}[$ with $k^j_i = k_i - 2$ indicates that packet $k_i - 2$ has been received, but neither packet $k_i - 1$ nor k_i. Using this knowledge, Agent i can evaluate the probability

$$p^j_{k_i,\ell|k^{j,i}_i,\rho^j_i} = \Pr\left(\delta^j_{i,\ell} = 1,\ \sum_{m=\ell+1}^{k_i} \delta^j_{i,m} = 0 | \delta^j_{i,k^j_i} = 1,\ \sum_{m=k^{j,i}_i+1}^{\rho^j_i} \delta^j_{i,m} = 0\right) \tag{36}$$

that the ℓ-th message sent by Agent i (with $k^j_i \leqslant \ell \leqslant k_i$) has been received by Agent j and that all following messages, including the k_i-th have been lost. By convention,

$$p^j_{k_i,k_i|k^{j,i}_i,\rho^j_i} = \Pr\left(\delta^j_{i,k_i} = 1 | \delta^j_{i,k^{j,i}_i} = 1,\ \sum_{m=k^{j,i}_i+1}^{\rho^j_i} \delta^j_{i,m} = 0\right).$$

Proposition 4 *As long as Agent i has not received any message from Agent j, then $p^j_{k_i+n,\ell|0,0}$ is evaluated for all ℓ as*

$$\begin{aligned} p^j_{k_i+n,\ell|0,0} &= (1-\pi)\,\pi^{k_i+n-\ell} \quad \textit{if } k_i+n-\ell \le \kappa \\ &= 0 \quad \textit{else.} \end{aligned} \tag{37}$$

Assume that Agent i receives a message from Agent j at $t_{j,k_j} \in [t_{i,k_i}, t_{i,k_i+1}[$ containing k^j_i. Then $k^{j,i}_i = k^j_i$, $\rho^j_i = k_i$, and

$$p^j_{k_i,k^{j,i}_i|k^{j,i}_i,k_i} = 1 \tag{38}$$

$$p^j_{k_i,\ell|k^{j,i}_i,k_i} = 0 \quad \forall \ell \in \left\{0, \ldots, k^{j,i}_i - 1, k^{j,i}_i + 1, \ldots, k_i\right\}. \tag{39}$$

Consider $t \in [t_{i,k_i+n}, t_{i,k_i+n+1}[$ with $n > 0$ and assume that the last message received by Agent i from Agent j has been at time $t_{j,k_j} \in [t_{i,k_i}, t_{i,k_i+1}[$. Consequently, $k_i^{j,i} \leqslant k_i$, and one has still $\rho_i^j = k_i$. Assume, moreover, that $p^j_{k_i+n-1,\ell|k_i^{j,i},k_i}$ is known for all $\ell = 0, \ldots, k_i + n - 1$. Then $p^j_{k_i+n,\ell|k_i^{j,i},k_i}$ can be evaluated recursively for all $\ell = 0, \ldots, k_i + n$ as follows

$$
\begin{aligned}
p^j_{k_i+n,k_i^{j,i}|k_i^{j,i},k_i} &= \pi p^j_{k_i+n-1,k_i^{j,i}|k_i^{j,i},k_i} \quad \text{if } k_i + n - k_i^{j,i} \leq \kappa && (40)\\
&= 0 \quad \text{else,} \\
p^j_{k_i+n,\ell|k_i^{j,i},k_i} &= \pi p^j_{k_i+n-1,\ell|k_i^{j,i},k_i} \quad \text{if } k_i + n - \ell \leq \kappa \text{ and } k_i < \ell < k_i + n && (41)\\
&= 0 \quad \text{if } \ell < k_i \text{ with } \ell \neq k_i^{j,i}, \text{ or } \ell < k_i + n - \kappa, \\
p^j_{k_i+n,k_i+n|k_i^{j,i},k_i} &= 1 - \sum_{\ell=k_i+1}^{k_i+n-1} p^j_{k_i+n,\ell|k_i^{j,i},k_i} - p^j_{k_i+n,k_i^{j,i}|k_i^{j,i},k_i}. && (42)
\end{aligned}
$$

When $k_i + n - k_i^{j,i} > \kappa$, (42) becomes

$$
p^j_{k_i+n,k_i+n|k_i^{j,i},k_i} = 1 - \sum_{\ell=k_i+1}^{k_i+n-1} p^j_{k_i+n,\ell|k_i^{j,i},k_i}. \tag{43}
$$

Finally, when $n > \kappa$, (42) becomes

$$
p^j_{k_i+n,k_i+n|k_i^{j,i},k_i} = 1 - \sum_{\ell=\max\{0,k_i+n-\kappa\}}^{k_i+n-1} p^j_{k_i+n,\ell|k_i^{j,i},k_i}. \tag{44}
$$

The proof of Proposition 4 is in Appendix A.2. A consequence of Proposition 4 is that at most κ terms $p^j_{k_i+n-1,\ell|k_i^{j,i},k_i}$, $\ell = \max\{0, k_i + n - \kappa\}, \ldots, k_i + n - 1$ are required to evaluate $p^j_{k_i+n,\ell|k_i^{j,i},k_i}$ all $\ell = 0, \ldots, k_i + n$.

Table 3 illustrates the evolution of $p_{k_i+n,\ell|k_i^{j,i},k_i}$ when $\kappa = 3$, $k_i = 5$, and $k_i^j = 3$.

Then Proposition 4 can be used with Assumption **A7** to evaluate $\mathbb{E}\left(||e_i^j(t)||^2\right)$, taking into account the feedback information provided by neighbors as follows.

Consider some Agent i and $k_i > 0$. Assume that Agent i knows the index k_i^j of the last message sent by Agent i and received by some neighbor Agent j. At time $t \in [t_{i,k_i+n}, t_{i,k_i+n+1}[$, the mean-square value of the estimation error (34) is

$$
\mathbb{E}\left(\left\|e_i^j(t)\right\|^2 |k_i^{j,i}\right) = \sum_{\ell=\max\{k_i-\kappa+1,0\}}^{k_i} p^j_{k_i+n,\ell|k_i^{j,i},k_i} \left\|\hat{q}_i^{i,\ell}(t) - q_i(t)\right\|^2. \tag{45}
$$

In what follows, the notation $\mathbb{E}(||e_i^j(t)||^2)$ is used in place of $\mathbb{E}(||e_i^j(t)||^2|k_i^{j,i})$. Contrary to (35), (45) depends now on the index of the neighbor Agent j via $k_i^{j,i}$, and so is updated each time Agent i receives a message from its neighbor, in addition to the update made each time Agent i broadcast a message as in (35).

5 Event-triggered communications accounting for packet losses

This section presents a CTC which may involve one of the state estimators introduced in Section 4.

Let $m_{\min} = \min_{i,j=1,\ldots,N} \{m_{ij} \neq 0\}$, $m_{\max} = \max_{i,j=1,\ldots,N} \{m_{ij}\}$, $N_{\min} = \min_{i=1\ldots N}\{N_i\}$, $\alpha_i = \sum_{j=1}^N m_{ij}$, and $\alpha_{\mathrm{M}} = \max_{i=1,\ldots,N} \alpha_i$. The distributed CTC (46) presented in Theorem 5 is designed to ensure an asymptotic convergence of the MAS to the target formation with a bounded MSE.

		k_i+n					
		5	6	7	8	9	10
ℓ	1	0	0	0	0	0	0
	2	0	0	0	0	0	0
	3	1	π	0	0	0	0
	4	0	0	0	0	0	0
	5	0	0	0	0	0	0
	6	$*$	$1-\pi$	$(1-\pi)\pi$	$(1-\pi)\pi^2$	$(1-\pi)\pi^3$	0
	7	$*$	$*$	$1-(1-\pi)\pi$	$\pi-(1-\pi)\pi^2$	$\pi^2-(1-\pi)\pi^3$	$\pi^3-(1-\pi)\pi^4$
	8	$*$	$*$	$*$	$1-\pi$	$(1-\pi)\pi$	$(1-\pi)\pi^2$
	9	$*$	$*$	$*$	$*$	$1-\pi$	$(1-\pi)\pi$
	10	$*$	$*$	$*$	$*$	$*$	$1-\pi+(1-\pi)\pi^4$

Table 3
Probabilities $p_{k_i+n,\ell|k_i^{j,i},k_i}$ that the ℓ-th message sent by Agent i has been received by Agent j and all following messages including the k_i-th one have been lost knowing the $k_i^{j,i}$-th message has been received, for $n \in [0,\ldots,5]$, $k_i = 5$, and $k_i^{j,i} = k_i^j = 3$; $*$ represents probabilities not defined.

Theorem 5 *Consider a MAS with Agent dynamics given by (1), the communication protocol defined in Section 2.2, and the control law (19). Assuming absence of communication delays, and a packet losses model satisfying Assumption A7. If the communications are triggered by each Agent i of the MAS with Agent dynamics (1) when the following condition is satisfied*

$$\begin{aligned}&\alpha_M\left[\sum_{j=1}^{N} m_{ij}\left(k_e\mathbb{E}\left(\left\|e_i^j\right\|^2\right)+k_pk_M\mathbb{E}\left(\left\|\dot{e}_i^j\right\|^2\right)\right)+k_pk_C^2\right.\\&\left.\times\sum_{j=1}^{N} m_{ij}\left(2\mathbb{E}\left(\left\|e_i^j\right\|^2\right)\left\|\dot{\hat{q}}_j^i\right\|^2+\mathbb{E}\left(\left\|e_i^j\right\|^4\right)+\mathbb{E}\left(\left\|\dot{e}_i^j\right\|^4\right)\right)\right]\\&+k_gb_i\left\|\dot{q}_i-\dot{q}_i^*\right\|^2\geq k_s\bar{s}_i^T\bar{s}_i+k_pk_g\bar{g}_i^T\bar{g}_i+\eta\end{aligned} \tag{46}$$

where $k_e = k_sk_p^2 + k_gk_p + \frac{k_g}{b_i}$, η *and* b_i *are design parameters such that*

$$\eta > 4\frac{(1-\pi)(1+\boldsymbol{a})^2}{1-\pi(1+\boldsymbol{a})^2}k_gb_i\dot{q}_{\max}^2 \tag{47}$$

for some $0 < \boldsymbol{a} < \sqrt{\frac{1}{\pi}} - 1$, *and* $0 < b_i < \frac{k_s}{k_sk_p+k_g}$, *then*

(a) The MAS asymptotically converges to the target formation with a bounded error such that $\lim_{t\to\infty}\mathbb{E}\left(\frac{1}{2}P(q,t)\right) \leq \xi$, *where* $\xi = \frac{N}{k_gc_3}\left[D_{\max}^2+\eta\right]$,

$$c_3 = \frac{\min\{k_1,k_p\}\min\left(1,\frac{N_{\min}m_{\min}}{m_{\max}}\right)}{\max\{1,k_M\}} \tag{48}$$

and $k_1 = k_s - (1 + k_p(k_M + 1))$.

(b) One has $t_{i,k_i+1} - t_{i,k_i} > \tau_{\min}$ *for some* $\tau_{\min} > 0$.

The proof of (a) in Theorem 5 is given in Appendix A. The absence of Zeno behavior is shown by the existence of a minimum inter-event time $\tau_{\min}$ (Theorem 5(b)) as shown in Appendix B . Each Agent i has to evaluate the expected values of $||e_i^j(t)||^2$ and $||\dot{e}_i^j(t)||^2$ for all $j \in \mathcal{N}_i$. This can be done using the expectation (35) or (45), as detailed in Section 4.3 and 4.4.

The CTC (46) is satisfied for Agent i mainly when $\mathbb{E}(||e_i^j(t)||^2)$ and $\mathbb{E}(||\dot{e}_i^j(t)||^2)$ become large. Thus, it is preferable to use the knowledge of $k_i^{j,i}$ provided by the proposed implicit feedback mechanism to calculate (45) rather than using (35). In the same way, a large packet loss probability π results in a large value of $\mathbb{E}(||e_i^j(t)||^2)$ and $\mathbb{E}(||\dot{e}_i^j(t)||^2)$, and therefore leads to an increase in the number of communications to compensate for the losses.

The right hand side of the CTC (46) is proportional to $\bar{g}_i(t)$ and $\bar{s}_i(t)$, *i.e.*, to the potential energy of the formation $P(q,t)$, which is large when agents are far from the target formation. Thus, the CTC is less frequently satisfied when agents are far from the target formation and satisfied more often when agents approach their objective. This behavior is consistent since agents require only a coarse knowledge of their relative positions when they are distant and try to group together. A more accurate knowledge of their position is needed when they are close to the target formation. A larger estimation error is thus tolerated at the beginning of the mission, allowing a reduced number of communication.

An analysis of the impact of the values of the parameters on the reduction of communications has been presented in [19] in absence of packet losses. These results can be extended to the case with packet losses. The choice of the parameters α_{M}, k_g, k_p and b_i also determines the number of broadcast messages. Choosing the coefficients m_{ij} such that $\alpha_i = \sum_{j=1}^{N} m_{ij}$ is small, leads to a reduction in the number of communications triggered resulting from the satisfaction of (46), at the cost of a less precise formation.

The following proposition introduces a condition on the initial estimation of agent states to guarantee that (46) in Theorem 5 is not satisfied at $t = 0$.

Proposition 6 *Condition (46) in Theorem 5 is not satisfied at $t = 0$, when a common initial value $\hat{x}_i^j(0)$ is known by all the neighbors j of each Agent i such that*

$$\left\|\dot{\hat{q}}_i^j(0)\right\| = 0,\ \left\|e_i^j(0)\right\|^2 \leq H_i,\ \left\|\dot{e}_i^j(0)\right\|^2 \leq H_i \tag{49}$$

where the bound $H_i \geq 0$ is defined for each Agent i as

$$H_i = \frac{\sqrt{(k_e + k_p k_M)^2 + k_p k_C^2 \xi_i} - (k_e + k_p k_M)}{2 k_p k_C^2} \tag{50}$$

$$\xi_i = \frac{k_p (k_s + k_g)}{\alpha_M \alpha_i} \bar{g}_i^T \bar{g}_i(0) + \frac{\eta}{\alpha_M \alpha_i}. \tag{51}$$

The proof of Proposition 6 is given in Appendix A.6. $H_i = 0$ corresponds to the case where the initial state $x_i(0)$ is known by all neighbors of Agent i, *i.e.* $\hat{x}_i^j(0) = x_i(0)\ \forall j \in \mathcal{N}_i$. The value of the bound H_i is proportional to $\bar{g}_i(0)$, *i.e.* the initial value of the potential energy of the formation. Thus, the most distant from the target formation agents are, the largest the initial error of the estimation $\hat{x}_i^j(0)$ can be tolerated.

We have assumed in Propriety 6 that all neighbors of Agent i share the same estimate $\hat{x}_i^j(0)$ of $x_i(0)$. This allows Agent i initializing the estimator of its own state by $\hat{x}_i^j(0)$ and avoids using a different estimator for each of its neighbors. When this hypothesis is not satisfied initially, in practice, after several communications, the local estimators of x_i and those performed by neighbors are likely to converge.

6 Example

Consider the dynamical model of N identical surface ships with coordinate vectors $q_i = [\, x_i \ y_i \ \psi_i \,]^T \in \mathbb{R}^3$, $i = 1 \ldots N$, in a local Earth-fixed frame. For Agent i, (x_i, y_i) represents its position and ψ_i its heading angle. The Agent dynamics are expressed in the body frame as

$$M_{\mathrm{b},i} \dot{\mathrm{v}}_i + C_{\mathrm{b},i}(\mathrm{v}_i)\, \mathrm{v}_i + D_{\mathrm{b},i} \mathrm{v}_i = \tau_{\mathrm{b},i} + d_{\mathrm{b},i}, \tag{52}$$

where v_i is the velocity vector in the body frame. The values of $M_{\mathrm{b},i}$, $D_{\mathrm{b},i}$, and $C_{\mathrm{b},i}(\mathrm{v}_i)$ are found in [9].

One takes $N = 6$. The model (52) may be expressed as (1) with $G = 0$ using an appropriate change of variables detailed in [9]. The parameters of (19) are $k_M = \|M_i\| = 33.8$, $k_C = \|C_i(1_N)\| = 43.96$, $k_p = 6$, $k_g = 20$, $k_s = 1 + k_p(k_M + 1)$, $b_i = \frac{1}{k_g}$.

6.1 Parameters

The initial value of the configuration vector is $q(0) = [x(0)^T, y(0)^T, \psi(0)^T]^T$, $\dot{q}(0) = 0_{3N\times 1}$, with $x(0) = [-0.35, 4.59, 4.72, 0.64, 3.53, -1.26]$, $y(0) = [-1.11, -4.59, 2.42, 1.36, 1.56, 3.36]$ and $\psi(0) = 0_N$. An hexagonal target formation is considered with $r^*(0) = [\, r_{(1)}^*(0)^T \ r_{(2)}^*(0)^T \ r_{(3)}^*(0)^T \,]^T$ where $r_{(1)}^*(0) = [0, 2, 3, 2, 0, -1]$, $r_{(2)}^*(0) = \left[0, 0, \sqrt{3},\ \ 2\sqrt{3}, 2\sqrt{3}, \sqrt{3}\right]$, and

$r^*_{(3)}(0) = 0_N$. Moreover, the target MAS velocity is $\dot{q}^*_1 = [1,\ 1,\ 0]^T$. Each Agent communicates with $N/2 = 3$ other agents. From [24], one obtains the coefficients matrix $S = [m_{ij}]_{i,j=1\ldots N}$ such

$$S = 0.1 \begin{bmatrix} 0 & 1.85 & 0 & 0.926 & 0 & 1.85 \\ 1.85 & 0 & 1.85 & 0 & 0.926 & 0 \\ 0 & 1.85 & 0 & 1.85 & 0 & 0.926 \\ 0.926 & 0 & 1.85 & 0 & 1.85 & 0 \\ 0 & 0.926 & 0 & 1.85 & 0 & 1.85 \\ 1.85 & 0 & 0.926 & 0 & 1.85 & 0 \end{bmatrix}.$$

One has $\alpha_i = \sum_{j=1}^{N} m_{ij} = 0.463$, for all $i = 1, \ldots, N$ and $\alpha_{\mathrm{M}} = 0.463$.

The simulation duration is $T = 4\,\mathrm{s}$, taken sufficiently large to reach a steady-state behavior, with an integration step size $\Delta t = 0.01$ s. Since time has been discretized, the minimum delay between the transmission of two messages by the same Agent is set to Δt. The perturbation $d_i(t)$ is assumed constant over each interval $[k\Delta t,\ (k+1)\,\Delta t[$. The components of $d_i(t)$ are independent realizations of zero-mean uniformly distributed noise $U\left(-D_{\max}/\sqrt{3},\ D_{\max}/\sqrt{3}\right)$ and are thus such that $\|d_i(t)\| \leq D_{\max}$. Let N_{m} be the total number of messages transmitted during a simulation. The performance of the proposed approach is evaluated with

$$R_{\mathrm{com}} = N_{\mathrm{m}}/\overline{N}_{\mathrm{m}} \tag{53}$$

where $\overline{N}_{\mathrm{m}} = NT/\Delta t \geq N_{\mathrm{m}}$. R_{com} is the ratio between the number of communications required using the proposed approach and the number of communications that would be obtained if a communication triggered at each sampling time instant. One takes $\kappa = 6$.

6.2 Simulations results

Figure 2 shows the performance of the proposed approach with the CTC (46) for different values of the packet loss probability π and disturbance bound $D_{\max}$. Results are averaged over 50 independent realizations of the noise and of the packet losses. As expected, the number of communications required for the MAS to converge increases with π and $D_{\max}$.

The influence of η on the number of communication is detailed in [20]. Increasing η leads to a reduction of R_{com} but increases the potential energy $P(q, T)$, and thus the discrepancy with respect to the target formation.

Figure 3 compares results of the proposed approach obtained without (a) and with (b) the exploitation of the index $k_i^{j,i}$ of the last message sent by Agent i and received by some neighbor Agent j. Using the implicit feedback from neighbors, and thus $\mathbb{E}\left(\left\|e_i^j(t)\right\|^2 |k_j^i\right)$ instead of $\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right)$ in the CTC, convergence is obtained with 75% less messages.

7 Conclusion

This paper addresses the problem of communication reduction in distributed formation control of a MAS with Euler-Lagrange dynamics in presence of packet losses and state perturbations. To evaluate its control input, each Agent maintains estimators of the states of the other agents as well as multiple estimators of its own state accounting for potentially lost packets in the communications with its neighbors. Using these estimators, each Agent is then able to compute an expected value of the estimation error of its own state as evaluated by its neighbors. An implicit feedback from other agents may be used to get a reduced estimation error. A distributed CTC is then proposed, involving these estimation errors, to reduce the number of communications. The behavior of the MAS is analyzed using stochastic Lyapunov functions in [21]. Convergence to the target formation and the absence of Zeno behavior have been proven. Simulations illustrate the effectiveness of the proposed approach. In future work, communication delays will also be considered along with packet losses.

References

[1] N. Abramson. The ALOHA system: Another alternative for computer communications. In *Proceedings of the November 17-19, fall joint computer conference*, pages 281–285, 1970.

[2] W. Chen, D. Ding, G. Wei, S. Zhang, and Y. Li. Event-based containment control for multi-agent systems with packet dropouts. *Journal of Systems Science*, 49(12):2658–2669, 2018.

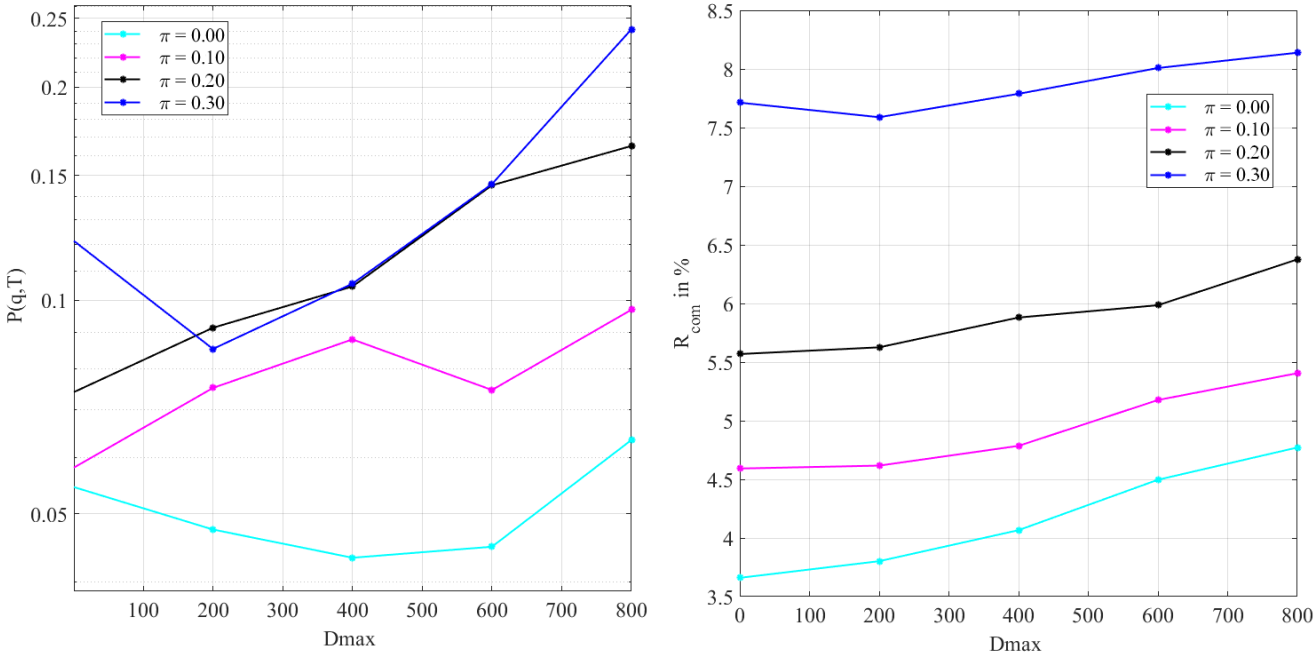
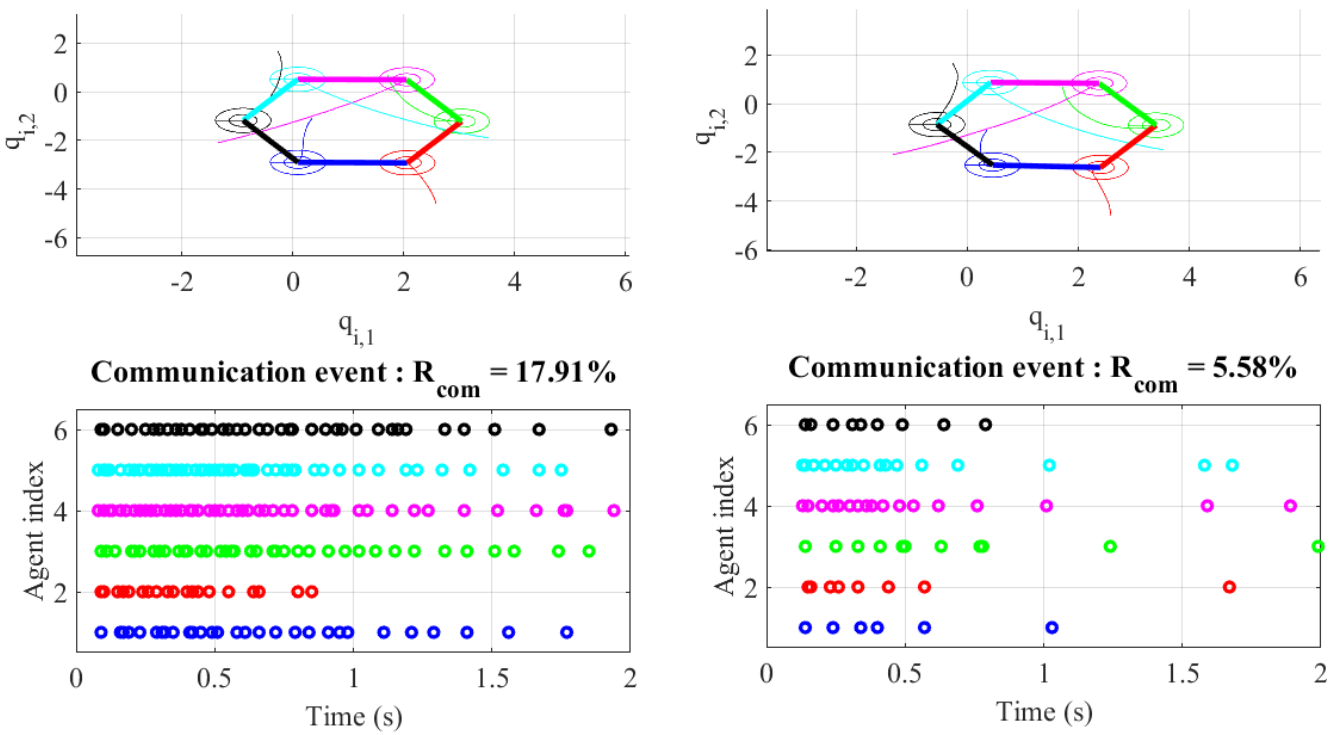


Fig. 2. Evolution of $P(q,T)$ and R_{com} for different values of $D_{\max}$, $\eta = 100$. The estimator (21) is considered, as well as $\mathbb{E}\left(\left\|e_i^j(t)\right\|^2 | k_j^{i,j}\right)$ from (45).



(a) without implicit feedback

(b) with implicit feedback

Fig. 3. Results of the proposed method using an estimated of the state error (a) via (35) (implicit feedback not used), (b) via (45) (implicit feed-back used). $D_{\max} = 200$, $\pi = 0.2$

[3] D. Ding, Z. Wang, B. Shen, and H. Dong. Event-triggered distributed h state estimation with packet dropouts through sensor networks. *IET Control Theory & Applications*, 9(13):1948–1955, 2015.

[4] V.S. Dolk, D. P. Borgers, and W Heemels. Output-based and decentralized dynamic event-triggered control with guaranteed l_p-gain performance and zeno-freeness. *IEEE Trans. on Automatic Control*, 62(1):34–49, 2017.

[5] W. Hu and L. Liu. Cooperative output regulation of heterogeneous linear multi-agent systems by event-triggered control. *IEEE trans. on cybernetics*, 47(1):105–116, 2017.

[6] Z. Hu, P. Shi, J. Zhang, and F. Deng. Control of discrete-time stochastic systems with packet loss by event-triggered approach. *IEEE Trans Systems, Man, and Cybernetics: Systems*, 1:1–10, 2018.

[7] A. Iacovazzi, D. Frassinelli, and Y. Elovici. On packet loss modeling: An empirical assessment. In *International Conference on the Network of the Future (NOF)*, pages 33–39, 2017.

[8] H. K. Khalil. *Nonlinear Systems, 3rd ed.* Upper Saddle River,NJ: Prentice-Hall, 2002.

[9] E KyrkjebÃž, K.Y. Pettersen, M. Wondergem, and H. Nijmeijer. Output synchronization control of ship replenishment operations: Theory and experiments. *Control Engineering Practice*, 15(6):741–755, 2007.

[10] Z Liu, W Chen, J Lu, H Wang, and J Wang. Formation control of mobile robots using distributed controller with sampled-data and communication delays. *IEEE Trans. Control Systems Technology*, 24(6):2125–2132, 2016.

[11] C. Makkar, G. Hu, W. G. Sawyer, and W.E. Dixon. Lyapunov-based tracking control in the presence of uncertain nonlinear parameterizable friction. *IEEE Trans. Automatic Control*, 52(10):1988–1994, 2007.

[12] J. Mei, W. Ren, and G. Ma. Distributed coordinated tracking with a dynamic leader for multiple euler-lagrange systems. *IEEE Trans. Automatic Control*, 56(6):1415–1421, 2011.

[13] B. Nabet and N.-E. Leonard. Tensegrity models and shape control of vehicle formations. In *arXiv:0902.3710*, pages 1–31, 2009.

[14] C. Nowzari, E. Garcia, and J. Cortés. Event-triggered communication and control of networked systems for multi-agent consensus. *Automatica*, 105:1–27, 2019.

[15] G. S. Seyboth, D. V. Dimarogonas, and K. H. Johansson. Event-based broadcasting for multi-agent average consensus. 49(1):245–252, 2013.

[16] D. Shi, T. Chen, and L. Shi. Event-triggered maximum likelihood state estimation. *Automatica*, 50(1):247–254, 2014.

[17] C. Viel, S. Bertrand, M. Kieffer, and H. Piet-Lahanier. Distributed event-triggered control for multi-agent formation stabilization. *Proc IFAC World Congress*, 50(1):8025–8030, 2017.

[18] C. Viel, S. Bertrand, M. Kieffer, and H. Piet-Lahanier. New state estimators and communication protocol for distributed event-triggered consensus of linear multi-agent systems with bounded perturbations. *IET Control Theory & Applications*, 11(11):1736–1748, 2017.

[19] C. Viel, S. Bertrand, M. Kieffer, and H. Piet-Lahanier. Distributed event-triggered control strategies for multi-agent formation stabilization and tracking. *Automatica*, 106:110–116, 2019.

[20] C. Viel, S. Bertrand, H. Piet-Lahanier, and M Kieffer. Distributed event-triggered control for multi-agent formation stabilization and tracking. *arXiv:1709.06652*, 2017.

[21] C. Viel, M. Kieffer, H. Piet-Lahanier, and S. Bertrand. Distributed event-triggered formation control for multi-agent systems in presence of packet losses. https://hal.archives-ouvertes.fr/hal-02892207, 2020.

[22] Z.-G. Wu, Y. Xu, Y.-J. Pan, H. Su, and Y. Tang. Event-triggered control for consensus problem in multi-agent systems with quantized relative state measurements and external disturbance. *IEEE Trans. Circuits and Systems I: Regular Papers*, 65(7):2232–2242, 2018.

[23] X. Xiao, J. H. Park, and L. Zhou. Event-triggered control of discrete-time switched linear systems with packet losses. *Applied Mathematics and Computation*, 333:344–352, 2018.

[24] Q. Yang, M. Cao, H. Fan, J. Chen, and J. Huang. Distributed formation stabilization for mobile agents using virtual tensegrity structures. In *Proc. Chinese Control Conference (CCC)*, pages 447–452, 2015.

[25] F. Zhou, Z. Huang, W. Liu, L. Li, and K. Gao. Periodic event-triggered condition design for the consensus of multiagent systems with communication delays. *Mathematical Problems in Engineering*, 1:1–9, 2016.

[26] W. Zhu, Z.P. Jiang, and G. Feng. Event-based consensus of multi-agent systems with general linear models. *Automatica*, 50(2):552–558, 2014.

A Appendix

A.1 Proof of Proposition 2

Assume that there exists $\varepsilon_2 > 0$ such that

$$\lim_{t\to\infty} \mathbb{E}\left(P\left(q,\, t\right)\right) \leqslant \varepsilon_2, \tag{A.1}$$

then

$$\lim_{t\to\infty} \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} m_{ij}\mathbb{E}\left(\left\|r_{ij}\left(t\right) - r_{ij}^{*}\left(t\right)\right\|^2\right) \leqslant \varepsilon_2. \tag{A.2}$$

Since

$$m_{ij}\mathbb{E}\left(\left\|r_{ij}\left(t\right) - r_{ij}^{*}\left(t\right)\right\|^2\right) \leqslant \sum_{i=1}^{N}\sum_{j=1}^{N} m_{ij}\mathbb{E}\left(\left\|r_{ij}\left(t\right) - r_{ij}^{*}\left(t\right)\right\|^2\right). \tag{A.3}$$

and $m_{ij} \geqslant 0$ for all (i, j) such that $m_{ij} > 0$, this implies

$$\lim_{t\to\infty} \mathbb{E}\left(\left\|r_{ij}\left(t\right) - r_{ij}^{*}\left(t\right)\right\|^2\right) \leqslant \frac{2\varepsilon_2}{m_{ij}}. \tag{A.4}$$

Consider now a pair (i, j) such that $m_{ij} = 0$. The communication graph has been assumed connected. Consequently, there exists a sequence $\left(i_1, \ldots, i_{N_{ij}}\right)$ of $N_{ij} \leqslant N$ nodes with $i_1 = i$ and $i_{N_{ij}} = j$ and such that $m_{i_k i_{k+1}} > 0$ for all $k = 1, \ldots, N_{ij} - 1$. Then

$$\lim_{t\to\infty} \mathbb{E}\left(\left\|r_{ij}\left(t\right) - r_{ij}^{*}\left(t\right)\right\|^2\right) \leqslant \lim_{t\to\infty} \sum_{k=1}^{N_{ij}-1} \mathbb{E}\left(\left\|r_{i_k i_{k+1}}\left(t\right) - r_{i_k i_{k+1}}^{*}\left(t\right)\right\|^2\right).$$

Using (A.4), one gets

$$\lim_{t\to\infty} \mathbb{E}\left(\left\|r_{ij}\left(t\right) - r_{ij}^{*}\left(t\right)\right\|^2\right) \leqslant \sum_{k=1}^{N_{ij}-1} \frac{2\varepsilon_2}{m_{i_k i_{k+1}}}. \tag{A.5}$$

Then introduce

$$\varepsilon_{11} = \max_{(i,j)\in\mathcal{N}^2, m_{ij}=0} \sum_{k=1}^{N_{ij}-1} \frac{2\varepsilon_2}{m_{i_k i_{k+1}}}, \tag{A.6}$$

$$\varepsilon_{12} = \max_{(i,j)\in\mathcal{N}^2, m_{ij}>0} \frac{2\varepsilon_2}{m_{ij}} \tag{A.7}$$

and

$$\varepsilon_1 = \max\left\{\varepsilon_{11}, \varepsilon_{12}\right\}. \tag{A.8}$$

Combining (A.4) and (A.5), one has for all $(i, j) \in \mathcal{N}^2$,

$$\lim_{t\to\infty} \mathbb{E}\left(\left\|r_{ij}\left(t\right) - r_{ij}^{*}\left(t\right)\right\|^2\right) \leqslant \varepsilon_1, \tag{A.9}$$

The converse is immediate: if there exists $\varepsilon_1 > 0$ such that (A.9) is satisfied for all $(i, j) \in \mathcal{N}^2$, then

$$\begin{aligned}\lim_{t\to\infty} \mathbb{E}\left(P\left(q,\, t\right)\right) &= \lim_{t\to\infty} \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} m_{ij}\mathbb{E}\left(\left\|r_{ij}\left(t\right) - r_{ij}^{*}\left(t\right)\right\|^2\right) \\ &\leqslant \varepsilon_2,\end{aligned}$$

with $\varepsilon_2 = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} m_{ij}\varepsilon_1$.

A.2 Proof of Proposition 3

Let define first $p_{k_i,\ell}$:

$$\begin{aligned} p^j_{k_i,\ell} &= \Pr\left(\delta^j_{i,\ell}=1,\delta^j_{i,\ell+1}=0,\dots,\delta^j_{i,k_i}=0\right)\\ &= \Pr\left(\delta^j_{i,k_i}=0|\delta^j_{i,\ell}=1,\delta^j_{i,\ell+1}=0,\dots,\delta^j_{i,k_i-1}=0\right)\\ &\times \Pr\left(\delta^j_{i,\ell}=1,\delta^j_{i,\ell+1}=0,\dots,\delta^j_{i,k_i-1}=0\right) \end{aligned} \tag{A.10}$$

so, using Assumption A7, one gets

$$\begin{aligned} p^j_{k_i,\ell} &= \pi p^j_{k_i-1,\ell} \text{ if } k_i-\ell\leqslant\kappa\\ &= 0 \text{ else.} \end{aligned} \tag{A.11}$$

Let study now $p^j_{k_i+n,k_i+n}$

$$\begin{aligned} p^j_{k_i+n,k_i+n} &= 1-\Pr\left(\delta^j_{i,k_i+n}=0\right)\\ &= 1-\Pr\left(\delta^j_{i,k_i+n}=0,\delta^j_{i,k_i+n-1}=1\right)\\ &- \Pr\left(\delta^j_{i,k_i+n}=0,\delta^j_{i,k_i+n-1}=0\right)\\ &= 1-\Pr\left(\delta^j_{i,k_i+n}=0|\delta^j_{i,k_i+n-1}=1\right)\\ &\times \Pr\left(\delta^j_{i,k_i+n-1}=1\right)\\ &- \Pr\left(\delta^j_{i,k_i+n}=0,\delta^j_{i,k_i+n-1}=0\right)\\ &= 1-\pi p^j_{k_i+n-1,k_i+n-1}\\ &- \Pr\left(\delta^j_{i,k_i+n}=0,\delta^j_{i,k_i+n-1}=0,\delta^j_{i,k_i+n-2}=1\right)\\ &- \Pr\left(\delta^j_{i,k_i+n}=0,\delta^j_{i,k_i+n-1}=0,,\delta^j_{i,k_i+n-2}=0\right)\\ &\dots\\ &= 1-\sum_{m=1}^{\kappa}\pi^m p^j_{k_i+n-m,k_i+n-m}\\ &- \Pr\left(\delta^j_{i,k_i+n}=0,\dots,\delta^j_{i,k_i+n-\kappa-1}=0\right)\\ &= 1-\sum_{m=1}^{\kappa}p^j_{k_i+n,k_i+n-m}. \end{aligned} \tag{A.12}$$

Remark $p^j_{k_i+n,k_i+n}$ is independent of j because there is no feedback for this proposition.

A.3 Proof of proposition 4

Before any reception from a packet for Agent j, (37) is evaluated as in proposition 3.

Consider $t\in[t_{i,k_i+n},t_{i,k_i+n+1}[$ with $n\geqslant 0$ and assume that the last message received by Agent i from Agent j was at time $t_{j,k_j}\in[t_{i,k_i},t_{i,k_i+1}[$. Thus $k_i^{j,i}=k_i^j\leqslant k_i$ and Agent i knows that Agent j has received the $k_i^{j,i}$-th message sent by Agent i, and has not received the following ones with index between $k_i^{j,i}$ and k_i. Agent i has no information about the reception by Agent j of the $k_i+1,\dots,k_i+n$-th messages, except if $k_i^{j,i}=k_i-\kappa$. Then, by definition of $p^j_{k_i,\ell|k_i^{j,i},k_i}$, one has

$$p^j_{k_i,k_i^{j,i}|k_i^{j,i},k_i}=\Pr\left(\delta^j_{i,k_i^{j,i}}=1|\delta^j_{i,k_i^{j,i}}=1,\sum_{m=k_i^{j,i}+1}^{k_i}\delta^j_{i,m}=0\right)=1$$

and for all $\ell \leq k_i$ with $\ell \neq k_i^{j,i}$

$$p^j_{k_i,\ell|k_i^{j,i},k_i} = \Pr\left(\sum_{m=\ell+1}^{k_i} \delta^j_{i,m} = 0, \delta^j_{i,\ell} = 1 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,m} = 0\right) = 0.$$

Consider now, with $n > 0$, the evaluation of

$$p^j_{k_i+n,k_i^{j,i}|k_i^{j,i},k_i} = \Pr\left(\sum_{m=1}^{n} \delta^j_{i,k_i+m} = 0, \delta^j_{i,k_i^{j,i}} = 1 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,m} = 0\right).$$

One has

$$\begin{aligned} p^j_{k_i+n,k_i^{j,i}|k_i^{j,i},k_i} &= \Pr\left(\delta^j_{i,k_i+n} = 0 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,m} = 0\right) \\ &\quad \Pr\left(\sum_{m=1}^{n-1} \delta^j_{i,k_i+m} = 0, \delta^j_{i,k_i^{j,i}} = 1 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,m} = 0\right) \qquad \text{(A.13)} \\ &= \Pr\left(\delta^j_{i,k_i+n} = 0 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i+n-1} \delta^j_{i,m} = 0\right) p^j_{k_i+n-1,k_i^{j,i}|k_i^{j,i},k_i}. \qquad \text{(A.14)} \end{aligned}$$

Using Assumption A7, if $k_i + n - k_i^{j,i} > \kappa$,

$$\Pr\left(\delta^j_{i,k_i+n} = 0 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i+n-1} \delta^j_{i,m} = 0\right) = 0$$

and if $k_i + n - k_i^{j,i} \leq \kappa$

$$\Pr\left(\delta^j_{i,k_i+n} = 0 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i+n-1} \delta^j_{i,m} = 0\right) = \pi.$$

Combining both equations with (A.14), one obtains (40).

Consider now the evaluation of $p^j_{k_i+n,\ell|k_i^{j,i},k_i}$ with $n > 0$. One has

$$p^j_{k_i+n,\ell|k_i^{j,i},k_i} = \Pr\left(\sum_{m=\ell+1}^{k_i+n} \delta^j_{i,m} = 0, \delta^j_{i,\ell} = 1 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,m} = 0\right).$$

If $\ell \leq k_i$ with $\ell \neq k_i^{j,i}$, one has clearly $p^j_{k_i+n,\ell|k_i^{j,i},k_i} = 0$. In what follows, we consider thus $k_i < \ell < k_i + n$ and

$$\begin{aligned} p^j_{k_i+n,\ell|k_i^{j,i},k_i} &= \Pr\left(\delta^j_{i,k_i+n} = 0 | \sum_{m=\ell+1}^{k_i+n-1} \delta^j_{i,m} = 0, \delta^j_{i,\ell} = 1, \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,m} = 0\right) \\ &\quad \Pr\left(\sum_{m=\ell+1}^{k_i+n-1} \delta^j_{i,m} = 0, \delta^j_{i,\ell} = 1 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,m} = 0\right) \\ &= \Pr\left(\delta^j_{i,k_i+n} = 0 | \delta^j_{i,\ell} = 1, \sum_{m=\ell+1}^{k_i+n-1} \delta^j_{i,m} = 0\right) p^j_{k_i+n-1,\ell|k_i^{j,i},k_i}. \qquad \text{(A.15)} \end{aligned}$$

Using again Assumption A7, if $k_i + n - \ell > \kappa$,

$$\Pr\left(\delta^j_{i,k_i+n} = 0 | \delta^j_{i,\ell} = 1, \sum_{m=\ell+1}^{k_i+n-1} \delta^j_{i,m} = 0\right) = 0$$

and if $k_i + n - \ell \leq \kappa$

$$\Pr\left(\delta^j_{i,k_i+n} = 0 | \delta^j_{i,\ell} = 1, \sum_{m=\ell+1}^{k_i+n-1} \delta^j_{i,m} = 0\right) = \pi.$$

Combining both equations with (A.15), one obtains (41).

Consider now the evaluation of $p^j_{k_i+n,k_i+n|k_i^{j,i},k_i}$ with $n > 0$.

$$\begin{aligned}
p^j_{k_i+n,k_i+n|k_i^{j,i},k_i} &= \Pr\left(\delta^j_{i,k_i+n} = 1 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,m} = 0\right) \\
&= 1 - \Pr\left(\delta^j_{i,k_i+n} = 0 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,m} = 0\right) \\
&= 1 - \Pr\left(\delta^j_{i,k_i+n} = 0, \delta^j_{i,k_i+n-1} = 1 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,m} = 0\right) \\
&- \Pr\left(\delta^j_{i,k_i+n} = 0, \delta^j_{i,k_i+n-1} = 0 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,m} = 0\right) \\
&= 1 - p^j_{k_i+n,k_i+n-1|k_i^{j,i},k_i} \\
&- \Pr\left(\delta^j_{i,k_i+n} = 0, \delta^j_{i,k_i+n-1} = 0, \delta^j_{i,k_i+n-2} = 0 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,m} = 0\right) \\
&- \Pr\left(\delta^j_{i,k_i+n} = 0, \delta^j_{i,k_i+n-1} = 0, \delta^j_{i,k_i+n-2} = 1 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,m} = 0\right) \\
&= 1 - p^j_{k_i+n,k_i+n-1|k_i^{j,i},k_i} - p^j_{k_i+n,k_i+n-2|k_i^{j,i},k_i} \\
&- \Pr\left(\delta^j_{i,k_i+n} = 0, \delta^j_{i,k_i+n-1} = 0, \delta^j_{i,k_i+n-2} = 0 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,m} = 0\right).
\end{aligned}$$

Proceeding similarly, one gets

$$\begin{aligned}
p^j_{k_i+n,k_i+n|k_i^{j,i},k_i} &= 1 - \sum_{\ell=k_i+1}^{k_i+n-1} p^j_{k_i+n,\ell|k_i^{j,i},k_i} \\
&\quad - \Pr\left(\sum_{m=k_i+1}^{k_i+n} \delta^j_{i,m} = 0 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,\ell} = 0\right) \\
&= 1 - \sum_{\ell=k_i+1}^{k_i+n-1} p^j_{k_i+n,\ell|k_i^{j,i},k_i} \\
&\quad - \Pr\left(\sum_{m=k_i+1}^{k_i+n} \delta^j_{i,m} = 0, \delta^j_{i,k_i^{j,i}} = 1 | \delta^j_{i,k_i^{j,i}} = 1, \sum_{m=k_i^{j,i}+1}^{k_i} \delta^j_{i,\ell} = 0\right) \\
&= 1 - \sum_{\ell=k_i+1}^{k_i+n-1} p^j_{k_i+n,\ell|k_i^{j,i},k_i} - p^j_{k_i+n,k_i^{j,i}|k_i^{j,i},k_i}.
\end{aligned}$$

Finally, (43) and (44) are obtained combining (40) and (41) with (42).

A.4 Proof of convergence with packet losses

To prove Theorem 5 a) one shows first that the MAS is converging with a bounded mean-square error. For that purpose, one will introduce a candidate Lyapunov function and show that it satisfies the conditions introduced in the Definition 1.

Consider some $D_{\max} \geqslant 0$, $\eta \geqslant 0$, and realizations $d_i(t)$, $i = 1, \ldots, N$ of the state perturbations.

Inspired by the proof developed in [16,3], consider the continuous positive-definite candidate Lyapunov function

$$V(t) = \mathbb{E}\left(\frac{1}{2}\sum_{i=1}^{N}\left(s_i(q(t,\delta))^T M_i s_i(q(t,\delta))\right) + \frac{k_g}{4} P(q(t,\delta),t)\right) \tag{A.16}$$

where the expectation is evaluated considering the random losses described by δ.

A.4.1 Continuity of the Lyapunov function

Assume that the first message is transmitted at time t_1, without loss of generality, by Agent 1 to N_1 neighbors. Consider some $t \in [t_1, t_2[$, where t_2 is the time at which the second message is transmitted, whatever the Agent. There are 2^{N_1} possible reception scenario, from no reception by all agents to a reception by all agents. Let σ represent the index of the σ-th scenario, $0 \leqslant \sigma \leqslant 2^{N_1}$ and $p_{\sigma,1}$ be the associated probability for the first communication. One may write

$$\begin{aligned}
V(t) &= \mathbb{E}\left(\frac{1}{2}\sum_{i=1}^{N}\left(s_i(q(t,\delta))^T M_i s_i(q(t,\delta))\right) + \frac{k_g}{4} P(q(t,\delta),t)\right) \\
&= \frac{1}{2}\sum_{\sigma=1}^{2^{N_1}} p_{\sigma,1}\left(\sum_{i=1}^{N} s_i(q(t,\delta_\sigma))^T M_i s_i(q(t,\delta_\sigma)) + \frac{k_g}{4} P(q(t,\delta_\sigma),t)\right)
\end{aligned} \tag{A.17}$$

where $\sum_{\sigma=1}^{2^{N_1}} p_{\sigma,1} = 1$.

For a given reception scenario σ of the first message, the time instant $t_{\sigma,2}$ of transmission of the second message and the index $i_{\sigma,2}$ of the transmitting Agent both depend on σ. More generally, at time $\underline{t}$, $S_{\underline{t}}$ different transmission and reception scenarios have to be considered. For a given scenario σ, let n_σ be the number of communications that have occurred. The associated loss vector is $\delta_\sigma = (\delta_{\sigma,1}, \ldots, \delta_{\sigma,n_\sigma})$, where $\delta_{\sigma,k}$ is the loss vector for the k-th communication. The probability associated to δ_σ is p_σ. The next communication time instant is $t_{\sigma,n_\sigma+1} > \underline{t}$ and the communicating Agent is $i_{\sigma,n_\sigma+1}$. Let

$$\bar{t} = \min_{\sigma=1,\ldots,S_{\underline{t}}} t_{\sigma,n_\sigma+1}$$

$$\overline{\sigma} = \arg \min_{\sigma=1,\ldots,S_{\underline{t}}} t_{\sigma,n_\sigma+1}$$

and $\bar{i}$ denote the index of the associated communicating Agent.

For all $t \in [\underline{t}, \bar{t}[$, one has

$$V(t) = \frac{1}{2} \sum_{\sigma=1}^{S_{\underline{t}}} p_\sigma \left(\sum_{i=1}^{N} s_i \left(q\left(t, \delta_\sigma\right)\right)^T M_i s_i \left(q\left(t, \delta_\sigma\right)\right) + \frac{k_g}{4} P\left(q\left(t, \delta_\sigma\right), t\right) \right).$$

In the scenario $\overline{\sigma}$, at time $\bar{t}$, Agent $\bar{i}$ is communicating. Consequently

$$\begin{aligned} V(\bar{t}) = & \frac{1}{2} \sum_{\sigma=1,\ldots,S_{\underline{t}},\sigma\neq\overline{\sigma}} p_\sigma \left(\sum_{i=1}^{N} s_i \left(q\left(\bar{t}, \delta_\sigma\right)\right)^T M_i s_i \left(q\left(\bar{t}, \delta_\sigma\right)\right) + \frac{k_g}{4} P\left(q\left(\bar{t}, \delta_\sigma\right), \bar{t}\right) \right) \\ & + \frac{1}{2} \sum_{\mu=1}^{2^{N_{\bar{i}}}} p_{(\overline{\sigma},\mu)} \left(\sum_{i=1}^{N} s_i \left(q\left(\bar{t}, \delta_{(\overline{\sigma},\mu)}\right)\right)^T M_i s_i \left(q\left(\bar{t}, \delta_{(\overline{\sigma},\mu)}\right)\right) + \frac{k_g}{4} P\left(q\left(\bar{t}, \delta_{(\overline{\sigma},\mu)}\right), \bar{t}\right) \right) \end{aligned} \tag{A.18}$$

where $p_{(\overline{\sigma},\mu)}$ denotes the probability of the μ-th loss scenario associated to the $n_{\overline{\sigma}}+1$ communication performed by Agent $\bar{i}$ at time $\bar{t}$, when the previous loss scenario is $\overline{\sigma}$. One has

$$\sum_{\mu=1}^{2^{N_{\bar{i}}}} p_{(\overline{\sigma},\mu)} = p_{\overline{\sigma}}. \tag{A.19}$$

Upon reception at time $\bar{t}^+$ of a message sent at time $\bar{t}$, only the estimators are updated according to (20). The state of agents receiving a message at time $\bar{t}^+$ from a neighbor is continuous, *i.e.*, $q_i\left(\bar{t}^+, \delta_{(\overline{\sigma},\mu)}\right) = q_i\left(\bar{t}^-, \delta_{\overline{\sigma}}\right)$, where $\bar{t}^-$ is a time instant immediately before transmission. This is also true for agents which do not receive the message sent at time $\bar{t}$. Thus, one gets $g_i\left(\bar{t}^+, \delta_{(\overline{\sigma},\mu)}\right) = g_i\left(\bar{t}^-, \delta_{\overline{\sigma}}\right)$, $s_i\left(q\left(\bar{t}^+, \delta_{(\overline{\sigma},\mu)}\right)\right) = s_i\left(q\left(\bar{t}^-, \delta_{\overline{\sigma}}\right)\right)$ for $i = 1, \ldots, N$, and consequently, $P\left(q\left(\bar{t}^+, \delta_{(\overline{\sigma},\mu)}\right), \bar{t}^+\right) = P\left(q\left(\bar{t}^-, \delta_{\overline{\sigma}}\right), \bar{t}^-\right)$ for all μ. Thus, at time $\bar{t}^+$, (A.18) becomes

$$\begin{aligned} V\left(\bar{t}^+\right) = & \frac{1}{2} \sum_{\sigma=1,\ldots,S_{\underline{t}},\sigma\neq\overline{\sigma}} p_\sigma \left(\sum_{i=1}^{N} s_i \left(q\left(\bar{t}^+, \delta_\sigma\right)\right)^T M_i s_i \left(q\left(\bar{t}^+, \delta_\sigma\right)\right) + \frac{k_g}{4} P\left(q\left(\bar{t}^+, \delta_\sigma\right), \bar{t}^+\right) \right) \\ & + \frac{1}{2} \sum_{\mu=1}^{2^{N_{\bar{i}}}} p_{(\overline{\sigma},\mu)} \left(\sum_{i=1}^{N} s_i \left(q\left(\bar{t}^+, \delta_{(\overline{\sigma},\mu)}\right)\right)^T M_i s_i \left(q\left(\bar{t}^+, \delta_{(\overline{\sigma},\mu)}\right)\right) + \frac{k_g}{4} P\left(q\left(\bar{t}^+, \delta_{(\overline{\sigma},\mu)}\right), \bar{t}^+\right) \right) \\ = & \frac{1}{2} \sum_{\sigma=1,\ldots,S_{\underline{t}},\sigma\neq\overline{\sigma}} p_\sigma \left(\sum_{i=1}^{N} s_i \left(q\left(\bar{t}^-, \delta_\sigma\right)\right)^T M_i s_i \left(q\left(\bar{t}^-, \delta_\sigma\right)\right) + \frac{k_g}{4} P\left(q\left(\bar{t}^-, \delta_\sigma\right), \bar{t}^-\right) \right) \\ & + \frac{1}{2} \sum_{\mu=1}^{2^{N_{\bar{i}}}} p_{(\overline{\sigma},\mu)} \left(\sum_{i=1}^{N} s_i \left(q\left(\bar{t}^-, \delta_{\overline{\sigma}}\right)\right)^T M_i s_i \left(q\left(\bar{t}^-, \delta_{\overline{\sigma}}\right)\right) + \frac{k_g}{4} P\left(q\left(\bar{t}^-, \delta_{\overline{\sigma}}\right), \bar{t}^-\right) \right) \end{aligned}$$

and using (A.19), one gets

$$\begin{aligned} V\left(\bar{t}^+\right) = & \frac{1}{2} \sum_{\sigma=1,\ldots,S_{\underline{t}},\sigma\neq\overline{\sigma}} p_\sigma \left(\sum_{i=1}^{N} s_i \left(q\left(\bar{t}^-, \delta_\sigma\right)\right)^T M_i s_i \left(q\left(\bar{t}^-, \delta_\sigma\right)\right) + \frac{k_g}{4} P\left(q\left(\bar{t}^-, \delta_\sigma\right), \bar{t}^-\right) \right) \\ & + \frac{1}{2} p_{\overline{\sigma}} \left(\sum_{i=1}^{N} s_i \left(q\left(\bar{t}^-, \delta_{\overline{\sigma}}\right)\right)^T M_i s_i \left(q\left(\bar{t}^-, \delta_{\overline{\sigma}}\right)\right) + \frac{k_g}{4} P\left(q\left(\bar{t}^-, \delta_{\overline{\sigma}}\right), \bar{t}^-\right) \right) \\ = & V\left(\bar{t}^-\right). \end{aligned}$$

Consequently, $V(t)$ is continuous at $\bar{t}$.

A.4.2 Differential inequality satisfied by the Lyapunov function

Using (A.18) from the previous section, the time derivative of V exists and can be evaluated for each $t \in [\underline{t}, \bar{t}[$ as follows

$$\dot{V}(t) = \sum_{\sigma=1}^{S_{\underline{t}}} p_\sigma \left(\sum_{i=1}^{N} \left(\frac{1}{2} s_i^T \left(q\left(t, \delta_\sigma\right)\right) \dot{M}_i s_i \left(q\left(t, \delta_\sigma\right)\right) + s_i^T \left(q\left(t, \delta_\sigma\right)\right) M_i \dot{s}_i \left(q\left(t, \delta_\sigma\right)\right) \right) + \frac{k_g}{4} \frac{d}{dt} P\left(q\left(t, \delta_\sigma\right), t\right) \right). \tag{A.20}$$

which may be written more concisely as

$$\dot{V} = \mathbb{E}\left(\sum_{i=1}^{N} \left[\frac{1}{2} s_i^T \dot{M}_i s_i + s_i^T M_i \dot{s}_i \right] + \frac{k_g}{4} \frac{d}{dt} P\left(q,\, t\right) \right), \tag{A.21}$$

where the expectation is to be taken over all possible transmission loss events.

Our aim, in what follows is to obtain a differential inequality satisfied by V. One starts considering the two terms in the right hand side of (A.21).

In (A.20), one has

$$\begin{aligned}
&\frac{1}{4} \frac{d}{dt} P\left(q,\, t\right) \\
&= \frac{1}{4} \frac{d}{dt} \sum_{i=1}^{N} \sum_{j=1}^{N} m_{ij} \left\| r_{ij} - r_{ij}^* \right\|^2 \\
&= \sum_{i=1}^{N} \left[\frac{1}{2} \sum_{j=1}^{N} m_{ij} \left(\dot{r}_{ij} - \dot{r}_{ij}^* \right)^T \left(r_{ij} - r_{ij}^* \right) \right] \\
&= \sum_{i=1}^{N} \frac{1}{2} \sum_{j=1}^{N} m_{ij} \left[\left(\dot{q}_i - \dot{q}_i^* \right)^T \left(r_{ij} - r_{ij}^* \right) - \left(\dot{q}_j - \dot{q}_j^* \right)^T \left(r_{ij} - r_{ij}^* \right) \right] \\
&= \sum_{i=1}^{N} \frac{1}{2} \sum_{j=1}^{N} m_{ij} \left[\left(\dot{q}_i - \dot{q}_i^* \right)^T \left(r_{ij} - r_{ij}^* \right) - \left(\dot{q}_i - \dot{q}_i^* \right)^T \left(r_{ji} - r_{ji}^* \right) \right]
\end{aligned} \tag{A.22}$$

Since $r_{ji} = -r_{ij}$, one gets

$$\begin{aligned}
\frac{1}{4} \frac{d}{dt} P\left(q,\, t\right) &= \sum_{i=1}^{N} \left(\dot{q}_i - \dot{q}_i^* \right)^T \sum_{j=1}^{N} m_{ij} \left(r_{ij} - r_{ij}^* \right) \\
&= \sum_{i=1}^{N} \left(\dot{q}_i - \dot{q}_i^* \right)^T g_i.
\end{aligned} \tag{A.23}$$

Combining (A.20) and (A.23), one obtains

$$\dot{V} = \mathbb{E}\left(\sum_{i=1}^{N} \left[\frac{1}{2} s_i^T \dot{M}_i s_i + s_i^T M_i \dot{s}_i + k_g \left(\dot{q}_i - \dot{q}_i^* \right)^T g_i \right] \right) \tag{A.24}$$

One focuses now on the term $M_i \dot{s}_i$. Using (16), one may write

$$M_i \dot{s}_i + C_i s_i = M_i \left(\ddot{q}_i - \ddot{q}_i^* + k_p \dot{g}_i \right) + C_i \left(\dot{q}_i - \dot{q}_i^* + k_p g_i \right)$$

and using (1), one gets

$$M_i \dot{s}_i + C_i s_i = u_i + d_i - G + M_i \left(k_p \dot{g}_i - \ddot{q}_i^* \right) + C_i \left(k_p g_i - \dot{q}_i^* \right). \tag{A.25}$$

Now, introducing (19), one gets

$$\begin{aligned} M_i\dot{s}_i + C_i s_i = &-k_s\bar{s}_i - k_g\bar{g}_i - Y_i\left(q_i,\, \dot{q}_i,\, k_p\dot{\bar{g}}_i - \ddot{q}_i^*, k_p\bar{g}_i - \dot{q}_i^*\right)\theta_i \\ &+M_i\left(k_p\dot{g}_i - \ddot{q}_i^*\right) + C_i\left(k_p g_i - \dot{q}_i^*\right) + d_i \end{aligned} \tag{A.26}$$

In what follows, one uses Y_i to represent $Y_i\left(q_i,\, \dot{q}_i,\, k_p\dot{\bar{g}}_i - \ddot{q}_i^*, k_p\bar{g}_i - \dot{q}_i^*\right)$. Assumption A3 leads to

$$-s_i^T Y_i\theta_i = -s_i^T\left(M_i\left(k_p\dot{\bar{g}}_i - \ddot{q}_i^*\right) + C_i\left(k_p\bar{g}_i - \dot{q}_i^*\right)\right). \tag{A.27}$$

Considering (2) and (A.26) in (A.24), one gets

$$\begin{aligned} \dot{V} = \mathbb{E}\Bigg(\sum_{i=1}^{N}\Bigg[&\frac{1}{2}s_i^T\dot{M}_i s_i - k_s s_i^T\bar{s}_i - k_g s_i^T\bar{g}_i - s_i^T C_i s_i + s_i^T\left(M_i\left(k_p\dot{g}_i - \ddot{q}_i^*\right) + C_i\left(k_p g_i - \dot{q}_i^*\right)\right) \\ &-s_i^T\left(M_i\left(k_p\dot{\bar{g}}_i - \ddot{q}_i^*\right) + C_i\left(k_p\bar{g}_i - \dot{q}_i^*\right)\right) + k_g\left(\dot{q}_i - \dot{q}_i^*\right)^T g_i + s_i^T d_i\Bigg]\Bigg). \end{aligned} \tag{A.28}$$

Now, introduce (14) in (16) to get

$$s_i = \dot{q}_i - \dot{q}_i^* + k_p\sum_{i=1}^{N} m_{ij}\left(q_i - q_j - r_{ij}^*\right). \tag{A.29}$$

Since $e_j^i = \hat{q}_j^i - q_j$, one gets

$$\begin{aligned} s_i &= \dot{q}_i - \dot{q}_i^* + k_p\sum_{i=1}^{N} m_{ij}\left(q_i - \hat{q}_j^i + e_j^i - r_{ij}^*\right) \\ &= \dot{q}_i - \dot{q}_i^* + k_p\sum_{i=1}^{N} m_{ij}\left(\bar{r}_{ij} - r_{ij}^*\right) + k_p\sum_{\substack{j=1\\ j\neq i}}^{N} m_{ij}e_j^i \\ &= \bar{s}_i + k_p E^i \end{aligned} \tag{A.30}$$

with

$$E^i = \sum_{i=1}^{N} m_{ij}e_j^i, \tag{A.31}$$

since $m_{ii} = 0$. Using similar derivations, one may show that

$$g_i = \bar{g}_i + E^i. \tag{A.32}$$

Replacing (A.30) and (A.32) in (A.28), one gets

$$\begin{aligned} \dot{V} = \mathbb{E}\Bigg(\sum_{i=1}^{N}\Bigg[&s_i^T\left[\frac{1}{2}\dot{M}_i - C_i\right]s_i - k_s s_i^T\bar{s}_i - k_g\left(\dot{q}_i - \dot{q}_i^* + k_p g_i\right)^T\bar{g}_i \\ &+k_p s_i^T\left(M_i\dot{E}^i + C_i E^i\right) + k_g\left(\dot{q}_i - \dot{q}_i^*\right)^T g_i + s_i^T d_i\Bigg]\Bigg). \end{aligned} \tag{A.33}$$

Let $\dot{V}_1 = \sum_{i=1}^{N} 2k_p s_i^T\left(M_i\dot{E}^i + C_i E^i\right)$. Using Assumption A2, $\frac{1}{2}\dot{M}_i - C_i$ is skew symmetric or definite negative thus $s_i^T\left[\frac{1}{2}\dot{M}_i - C_i\right]s_i \leq 0$. For all $b > 0$ and all vectors x and y of similar size, one has

$$x^T y \leq \frac{1}{2}\left(bx^T x + \frac{1}{b}y^T y\right). \tag{A.34}$$

Using (A.34) with $b=1$, and the fact that $d_i^T d_i \leq D_{\max}^2$, one deduces that $d_i^T s_i \leq \frac{1}{2}\left(D_{\max}^2 + s_i^T s_i\right)$ and that

$$\dot{V} \leq \mathbb{E}\left(\sum_{i=1}^{N}\left[-k_s s_i^T \bar{s}_i - k_g k_p g_i^T \bar{g}_i + \frac{1}{2} s_i^T s_i + \frac{1}{2} D_{\max}^2 \right.\right.$$
$$\left.\left. + k_g \left(\dot{q}_i - \dot{q}_i^*\right)^T \left(g_i - \bar{g}_i\right)\right] + \frac{1}{2}\dot{V}_1\right). \tag{A.35}$$

One notices that $r_{ij} = q_i - q_j = q_i - \hat{q}_j^i + e_j^i = \bar{r}_{ij} + e_j^i$, thus

$$\begin{aligned}
\left\|s_i - \bar{s}_i\right\|^2 &= s_i^T s_i - 2 s_i^T \bar{s}_i + \bar{s}_i^T \bar{s}_i \\
\left\|k_p E^i\right\|^2 &= s_i^T s_i - 2 s_i^T \bar{s}_i + \bar{s}_i^T \bar{s}_i \\
s_i^T \bar{s}_i &= -\frac{1}{2}\left\|k_p E^i\right\|^2 + \frac{1}{2} s_i^T s_i + \frac{1}{2} \bar{s}_i^T \bar{s}_i
\end{aligned} \tag{A.36}$$

Using similar derivations, from (A.36), one shows that $g_i^T \bar{g}_i = -\frac{1}{2}\left\|E^i\right\|^2 + \frac{1}{2} g_i^T g_i + \frac{1}{2} \bar{g}_i^T \bar{g}_i$. Injecting the latter expression in (A.35), one gets

$$\begin{aligned}
\dot{V} &\leq \mathbb{E}\left(\sum_{i=1}^{N}\left[\frac{k_s}{2}\left(k_p^2\left\|E^i\right\|^2 - s_i^T s_i - \bar{s}_i^T \bar{s}_i\right) + k_p k_g \frac{1}{2}\left(\left\|E^i\right\|^2 - g_i^T g_i - \bar{g}_i^T \bar{g}_i\right) + \frac{1}{2} s_i^T s_i + \frac{1}{2} D_{\max}^2 \right.\right. \\
&\quad \left.\left. + k_g \left(\dot{q}_i - \dot{q}_i^*\right)^T \left(g_i - \bar{g}_i\right)\right] + \frac{1}{2}\dot{V}_1\right) \\
&\leq \mathbb{E}\left(\sum_{i=1}^{N}\left[-\frac{(k_s - 1)}{2} s_i^T s_i - \frac{k_s}{2} \bar{s}_i^T \bar{s}_i + \frac{k_s k_p^2 + k_g k_p}{2}\left\|E^i\right\|^2 - \frac{1}{2} k_p k_g \left(g_i^T g_i + \bar{g}_i^T \bar{g}_i\right) + \frac{1}{2} D_{\max}^2 \right.\right. \\
&\quad \left.\left. + k_g \left(\dot{q}_i - \dot{q}_i^*\right)^T \left(g_i - \bar{g}_i\right)\right] + \frac{1}{2}\dot{V}_1\right).
\end{aligned} \tag{A.37}$$

Using (A.34) with $b = b_i > 0$, one shows that $2\dot{q}_i^T\left(g_i - \bar{g}_i\right) \leq \left(b_i \left\|\dot{q}_i\right\|^2 + \frac{1}{b_i}\left\|E^i\right\|^2\right)$. Using this result in (A.37), one gets

$$\begin{aligned}
\dot{V} \leq \frac{1}{2}\sum_{i=1}^{N}\Big[&-(k_s - 1)\,\mathbb{E}\left(s_i^T s_i\right) - k_s \mathbb{E}\left(\bar{s}_i^T \bar{s}_i\right) + \left(k_s k_p^2 + k_g k_p + \frac{k_g}{b_i}\right)\mathbb{E}\left(\left\|E^i\right\|^2\right) + b_i k_g \mathbb{E}\left(\left\|\dot{q}_i - \dot{q}_i^*\right\|^2\right) \\
&- k_p k_g \mathbb{E}\left(g_i^T g_i + \bar{g}_i^T \bar{g}_i\right) + D_{\max}^2\Big] + \frac{1}{2}\mathbb{E}\left(\dot{V}_1\right)
\end{aligned} \tag{A.38}$$

Consider now $\dot{V}_1$. Using (A.34) with $b=1$ and Assumption A1, one obtains

$$\begin{aligned}
\sum_{i=1}^{N} 2 k_p s_i^T\left(M_i \dot{E}^i + C_i E^i\right) &\leq \sum_{i=1}^{N} k_p\left(s_i^T M_i s_i + s_i^T s_i + \left[\dot{E}^{iT} M_i \dot{E}^i + E^{iT} C_i^T C_i E^i\right]\right) \\
&\leq \sum_{i=1}^{N} k_p\left((k_M + 1)\, s_i^T s_i + \left[k_M \dot{E}^{iT} \dot{E}^i + E^{iT} C_i^T C_i E^i\right]\right)
\end{aligned} \tag{A.39}$$

Focus now on the terms $E^{iT} C_i^T C_i E^i$. Using Assumption A2, one has

$$\begin{aligned}
\sum_{i=1}^{N} E^{iT} C_i^T C_i E^i &= \sum_{i=1}^{N}\left(\sum_{j=1}^{N} m_{ij} e_j^i\right)^T C_i^T C_i \left(\sum_{\ell=1}^{N} m_{i\ell} e_\ell^i\right) \\
&\leq \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{\ell=1}^{N} m_{i\ell} m_{ij} \left\|C_i\right\|^2 e_j^{iT} e_\ell^i.
\end{aligned} \tag{A.40}$$

Using again (A.34) with $b = 1$, one gets

$$\begin{aligned}\sum_{i=1}^{N} E^{iT} C_i^T C_i E^i &\leq \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{\ell=1}^{N} m_{i\ell} m_{ij} \left\| C_i \right\|^2 \left(e_j^{iT} e_j^i + e_\ell^{iT} e_\ell^i \right) \\ &\leq \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{\ell=1}^{N} m_{i\ell} m_{ij} \left\| C_i \right\|^2 \left(e_j^{iT} e_j^i \right) \\ &\leq \sum_{i=1}^{N} \alpha_i \sum_{j=1}^{N} m_{ij} \left\| C_i \right\|^2 \left(e_j^{iT} e_j^i \right). \end{aligned} \tag{A.41}$$

Since $m_{ij} = m_{ji}$ and $m_{ij} = 0$ if $(i, j) \notin \mathcal{N}_j$, one may write

$$\sum_{i=1}^{N} \sum_{j=1}^{N} m_{ij} \left\| e_j^i \right\|^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} m_{ji} \left\| e_i^j \right\|^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} m_{ij} \left\| e_i^j \right\|^2 \tag{A.42}$$

and using Assumption A2, one gets

$$\begin{aligned}\sum_{i=1}^{N} E^{iT} C_i^T C_i E^i &\leq \sum_{i=1}^{N} \left(\alpha_{\mathrm{M}} \sum_{j=1}^{N} \left[m_{ij} \left\| e_i^j \right\|^2 \left\| C_j \right\|^2 \right] \right) \\ &\leq \sum_{i=1}^{N} \left(\alpha_{\mathrm{M}} \sum_{j=1}^{N} \left[m_{ij} \left\| e_i^j \right\|^2 k_C^2 \left\| \dot{q}_j \right\|^2 \right] \right). \end{aligned} \tag{A.43}$$

Observing that

$$\begin{aligned}\left\| \dot{q}_j \right\|^2 &= \left\| \dot{\hat{q}}_j^i + \dot{e}_j^i \right\|^2 \\ &= \left\| \dot{\hat{q}}_j^i \right\|^2 + \left\| \dot{e}_j^i \right\|^2 + 2 \dot{\hat{q}}_j^{iT} \dot{e}_j^i \\ &\leq 2 \left\| \dot{\hat{q}}_j^i \right\|^2 + 2 \left\| \dot{e}_j^i \right\|^2, \end{aligned}$$

(A.42) can be rewritten as

$$\begin{aligned}\sum_{i=1}^{N} E^{iT} C_i^T C_i E^i &\leq 2 \alpha_{\mathrm{M}} k_C^2 \sum_{i=1}^{N} \sum_{j=1}^{N} m_{ij} \left\| e_i^j \right\|^2 \left(\left\| \dot{\hat{q}}_j^i \right\|^2 + \left\| \dot{e}_j^i \right\|^2 \right) \\ &\leq 2 \alpha_{\mathrm{M}} k_C^2 \sum_{i=1}^{N} \sum_{j=1}^{N} m_{ij} \left(\left\| e_i^j \right\|^2 \left\| \dot{\hat{q}}_j^i \right\|^2 + \left\| e_i^j \right\|^2 \left\| \dot{e}_j^i \right\|^2 \right). \end{aligned}$$

Using (A.34) with $b = 1$ and $m_{ij} = m_{ji}$, one gets

$$\begin{aligned}\sum_{i=1}^{N} E^{iT} C_i^T C_i E^i &\leq 2 \alpha_{\mathrm{M}} k_C^2 \sum_{i=1}^{N} \sum_{j=1}^{N} m_{ij} \left(\left\| e_i^j \right\|^2 \left\| \dot{\hat{q}}_j^i \right\|^2 + \frac{1}{2} \left\| e_i^j \right\|^4 + \frac{1}{2} \left\| \dot{e}_j^i \right\|^4 \right) \\ &\leq 2 \alpha_{\mathrm{M}} k_C^2 \sum_{i=1}^{N} \sum_{j=1}^{N} m_{ij} \left(\left\| e_i^j \right\|^2 \left\| \dot{\hat{q}}_j^i \right\|^2 + \frac{1}{2} \left\| e_i^j \right\|^4 + \frac{1}{2} \left\| \dot{e}_i^j \right\|^4 \right). \end{aligned} \tag{A.44}$$

Similarly, one shows that $\sum_{i=1}^{N} E^{iT} E^i \leq \sum_{i=1}^{N} \alpha_{\mathrm{M}} \sum_{j=1}^{N} m_{ij} \left\| e_i^j \right\|^2$ and $\sum_{i=1}^{N} \dot{E}^{iT} \dot{E}^i \leq \sum_{i=1}^{N} \alpha_{\mathrm{M}} \sum_{j=1}^{N} m_{ij} \left\| \dot{e}_i^j \right\|^2$.

Injecting (A.39) and (A.44) in (A.38), one gets

$$\begin{aligned}\dot{V} \leq \frac{1}{2}\sum_{i=1}^{N}\Big[&-\left(k_s-1-k_p\left(k_M+1\right)\right)\mathbb{E}\left(s_i^T s_i\right)-k_s\mathbb{E}\left(\bar{s}_i^T\bar{s}_i\right)+D_{\max}^2\\ &-k_pk_g\mathbb{E}\left(g_i^T g_i\right)-k_pk_g\mathbb{E}\left(\bar{g}_i^T\bar{g}_i\right)+k_gb_i\mathbb{E}\left(\|\dot{q}_i-\dot{q}_i^*\|^2\right)+k_pk_M\sum_{j=1}^{N}\alpha_{\mathrm{M}}m_{ij}\mathbb{E}\left(\left\|\dot{e}_i^j\right\|^2\right)\\ &+\left(k_sk_p^2+k_gk_p+\frac{k_g}{b_i}\right)\sum_{j=1}^{N}\alpha_{\mathrm{M}}m_{ij}\mathbb{E}\left(\left\|e_i^j\right\|^2\right)\\ &+2\alpha_{\mathrm{M}}k_pk_C^2\sum_{j=1}^{N}m_{ij}\left(\mathbb{E}\left(\left\|e_i^j\right\|^2\right)\left\|\dot{\hat{q}}_j^i\right\|^2+\frac{1}{2}\mathbb{E}\left(\left\|e_i^j\right\|^4\right)+\frac{1}{2}\mathbb{E}\left(\left\|\dot{e}_i^j\right\|^4\right)\right)\Big]\end{aligned} \tag{A.45}$$

The CTC (46) leads to

$$\begin{aligned}\dot{V} &\leq \frac{1}{2}\sum_{i=1}^{N}\mathbb{E}\left[-\left(k_s-1-k_p\left(k_M+1\right)\right)s_i^T s_i-k_gk_pg_i^T g_i+D_{\max}^2+\eta\right]\\ \dot{V} &\leq \frac{1}{2}\sum_{i=1}^{N}\mathbb{E}\left[-k_1s_i^T s_i-k_gk_pg_i^T g_i+D_{\max}^2+\eta\right]\end{aligned} \tag{A.46}$$

with $k_1 = k_s - 1 - k_p\left(k_M+1\right)$.

Introducing $k_{\mathrm{m}} = \min\left\{k_1, k_p\right\}$, from (A.46), one gets

$$\dot{V} \leq \frac{1}{2}\sum_{i=1}^{N}\mathbb{E}\left[-k_{\mathrm{m}}\left(s_i^T s_i+k_gg_i^T g_i\right)+D_{\max}^2+\eta\right]. \tag{A.47}$$

A lower bound of $\sum_{i=1}^{N} g_i^T g_i$ has now to be introduced using the following lemma, which proof is given in Appendix A.5.1.

Lemma 7 *For all t, one has*

$$\sum_{i=1}^{N} g_i^T g_i \geq \frac{N_{\min}m_{\min}}{m_{\max}}P\left(q,t\right), \tag{A.48}$$

where $m_{\min} = \min_{i,j=1\ldots N}\left(m_{ij}\neq 0\right)$, $m_{\max} = \max\limits_{i,\ell=1\ldots N}\left(m_{ij}\right)$ and $N_{\min} = \min_{i=1\ldots N}\left(N_i\right)$.

Using Lemma 7 and introducing $k_3 = \frac{N_{\min}m_{\min}}{m_{\max}}$, one may write

$$\begin{aligned}\dot{V} &\leq \mathbb{E}\left(-\frac{k_{\mathrm{m}}}{2}\left[\sum_{i=1}^{N}s_i^T s_i+\frac{k_3k_g}{4}P\left(q,t\right)\right]+\frac{N}{2}\left(D_{\max}^2+\eta\right)\right)\\ &\leq \mathbb{E}\left(-\frac{k_{\mathrm{m}}}{k_M^*}\left[\frac{1}{2}\sum_{i=1}^{N}\left(k_Ms_i^T s_i\right)+\frac{k_3k_g}{4}P\left(q,t\right)\right]+\frac{N}{2}\left(D_{\max}^2+\eta\right)\right)\\ &\leq \mathbb{E}\left(-\frac{k_4}{k_M^*}\left[\frac{1}{2}\sum_{i=1}^{N}\left(k_Ms_i^T s_i\right)+\frac{k_g}{4}P\left(q,t\right)\right]+\frac{N}{2}\left(D_{\max}^2+\eta\right)\right)\end{aligned} \tag{A.49}$$

with $k_M^* = 1$ if $k_M < 1$ and $k_M^* = k_M$ else, and $k_4 = k_{\mathrm{m}}\min\left(1,k_3\right)$. Introducing $c_3 = \frac{k_4}{k_M^*}$, one gets

$$\begin{aligned}\dot{V} &\leq \mathbb{E}\left(-c_3\left[\frac{1}{2}\sum_{i=1}^{N}\left(s_i^T M_i s_i\right)+\frac{k_g}{4}P\left(q,t\right)\right]+\frac{N}{2}\left[D_{\max}^2+\eta\right]\right)\\ \dot{V} &\leq -c_3V+\frac{N}{2}\left[D_{\max}^2+\eta\right].\end{aligned} \tag{A.50}$$

A.4.3 Upper bound of the Lyapunov function

Consider $t \in [\underline{t}, \bar{t}[$ and the function W satisfying

$$\dot{W} = -c_3 W + \frac{N}{2}\left[D_{\max}^2 + \eta\right]. \tag{A.51}$$

The solution of (A.51) with initial condition $W(\underline{t}) = V(\underline{t})$ is

$$W(t) = \exp\left(-c_3\left(t - \underline{t}\right)\right) V(\underline{t}) + \left(1 - \exp\left(-c_3\left(t - \underline{t}\right)\right)\right) \frac{N}{2c_3}\left[D_{\max}^2 + \eta\right]. \tag{A.52}$$

Then, using [8, Lemma 3.4] (Comparison lemma), one has $V(t) \leq W(t)\ \forall t \in [\underline{t}, \bar{t}[$, so

$$V(t) \leq \exp\left(-c_3\left(t - \underline{t}\right)\right) V(\underline{t}) + \left(1 - \exp\left(-c_3\left(t - \underline{t}\right)\right)\right) \frac{N}{2c_3}\left[D_{\max}^2 + \eta\right] \tag{A.53}$$

$$\leq \exp\left(-c_3\left(t - \underline{t}\right)\right)\left[V(\underline{t}) - \frac{N}{2c_3}\left[D_{\max}^2 + \eta\right]\right] + \frac{N}{2c_3}\left[D_{\max}^2 + \eta\right] \tag{A.54}$$

Then, since $V(\underline{t}) > \frac{N}{2c_3}\left[D_{\max}^2 + \eta\right]$, $V(t)$ is decreasing over the interval $[\underline{t}, \bar{t}[$.

Using (A.53), one may write $\forall t > 0$

$$V(t) \leq \exp\left(-c_3 t\right) V(0) + \left(1 - \exp\left(-c_3 t\right)\right) \frac{N}{2c_3}\left[D_{\max}^2 + \eta\right] \tag{A.55}$$

and from (A.55), one has

$$\begin{aligned}
\lim_{t\to\infty} V(t) &\leq \frac{N}{2c_3}\left[D_{\max}^2 + \eta\right] \\
\lim_{t\to\infty} \mathbb{E}\left(\frac{1}{2}\sum_{i=1}^{N}\left(s_i^T M_i s_i\right) + \frac{k_g}{4} P(q,t)\right) &\leq \frac{N}{2c_3}\left[D_{\max}^2 + \eta\right] \\
\lim_{t\to\infty} \mathbb{E}\left(\frac{1}{2} P(q,t)\right) &\leq \frac{N}{k_g c_3}\left[D_{\max}^2 + \eta\right].
\end{aligned} \tag{A.56}$$

Asymptotically, the formation error is bounded and according to Definition 1, the system is asymptotically converging to the target formation with a bounded mean-square error.

A.5 Additional proof elements

A.5.1 Upper-bound on $\sum_{i=1}^{N} g_i^T g_i$

From (14), one may write

$$\begin{aligned}
\sum_{i=1}^{N} g_i^T g_i &= \sum_{i=1}^{N}\left(\sum_{j=1}^{N} m_{ij}\left(r_{ij} - r_{ij}^*\right)\right)^T \left(\sum_{\ell=1}^{N} m_{i\ell}\left(r_{i\ell} - r_{i\ell}^*\right)\right) \\
&= \sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell} m_{ij}\left(r_{ij} - r_{ij}^*\right)^T \left(r_{i\ell} - r_{i\ell}^*\right)
\end{aligned} \tag{A.57}$$

Using the fact that

$$\left(a - b\right)^T\left(a - b\right) = a^T a + b^T b - 2a^T b, \tag{A.58}$$

one gets

$$\sum_{i=1}^{N} g_i^T g_i = \sum_{i=1}^{N}\left[\frac{1}{2}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell} m_{ij}\left[\left\|r_{ij} - r_{ij}^*\right\|^2 + \left\|r_{i\ell} - r_{i\ell}^*\right\|^2 - \left\|r_{ij} - r_{ij}^* - \left(r_{i\ell} - r_{i\ell}^*\right)\right\|^2\right]\right]. \tag{A.59}$$

One has

$$\begin{aligned}\left(r_{ij}-r_{ij}^{*}\right)-\left(r_{i\ell}-r_{i\ell}^{*}\right) &= \left(r_{ij}-r_{i\ell}\right)-\left(r_{ij}^{*}-r_{i\ell}^{*}\right)\\ &= r_{\ell j}-r_{\ell j}^{*}\end{aligned}$$

Injecting this result in (A.59) leads to

$$\sum_{i=1}^{N} g_i^T g_i = \sum_{i=1}^{N}\left[\frac{1}{2}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell}m_{ij}\left[\left\|r_{ij}-r_{ij}^{*}\right\|^2+\left\|r_{i\ell}-r_{i\ell}^{*}\right\|^2-\left\|r_{\ell j}-r_{\ell j}^{*}\right\|^2\right]\right] \tag{A.60}$$

with $m_{\max}=\max\limits_{i,j=1\ldots N}\left(m_{ij}\right)$

$$\begin{aligned}
m_{\max}\sum_{i=1}^{N} g_i^T g_i &\geq \sum_{i=1}^{N}\left[\frac{1}{2}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell}m_{ij}m_{\ell j}\left[\left\|r_{ij}-r_{ij}^{*}\right\|^2+\left\|r_{i\ell}-r_{i\ell}^{*}\right\|^2-\left\|r_{\ell j}-r_{\ell j}^{*}\right\|^2\right]\right]\\
m_{\max}\sum_{i=1}^{N} g_i^T g_i &\geq \frac{1}{2}\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^{*}\right\|^2+\frac{1}{2}\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell}m_{ij}m_{\ell j}\left\|r_{i\ell}-r_{i\ell}^{*}\right\|^2\\
&\quad-\frac{1}{2}\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell}m_{ij}m_{\ell j}\left\|r_{\ell j}-r_{\ell j}^{*}\right\|^2\\
m_{\max}\sum_{i=1}^{N} g_i^T g_i &\geq \frac{1}{2}\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^{*}\right\|^2+\frac{1}{2}\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^{*}\right\|^2\\
&\quad-\frac{1}{2}\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^{*}\right\|^2\\
m_{\max}\sum_{i=1}^{N} g_i^T g_i &\geq \frac{1}{2}\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^{*}\right\|^2 .
\end{aligned} \tag{A.61}$$

According $m_{i\ell}=0$ if $\ell\notin\mathcal{N}_i$, one gets

$$\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^{*}\right\|^2=\sum_{i=1}^{N}\sum_{\ell\in\mathcal{N}_i} m_{i\ell}\sum_{j=1}^{N} m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^{*}\right\|^2$$

Let $m_{\min}=\min_{i,j=1\ldots N}\left(m_{ij}\neq 0\right)$ and $N_{\min}=\min_{i=1\ldots N}\left(N_i\right)$. One may write

$$\begin{aligned}
\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^{*}\right\|^2 &= \sum_{i=1}^{N}\sum_{\ell\in\mathcal{N}_i} m_{\min}\sum_{j=1}^{N} m_{ij}\left\|r_{ij}-r_{ij}^{*}\right\|^2 .\\
\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^{*}\right\|^2 &\geq N_{\min}m_{\min}\sum_{i=1}^{N}\sum_{j=1}^{N} m_{ij}\left\|r_{ij}-r_{ij}^{*}\right\|^2
\end{aligned}$$

and so

$$\begin{aligned}
m_{\max}\sum_{i=1}^{N} g_i^T g_i &\geq N_{\min}m_{\min}\sum_{i=1}^{N}\sum_{j=1}^{N} m_{ij}\left\|r_{ij}-r_{ij}^{*}\right\|^2\\
\sum_{i=1}^{N} g_i^T g_i &\geq \frac{N_{\min}m_{\min}}{m_{\max}}P\left(q,t\right)
\end{aligned} \tag{A.62}$$

A.5.2 Evaluation of c_3

$$
\begin{aligned}
c_3 &= \frac{k_4}{k_M^*} \\
&= \frac{k_{\mathrm{m}} \min(1, k_3)}{\max\{1, k_M\}} \\
&= \frac{\min\{k_1, k_p\} \min\left(1, \frac{N_{\min} m_{\min}}{m_{\max}}\right)}{\max\{1, k_M\}}.
\end{aligned} \tag{A.63}
$$

where $k_1 = k_s - 1 - k_p (k_M + 1)$.

A.5.3 Evaluation of $\mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i+1}^+\right)\right\|^2\right)$ and $\mathbb{E}\left(\left\|\dot{e}_i^j\left(t_{i,k_i+1}^+\right)\right\|^2\right)$

$\mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i+1}^+\right)\right\|^2\right)$ and $\mathbb{E}\left(\left\|\dot{e}_i^j\left(t_{i,k_i+1}^+\right)\right\|^2\right)$ are evaluated assuming that the implicit feedback is not employed. A similar evaluation may be performed considering this information. Using results of (35), one may write immediately after the transmission of the $k_i + 1$ message by Agent i

$$
\begin{aligned}
\mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i+1}^+\right)\right\|^2\right) &= \sum_{\ell=k_i-\kappa+2}^{k_i+1} p_{k_i+1,\ell} \left\|\hat{q}_i^{i,\ell}\left(t_{i,k_i+1}^+\right) - q_i\left(t_{i,k_i+1}^+\right)\right\|^2 \\
&= \sum_{\ell=k_i-\kappa+1}^{k_i} \pi p_{k_i,\ell} \left\|\hat{q}_i^{i,\ell}\left(t_{i,k_i+1}^+\right) - q_i\left(t_{i,k_i+1}^+\right)\right\|^2 \\
&+ p_{k_i+1,k_i+1} \left\|\hat{q}_i^{i,k_i+1}\left(t_{i,k_i+1}^+\right) - q_i\left(t_{i,k_i+1}^+\right)\right\|^2 \\
&- p_{k_i+1,k_i-\kappa} \left\|\hat{q}_i^{i,k_i-\kappa+1}\left(t_{i,k_i+1}^+\right) - q_i\left(t_{i,k_i+1}^+\right)\right\|^2,
\end{aligned}
$$

where $p_{k_i+1,\ell} = \pi p_{k_i,\ell}$ has been shown in (40). Since $\hat{q}_i^{i,\ell}\left(t_{i,k_i+1}^+\right) = \hat{q}_i^{i,\ell}\left(t_{i,k_i+1}^-\right)$ for all $\ell = k_i - \kappa + 1, \ldots, k_i$ and $q_i\left(t_{i,k_i+1}^+\right) = q_i\left(t_{i,k_i+1}^-\right)$, one deduces

$$
\begin{aligned}
\mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i+1}^+\right)\right\|^2\right) &= \pi \mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i+1}^-\right)\right\|^2\right) \\
&+ p_{k_i+1,\ell} \left\|\hat{q}_i^{i,k_i+1}\left(t_{i,k_i+1}^+\right) - q_i\left(t_{i,k_i+1}^+\right)\right\|^2 \\
&- p_{k_i+1,k_i-\kappa} \left\|\hat{q}_i^{i,k_i-\kappa+1}\left(t_{k_i+1}\right) - q_i\left(t_{k_i+1}\right)\right\|^2.
\end{aligned}
$$

According to Proposition 4, $p_{k_i+1,k_i-\kappa} = 0$. Moreover, at $t = t_{k+1}^+$, $\hat{q}_i^{i,k_i+1}\left(t_{i,k_i+1}^+\right) = q_i\left(t_{i,k_i+1}^+\right)$. Consequently,

$$
\mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i+1}^+\right)\right\|^2\right) = \pi \mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i+1}^-\right)\right\|^2\right).
$$

Similarly, one has

$$
\mathbb{E}\left(\left\|\dot{e}_i^j\left(t_{i,k_i+1}^+\right)\right\|^2\right) = \pi \mathbb{E}\left(\left\|\dot{e}_i^j\left(t_{i,k_i+1}^-\right)\right\|^2\right)
$$

since $\dot{\hat{q}}_i^{i,\ell}\left(t_{i,k_i+1}^+\right) = \dot{\hat{q}}_i^{i,\ell}\left(t_{i,k_i+1}^-\right)$ for all $\ell = k_i - \kappa + 1, \ldots, k_i$ and $\dot{q}_i\left(t_{i,k_i+1}^+\right) = \dot{q}_i\left(t_{i,k_i+1}^-\right)$.

A.6 Proof of upper-bounded H_i

The CTC (46) is not triggering at $t = 0$ if

$$\begin{aligned} k_s \bar{s}_i^T \bar{s}_i(0) + k_p k_g \bar{g}_i^T \bar{g}_i(0) + \eta > \alpha_{\mathrm{M}} \Bigg[& \sum_{j=1}^{N} m_{ij} \left(k_e \left\| e_i^j \right\|^2 + k_p k_M \left\| \dot{e}_i^j \right\|^2 \right) \\ & + 2 k_p k_C^2 \sum_{j=1}^{N} m_{ij} \left(\left\| e_i^j \right\|^2 \left\| \dot{\hat{q}}_j^i \right\|^2 + \frac{1}{2} \left\| e_i^j \right\|^4 + \frac{1}{2} \left\| \dot{e}_i^j \right\|^4 \right) \Bigg] \\ & + k_g b_i \left\| \dot{q}_i - \dot{q}_i^* \right\|^2 . \end{aligned}$$

Remind $\bar{s}_i^T \bar{s}_i = k_p \bar{g}_i^T \bar{g}_i + \left\| \dot{q}_i - \dot{q}_i^* \right\|^2$, so

$$\begin{aligned} k_s \left\| \dot{q}_i - \dot{q}_i^* \right\|^2 (0) + k_p (k_s + k_g) \bar{g}_i^T \bar{g}_i(0) + \eta > \alpha_{\mathrm{M}} \Bigg[& \sum_{j=1}^{N} m_{ij} \left(k_e \left\| e_i^j \right\|^2 + k_p k_M \left\| \dot{e}_i^j \right\|^2 \right) \\ & + 2 k_p k_C^2 \sum_{j=1}^{N} m_{ij} \left(\left\| e_i^j \right\|^2 \left\| \dot{\hat{q}}_j^i \right\|^2 + \frac{1}{2} \left\| e_i^j \right\|^4 + \frac{1}{2} \left\| \dot{e}_i^j \right\|^4 \right) \Bigg] \\ & + k_g b_i \left\| \dot{q}_i - \dot{q}_i^* \right\|^2 . \end{aligned} \tag{A.64}$$

Since $b_i < \frac{k_s}{k_s k_p + k_g}$, one has

$$\begin{aligned} k_s - k_g \left(\frac{k_s}{k_s k_p + k_g} \right) &= k_s \frac{k_s k_p + k_g - k_g}{k_s k_p + k_g} \\ &= k_s \frac{k_s k_p}{k_s k_p + k_g} > 0 \end{aligned}$$

so (A.64) is satisfied if

$$\begin{aligned} k_p (k_s + k_g) \bar{g}_i^T \bar{g}_i(0) + \eta > \alpha_{\mathrm{M}} \Bigg[& \sum_{j=1}^{N} m_{ij} \left(k_e \left\| e_i^j \right\|^2 + k_p k_M \left\| \dot{e}_i^j \right\|^2 \right) \\ & + 2 k_p k_C^2 \sum_{j=1}^{N} m_{ij} \left(\left\| e_i^j \right\|^2 \left\| \dot{\hat{q}}_j^i \right\|^2 + \frac{1}{2} \left\| e_i^j \right\|^4 + \frac{1}{2} \left\| \dot{e}_i^j \right\|^4 \right) \Bigg] . \end{aligned} \tag{A.65}$$

We choose the condition $\left\| \dot{\hat{q}}_j^i \right\|^2 = 0$, $\left\| e_i^j \right\|^2 < H_i$ and $\left\| \dot{e}_i^j \right\|^2 < H_i$ where $H_i \geq 0$ is a constant. Remind $\alpha_i = \sum_{j=1}^{N} m_{ij}$. Thus, (A.65) may be rewritten as

$$\begin{aligned} k_p (k_s + k_g) \bar{g}_i^T \bar{g}_i(0) + \eta > \alpha_{\mathrm{M}} \Bigg[& \sum_{j=1}^{N} m_{ij} (k_e + k_p k_M) H_i \\ & + 2 k_p k_C^2 \sum_{j=1}^{N} m_{ij} \left(\frac{1}{2} H_i^2 \right) \Bigg] \\ k_p (k_s + k_g) \bar{g}_i^T \bar{g}_i(0) + \eta > \alpha_{\mathrm{M}} \alpha_i & \left[(k_e + k_p k_M) H_i + k_p k_C^2 H_i^2 \right] \end{aligned} \tag{A.66}$$

and so

$$\left(\frac{k_p (k_s + k_g)}{\alpha_{\mathrm{M}} \alpha_i} \bar{g}_i^T \bar{g}_i(0) + \frac{\eta}{\alpha_{\mathrm{M}} \alpha_i} \right) - (k_e - k_p k_M) H_i - \left(k_p k_C^2 \right) H_i^2 > 0. \tag{A.67}$$

By solving (A.67), we find

$$H_i < \frac{\sqrt{(k_e + k_p k_M)^2 + k_p k_C^2 \xi_i} - (k_e + k_p k_M)}{2 k_p k_C^2}$$

where $\xi_i = \frac{k_p \left(k_s + k_g \right)}{\alpha_{\mathrm{M}} \alpha_i} \bar{g}_i^T \bar{g}_i(0) + \frac{\eta}{\alpha_{\mathrm{M}} \alpha_i}$.

B Proof of absence of Zeno behavior

To prove the abscence of Zeno behavior, we have to define a minimum inter-event time $\tau_{\min}$ such $t_{i,k_i+1} - t_{i,k_i} \geq \tau_{\min}$. To obtain it, we study the evolution of the right-hand side $C_R(t)$ and the left-hand side $C_L(t)$ of the CTC exposed in Theorem 5 to find a condition such $C_R(t_{i,k_i} + \Delta t) < C_L(t_{i,k_i} + \Delta t)$ for $\Delta t < \tau_{\min}$. Lipschitzien hypotheses described in Assumptions A6, A8 and A9 will be used to obtain an estimation of $C_R(t_{i,k_i} + \Delta t)$ and $C_L(t_{i,k_i} + \Delta t)$ before be used to estimate $\tau_{\min}$.

B.1 Proof that $t_{i,k} < t_{i,k+1}$

In a first time, let show the CTC is not satisfied immediately after a communication, *i.e.* $t_{i,k} < t_{i,k+1}$, and find conditions on design parameters to guarantee it. Consider

$$C_{\mathrm{L}}(t) = k_s \bar{s}_i^T(t)\,\bar{s}_i(t) + k_p k_g \bar{g}_i^T(t)\,\bar{g}_i(t) + \eta$$

and

$$\begin{aligned} C_{\mathrm{R}}(t) = \alpha_{\mathrm{M}} \Bigg[& \sum_{j=1}^{N} m_{ij} \left(k_e \mathbb{E}\left(\left\| e_i^j(t) \right\|^2 \right) + k_p k_{\mathrm{M}} \mathbb{E}\left(\left\| \dot{e}_i^j(t) \right\|^2 \right) \right) \\ & + 2 k_p k_C^2 \sum_{j=1}^{N} m_{ij} \left(\mathbb{E}\left(\left\| e_i^j(t) \right\|^2 \right) \left\| \dot{\hat{q}}_j^i(t) \right\|^2 + \frac{1}{2}\mathbb{E}\left(\left\| e_i^j(t) \right\|^4 \right) + \frac{1}{2}\mathbb{E}\left(\left\| \dot{e}_i^j(t) \right\|^4 \right) \right) \Bigg] \\ & + k_g b_i \left\| \dot{q}_i(t) - \dot{q}_i^*(t) \right\|^2, \end{aligned}$$

with $k_e = \left(k_s k_p^2 + k_g k_p + \frac{k_g}{b_i} \right)$.

According to (46), no communication is triggered as long as $C_{\mathrm{L}}(t) > C_{\mathrm{R}}(t)$. A communication is triggered at $t = t_{i,k}$ when

$$C_{\mathrm{L}}(t_{i,k}) = C_{\mathrm{R}}(t_{i,k}). \tag{B.1}$$

The message sent at time $t = t_{i,k}$ by Agent i implies an update of the estimates $\hat{q}_i^j$, $j \in \mathcal{N}_i$ of that state q_i run by the neighbors of Agent i. Introduce t_{i,k_i}^- as the instant t_{i,k_i} before the update and t_{i,k_i}^+ be the instant t_{i,k_i} after the update. Consequently, the expected state estimation error will be such that

$$\mathbb{E}\left(\left\| e_i^j\left(t_{i,k_i}^+\right) \right\|^2 \right) = \pi \mathbb{E}\left(\left\| e_i^j\left(t_{i,k_i}^-\right) \right\|^2 \right), \tag{B.2}$$

see in Appendix A.5.3. Nevertheless, $k_s \bar{s}_i^T \bar{s}_i + k_p k_g \bar{g}_i^T \bar{g}_i + \eta$ and $\|\dot{q}_i - \dot{q}_i^*\|^2$, which are not updated by the communication, stay unchanged. Consequently

$$C_{\mathrm{L}}\left(t_{i,k_i}^+\right) = C_{\mathrm{L}}\left(t_{i,k_i}^-\right). \tag{B.3}$$

To prove that $t_{i,k} < t_{i,k+1}$, one has to show that $C_{\mathrm{L}}\left(t_{i,k_i}^+\right) > C_{\mathrm{R}}\left(t_{i,k_i}^+\right)$.

Using Appendix A.5.3 and the continuity of $\dot{q}_i(t)$ and $\dot{q}_i^*$, one may write

$$\begin{aligned} C_{\mathrm{R}}\left(t_{i,k_i}^+\right) = k_g b_i \left\| \dot{q}_i\left(t_{i,k_i}^-\right) - \dot{q}_i^*\left(t_{i,k_i}^-\right) \right\|^2 + \pi \alpha_{\mathrm{M}} \Bigg[& \sum_{j=1}^{N} m_{ij} \left(k_e \mathbb{E}\left(\left\| e_i^j\left(t_{i,k_i}^-\right) \right\|^2 \right) + k_p k_M \mathbb{E}\left(\left\| \dot{e}_i^j\left(t_{i,k_i}^-\right) \right\|^2 \right) \right) \\ & + 2 k_p k_C^2 \sum_{j=1}^{N} m_{ij} \left(\mathbb{E}\left(\left\| e_i^j\left(t_{i,k_i}^-\right) \right\|^2 \right) \left\| \dot{\hat{q}}_j^i \right\|^2 + \frac{1}{2}\mathbb{E}\left(\left\| e_i^j\left(t_{i,k_i}^-\right) \right\|^4 \right) + \frac{1}{2}\mathbb{E}\left(\left\| \dot{e}_i^j\left(t_{i,k_i}^-\right) \right\|^4 \right) \right) \Bigg] \end{aligned}$$

Using (B.1) and (B.3), one gets

$$\begin{aligned} C_{\mathrm{R}}\left(t_{i,k_i}^+\right) &= k_g b_i \left\| \dot{q}_i\left(t_{i,k_i}^-\right) - \dot{q}_i^*\left(t_{i,k_i}^-\right) \right\|^2 + \pi \left[k_s \bar{s}_i^T \bar{s}_i\left(t_{i,k_i}^-\right) + k_p k_g \bar{g}_i^T \bar{g}_i\left(t_{i,k_i}^-\right) + \eta - k_g b_i \left\| \dot{q}_i\left(t_{i,k_i}^-\right) - \dot{q}_i^*\left(t_{i,k_i}^-\right) \right\|^2 \right] \\ &= (1-\pi)\, k_g b_i \left\| \dot{q}_i\left(t_{i,k_i}^-\right) - \dot{q}_i^*\left(t_{i,k_i}^-\right) \right\|^2 + \pi \left[k_s \bar{s}_i^T \bar{s}_i\left(t_{i,k_i}^-\right) + k_p k_g \bar{g}_i^T \bar{g}_i\left(t_{i,k_i}^-\right) + \eta \right]. \end{aligned} \tag{B.4}$$

The CTC is not satisfied if

$$\begin{aligned}
C_{\mathrm{L}}\left(t_{i,k_i}^{+}\right) &> C_{\mathrm{R}}\left(t_{i,k_i}^{+}\right) \\
k_s \bar{s}_i^T \bar{s}_i\left(t_{i,k_i}^{-}\right) + k_p k_g \bar{g}_i^T \bar{g}_i\left(t_{i,k_i}^{-}\right) + \eta &> (1-\pi)\, k_g b_i \left\|\dot{q}_i\left(t_{i,k_i}^{-}\right) - \dot{q}_i^*\left(t_{i,k_i}^{-}\right)\right\|^2 \\
&\quad + \pi\left[k_s \bar{s}_i^T \bar{s}_i\left(t_{i,k_i}^{-}\right) + k_p k_g \bar{g}_i^T \bar{g}_i\left(t_{i,k_i}^{-}\right) + \eta\right] \qquad \text{(B.5)} \\
(1-\pi)\left(k_s \bar{s}_i^T \bar{s}_i\left(t_{i,k_i}^{-}\right) + k_p k_g \bar{g}_i^T \bar{g}_i\left(t_{i,k_i}^{-}\right) + \eta\right) &> (1-\pi)\, k_g b_i \left\|\dot{q}_i\left(t_{i,k_i}^{-}\right) - \dot{q}_i^*\left(t_{i,k_i}^{-}\right)\right\|^2 \\
k_s \bar{s}_i^T \bar{s}_i\left(t_{i,k_i}^{-}\right) + k_p k_g \bar{g}_i^T \bar{g}_i\left(t_{i,k_i}^{-}\right) + \eta &> k_g b_i \left\|\dot{q}_i\left(t_{i,k_i}^{-}\right) - \dot{q}_i^*\left(t_{i,k_i}^{-}\right)\right\|^2 \qquad \text{(B.6)}
\end{aligned}$$

and let show now that (B.6) is always satisfied.

Using the property $x^T y \geq -\frac{1}{2}\left(b_{i2} x^T x + \frac{1}{b_{i2}} y^T y\right)$ for some $b_{i2} > 0$, one deduces that

$$\begin{aligned}
\bar{s}_i^T \bar{s}_i &= k_p^2 \bar{g}_i^T \bar{g}_i + \left\|\dot{q}_i - \dot{q}_i^*\right\|^2 + 2 k_p \bar{g}_i^T \left(\dot{q}_i - \dot{q}_i^*\right) \\
&\geq \left(k_p^2 - k_p b_{i2}\right) \bar{g}_i^T \bar{g}_i + \left(1 - \frac{k_p}{b_{i2}}\right) \left\|\dot{q}_i - \dot{q}_i^*\right\|^2 . \qquad \text{(B.7)}
\end{aligned}$$

Using (B.7), a sufficient condition for (B.6) to be satisfied is

$$\begin{aligned}
k_s\left(k_p^2 - k_p b_{i2}\right) \bar{g}_i^T \bar{g}_i + k_s\left(1 - \frac{k_p}{b_{i2}}\right) \left\|\dot{q}_i - \dot{q}_i^*\right\|^2 + k_p k_g \bar{g}_i^T \bar{g}_i + \eta &> k_g b_i \left\|\dot{q}_i - \dot{q}_i^*\right\|^2 \\
k_s\left(1 - \frac{k_p}{b_{i2}}\right) \left\|\dot{q}_i - \dot{q}_i^*\right\|^2 + \left[k_p k_g + k_s\left(k_p^2 - k_p b_{i2}\right)\right] \bar{g}_i^T \bar{g}_i + \eta &> k_g b_i \left\|\dot{q}_i - \dot{q}_i^*\right\|^2 \\
k_1 \bar{g}_i^T \bar{g}_i + \eta &> k_2 \left\|\dot{q}_i - \dot{q}_i^*\right\|^2 \qquad \text{(B.8)}
\end{aligned}$$

where $k_1 = \left[k_p k_g + k_s\left(k_p^2 - k_p b_{i2}\right)\right]$ and $k_2 = \left[k_g b_i - k_s\left(1 - \frac{k_p}{b_{i2}}\right)\right]$. To ensure that the inequality (B.8) is satisfied independently of the values of $\bar{g}_i$ and $\dot{q}_i$, it is sufficient to find b_i and b_{i2} such that $k_1 > 0$ and $k_2 < 0$. Consider first k_1.

$$\begin{aligned}
k_p k_g + k_s\left(k_p^2 - k_p b_{i2}\right) &> 0 \\
\frac{k_g}{k_s} &> \left(-k_p + b_{i2}\right) \\
\frac{k_s k_p + k_g}{k_s} &> b_{i2}. \qquad \text{(B.9)}
\end{aligned}$$

Focus now on k_2

$$\begin{aligned}
k_g b_i - k_s\left(1 - \frac{k_p}{b_{i2}}\right) &< 0 \\
\frac{k_g b_i}{k_s} &< 1 - \frac{k_p}{b_{i2}} \qquad \text{(B.10)}
\end{aligned}$$

Since $b_{i2} > 0$, one has $\frac{k_g b_i}{k_s} < 1$ and so $b_i < \frac{k_s}{k_g}$. Then

$$\frac{k_s k_p}{k_s - k_g b_i} < b_{i2}. \qquad \text{(B.11)}$$

Finally, one has to find a condition on b_i such that (B.9) and (B.10) can be satisfied simultaneously

$$\frac{k_s k_p + k_g}{k_s} > b_{i2} > \frac{k_s k_p}{k_s - k_g b_i}. \qquad \text{(B.12)}$$

One may find b_{i2} if

$$\begin{aligned} k_s - k_g b_i &> \frac{k_s^2 k_p}{k_s k_p + k_g} \\ \frac{1}{k_g}\left(k_s - \frac{k_s^2 k_p}{k_s k_p + k_g}\right) &> b_i \\ b_i &< \frac{k_s}{k_s k_p + k_g}. \end{aligned} \tag{B.13}$$

which also ensures that $b_i < \frac{k_s}{k_g}$. Thus, once $b_i < \frac{k_s}{k_s k_p + k_g}$, there exists some b_{i2} such that (B.12) is satisfied. As a consequence, (46) stops to be satisfied when $t = t_{i,k}^+$.

B.2 Study of upper-bound

From the Lipschitzien hypotheses of Assumptions A6, A8 and A9, we can deduce there exist K, $\hat{K}$ and K^* such

$$\|q_i(t+\Delta t) - q_i(t)\| \leq K\Delta t \tag{B.14}$$

$$\left\|\hat{q}_i^j(t+\Delta t) - \hat{q}_i^j(t)\right\| \leq \hat{K}\Delta t \tag{B.15}$$

$$\|q_i^*(t+\Delta t) - q_i^*(t)\| \leq K^*\Delta t \tag{B.16}$$

These conditions are now used to find an upper bound for all the terms inside the CTC.

B.2.1 Upper-bound on $\mathbb{E}\left(\left\|e_i^j(t+\Delta t)\right\|^2\right)$ *and* $\mathbb{E}\left(\left\|e_i^j(t+\Delta t)\right\|^2\right)$

Define $I = [t,\, t+\Delta t]$ such that Agent i broadcasts no communication inside I.

Let's study first an estimate $\hat{q}_i^{i,\ell}$ for $\ell \in \{1 \dots \kappa\}$. Using (B.14) and (B.15)

$$\begin{aligned} \left\|e_i^{i,\ell}(t+\Delta t)\right\| &= \left\|\hat{q}_i^{i,\ell}(t+\Delta t) - q_i(t+\Delta t)\right\| \\ &= \left\|\hat{q}_i^{(\ell)}(t+\Delta t) - \hat{q}_i^{i,\ell}(t) + \hat{q}_i^{i,\ell}(t) - q_i(t+\Delta t) - q_i(t) + q_i(t)\right\| \\ &\leq \left\|e_i^{i,\ell}(t)\right\| + \|q_i(t+\Delta t) - q_i(t)\| + \left\|\hat{q}_i^{i,\ell}(t+\Delta t) - \hat{q}_i^{i,\ell}(t)\right\| \\ &\leq \left\|e_i^{i,\ell}(t)\right\| + \left(\hat{K} + K\right)\Delta t. \end{aligned} \tag{B.17}$$

Consequently

$$\left\|e_i^{i,\ell}(t+\Delta t)\right\|^2 \leq \left\|e_i^{i,\ell}(t)\right\|^2 + \left(\left(\hat{K}+K\right)\Delta t\right)^2 + 2\left\|e_i^{i,\ell}(t)\right\|\left(\hat{K}+K\right)\Delta t \tag{B.18}$$

Using the Young relation $xy \leq \frac{1}{2\boldsymbol{a}}x^2 + \frac{\boldsymbol{a}}{2}y^2$ with $\boldsymbol{a} > 0$, one gets

$$\left\|e_i^{i,\ell}(t+\Delta t)\right\|^2 \leq (1+\boldsymbol{a})\left\|e_i^{i,\ell}(t)\right\|^2 + \left(1+\frac{1}{\boldsymbol{a}}\right)\left(\left(\hat{K}+K\right)\Delta t\right)^2. \tag{B.19}$$

Using (B.19), one may write

$$\begin{aligned} \mathbb{E}\left(\left\|e_i^j(t+\Delta t)\right\|^2\right) &= \sum_{\ell=\max\{k_i-\kappa+1,0\}}^{k_i} p_{k_i,\ell|k_i^{j,i}} \left\|\hat{q}_i^{i,\ell}(t+\Delta t) - q_i(t+\Delta t)\right\|^2 \\ &= \sum_{\ell=\max\{k_i-\kappa+1,0\}}^{k_i} p_{k_i,\ell|k_i^{j,i}} \left\|e_i^{i,\ell}(t+\Delta t)\right\|^2 \\ &\leq \sum_{\ell=\max\{k_i-\kappa+1,0\}}^{k_i} p_{k_i,\ell|k_i^{j,i}} \left[(1+\boldsymbol{a})\left\|e_i^{i,\ell}(t)\right\|^2 + \left(1+\frac{1}{\boldsymbol{a}}\right)\left(\left(\hat{K}+K\right)\Delta t\right)^2\right] \\ &= (1+\boldsymbol{a})\,\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right) + \left(1+\frac{1}{\boldsymbol{a}}\right)\left(\left(\hat{K}+K\right)\Delta t\right)^2 \sum_{\ell=\max\{k_i-\kappa+1,0\}}^{k_i} p_{k_i,\ell|k_i^{j,i}} \end{aligned} \tag{B.20}$$

and since $\sum_{\ell=\max\{k_i-\kappa+1,0\}}^{k_i} p_{k_i,\ell|k_i^{j,i}} = 1$, one can obtain

$$\mathbb{E}\left(\left\|e_i^j\left(t+\Delta t\right)\right\|^2\right) \leq (1+\boldsymbol{a})\,\mathbb{E}\left(\left\|e_i^j\left(t\right)\right\|^2\right) + \left(1+\frac{1}{\boldsymbol{a}}\right)\left(\left(\hat{K}+K\right)\Delta t\right)^2 \tag{B.21}$$

In the same way, using (4) and (24), one has

$$\left\|\dot{e}_i^j\left(t+\Delta t\right)\right\|^2 \leq (1+\boldsymbol{a})\left\|\dot{e}_i^j\left(t\right)\right\|^2 + \left(1+\frac{1}{\boldsymbol{a}}\right)\left(\left(\hat{K}_d+K_d\right)\Delta t\right)^2 \tag{B.22}$$

and

$$\mathbb{E}\left(\left\|\dot{e}_i^j\left(t+\Delta t\right)\right\|^2\right) \leq (1+\boldsymbol{a})\,\mathbb{E}\left(\left\|\dot{e}_i^j\left(t\right)\right\|^2\right) + \left(1+\frac{1}{\boldsymbol{a}}\right)\left(\left(\hat{K}_d+K_d\right)\Delta t\right)^2. \tag{B.23}$$

The value of $\boldsymbol{a}$ will be defined later.

B.2.2 Upper-bound on $\mathbb{E}\left(\left\|e_i^j\left(t+\Delta t\right)\right\|^4\right)$ *and* $\mathbb{E}\left(\left\|\dot{e}_i^j\left(t+\Delta t\right)\right\|^4\right)$

Using (B.19), one may write

$$\begin{aligned}
\left\|e_i^{i,\ell}\left(t+\Delta t\right)\right\|^4 &\leq \left(\left\|e_i^{i,\ell}\left(t\right)\right\|^2 + \left(\left(\hat{K}+K\right)\Delta t\right)^2 + 2\left\|e_i^{i,\ell}\left(t\right)\right\|\left(\hat{K}+K\right)\Delta t\right)^2 \\
&\leq \left\|e_i^{i,\ell}\left(t\right)\right\|^4 + \left(\left(\hat{K}+K\right)\Delta t\right)^4 + 4\left\|e_i^{i,\ell}\left(t\right)\right\|^2\left(\left(\hat{K}+K\right)\Delta t\right)^2 \\
&+ 2\left\|e_i^{i,\ell}\left(t\right)\right\|^2\left(\left(\hat{K}+K\right)\Delta t\right)^2 + 4\left\|e_i^{i,\ell}\left(t\right)\right\|^3\left(\hat{K}+K\right)\Delta t \\
&+ 4\left\|e_i^{i,\ell}\left(t\right)\right\|\left(\left(\hat{K}+K\right)\Delta t\right)^3 \\
&\leq \left\|e_i^{i,\ell}\left(t\right)\right\|^4 + \left(\left(\hat{K}+K\right)\Delta t\right)^4 + 6\left\|e_i^{i,\ell}\left(t\right)\right\|^2\left(\left(\hat{K}+K\right)\Delta t\right)^2 \\
&+ 4\left\|e_i^{i,\ell}\left(t\right)\right\|\left(\hat{K}+K\right)\Delta t\left(\left\|e_i^{i,\ell}\left(t\right)\right\|^2 + \left(\left(\hat{K}+K\right)\Delta t\right)^2\right).
\end{aligned} \tag{B.24}$$

Using Young's relation with $\boldsymbol{c} > 0$ and $\boldsymbol{d} > 0$, one gets

$$\begin{aligned}
\left\|e_i^{i,\ell}\left(t+\Delta t\right)\right\|^4 &\leq \left\|e_i^{i,\ell}\left(t\right)\right\|^4 + \left(\left(\hat{K}+K\right)\Delta t\right)^4 + 3\left(\boldsymbol{c}\left\|e_i^{i,\ell}\left(t\right)\right\|^4 + \frac{1}{\boldsymbol{c}}\left(\left(\hat{K}+K\right)\Delta t\right)^4\right) \\
&+ 2\left(\boldsymbol{d}\left\|e_i^{i,\ell}\left(t\right)\right\|^2 + \frac{1}{\boldsymbol{d}}\left(\left(\hat{K}+K\right)\Delta t\right)^2\right)\left(\left\|e_i^{i,\ell}\left(t\right)\right\|^2 + \left(\left(\hat{K}+K\right)\Delta t\right)^2\right) \\
&\leq (1+3\boldsymbol{c})\left\|e_i^{i,\ell}\left(t\right)\right\|^4 + \left(1+3\frac{1}{\boldsymbol{c}}\right)\left(\left(\hat{K}+K\right)\Delta t\right)^4 \\
&+ 2\left(\boldsymbol{d}\left\|e_i^{i,\ell}\left(t\right)\right\|^4 + \frac{1}{\boldsymbol{d}}\left(\left(\hat{K}+K\right)\Delta t\right)^4 + \left(\boldsymbol{d}+\frac{1}{\boldsymbol{d}}\right)\left\|e_i^{i,\ell}\left(t\right)\right\|^2\left(\left(\hat{K}+K\right)\Delta t\right)^2\right).
\end{aligned} \tag{B.25}$$

Using Young relation $xy \leq \frac{1}{2\boldsymbol{f}}x^2 + \frac{\boldsymbol{f}}{2}y^2$ with $\boldsymbol{f} > 0$, one gets

$$\begin{aligned}
\left\|e_i^{i,\ell}\left(t+\Delta t\right)\right\|^4 &\leq (1+3\boldsymbol{c}+2\boldsymbol{d})\left\|e_i^{i,\ell}\left(t\right)\right\|^4 + \left(1+3\frac{1}{\boldsymbol{c}}+2\frac{1}{\boldsymbol{d}}\right)\left(\left(\hat{K}+K\right)\Delta t\right)^4 \\
&+ \left(\boldsymbol{d}+\frac{1}{\boldsymbol{d}}\right)\left(\boldsymbol{f}\left\|e_i^{i,\ell}\left(t\right)\right\|^4 + \frac{1}{\boldsymbol{f}}\left(\left(\hat{K}+K\right)\Delta t\right)^4\right) \\
&\leq \left(1+3\boldsymbol{c}+2\boldsymbol{d}+\left(\boldsymbol{d}+\frac{1}{\boldsymbol{d}}\right)\boldsymbol{f}\right)\left\|e_i^{i,\ell}\left(t\right)\right\|^4 + \left(1+3\frac{1}{\boldsymbol{c}}+2\frac{1}{\boldsymbol{d}}+\left(\boldsymbol{d}+\frac{1}{\boldsymbol{d}}\right)\frac{1}{\boldsymbol{f}}\right)\left(\left(\hat{K}+K\right)\Delta t\right)^4
\end{aligned} \tag{B.26}$$

We would like to impose $\left(1+3\boldsymbol{c}+2\boldsymbol{d}+\left(\boldsymbol{d}+\frac{1}{\boldsymbol{d}}\right)\boldsymbol{f}\right)=1+\boldsymbol{a}$. A possible solution, detailed below in Section B.4, is

$$\boldsymbol{f}=1+\frac{\sqrt{4+\boldsymbol{a}^2}-2}{2} \tag{B.27}$$

$$\boldsymbol{d}=\sqrt{\frac{\boldsymbol{f}}{\boldsymbol{f}+2}} \tag{B.28}$$

$$\boldsymbol{c}=\frac{\boldsymbol{a}-2\sqrt{\boldsymbol{f}\left(\boldsymbol{f}+2\right)}}{3}, \tag{B.29}$$

where Section B.4 shows that $\boldsymbol{c}>0$. So (B.26) can be rewritten such

$$\left\|e_i^{i,\ell}\left(t+\Delta t\right)\right\|^4\leq\left(1+\boldsymbol{a}\right)\left\|e_i^{i,\ell}\left(t\right)\right\|^4+\boldsymbol{F}\left(\left(\hat{K}+K\right)\Delta t\right)^4 \tag{B.30}$$

with $\boldsymbol{F}=\left(1+3\frac{1}{\boldsymbol{c}}+2\frac{1}{\boldsymbol{d}}+\left(\boldsymbol{d}+\frac{1}{\boldsymbol{d}}\right)\frac{1}{\boldsymbol{f}}\right)$. Finally, using a method similar to that of Section B.2.1, one obtains

$$\mathbb{E}\left(\left\|e_i^j\left(t+\Delta t\right)\right\|^4\right)\leq\left(1+\boldsymbol{a}\right)\mathbb{E}\left(\left\|e_i^j\left(t\right)\right\|^4\right)+\boldsymbol{F}\left(\left(\hat{K}+K\right)\Delta t\right)^4 \tag{B.31}$$

Similarly, one has

$$\mathbb{E}\left(\left\|\dot{e}_i^j\left(t+\Delta t\right)\right\|^4\right)\leq\left(1+\boldsymbol{a}\right)\mathbb{E}\left(\left\|\dot{e}_i^j\left(t\right)\right\|^4\right)+\boldsymbol{F}\left(\left(\hat{K}_d+K_d\right)\Delta t\right)^4. \tag{B.32}$$

B.2.3 Upper-bound on $\left\|\dot{\hat{q}}_j^i\left(t+\Delta t\right)\right\|$

Using (24) and (10), one may write

$$\begin{aligned}\left\|\dot{\hat{q}}_j^i\left(t+\Delta t\right)\right\|&=\left\|\dot{\hat{q}}_j^i\left(t+\Delta t\right)-\dot{\hat{q}}_j^i\left(t\right)+\dot{\hat{q}}_j^i\left(t\right)\right\|\\&\leq\left\|\dot{\hat{q}}_j^i\left(t\right)\right\|+\hat{K}_d\Delta t.\end{aligned} \tag{B.33}$$

Using Young's relation with $\boldsymbol{a}>0$, one gets

$$\begin{aligned}\left\|\dot{\hat{q}}_j^i\left(t+\Delta t\right)\right\|^2&\leq\left\|\dot{\hat{q}}_j^i\left(t\right)\right\|^2+\left(\hat{K}_d\Delta t\right)^2+2\left\|\dot{\hat{q}}_j^i\left(t\right)\right\|\hat{K}_d\Delta t\\&\leq\left(1+\boldsymbol{a}\right)\left\|\dot{\hat{q}}_j^i\left(t\right)\right\|^2+\left(1+\frac{1}{\boldsymbol{a}}\right)\left(\hat{K}_d\Delta t\right)^2\end{aligned} \tag{B.34}$$

B.2.4 Upper-bound on $\|\bar{g}_i\left(t\right)\|$ *and* $\|\bar{s}_i\left(t\right)\|$

Consider now $\|\bar{g}_i\left(t\right)\|$

$$
\begin{aligned}
\|\bar{g}_i(t)\| &= \left\| \sum_{j=1}^{N} m_{ij} \left(\left(q_i(t) - \hat{q}_j^i(t) \right) - \left(q_i^*(t) - q_j^*(t) \right) \right) \right\| \\
&= \Bigg\| \sum_{j=1}^{N} m_{ij} \left(\left(q_i(t) - \hat{q}_j^i(t) \right) - \left(q_i^*(t) - q_j^*(t) \right) \right) \\
&- \sum_{j=1}^{N} m_{ij} \left(\left(q_i(t+\Delta t) - \hat{q}_j^i(t+\Delta t) \right) - \left(q_i^*(t+\Delta t) - q_j^*(t+\Delta t) \right) \right) \\
&+ \sum_{j=1}^{N} m_{ij} \left(\left(q_i(t+\Delta t) - \hat{q}_j^i(t+\Delta t) \right) - \left(q_i^*(t+\Delta t) - q_j^*(t+\Delta t) \right) \right) \Bigg\| \\
&= \Bigg\| \sum_{j=1}^{N} m_{ij} \left(\left(q_i(t) - q_i(t+\Delta t) \right) - \left(q_i^*(t) - q_i^*(t+\Delta t) \right) \right) \\
&- \sum_{j=1}^{N} m_{ij} \left(\left(\hat{q}_j^i(t) - \hat{q}_j^i(t+\Delta t) \right) - \left(q_j^*(t) - q_j^*(t+\Delta t) \right) \right) \\
&+ \bar{g}_i(t+\Delta t) \| \qquad \text{(B.35)}
\end{aligned}
$$

In (B.35), using (B.14) and Assumption A8, one may write

$$
\begin{aligned}
\|\bar{g}_i(t)\| &\leq \|\bar{g}_i(t+\Delta t)\| + \sum_{j=1}^{N} m_{ij} \left(\|q_i(t+\Delta t) - q_i(t)\| + \|q_i^*(t+\Delta t) - q_i^*(t)\| \right. \\
&+ \left. \left\| \hat{q}_j^i(t+\Delta t) - \hat{q}_j^i(t) \right\| + \left\| q_j^*(t+\Delta t) - q_j^*(t) \right\| \right) \\
&\leq \|\bar{g}_i(t+\Delta t)\| + \Delta t \sum_{j=1}^{N} m_{ij} \left(K + \hat{K} + 2K^* \right) \\
&\leq \|\bar{g}_i(t+\Delta t)\| + \alpha_i \Delta t \left(K + \hat{K} + 2K^* \right). \qquad \text{(B.36)}
\end{aligned}
$$

Using the same steps of (B.36), it can be shown than

$$
\|\bar{g}_i(t+\Delta t)\| \leq \|\bar{g}_i(t)\| + \alpha_i \Delta t \left(K + \hat{K} + 2K^* \right). \qquad \text{(B.37)}
$$

Thus, one can deduce from (B.36) and (B.37) that

$$
\left| \|\bar{g}_i(t)\| - \|\bar{g}_i(t+\Delta t)\| \right| \leq \alpha_i \Delta t \left(K + \hat{K} + 2K^* \right). \qquad \text{(B.38)}
$$

From (B.38), one may write

$$
\begin{aligned}
\left| \|\bar{g}_i(t)\| - \|\bar{g}_i(t+\Delta t)\| \right| &\leqslant \alpha_i \Delta t \left(K + \hat{K} + 2K^* \right) \\
\left(\|\bar{g}_i(t)\| - \|\bar{g}_i(t+\Delta t)\| \right)^2 &\leqslant \left(\alpha_i \Delta t \left(K + \hat{K} + 2K^* \right) \right)^2 \\
\|\bar{g}_i(t+\Delta t)\|^2 - 2\|\bar{g}_i(t)\| \|\bar{g}_i(t+\Delta t)\| + \|\bar{g}_i(t)\|^2 &\leqslant \left(\alpha_i \Delta t \left(K + \hat{K} + 2K^* \right) \right)^2 \\
- \left(\alpha_i \Delta t \left(K + \hat{K} + 2K^* \right) \right)^2 + \|\bar{g}_i(t)\|^2 &\leqslant - \|\bar{g}_i(t+\Delta t)\|^2 + 2\|\bar{g}_i(t)\| \|\bar{g}_i(t+\Delta t)\| \qquad \text{(B.39)}
\end{aligned}
$$

Young relation $xy \leq \frac{1}{2\boldsymbol{d}}x^2 + \frac{\boldsymbol{d}}{2}y^2$ with $\boldsymbol{d} > 1$, one gets

$$
\begin{aligned}
-\left(\alpha_i \Delta t \left(K + \hat{K} + 2K^*\right)\right)^2 + \|\bar{g}_i(t)\|^2 &\leqslant -\|\bar{g}_i(t+\Delta t)\|^2 + \frac{1}{\boldsymbol{d}}\|\bar{g}_i(t)\|^2 + \boldsymbol{d}\|\bar{g}_i(t+\Delta t)\|^2 \\
-\left(\alpha_i \Delta t \left(K + \hat{K} + 2K^*\right)\right)^2 + \left(1 - \frac{1}{\boldsymbol{d}}\right)\|\bar{g}_i(t)\|^2 &\leqslant (\boldsymbol{d}-1)\|\bar{g}_i(t+\Delta t)\|^2 \\
-\frac{1}{(\boldsymbol{d}-1)}\left(\alpha_i \Delta t \left(K + \hat{K} + 2K^*\right)\right)^2 + \frac{\left(1-\frac{1}{\boldsymbol{d}}\right)}{(\boldsymbol{d}-1)}\|\bar{g}_i(t)\|^2 &\leqslant \|\bar{g}_i(t+\Delta t)\|^2 .
\end{aligned} \tag{B.40}
$$

By taking $\boldsymbol{d} = \frac{1}{1-\boldsymbol{b}}$ with $1 > \boldsymbol{b} > 0$, one gets $\frac{\left(1-\frac{1}{\boldsymbol{b}}\right)}{(\boldsymbol{b}-1)} = 1 - \boldsymbol{b}$ and $\frac{1}{(\boldsymbol{d}-1)} = \frac{1-\boldsymbol{b}}{\boldsymbol{b}}$:

$$
\left(1 - \frac{1}{\boldsymbol{b}}\right)\left(\alpha_i \Delta t \left(K + \hat{K} + 2K^*\right)\right)^2 + (1-\boldsymbol{b})\|\bar{g}_i(t)\|^2 \leqslant \|\bar{g}_i(t+\Delta t)\|^2 \tag{B.41}
$$

Using the same steps that for $\|\bar{g}_i(t)\|$, one gets for $\|\bar{s}_i(t)\|$ the following expression:

$$
\begin{aligned}
\|\bar{s}_i(t)\| &= \|\dot{q}_i(t) - \dot{q}_i^*(t) + k_p \bar{g}_i(t)\| \\
\|\bar{s}_i(t)\| &= \Bigg\| k_p \sum_{j=1}^{N} m_{ij}\left((q_i(t) - q_i(t+\Delta t)) - (q_i^*(t) - q_i^*(t+\Delta t))\right) \\
&\quad - k_p \sum_{j=1}^{N} m_{ij}\left(\left(\hat{q}_j^i(t) - \hat{q}_j^i(t+\Delta t)\right) - \left(q_j^*(t) - q_j^*(t+\Delta t)\right)\right) \\
&\quad + \dot{q}_i(t) - \dot{q}_i(t+\Delta t) - \dot{q}_i^*(t) + \dot{q}_i^*(t+\Delta t) + \bar{s}_i(t+\Delta t)\| \\
\|\bar{s}_i(t)\| &\leq \|\bar{s}_i(t+\Delta t)\| + \sum_{j=1}^{N} m_{ij}\left(\|q_i(t+\Delta t) - q_i(t)\| + \|q_i^*(t+\Delta t) - q_i^*(t)\|\right. \\
&\quad \left. + \left\|\hat{q}_j^i(t+\Delta t) - \hat{q}_j^i(t)\right\| + \left\|q_j^*(t+\Delta t) - q_j^*(t)\right\|\right) \\
&\quad + \|\dot{q}_i(t) - \dot{q}_i(t+\Delta t)\| + \|\dot{q}_i^*(t) - \dot{q}_i^*(t+\Delta t)\| \\
\|\bar{s}_i(t)\| &\leq \|\bar{s}_i(t+\Delta t)\| + \Delta t\left[\alpha_i\left(K + \hat{K} + 2K^*\right) + 2K_d^*\right]
\end{aligned} \tag{B.42}
$$

In a similar way, it can be shown that

$$
\|\bar{s}_i(t+\Delta t)\| \leq \|\bar{s}_i(t)\| + \Delta t\left[\alpha_i\left(K + \hat{K} + 2K^*\right) + 2K_d^*\right] \tag{B.43}
$$

Using (B.42) and (B.43), one can deduce

$$
\left|\|\bar{s}_i(t+\Delta t)\| - \|\bar{s}_i(t)\|\right| \leq \Delta t\left[\alpha_i\left(K + \hat{K} + 2K^*\right) + 2K_d^*\right]
$$

Following the same steps from (B.39) to (B.41), one gets

$$
\left(1 - \frac{1}{\boldsymbol{b}}\right)(\Delta t)^2\left[\alpha_i\left(K + \hat{K} + 2K^*\right) + 2K_d^*\right]^2 + (1-\boldsymbol{b})\|\bar{s}_i(t)\|^2 \leq \|\bar{s}_i(t+\Delta t)\|^2 \tag{B.44}
$$

with $1 > \boldsymbol{b} > 0$.

B.2.5 Upper-bound on $\left\|\dot{\hat{q}}_j^i(t+\Delta t)\right\|^2 \mathbb{E}\left(\left\|e_i^j(t+\Delta t)\right\|^2\right)$

Using (B.21) and (B.34), one gets

$$\begin{aligned}\left\|\dot{\hat{q}}_j^i(t+\Delta t)\right\|^2 \mathbb{E}\left(\left\|e_i^j(t+\Delta t)\right\|^2\right) &\leq \left((1+\boldsymbol{a})\left\|\dot{\hat{q}}_j^i(t)\right\|^2 + \left(1+\frac{1}{\boldsymbol{a}}\right)\left(\hat{K}_d\Delta t\right)^2\right) \\ &\times \left((1+\boldsymbol{a})\,\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right) + \left(1+\frac{1}{\boldsymbol{a}}\right)\left(\left(\hat{K}+K\right)\Delta t\right)^2\right) \\ &\leq (1+\boldsymbol{a})^2\left\|\dot{\hat{q}}_j^i(t)\right\|^2 \mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right) \\ &+ (1+\boldsymbol{a})\left(1+\frac{1}{\boldsymbol{a}}\right)\left[\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right)\left(\hat{K}_d\right)^2 + \left\|\dot{\hat{q}}_j^i(t)\right\|^2\left(\hat{K}+K\right)^2\right](\Delta t)^2 \\ &+ \left(1+\frac{1}{\boldsymbol{a}}\right)^2\left(\left(\hat{K}+K\right)\hat{K}_d\right)^2(\Delta t)^4. \end{aligned} \tag{B.45}$$

Introducing $f(x,y)$ such that

$$f\left(\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right), \left\|\dot{\hat{q}}_j^i(t)\right\|^2\right) = (1+\boldsymbol{a})\left(1+\frac{1}{\boldsymbol{a}}\right)\left[\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right)\left(\hat{K}_d\right)^2 + \left\|\dot{\hat{q}}_j^i(t)\right\|^2\left(\hat{K}+K\right)^2\right] \tag{B.46}$$

and B.45 can be rewritten as

$$\begin{aligned}\left\|\dot{\hat{q}}_j^i(t+\Delta t)\right\|^2 \mathbb{E}\left(\left\|e_i^j(t+\Delta t)\right\|^2\right) &\leq (1+\boldsymbol{a})^2\left\|\dot{\hat{q}}_j^i(t)\right\|^2 \mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right) \\ &+ f\left(\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right), \left\|\dot{\hat{q}}_j^i(t)\right\|^2\right)(\Delta t)^2 \\ &+ \left(1+\frac{1}{\boldsymbol{a}}\right)^2\left(\left(\hat{K}+K\right)\hat{K}_d\right)^2(\Delta t)^4. \end{aligned} \tag{B.47}$$

Let's show now $f\left(\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right), \left\|\dot{\hat{q}}_j^i(t)\right\|^2\right)$ is bounded. From (23), one has $\left\|\dot{\hat{q}}_j^i(t)\right\|^2 \leq \dot{q}_{\max}^2$. From the CTC, one has

$$\alpha_{\mathrm{M}} k_e \mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right) \leq k_s\left\|\bar{s}_i(t)\right\|^2 + k_p k_g\left\|\bar{g}_i(t)\right\|^2 + \eta. \tag{B.48}$$

Following steps described in Section B.5, one may show $\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right) \leq E_{\max}$. Thus, $\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right)$ is bounded and so $f\left(\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right), \left\|\dot{\hat{q}}_j^i(t)\right\|^2\right) \leq f_{\max}$.

B.3 Minimum inter-event time

Let's consider $t = t_{i,k_i}^+$, where t_{i,k_i}^+ correspond to the instant t_{i,k_i} after the update of the estimate values. The left side of the CTC

$$C_{\mathrm{L}}\left(t_{i,k_i}^+ + \Delta t\right) = k_s\left\|\bar{s}_i\left(t_{i,k_i}^+ + \Delta t\right)\right\|^2 + k_p k_g\left\|\bar{g}_i\left(t_{i,k_i}^+ + \Delta t\right)\right\|^2 + \eta \tag{B.49}$$

becomes using results of Section B.2 and $\eta = (1-\boldsymbol{b}+\boldsymbol{b})\eta$,

$$\begin{aligned} C_{\mathrm{L}}\left(t_{i,k_i}^+ + \Delta t\right) &\geq (1-\boldsymbol{b})\left[k_s\left\|\bar{s}_i\left(t_{i,k_i}^+\right)\right\|^2 + k_p k_g\left\|\bar{g}_i\left(t_{i,k_i}^+\right)\right\|^2 + \eta\right] + \boldsymbol{b}\eta \\ &+ \left(1-\frac{1}{\boldsymbol{b}}\right)(\Delta t)^2\left[k_s\left[\alpha_i\left(K+\hat{K}+2K^*\right)+2K_d^*\right]^2\right. \\ &\left. + k_p k_g\left(\alpha_i\Delta t\left(K+\hat{K}+2K^*\right)\right)^2\right] \\ &\geq (1-\boldsymbol{b})\,C_{\mathrm{L}}\left(t_{i,k_i}^+\right) + \left(1-\frac{1}{\boldsymbol{b}}\right)(\Delta t)^2 K_1 + \boldsymbol{b}\eta \end{aligned} \tag{B.50}$$

with

$$K_1 = k_s \left[\alpha_i \left(K + \hat{K} + 2K^*\right) + 2K_d^*\right]^2 + k_p k_g \left(\alpha_i \Delta t \left(K + \hat{K} + 2K^*\right)\right)^2 \tag{B.51}$$

In the same way, the right side of the CTC

$$\begin{aligned} C_{\mathrm{R}}\left(t_{i,k_i}^+ + \Delta t\right) = \alpha_{\mathrm{M}} \Bigg[& \sum_{j=1}^{N} m_{ij} \left(k_e \mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i}^+ + \Delta t\right)\right\|^2\right) + k_p k_{\mathrm{M}} \mathbb{E}\left(\left\|\dot{e}_i^j\left(t_{i,k_i}^+ + \Delta t\right)\right\|^2\right)\right) \\ & + 2k_p k_C^2 \sum_{j=1}^{N} m_{ij} \left(\mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i}^+ + \Delta t\right)\right\|^2\right) \left\|\dot{\hat{q}}_j^i\left(t_{i,k_i}^+ + \Delta t\right)\right\|^2\right. \\ & \left. + \frac{1}{2}\mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i}^+ + \Delta t\right)\right\|^4\right) + \frac{1}{2}\mathbb{E}\left(\left\|\dot{e}_i^j\left(t_{i,k_i}^+ + \Delta t\right)\right\|^4\right)\right)\Bigg] \end{aligned} \tag{B.52}$$

becomes using results of Section B.2

$$\begin{aligned} C_{\mathrm{R}}\left(t_{i,k_i}^+ + \Delta t\right) \leq \alpha_{\mathrm{M}} \Bigg[& (1+\boldsymbol{a}) \sum_{j=1}^{N} m_{ij} \left(k_e \mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i}^+\right)\right\|^2\right) + k_p k_{\mathrm{M}} \mathbb{E}\left(\left\|\dot{e}_i^j\left(t_{i,k_i}^+\right)\right\|^2\right)\right) \\ & + 2k_p k_C^2 \sum_{j=1}^{N} m_{ij} \left((1+\boldsymbol{a})^2 \mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i}^+\right)\right\|^2\right) \left\|\dot{\hat{q}}_j^i\left(t_{i,k_i}^+\right)\right\|^2\right. \\ & \left. + \frac{1}{2}(1+\boldsymbol{a}) \mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i}^+\right)\right\|^4\right) + \frac{1}{2}\mathbb{E}\left(\left\|\dot{e}_i^j\left(t_{i,k_i}^+\right)\right\|^4\right)\right)\Bigg] \\ & + \alpha_{\mathrm{M}} \Bigg[\left(1 + \frac{1}{\boldsymbol{a}}\right) \alpha_i \left(k_e \left(\hat{K} + K\right)^2 + k_p k_{\mathrm{M}} \left(\hat{K}_d + K_d\right)^2\right) (\Delta t)^2 \\ & + 2k_p k_C^2 \sum_{j=1}^{N} m_{ij} \left(f\left(\mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i}^+\right)\right\|^2\right), \left\|\dot{\hat{q}}_j^i(t)\right\|^2\right) (\Delta t)^2\right. \\ & \left. + \left[\boldsymbol{F}\left(\left(\hat{K} + K\right)^2 + \left(\hat{K}_d + K_d\right)^4\right) + \left(1 + \frac{1}{\boldsymbol{a}}\right)^2 \left(\left(\hat{K} + K\right)\hat{K}_d\right)^2\right] (\Delta t)^4\right)\Bigg] \end{aligned} \tag{B.53}$$

and since $(1+\boldsymbol{a}) > 1$, one can upper-bound (B.53) such that

$$\begin{aligned} C_{\mathrm{R}}\left(t_{i,k_i}^+ + \Delta t\right) \leq & (1+\boldsymbol{a})^2 C_{\mathrm{R}}(t) \\ & + \alpha_{\mathrm{M}} \Bigg[\left(1 + \frac{1}{\boldsymbol{a}}\right) \alpha_i \left(k_e \left(\hat{K} + K\right)^2 + k_p k_{\mathrm{M}} \left(\hat{K}_d + K_d\right)^2\right) (\Delta t)^2 \\ & + 2k_p k_C^2 \sum_{j=1}^{N} m_{ij} \left(f\left(\mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i}^+\right)\right\|^2\right), \left\|\dot{\hat{q}}_j^i(t)\right\|^2\right) (\Delta t)^2\right. \\ & \left. + \left[\boldsymbol{F}\left(\left(\hat{K} + K\right)^2 + \left(\hat{K}_d + K_d\right)^4\right) + \left(1 + \frac{1}{\boldsymbol{a}}\right)^2 \left(\left(\hat{K} + K\right)\hat{K}_d\right)^2\right] (\Delta t)^4\right)\Bigg] \\ & \leq (1+\boldsymbol{a})^2 C_{\mathrm{R}}\left(t_{i,k_i}^+\right) + R\left(t_{i,k_i}^+\right)(\Delta t)^2 + T(\Delta t)^4 \end{aligned} \tag{B.54}$$

with

$$\begin{aligned} R\left(t_{i,k_i}^+\right) = & \alpha_{\mathrm{M}} \left(1 + \frac{1}{\boldsymbol{a}}\right) \alpha_i \left(k_e \left(\hat{K} + K\right)^2 + k_p k_{\mathrm{M}} \left(\hat{K}_d + K_d\right)^2\right) \\ & + 2\alpha_{\mathrm{M}} k_p k_C^2 \sum_{j=1}^{N} m_{ij} f\left(\mathbb{E}\left(\left\|e_i^j\left(t_{i,k_i}^+\right)\right\|^2\right), \left\|\dot{\hat{q}}_j^i\left(t_{i,k_i}^+\right)\right\|^2\right) \end{aligned} \tag{B.55}$$

$$Q = \boldsymbol{F}\left(\left(\hat{K} + K\right)^2 + \left(\hat{K}_d + K_d\right)^4\right) + \left(1 + \frac{1}{\boldsymbol{a}}\right)^2 \left(\left(\hat{K} + K\right)\hat{K}_d\right)^2 \tag{B.56}$$

where $R\left(t_{i,k_i}^+\right)$ is bounded such $R\left(t_{i,k_i}^+\right) \leq R_{\max}$ with

$$R_{\max} = \alpha_{\mathrm{M}} \left(1 + \frac{1}{\boldsymbol{a}}\right) \alpha_i \left(k_e \left(\hat{K} + K\right)^2 + k_p k_{\mathrm{M}} \left(\hat{K}_d + K_d\right)^2\right) + 2\alpha_{\mathrm{M}} k_p k_C^2 \sum_{j=1}^{N} m_{ij} f_{\max}. \tag{B.57}$$

At $t = t^+_{i,k_i} + \Delta t$, the CTC is not triggering if

$$C_{\mathrm{R}}\left(t^+_{i,k_i} + \Delta t\right) < C_{\mathrm{L}}\left(t^+_{i,k_i} + \Delta t\right) \tag{B.58}$$

where (B.58) is satisfied if

$$\begin{aligned}
(1+\boldsymbol{a})^2 C_{\mathrm{R}}\left(t^+_{i,k_i}\right) + R_{\max}(\Delta t)^2 + Q(\Delta t)^4 &< (1-\boldsymbol{b}) C_{\mathrm{L}}\left(t^+_{i,k_i}\right) + \left(1-\frac{1}{\boldsymbol{b}}\right)(\Delta t)^2 K_1 + \boldsymbol{b}\eta \\
-\boldsymbol{b}\eta + \left(R_{\max} - \left(1-\frac{1}{\boldsymbol{b}}\right)K_1\right)(\Delta t)^2 + Q(\Delta t)^4 &< (1-\boldsymbol{b}) C_{\mathrm{L}}\left(t^+_{i,k_i}\right) - (1+\boldsymbol{a})^2 C_{\mathrm{R}}\left(t^+_{i,k_i}\right) \\
-\boldsymbol{b}\eta + \left(R_{\max} - \left(1-\frac{1}{\boldsymbol{b}}\right)K_1\right)(\Delta t)^2 + Q(\Delta t)^4 &< (1+\boldsymbol{a})^2 \left(\frac{(1-\boldsymbol{b})}{(1+\boldsymbol{a})^2} C_{\mathrm{L}}\left(t^+_{i,k_i}\right) - C_{\mathrm{R}}\left(t^+_{i,k_i}\right)\right)
\end{aligned} \tag{B.59}$$

From (B.4), one has $C_{\mathrm{R}}\left(t^+_{i,k_i}\right) = \pi C_{\mathrm{L}}\left(t^+_{i,k_i}\right) + (1-\pi) k_g b_i \left\|\dot{q}_i\left(t^+_{i,k_i}\right) - \dot{q}_i^*\left(t^+_{i,k_i}\right)\right\|^2$. We choose $\boldsymbol{a}$ and $\boldsymbol{b}$ such $\frac{(1-\boldsymbol{b})}{(1+\boldsymbol{a})^2} = \pi$

$$\begin{aligned}
\frac{(1-\boldsymbol{b})}{(1+\boldsymbol{a})^2} &= \pi \\
(1-\boldsymbol{b}) &= \pi(1+\boldsymbol{a})^2 \\
\boldsymbol{b} &= 1 - \pi(1+\boldsymbol{a})^2
\end{aligned} \tag{B.60}$$

and

$$\begin{aligned}
1 - \pi(1+\boldsymbol{a})^2 &> 0 \\
\sqrt{\frac{1}{\pi}} &> 1 + \boldsymbol{a} \\
\sqrt{\frac{1}{\pi}} - 1 &> \boldsymbol{a}
\end{aligned} \tag{B.61}$$

and since $\pi < 1$, $a > 0$. So (B.59) becomes

$$-\boldsymbol{b}\eta + \left(R_{\max} - \left(1-\frac{1}{\boldsymbol{b}}\right)K_1\right)(\Delta t)^2 + Q(\Delta t)^4 < -(1+\boldsymbol{a})^2(1-\pi) k_g b_i \left\|\dot{q}_i\left(t^+_{i,k_i}\right) - \dot{q}_i^*\left(t^+_{i,k_i}\right)\right\|^2 \tag{B.62}$$

and so

$$\left((1+\boldsymbol{a})^2(1-\pi) k_g b_i \left\|\dot{q}_i\left(t^+_{i,k_i}\right) - \dot{q}_i^*\left(t^+_{i,k_i}\right)\right\|^2 - \boldsymbol{b}\eta\right) + \left(R_{\max} - \left(1-\frac{1}{\boldsymbol{b}}\right)K_1\right)(\Delta t)^2 + Q(\Delta t)^4 < 0 \tag{B.63}$$

By solving (B.63) for Δ as unknown, one gets

$$\Delta t < \tau_{\min} \tag{B.64}$$

with

$$\tau_{\min} = \sqrt{\frac{\Psi + \sqrt{\Psi^2 + 4Q\Pi}}{2T}} \tag{B.65}$$

$$\Psi = \left(1-\frac{1}{\boldsymbol{b}}\right)K_1 - R_{\max} \tag{B.66}$$

$$\Pi = \boldsymbol{b}\eta - (1+\boldsymbol{a})^2(1-\pi) k_g b_i \left\|\dot{q}_i\left(t^+_{i,k_i}\right) - \dot{q}_i^*\left(t^+_{i,k_i}\right)\right\|^2$$

where $\Pi > 0$ since (47) and $\boldsymbol{a}$ chosen such $\sqrt{\frac{1}{\pi}} - 1 > \boldsymbol{a} > 0$. Remark Π is also bounded as

$$\Pi \leq \boldsymbol{b}\eta - 4(1+\boldsymbol{a})^2(1-\pi) k_g b_i \dot{q}_{\max}^2. \tag{B.67}$$

Moreover, since $Q > 0$, $\Psi + \sqrt{\Psi^2 + 4\Pi Q} > 0$ for all values of Ψ, so $\tau_{\min}$ exists and is positive.

From (B.64), we deduce than for $t > t_{i,k_i}$,the CTC is not triggering since $t < t_{i,k_i} + \Delta t$. Thus, we can conclude that

$$t_{i,k_i+1} - t_{i,k_i} \geq \tau_{\min} \tag{B.68}$$

B.4 Calculation of coefficient $\boldsymbol{c}, \boldsymbol{d}, \boldsymbol{f}$

We want to determine potential values of $\boldsymbol{c}$, $\boldsymbol{d}$ and $\boldsymbol{f}$ satisfying $1+3\boldsymbol{c}+2\boldsymbol{d}+\left(\boldsymbol{d}+\frac{1}{\boldsymbol{d}}\right)\boldsymbol{f}=(1+\boldsymbol{a})$. Thus

$$\begin{aligned}
1+3\boldsymbol{c}+2\boldsymbol{d}+\left(\boldsymbol{d}+\frac{1}{\boldsymbol{d}}\right)\boldsymbol{f} &= (1+\boldsymbol{a}) \\
3\boldsymbol{c}\boldsymbol{d}+2\boldsymbol{d}^2+\left(\boldsymbol{d}^2+1\right)\boldsymbol{f} &= \boldsymbol{d}\boldsymbol{a} \\
\boldsymbol{d}^2\left(\boldsymbol{f}+2\right)+\boldsymbol{d}\left(3\boldsymbol{c}-\boldsymbol{a}\right)+\boldsymbol{f} &= 0
\end{aligned} \tag{B.69}$$

We solve(B.69) using $\boldsymbol{d}$ as the unknown by calculating the discriminant

$$\Delta_d = \left(3\boldsymbol{c}-\boldsymbol{a}\right)^2 - 4\boldsymbol{f}\left(\boldsymbol{f}+2\right) \tag{B.70}$$

and choose the value to have $\Delta_f = 0$

$$\begin{aligned}
\left(3\boldsymbol{c}-\boldsymbol{a}\right)^2 - 4\boldsymbol{f}\left(\boldsymbol{f}+2\right) &= 0 \\
\left(3\boldsymbol{c}-\boldsymbol{a}\right)^2 &= 4\boldsymbol{f}\left(\boldsymbol{f}+2\right)
\end{aligned}$$

and to find the smallest $\boldsymbol{c}$, we choose

$$\begin{aligned}
a-3\boldsymbol{c} &= 2\sqrt{\boldsymbol{f}\left(\boldsymbol{f}+2\right)} \\
\boldsymbol{c} &= \frac{\boldsymbol{a}-2\sqrt{\boldsymbol{f}\left(\boldsymbol{f}+2\right)}}{3}
\end{aligned} \tag{B.71}$$

so

$$\begin{aligned}
\boldsymbol{d} &= \frac{-\left(3\boldsymbol{c}-\boldsymbol{a}\right)}{2\left(\boldsymbol{f}+2\right)} \\
&= \frac{-\left(\boldsymbol{a}-2\sqrt{\boldsymbol{f}\left(\boldsymbol{f}+2\right)}-\boldsymbol{a}\right)}{2\left(\boldsymbol{f}+2\right)} \\
&= \sqrt{\frac{\boldsymbol{f}}{\boldsymbol{f}+2}}
\end{aligned} \tag{B.72}$$

and since $\boldsymbol{c} > 0$

$$\begin{aligned}
\boldsymbol{c} &> 0 \\
\frac{\boldsymbol{a}-2\sqrt{\boldsymbol{f}\left(\boldsymbol{f}+2\right)}}{3} &> 0 \\
\boldsymbol{a} &> 2\sqrt{\boldsymbol{f}\left(\boldsymbol{f}+2\right)} \\
\frac{\boldsymbol{a}^2}{4} &> \boldsymbol{f}^2+2\boldsymbol{f} \\
0 &> \boldsymbol{f}^2+2\boldsymbol{f}-\frac{\boldsymbol{a}^2}{4}
\end{aligned} \tag{B.73}$$

we solve it and obtain

$$\boldsymbol{f} > \frac{-2+\sqrt{4+\boldsymbol{a}^2}}{2} \tag{B.74}$$

and we choose so

$$\boldsymbol{f} = 1+\frac{-2+\sqrt{4+\boldsymbol{a}^2}}{2} \tag{B.75}$$

$$\boldsymbol{d} = \sqrt{\frac{\boldsymbol{f}}{\boldsymbol{f}+2}} \tag{B.76}$$

$$\boldsymbol{c} = \frac{\boldsymbol{a}-2\sqrt{\boldsymbol{f}\left(\boldsymbol{f}+2\right)}}{3} \tag{B.77}$$

B.5 Bound on $\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right)$

The CTC imposes that

$$\alpha_{\mathrm{M}} k_e \mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right) \leq k_s \bar{s}_i^T(t)\,\bar{s}_i(t) + k_p k_g \bar{g}_i^T(t)\,\bar{g}_i(t) + \eta. \tag{B.78}$$

Since $\bar{s}_i(t)$ and $\bar{g}_i(t)$ are perfectly known by Agent i, one may write

$$\sum_{i=1}^{N} \alpha_{\mathrm{M}} k_e \mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right) \leq \sum_{i=1}^{N} \mathbb{E}\left(k_s \bar{s}_i^T(t)\,\bar{s}_i(t) + k_p k_g \bar{g}_i^T(t)\,\bar{g}_i(t) + \eta\right). \tag{B.79}$$

Using the Young relation $xy \leq \frac{1}{2\boldsymbol{h}}x^2 + \frac{\boldsymbol{h}}{2}y^2$ with $\boldsymbol{h} > 0$, one gets

$$\begin{aligned}
\|\bar{s}_i\|^2 &= \left\|s_i - k_p E^i\right\|^2 \\
&= \|s_i\|^2 + \left\|k_p E^i\right\|^2 - 2k_p E^{iT}\bar{s}_i \\
&\leq \left(1 + \frac{1}{\boldsymbol{h}}\right)\|s_i\|^2 + (1+\boldsymbol{h})\,k_p^2\left\|E^i\right\|^2.
\end{aligned} \tag{B.80}$$

In the same way, one gets

$$\|\bar{g}_i\|^2 \leq \left(1 + \frac{1}{\boldsymbol{f}}\right)\|g_i\|^2 + (1+\boldsymbol{f})\left\|E^i\right\|^2 \tag{B.81}$$

with $\boldsymbol{f} > 0$. Then (B.79) becomes

$$\begin{aligned}
\sum_{i=1}^{N} \alpha_{\mathrm{M}} k_e \mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right) &\leq \sum_{i=1}^{N} k_s \mathbb{E}\left(\left(1 + \frac{1}{\boldsymbol{h}}\right)\|s_i(t)\|^2 + (1+\boldsymbol{h})\,k_p^2\left\|E^i\right\|^2\right) \\
&+ \sum_{i=1}^{N} k_p k_g \mathbb{E}\left(\left(1 + \frac{1}{\boldsymbol{f}}\right)\|g_i(t)\|^2 + (1+\boldsymbol{f})\left\|E^i\right\|^2\right) + N\eta \\
&\leq \sum_{i=1}^{N} \mathbb{E}\left(k_s\left(1 + \frac{1}{\boldsymbol{h}}\right)\|s_i(t)\|^2 + k_p k_g\left(1 + \frac{1}{\boldsymbol{f}}\right)\|g_i(t)\|^2\right) + N\eta \\
&\left((1+\boldsymbol{h})\,k_p^2 k_s + k_p k_g\,(1+\boldsymbol{f})\right)\sum_{i=1}^{N}\mathbb{E}\left(\left\|E^i\right\|^2\right).
\end{aligned} \tag{B.82}$$

In (B.82),

$$\begin{aligned}
\sum_{i=1}^{N}\mathbb{E}\left(\left\|E^i\right\|^2\right) &= \sum_{i=1}^{N}\mathbb{E}\left(\left\|\sum_{j=1}^{N} m_{ij} e_j^i(t)\right\|^2\right) \\
&\leq \mathbb{E}\left(\sum_{i=1}^{N}\sum_{j=1}^{N} m_{ij}\left\|e_j^i(t)\right\|^2\right) \\
&\leq \mathbb{E}\left(\sum_{j=1}^{N}\sum_{i=1}^{N} m_{ji}\left\|e_i^j(t)\right\|^2\right) \\
&\leq \sum_{i=1}^{N}\alpha_{\mathrm{M}}\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right).
\end{aligned} \tag{B.83}$$

Thus

$$\sum_{i=1}^{N}\alpha_{\mathrm{M}}k_e\mathbb{E}\left(\left\|e_i^j\left(t\right)\right\|^2\right)\leq\sum_{i=1}^{N}\mathbb{E}\left(k_s\left(1+\frac{1}{\boldsymbol{h}}\right)\left\|s_i\left(t\right)\right\|^2+k_pk_g\left(1+\frac{1}{\boldsymbol{f}}\right)\left\|g_i\left(t\right)\right\|^2\right)+N\eta$$
$$+\left(\left(1+\boldsymbol{h}\right)k_p^2k_s+k_pk_g\left(1+\boldsymbol{f}\right)\right)\sum_{i=1}^{N}\alpha_{\mathrm{M}}\mathbb{E}\left(\left\|e_i^j\left(t\right)\right\|^2\right)$$
$$\sum_{i=1}^{N}\alpha_{\mathrm{M}}K_A\mathbb{E}\left(\left\|e_i^j\left(t\right)\right\|^2\right)\leq\sum_{i=1}^{N}\mathbb{E}\left(k_s\left(1+\frac{1}{\boldsymbol{h}}\right)\left\|s_i\left(t\right)\right\|^2+k_pk_g\left(1+\frac{1}{\boldsymbol{f}}\right)\left\|g_i\left(t\right)\right\|^2\right)+N\eta \tag{B.84}$$

where $K_A=\left(k_e-\left(\left(1+\boldsymbol{h}\right)k_p^2k_s+k_pk_g\left(1+\boldsymbol{f}\right)\right)\right)$. We desire take $\boldsymbol{f}$ and $\boldsymbol{h}$ such that $K_A>0$. Remind $k_e=k_sk_p^2+k_gk_p+\frac{k_g}{b_i}$ and $b_i<\frac{k_s}{k_sk_p+k_g}$ so $k_e>k_sk_p^2+2k_gk_p+\frac{k_g^2}{k_s}$. Thus

$$K_A\geq k_sk_p^2+2k_gk_p+\frac{k_g^2}{k_s}-\left(\left(1+\boldsymbol{h}\right)k_p^2k_s+k_gk_p\left(1+\boldsymbol{f}\right)\right)$$
$$K_A\geq\frac{k_g^2}{k_s}-\left(\boldsymbol{h}k_p^2k_s+k_gk_p\left(\boldsymbol{f}-1\right)\right) \tag{B.85}$$

Thus, by taking $\boldsymbol{f}<1$ and $\boldsymbol{h}<\frac{k_g^2}{k_p^2k_s^2}$, the right-hand side of (B.85) is strictly positive and so $K_A>0$.

Go back to (B.84). Let define $\underline{\lambda}_M x^Tx\leq x^TM_ix$. Using $\underline{\lambda}_M$ and (B.84), one may writte

$$\sum_{i=1}^{N}\alpha_{\mathrm{M}}K_A\mathbb{E}\left(\left\|e_i^j\left(t\right)\right\|^2\right)\leq\mathbb{E}\left(\sum_{i=1}^{N}\left(s_i^T\left(t\right)M_is_i\left(t\right)\frac{\left(1+\frac{1}{\boldsymbol{h}}\right)k_s}{\underline{\lambda}_M}+k_pk_g\left(1+\frac{1}{\boldsymbol{f}}\right)\left\|g_i\left(t\right)\right\|^2\right)\right)+N\eta \tag{B.86}$$

Let's study now $\left\|g_i\right\|^2$. Using (A.59), one has

$$\sum_{i=1}^{N}g_i^Tg_i=\sum_{i=1}^{N}\left[\frac{1}{2}\sum_{\ell=1}^{N}\sum_{j=1}^{N}m_{i\ell}m_{ij}\left[\left\|r_{ij}-r_{ij}^*\right\|^2+\left\|r_{i\ell}-r_{i\ell}^*\right\|^2-\left\|r_{\ell j}-r_{\ell j}^*\right\|^2\right]\right] \tag{B.87}$$

Since $m_{\min}=\min\limits_{i,j=1\ldots N}\left(m_{ij}\neq 0\right)$,

$$m_{\min}\sum_{i=1}^{N}g_i^Tg_i\leq\sum_{i=1}^{N}\left[\frac{1}{2}\sum_{\ell=1}^{N}\sum_{j=1}^{N}m_{i\ell}m_{ij}m_{\ell j}\left[\left\|r_{ij}-r_{ij}^*\right\|^2+\left\|r_{i\ell}-r_{i\ell}^*\right\|^2-\left\|r_{\ell j}-r_{\ell j}^*\right\|^2\right]\right]$$
$$m_{\min}\sum_{i=1}^{N}g_i^Tg_i\leq\frac{1}{2}\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N}m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^*\right\|^2+\frac{1}{2}\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N}m_{i\ell}m_{ij}m_{\ell j}\left\|r_{i\ell}-r_{i\ell}^*\right\|^2$$
$$-\frac{1}{2}\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N}m_{i\ell}m_{ij}m_{\ell j}\left\|r_{\ell j}-r_{\ell j}^*\right\|^2$$
$$m_{\min}\sum_{i=1}^{N}g_i^Tg_i\leq\frac{1}{2}\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N}m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^*\right\|^2+\frac{1}{2}\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N}m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^*\right\|^2$$
$$-\frac{1}{2}\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N}m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^*\right\|^2$$
$$m_{\min}\sum_{i=1}^{N}g_i^Tg_i\leq\frac{1}{2}\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N}m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^*\right\|^2.$$

Since $m_{i\ell}=0$ if $\ell\notin\mathcal{N}_i$, one gets

$$\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N}m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^*\right\|^2=\sum_{i=1}^{N}\sum_{\ell\in\mathcal{N}_i}m_{i\ell}\sum_{j=1}^{N}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^*\right\|^2 \tag{B.88}$$

As $m_{\max} = \max_{i,j=1\ldots N}(m_{ij})$ and $N_{\max} = \max_{i=1\ldots N}(N_i)$. One may write

$$\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^{*}\right\|^2 \leq \sum_{i=1}^{N}\sum_{\ell\in\mathcal{N}_i} m_{\max}\sum_{j=1}^{N} m_{ij}\left\|r_{ij}-r_{ij}^{*}\right\|^2.$$

$$\sum_{i=1}^{N}\sum_{\ell=1}^{N}\sum_{j=1}^{N} m_{i\ell}m_{ij}m_{\ell j}\left\|r_{ij}-r_{ij}^{*}\right\|^2 \leq N_{\max}m_{\max}\sum_{i=1}^{N}\sum_{j=1}^{N} m_{ij}\left\|r_{ij}-r_{ij}^{*}\right\|^2 \tag{B.89}$$

and so

$$\begin{aligned} m_{\min}\sum_{i=1}^{N} g_i^T g_i &\leq N_{\max}m_{\max}\sum_{i=1}^{N}\sum_{j=1}^{N} m_{ij}\left\|r_{ij}-r_{ij}^{*}\right\|^2 \\ \sum_{i=1}^{N} g_i^T g_i &\leq \frac{N_{\max}m_{\max}}{m_{\min}}P(q,t) \\ \frac{m_{\min}}{N_{\max}m_{\max}}\sum_{i=1}^{N} g_i^T g_i &\leq P(q,t) \end{aligned} \tag{B.90}$$

Using (B.90), (B.86) becomes

$$\begin{aligned} \sum_{i=1}^{N}\alpha_{\mathrm{M}}K_A\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right) &\leq \mathbb{E}\left(\sum_{i=1}^{N}\left(\frac{1}{2}s_i^T(t)M_i s_i(t)\left(\frac{2\left(1+\frac{1}{\boldsymbol{h}}\right)k_s}{\underline{\lambda}_M}\right)\right)+\left(4\frac{N_{\max}m_{\max}}{m_{\min}}k_p\left(1+\frac{1}{\boldsymbol{f}}\right)\right)\frac{k_g}{4}P(q,t)\right)+N\eta \\ &\leq K_B\mathbb{E}\left(\sum_{i=1}^{N}\left(\frac{1}{2}s_i^T(t)M_i s_i(t)\right)+\frac{k_g}{4}P(q,t)\right)+N\eta \end{aligned}$$

where $K_B = \max\left(\frac{2\left(1+\frac{1}{\boldsymbol{h}}\right)k_s}{\underline{\lambda}_M}, 4\frac{N_{\max}m_{\max}}{m_{\min}}k_p\left(1+\frac{1}{\boldsymbol{f}}\right)\right)$. From (A.19), one has $V(t) = \mathbb{E}\left(\sum_{i=1}^{N}\left[\frac{1}{2}s_i^T(t)M_i s_i(t)\right]+\frac{k_g}{4}P(q,t)\right)$. Thus

$$\sum_{i=1}^{N}\alpha_{\mathrm{M}}K_A\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right) \leq K_B V(t)+N\eta \tag{B.91}$$

$$\alpha_{\mathrm{M}}K_A\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right) \leq K_B V(t)+N\eta. \tag{B.92}$$

From (A.55), one has $V(t) \leq V(0)+\frac{N}{2}\left[D_{\max}^2+\eta\right]$. So

$$\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right) \leq \frac{K_B\left[V(0)+\frac{N}{2}\left[D_{\max}^2+\eta\right]\right]+N\eta}{\alpha_{\mathrm{M}}K_A}. \tag{B.93}$$

So $\mathbb{E}\left(\left\|e_i^j(t)\right\|^2\right)$ is bound using constant parameters, independently of the instant t.