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# Geometric tomography for measuring rectangular radiotherapy fields from six projections 

Laurent Desbat, Simon Rit, Rolf Clackdoyle, Patrice Jalade, Julien Ribouton and Patrick Pittet


#### Abstract

Quality assurance (QA) of x-ray radiotherapy is crucial but the dosimetry of small fields is still a challenge. We are investigating the use of scintillating parallel fibers which measure the integral dose along a line, thus providing a onedimensional (1D) parallel projection of the dose of the x-ray field in a plane of a tissue equivalent medium. By stacking a few layers of scintillating fibers, each with a different orientation, one obtains a sinogram of the dose of the $x$-ray field sampled with a few angles and sub-millimetric resolution of each projection. We propose a geometric approach based on projections moments to identify the few parameters defining the irradiation field (e.g., center, radius, orientation, width, length and intensity for a rectangle). The order- 0 moment of at least one projection provides an estimate of the integral dose. The order- 1 moments of at least two projections gives the center of mass, i.e., order- 1 moments of the dose map. The order- 2 moments of at least three projections identify order2 moments of the dose map. In the specific case of a uniform rectangular field, we show that the length, the width and the orientation of the rectangle can be derived from the order-2 moments of the dose map. The formulas are extended to the case of a non-uniform field due to a lateral Gaussian penumbra. Our approach is validated on simulated data of rectangular fields with increasing levels of noise and it demonstrates an accurate, robust and rapid estimation of the rectangle parameters from six projections only. A proof of concept experiment with scintillating fibers imaged with a sCMOS camera irradiated under a $30 \times 30 \mathbf{m m}^{2}$ square field of a 6 MV linear accelerator gave, from six projections, $31.7 \times 31.4 \mathbf{~ m m}^{2}$ without accounting for the penumbra and $28.6 \times 28.3 \mathrm{~mm}^{2}$ with an estimated $\sigma=4 \mathrm{~mm}$ penumbra.


## I. Introduction

QUALITY assurance (QA) of x-ray radiotherapy is crucial but the dosimetry of small fields is still a challenge. Reference measurements are currently obtained with passive dosimeters, e.g., gafchromic films [1]. These techniques require long processing time and cannot be used immediately before a treatment fraction, which would be preferable, e.g., for stereotactic treatments with a few high dose fractions. There is currently no obvious alternative device which could rapidly measure the dose, particularly when the linear accelerator is combined with a magnetic resonance imager (MRI-linac).

We are investigating the use of scintillating fibers which measure the integral dose along a line, thus providing a onedimensional (1D) parallel projection of the dose of the x-ray
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field in a plane of a tissue equivalent medium, see Fig. 1.Unlike solid state detectors, these fibers are tissue equivalent. By stacking a few layers of scintillating fibers, each with a different orientation, one obtains a sinogram of the dose of the x-ray field which can then be reconstructed. A similar concept has been developed previously in [2] with a rotation sytem. Without rotation, hardware constraints will limit the sinograms to a few projection angles with sub-millimetric resolution of each projection.

Convex bodies and polygons may be identified from a few xray projections with geometric tomography [3]. Our approach is designed for the QA of simple radiotherapy fields. We propose a moment-based method to identify the few parameters defining the irradiation field (e.g., center, orientation, width, length and intensity for a rectangle) and then we verify in the projections that the estimated parameters are consistent with all the collected projection data.

## II. Theory

We assume that the scintillating fibers yield measurements of the parallel projection $p$, see Fig. 2, equivalently the Radon transform $\mathcal{R} f$, of $f$, see [4],

$$
\begin{equation*}
p_{\phi}(s):=p(\phi, s):=\mathcal{R} f(\phi, s):=\int_{\mathbb{R}} f\left(s \vec{\theta}_{\phi}+l \vec{\zeta}_{\phi}\right) \mathrm{d} l \tag{1}
\end{equation*}
$$

where $\phi \in[0, \pi), s \in \mathbb{R}, \vec{\theta}_{\phi}=(\cos \phi, \sin \phi), \vec{\zeta}_{\phi}=$ $(-\sin \phi, \cos \phi)$, for a limited number of angles $\phi_{n}, n=$ $1, \ldots, n_{\phi}$ and a relatively large number of samples of $s_{n}, n=$ $1, \ldots, n_{s}$.

## A. Moments of $f$ from moments of its projections $p$

Let $\Pi_{n}(\phi)$ be the moment of order $n \in \mathbb{N}$ of the projection $p_{\phi}$,

$$
\begin{equation*}
\Pi_{n}(\phi)=\int_{\mathbb{R}} s^{n} p_{\phi}(s) \mathrm{d} s \tag{2}
\end{equation*}
$$

The order- 0 moment $\Pi_{0}(\phi)$ is the mass of $f$ (independenly of $\phi$ ) or, in our context, the integral dose.
$\Pi_{0}(\phi)=\int_{\mathbb{R}} p_{\phi}(s) \mathrm{d} s=\int_{\mathbb{R}^{2}} f\left(s \vec{\theta}_{\phi}+l \vec{\zeta}_{\phi}\right) \mathrm{d} l \mathrm{~d} s=\int_{\mathbb{R}^{2}} f(\vec{x}) \mathrm{d} \vec{x}$.
It can be robustly estimated from all projections as

$$
\begin{equation*}
A_{0,0}=\frac{\sum_{m=1}^{n_{\phi}} \Pi_{0}\left(\phi_{m}\right)}{n_{\phi}} \tag{3}
\end{equation*}
$$

The order-1 moment can be used to estimate the center of mass $\vec{c} \in \mathbb{R}^{2}$ of $f$. One can show that

$$
\begin{equation*}
\vec{c} \cdot \vec{\theta}_{\phi}=\frac{\Pi_{1}(\phi)}{A_{0,0}} \tag{4}
\end{equation*}
$$



Fig. 1. Schematic view of the QA system based on scintillating wave-guides (integrating the dose).


Fig. 2. Radon transform (parallel geometry): $s \vec{\theta}_{\phi}+\mathbb{R} \vec{\zeta}_{\phi}$ is the (blue) line of integration.
i.e. that $\frac{\Pi_{1}(\phi)}{A_{0,0}}$ is the projection of $\vec{c}$ onto the axis of direction $\vec{\theta}_{\phi}$. If at least two projections are known at different angle directions $\phi_{1}, \phi_{2}$, the center of mass $\vec{c}$ of $f$ can be estimated by solving the linear system

$$
\left\{\begin{array}{l}
\vec{c} \cdot \vec{\theta}_{\phi_{1}}=\frac{\Pi_{1}\left(\phi_{1}\right)}{A_{0,0}} \\
\vec{c} \cdot \vec{\theta}_{\phi_{2}}=\frac{\Pi_{1}\left(\phi_{2}\right)}{A_{0,0}} .
\end{array}\right.
$$

If more than two projection angles are measured, $\vec{c}$ can be estimated more robustly in the least squares sense.

The order- 2 moment, shifted using $\vec{c}$, can then be used to calculate order- 2 moments of $f$

$$
\begin{align*}
\int_{\mathbb{R}}(s- & \left.\vec{c} \cdot \vec{\theta}_{\phi}\right)^{2} p_{\phi}(s) \mathrm{d} s \\
& =\cos ^{2} \phi A_{2,0}+2 \cos \phi \sin \phi A_{1,1}+\sin ^{2} \phi A_{0,2} \tag{5}
\end{align*}
$$

where

$$
A_{i, j}:=\int_{\mathbb{R}^{2}} x_{1}^{i} x_{2}^{j} f(\vec{x}+\vec{c}) \mathrm{d} \vec{x}
$$

If we can compute $\int_{\mathbb{R}}\left(s-c_{\phi}\right)^{2} p_{\phi}(s) \mathrm{d} s$ for three different projection angles, Eq. (5) yields a $3 \times 3$ linear system with a unique solution for $A_{2,0}, A_{1,1}$ and $A_{0,2}$. If more projections are measured, a least squares estimation can be used.

## B. Estimation of a rectangle field

The previous section provides robust estimators of moments of the sought $f$. We now examine how to estimate a rectangular field from these moments, assuming first no penumbra of the x-ray field. The unknowns are its center of mass $\vec{c}$, its length $l$, its width $w$, its orientation $\alpha$ and the dose $d$ in every point of the rectangle. Their estimation uses the (symmetric) covariance matrix

$$
\mathbf{M}=\frac{1}{A_{0,0}}\left[\begin{array}{ll}
A_{2,0} & A_{1,1} \\
A_{1,1} & A_{0,2}
\end{array}\right]
$$

which can be diagonalized in an orthonormal basis $\vec{v}_{1}, \vec{v}_{2}$ such that $\mathbf{M} \vec{v}_{1}=\lambda_{1} \vec{v}_{1}$ and $\mathbf{M} \vec{v}_{2}=\lambda_{2} \vec{v}_{2}$ with $\lambda_{1} \geq \lambda_{2}>0$. It can then be shown that $l=\sqrt{12 \lambda_{1}}, w=\sqrt{12 \lambda_{2}}, \alpha$ is the angle between the vector $(1,0)$ and $\vec{v}_{1}$ and $d=A_{0,0} / h l$.

Megavoltage x-ray rectangular fields are blurred by the transport of electrons and it is common to model this penumbra by a Gaussian convolution (see, e.g., the margin recipe in [5]). If the penumbra is characterized by a standard deviation $\sigma$, then the length and the width are $l=\sqrt{12\left(\lambda_{1}-\sigma^{2}\right)}$, $w=\sqrt{12\left(\lambda_{2}-\sigma^{2}\right)}$. Calculation of the other parameters is the same.

## III. EXPERIMENTS

## A. Simulations

Following the design of an initial prototype, we simulated 6 views equally distributed over $\pi, 128$ pixels per view and 0.4 mm pixel spacing. Projections of 1000 random rectangles were simulated and the parameters were automatically estimated. The sampling was uniformly distributed in the interval $[1,10]$ for the dose level $d,[-5,5] \mathrm{mm}$ for the two coordinates of the center of mass $\vec{c}$, $[15,20] \mathrm{mm}$ for the length $l,[5,15] \mathrm{mm}$ for the width $w$ and $[-90,90]$ degrees for the orientation $\alpha$. The experiment was reproduced with several levels of multiplicative random Gaussian noise in the sinogram.

## B. Real data

Preliminary data were measured using a scintillating fiber detector ( SciFi ) designed for the LHCb experiment [6]. The detector was irradiated with a 6 MV linear accelerator using a $30 \times 30 \mathrm{~mm}^{2}$ field. The scintillating signal at the detector's output was captured using a sCMOS camera achieving a $22 \mu \mathrm{~m}$ effective pixel resolution. The signal across the detector was averaged in the beam direction to yield a 1D projection. Several orientations were recorded by rotating the head of the linear accelerator in 30 or 15 degree steps. The estimation of the field was first realized assuming no penumbra. In a second step, we estimated the penumbra by using the $\sigma$ value of a Gaussian convolution that yielded the best fit of the projections in the least squares sense with Nelder-Mead downhill simplex descent. Only the length and the width are reported since the position and the pixel values of the camera were not calibrated in this proof-of-concept.

## IV. Results

Table I demonstrates sub-millimetric estimation of the rectangle parameters even in the presence of noise.

TABLE I
MEAN (STANDARD DEVIATION) OF THE SIGNED DIFFERENCE BETWEEN THE ESTIMATED PARAMETERS OF THE RECTANGLES AND THEIR REFERENCE VALUES FOR THE 1000 RANDOM RECTANGLES.

| Noise | Dose <br> $($ a.u. $)$ | Center of <br> mass $(\mathrm{mm})$ | Length <br> $(\mathrm{mm})$ | Width <br> $(\mathrm{mm})$ | Angle <br> $($ degrees $)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $0 \%$ | $0.00(0.02)$ | $-0.00(0.01)$ | $-0.00(0.03)$ | $0.00(0.04)$ | $-0.00(0.02)$ |
| $5 \%$ | $0.00(0.05)$ | $0.00(0.02)$ | $0.00(0.06)$ | $-0.00(0.07)$ | $0.01(0.25)$ |
| $10 \%$ | $0.00(0.10)$ | $0.00(0.04)$ | $0.00(0.11)$ | $-0.00(0.12)$ | $-0.00(0.50)$ |
| $20 \%$ | $0.01(0.19)$ | $0.00(0.08)$ | $-0.01(0.20)$ | $0.00(0.22)$ | $0.01(0.98)$ |
| $50 \%$ | $0.07(0.56)$ | $-0.00(0.19)$ | $-0.00(0.48)$ | $-0.04(0.55)$ | $-0.11(2.41)$ |

Fig. 3 shows the 6 raw projections ( 30 degree step) before processing and after a 0.7 mm smoothing. They were used to estimate the parameters of the rectangle. The estimation gave $\mathrm{l}=31.7 \mathrm{~mm}$ and $\mathrm{w}=31.4 \mathrm{~mm}$ without penumbra and $\mathrm{l}=28.6 \mathrm{~mm}$ and $\mathrm{w}=28.3 \mathrm{~mm}$ with the estimated $\sigma=4 \mathrm{~mm}$ penumbra. Using 12 projections ( 15 degree step), we improved the estimation with $\mathrm{l}=29.6 \mathrm{~mm}, \mathrm{w}=29.2 \mathrm{~mm}$ and $\sigma=3.4 \mathrm{~mm}$, see Fig. 4 .

## V. DISCUSSION AND CONCLUSIONS

We have proposed a procedure for robust and accurate estimation of the parameters of a rectangular field from at least three projections. This approach is also much faster than iterative reconstruction algorithms used in previous similar works [2]. Other shapes depending on a very few parameters could also be considered, e.g., disks, but the moment-based approach is probably not suited to more complex field shapes such as the ones obtained with multileaf collimators [7].

A proof-of-concept experiment demonstrated the accuracy of the method on real data. Improving the pre-processing of the measurements with flat field and dark field corrections and pre-characterizing the penumbra should also improve the quantification. For QA, the final result should be validated by comparing the estimated sinogram with the measurements as in Fig. 3 or in Fig. 4.

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Fig. 3. Six measured and estimated profiles of a $30 \times 30 \mathrm{~mm}^{2}$ square field on a clinical linear accelerator. Top: smoothed data. Middle: projection profiles of the estimated square without a penumbra model. Down: projection profiles of the estimated square with a penumbra model



Estimations with penumbra


Fig. 4. Twelve measured and estimated profiles of a $30 \times 30 \mathrm{~mm}^{2}$ square field on a clinical linear accelerator. Top: smoothed data. Middle: projection profiles of the estimated square without a penumbra model. Down: projection profiles of the estimated square with a penumbra model

