Statistical modelling of ocean SAR images

Yves Delignon, René Garello, Alain Hillion

Statistical modelling of ocean SAR images

Y. Delignon
R. Garello
A. Hillion

Indexing terms: Modelling, Statistical analysis, Imaging, Backscattering, Synthetic-aperture radar

Abstract: The paper considers the statistical modelling of fully developed backscattering in the case of SAR images of the ocean surface. According to the random-walk theory, the SAR image grey level is modelled as the product of a speckle noise and a variable which is dependent on the reflectivity of the illuminated surface and the radar-point-spread function. The purpose of the study is the statistical modelling of the latter variable. As nothing is known about these statistics, the authors propose the use of an estimation method based on a system of distributions. The set contains known density-probability functions with very flexible shapes that are supposed to fit its distribution. The associated image intensity distributions are processed and form a new system called KUBW, referring to the special functions used to generate the distributions. The classical K law belongs to the new system of distributions. By using a statistical test on the intensity distribution, the authors assess the relevance of the system of distributions in comparison with the classical model. The paper concludes with a discussion of the merits of the method and its extension to the case of ocean SAR image applications.

1 Introduction

Synthetic aperture radar images are known to be affected by speckle noise effect because of the coherent technique used in their generation. This noise effect limits the performance of airborne sensors. The statistical formulation is therefore of interest for designing detection, segmentation, filtering and pattern recognition algorithms and for optimum radar performance analysis. These analyses need, in order to be performed, a statistical image model that is usually obtained by two approaches.

The first consists of a parametric estimation of the SAR image-intensity distribution. Among the known distributions are the lognormal [1], Weibull [2], gamma [3, 4] and more recently the beta of the first and second kind [5, 6] distributions. The lognormal or Weibull distributions sometimes fit correctly with the SAR image histograms. In a previous work [6, 7], we have proposed modelling the grey level probability density function using the Pearson system of distributions. By using the estimation of higher moments, we have shown that the selected distribution in the Pearson system (beta of the first kind and beta of the second kind distributions) fits the histograms [6, 7] better than the usual parametric distributions. Although relevant, these functions are not derived from a physical model or clutter scattering mechanism.

The second approach to radar image-intensity distribution consists of considering the physical mechanism responsible for the backscattering from the surface. The mathematical foundations for this approach were first established in relation to the random-walk problem. The main result of this theory models the speckle texture by a multiplying noise following the exponential distribution [8, 9] for the images in intensity.

The gamma law of observation without noise is most commonly used and corresponds to the K law for the intensity [10–15]. Indeed, this distribution was used for modelling the statistical properties of radiation scattered by land [13], sea [11] and extended area [14] in the case of radar images, and also for the optical scattering of the random phase [15].

This paper introduces and studies a new set of parametric distributions for modelling ocean radar images.

The statistical method developed is based on the system of distributions. It comprises collecting in a system many well known distributions of varied shapes, and selecting, for a given sample, the parametric distribution which fits its histogram. This statistical method presents two main advantages : it avoids the subjective choice of a parametric law and provides a more reliable estimation than the classical one.

The implications of this programme are extensive because we will be able to propose more efficient segmentation, filtering and detection algorithms.

2 Compound distribution model

The compound distribution model for sea clutter was introduced by Ward [10]. This model represents the clutter as a noise-like speckle signal modulated by a function, which is derived from the radar reflectivity of the resolution cell and the radar point-spread function. In the case of SAR imagery, the point spread function depends on motion within the resolution cell and includes the effects of this motion, such as the well

known 'velocity bunching' phenomenon for ocean waves. For single-look radar data, the speckle signal may be described by an exponential distribution. For multi-look radar data, this becomes a χ-squared distribution, which is a special case of the gamma distribution.

The distribution of the intensity in the image is the compound distribution which is obtained by

$$f_I(x) = \int_0^{+\infty} f_{I/S=s}(x) f_S(s) ds \qquad (1)$$

with

$$f_{I/S=s}(x) = \frac{N^N}{s^N \Gamma(N)} x^{N-1} \exp\left(-\frac{Nx}{s}\right)$$
$$x \geq 0 \qquad (2)$$

where N is the number of looks of the image, s depends on the reflectivity and the radar point-spread function, and $\Gamma(.)$ is the gamma function.

As S is not observable, Wackerman [16] has proposed a nonparametric method for estimating its distribution. We propose a set of distributions which contains U, J, and bell-shaped probability density functions, and a large variety of unimodal distributions such as gamma, beta of the first kind, beta of the second kind, inverse gamma distributions, etc. The proposed system is a subset of the Pearson system [6, 7]. All the parametric distributions are linked by the following differential equation :

$$\frac{df_S(s)}{f_S(s)} = -\frac{s+a}{s(c_1 + c_2 s)} ds \qquad (3)$$

with a, c_1, c_2 parameters of the distributions.

By solving this differential equation (eqn. 3), we obtain the Pearson distribution of types I, III, V and VI.

2.1 Pearson distribution of type I or beta law of the first kind

$$f_S(s) = \frac{1}{\beta^{p+q-1} B(p,q)} s^{p-1} (\beta - s)^{q-1}, \quad s \in [0, \beta] \qquad (4)$$

where p and q are the shape parameters

$$p = -\frac{a}{c_1} + 1, \quad q = \frac{a}{c_1} - \frac{1}{c_2} + 1 \quad \text{so that } p \geq 0,\ q \geq 0$$

β is the scaling parameter such that

$$\beta = -\frac{c_1}{c_2}, \quad \beta \geq 0$$

and $B(.,.)$ is the beta function:

$$B(p,q) = \int_0^1 x^{p-1} (1-x)^{q-1} dx$$

In the Pearson system, the beta distribution of the first kind presents the largest variety of shapes among the Pearson distributions (Fig. 1). U-shape density for $p < 1$ and $q < 1$, J-shape density for $p < 1$ and $q > 1$, L-shape density for $p > 1$ and $q < 1$, and bell-shape density for $p > 1$ and $q > 1$.

The normal distribution is a limit form of the beta distribution of the second kind, when c_1 and c_2 tend to zero and the gamma distribution is the limit form of the beta distribution as c_2 tends to zero.

2.2 Pearson distribution of type III or gamma law

$$f_S(s) = \frac{\beta^\alpha s^{\alpha-1}}{\Gamma(\alpha)} e^{-\beta s}, \quad s \in [0, +\infty[\qquad (5)$$

where α is the shape parameter,

$$\alpha = -\frac{a}{c_1} + 1, \quad \alpha \geq 0$$

β is the scaling parameter,

$$\beta = \frac{1}{c_1}, \quad \beta \geq 0$$

and $\Gamma(.)$ is the gamma function,

$$\Gamma(\alpha) = \int_0^{+\infty} x^{\alpha-1} e^{-x} dx$$

If $\alpha \leq 1$, L the probability-density function is L-shaped. If $\alpha > 1$, gamma density is bell-shaped.

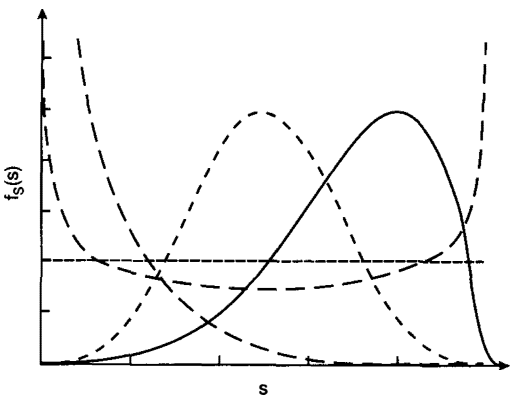

Fig. 1 *S-distribution system: type-I density*
——— $p = 5, q = 2$
—·—· $p = 5, q = 5$
····· $p = 1, q = 1$
— — $p = 0.6, q = 0.6$
– – – $p = 0.6, q = 5$

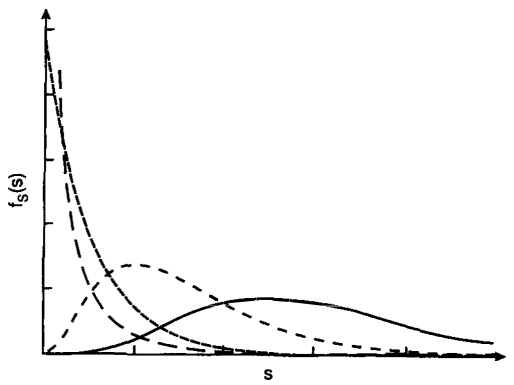

Fig. 2 *S-distribution system: type-III density*
——— $\alpha = 6$
—·—· $\alpha = 3$
····· $\alpha = 1$
— — $\alpha = 0.5$

Fig. 2 shows the gamma distribution for various values of α. It may be shown that the gamma distribution tends to the normal distribution when c_1 tends to zero.

2.3 Pearson distribution of type V or inverse gamma law

$$f_S(s) = \frac{s^{-\alpha-1}}{\beta^\alpha \Gamma(\alpha)} e^{-1/\beta s}, \quad s \in]0, +\infty[\qquad (6)$$

with α shape parameter,

$$\alpha = \frac{1}{c_2}, \quad \alpha \geq 0$$

and β is the scale parameter,

$$\beta = -\frac{c_2}{a}, \quad \beta \geq 0$$

The type-V density is bell-shaped (Fig. 3) and tends to the normal distribution when c_2 tends to zero.

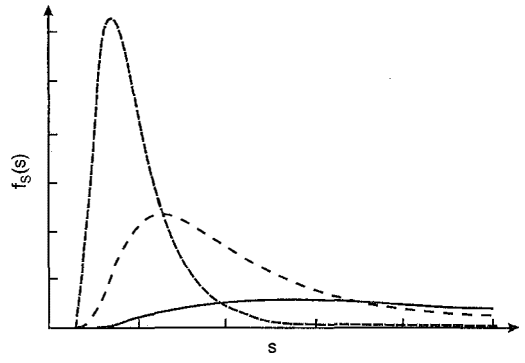

Fig. 3 *S-distribution system: type-V density*
——— $\alpha = 1$
—·—· $\alpha = 3$
····· $\alpha = 6$

2.4 Pearson distribution of type VI or beta law of the second kind

$$f_S(s) = \frac{\beta^q}{B(p,q)} \frac{s^{p-1}}{(\beta+s)^{p+q}}, \quad s \in]0, +\infty[\qquad (7)$$

where p and q are shape parameters,

$$p = -\frac{a}{c_1} + 1, \quad q = \frac{1}{c_2} - 1, \quad \text{so that } p \geq 0,\ q \geq 0$$

and β is the scale parameter,

$$\beta = \frac{c_1}{c_2}, \quad \beta \geq 0$$

The beta distribution of the second kind is well known by statisticians because it approaches many distributions, such as the normal, lognormal, exponential and gamma distributions. Actually, this flexibility is due to the two shape parameters (Fig. 4). Note that the gamma distribution is the limit form of the beta distribution of the second kind when c_2 tends to zero, that the normal distribution is the limit form when c_1 and c_2 tend to zero, and that the inverse gamma distribution can be obtained from the beta distribution of the second kind when c_1 tends to zero.

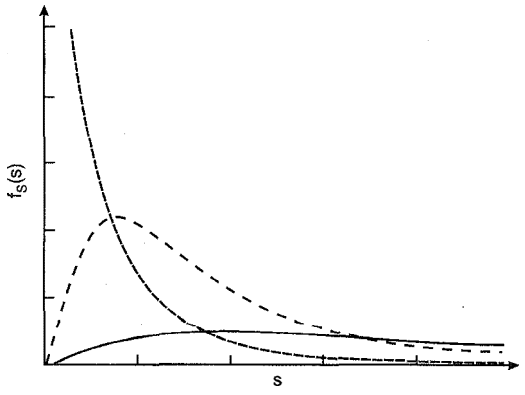

Fig. 4 *S-distribution system: type-VI density*
——— $p = 3,\ q = 0.8$
—·—· $p = 3,\ q = 3$
····· $p = 1,\ q = 3$

3 Parametric intensity distribution

For each distribution of S, we process the corresponding intensity distribution. The new set of distributions forms a new system called KUBW which is explained in this Section. The corresponding intensity distributions are obtained by solving the integral eqn. 1, where $f_{I/S=s}(x)$ is given by eqn. 2 and $f_S(s)$ belongs to the system described in the preceding Section.

3.1 f_S type-I Pearson density or beta law of the first kind

The intensity distribution [17] is expressed by :

$$f_I(x) = \frac{\Gamma(q)}{\Gamma(N)B(p,q)} \frac{1}{x} \left(\frac{Nx}{\beta}\right)^{(p+N-1)/2} e^{-Nx/2\beta} \times W_{(-p-2q+N+1)/2,(N-p)/2}\left(\frac{Nx}{\beta}\right)$$

$$x \in [0, +\infty[\qquad (8)$$

with $W_{\lambda,\mu}$ (λ,μ real numbers) as the Whittaker function, p and q are the shape parameters :

$$p = -\frac{a}{c_1} + 1, \quad q = \frac{a}{c_1} - \frac{1}{c_2} + 1 \quad \text{so that } p \geq 0,\ q \geq 0$$

and β is the scaling parameter,

$$\beta = \frac{c_1}{c_2}, \quad \beta \geq 0$$

This distribution, which depends on the Whittaker function, is called the W probability-density function. Note that the W distribution tends to the K distribution when c_2 tends to zero. Various W densities are shown in Fig. 5.

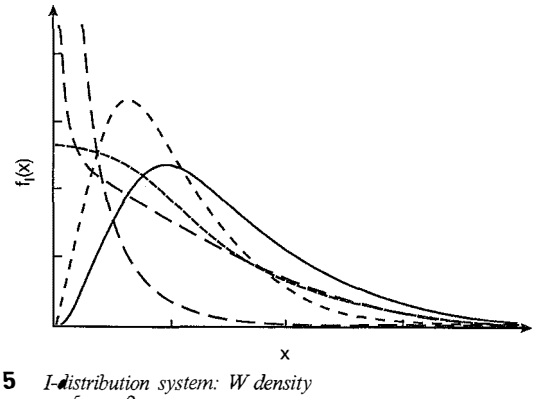

Fig. 5 *I-distribution system: W density*
——— $p = 5,\ q = 2$
—·—· $p = 5,\ q = 5$
····· $p = 1,\ q = 1$
— — $p = 0.6,\ q = 0.6$
– – – $p = 0.6,\ q = 5$

3.2 f_S type-III Pearson density or gamma law

We obtain the K law for the intensity [10–15]:

$$f_I(x) = \frac{1}{\Gamma(N)\Gamma(\alpha)} \frac{b}{\sqrt{x}} \left(\frac{b\sqrt{x}}{2}\right)^{\alpha+N-1} K_{\alpha-N}(b\sqrt{x})$$

$$x \in [0, +\infty[\qquad (9)$$

with scale parameter,

$$\beta = 2\sqrt{\frac{N}{\beta}}$$

shape parameter,

$$\alpha = -\frac{a}{c_1} + 1$$

and K is the modified Bessel function of the second kind.

Fig. 6 shows the K probability-density function for various values of α. Recalling that the K distribution tends to the exponential distribution as α tends to infinity, we obtain the speckle distribution which corresponds to a sea with no swell modulation.

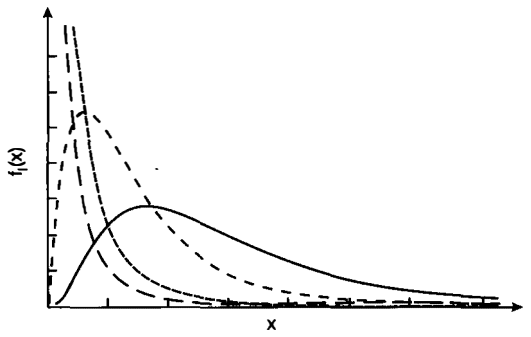

Fig. 6 *I-distribution system: K density*
——— $\alpha = 6$
—·—· $\alpha = 3$
····· $\alpha = 1$
— — $\alpha = 0.5$

3.3 f_S type-V Pearson density or inverse gamma law

For the intensity, the beta law of the second kind is obtained:

$$f_I(x) = \frac{N\beta}{B(N,\alpha)} \frac{(N\beta x)^{N-1}}{(N\beta x+1)^{N+\alpha}}, \quad x \in [0,+\infty[\tag{10}$$

with α shape parameter,

$$\alpha = -\frac{a}{c_1} + 1$$

and β scale parameter,

$$\beta = -\frac{c_2}{a}, \quad s > 0$$

The distribution of type-I is represented in Fig. 7 for the same values that were used for the type-V distribution shown in Fig. 3.

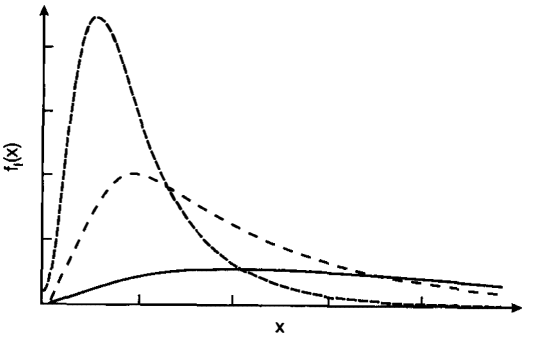

Fig. 7 *I-distribution system: B density*
——— $\alpha = 1$
—·—· $\alpha = 3$
····· $\alpha = 6$

3.4 f_S type-VI Pearson density or beta law of the second kind

The intensity distribution, processed by eqns. 1, 2 and 7, is expressed by the following equation:

$$f_I(x) = \frac{\Gamma(N+q)}{\Gamma(N)B(p,q)} \frac{1}{x} \left(\frac{Nx}{\beta}\right)^N U_{p+q,1-N+p}\left(\frac{Nx}{\beta}\right)$$
$$x \in [0,+\infty[\tag{11}$$

with $U_{\lambda,\mu}(z)$ as the hypergeometric function of λ and μ, p and q as shape parameters:

$$p = -\frac{a}{c_1} + 1, \quad q = \frac{1}{c_2} + 1$$

and β as the scale parameter:

$$\beta = \frac{c_1}{c_2}$$

As this density depends on the U function, it is called the U probability-density function. This last distribution is illustrated by Fig. 8 for the same parameter values as the S distribution given in Fig. 4.

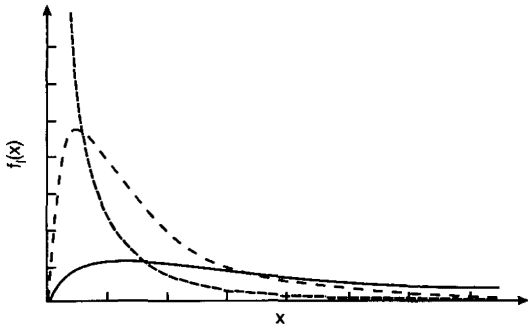

Fig. 8 *I-distribution system: U density*
——— $p = 3, q = 0.8$
—·—· $p = 3, q = 3$
····· $p = 1, q = 3$

The U distribution tends to the K distribution when c_2 tends to zero, whereas it will tend to the beta distribution when c_1 tends to zero.

As the K, U, B and W densities of the intensity depend on the Bessel modified of the second kind, the hypergeometric, the beta and the Whittaker special functions, respectively, the system of distributions is called the KUBW system.

4 Estimation

The parameters a, c_1 and c_2 in eqn. 3 can be expressed in terms of the moments of the distribution. Eqn. 3 may be written (after multiplying both sides by x^r):

$$s^r(c_1 s + c_2 s^2)\frac{df_S(s)}{ds} + s^r(a+s)f_S(s) = 0 \tag{12}$$

By integrating eqn. 12 between $-\infty$ and $+\infty$, and assuming $x^r.f_s(s) \to 0$ as $s \to +\infty$ for $r \leq 4$, we obtain the equation:

$$(-c_2(r+2)+1)m'_{r+1} + (-(r+1)c_1 + a)m'_r = 0 \tag{13}$$

where $(m'_\gamma)_{\gamma=1,3}$ are the first, second and third-order moments about zero of S.

Putting $r = 0$, 1, 2 in eqn. 13, and noting $m'_0 = 1$, we obtain three simultaneous equations for a, c_1 and c_2. These equations depend on the three first moments of S (m'_1, m'_2 and m'_3). The formulas for a, c_1 and c_2 are

$$a = \frac{m'_3m'_2 - 4m'_3m'^2_1 + 3m'^2_2m'_1}{2(2m'_3m'_1 + m'_2m'^2_1 - 3m'^2_2)} \tag{14}$$

$$c_1 = \frac{m'_3m'_2 - 2m'_3m'^2_1 + m'^2_2m'_1}{2(2m'_3m'_1 + m'_2m'^2_1 - 3m'^2_2)} \tag{15}$$

$$c_2 = \frac{m'_3m'_1 + m'_2m'^2_1 - 2m'^2_2}{2(2m'_3m'_1 + m'_2m'^2_1 - 3m'^2_2)} \tag{16}$$

Note that β_1 and β_2 are, respectively, the square of the skewness and the kurtosis of the reflectivity,

$$\beta_1 = \frac{m_3^2}{m_2^3}, \quad \beta_2 = \frac{m_4}{m_2^2} \tag{17}$$

where $(m_\gamma)_{\gamma=1,3}$ are the first, second and third-order centred moments of S.

Each S distribution can be represented by a separate subspace in the (β_1, β_2) plane (Fig. 9). β_1 and β_2 describe, respectively, the flatness and the dissymmetry of the distribution, so they characterise the distribution shape.

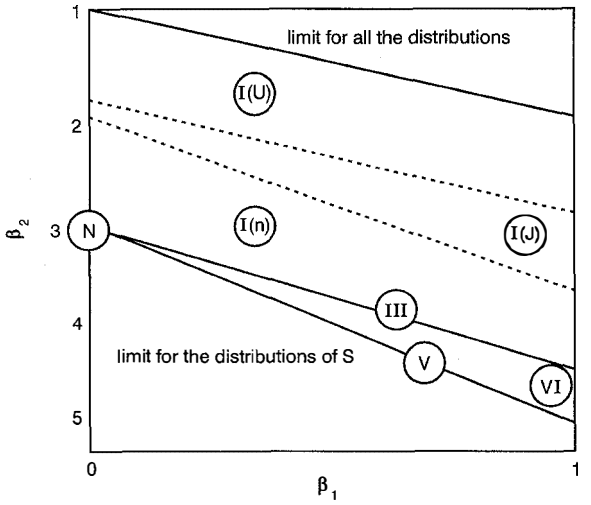

Fig. 9 *S distributions in the (β_1,β_2) plane*
N = normal distribution
I = beta distribution of the first kind
III = gamma distribution
V = inverse gamma distribution
VI = beta distribution of the second kind

The subspace dimension depends on the number of shape parameters in the associated distribution. Thus, the beta of the first kind and beta of the second kind distributions are represented by an area, because they have two shape parameters. The gamma distribution, which depends on one shape parameter, is represented in the (β_1,β_2) plane by a line. a, c_1, c_2 parameters are estimated by the method of moments. The estimated γ-order moment of S is the following:

$$\hat{m}'_\gamma = \frac{\hat{\mu}'_\gamma}{E[I^\gamma / S = s]} \tag{18}$$

where the γth moment of the intensity μ'_γ is estimated with the intensity image.

$E[I^\gamma/S=s]$ is calculated using eqn. 2:

$$E[I^\gamma / S = s] = \frac{\Gamma(n_1 + \gamma)}{n_1^\gamma \Gamma(n_1)} \tag{19}$$

So, the moment estimators of S are obtained by the following:

$$\hat{m}'_\gamma = n_1^\gamma \frac{\Gamma(n_1)}{\Gamma(n_1 + \gamma)} \hat{\mu}'_\gamma \tag{20}$$

By using eqn. 17, we obtain an estimation of the $(\hat{\beta}_1, \hat{\beta}_2)$ of the S random variable, for a given sample. The automatic selection of a parametric distribution in the KUBW system is, therefore, performed according to the location $(\hat{\beta}_1, \hat{\beta}_2)$ in the (β_1, β_2) graph (Fig. 9). Finally, the estimation of a, c_0 and c_1 is obtained by using eqns. 14, 15 and 16.

5 Results

As the histogram of S is not known, the parametric distribution cannot be compared with it. Consequently, we analyse the results of adjusting S with regard to the test result of the intensity.

The selected distribution in the KUBW system is compared to the histogram using a fitting Kolmogorov–Smirnov test. This test is based on the Kolmogorov statistic λ_n which depends on the empirical cumulative histogram $F_n(x)$ and the parametric cumulative density function $F(x)$:

$$\lambda_n = \sqrt{n} \sup_{x \in \vartheta} |F_n(x) - F(x)| \tag{21}$$

where θ is the grey-level support and n the size of the sample.

Four selected Seasat radar images of various sea states were used to present the results of the statistical method developed in this paper.

OLERON is an image obtained near France's Oleron island. The image shows only a speckle texture, and so the swell does not appear in the image.

JASIN1 was obtained during an oceanographic survey in the North of Scotland. A periodic modulation of the backscattering by the swell can be seen.

JASIN4 comes from the same survey. The period of the swell is very short and the texture is very spiky.

ATLANTIQUE is an image taken over the Atlantic Ocean. The organised texture component has a wavelength in between the JASIN4 to JASIN1 examples.

Table 1 shows the K, U, B and W distributions fitting results for the Seasat images. The Kolmogorov–Smirnov statistic, as defined in the preceding test, is below 1.31, and so the statistical method developed in this paper offers good results.

Table 1: Estimation results of KUBW system of distributions on the Seasat images

	OLERON	ATLANTIQUE	JASIN1	JASIN4
S-distribution type	I	I	I	VI
I-distribution type	*W*	*W*	*W*	*U*
β_1 reflectivity	0.02	1.44	0.89	4.66
β_2 reflectivity	3.13	5	5.16	11.96
λ_n	0.82	1.31	0.858	0.97

Estimation of the β_1 and β_2 parameters illustrates their dependence on the sea surface. As the (0,3) couple is a normal distribution, we can conclude that the quieter the sea is, the more the distribution will resemble a normal distribution. The estimation method retains, for S, the beta of the first kind distribution for a relatively quiet sea surface and the beta of the second kind for other sea states. For the intensity, the corresponding distributions are the W and U laws.

The S distribution is also modelled by the gamma probability-density function, which provides the K law for the intensity. Table 2 shows the fitting results. It may be noted that the results are worse than those obtained by the KUBW system of distributions (Table 1). In fact, the gamma law offers a compromise between the beta of the first kind and the beta of the second kind distributions. This compromise is justified in the (β_1,β_2) plane because the gamma curve is a

transition line between the beta laws' subspaces (Fig. 10). This explains the fitting results obtained with the K law.

Table 2: Estimation results of the K distributions on the Seasat images

	OLERON	ATLANTIQUE	JASIN1	JASIN4
S-distribution type	III	III	III	III
I-distribution type	K	K	K	K
λ_n	32	2.45	8.67	9

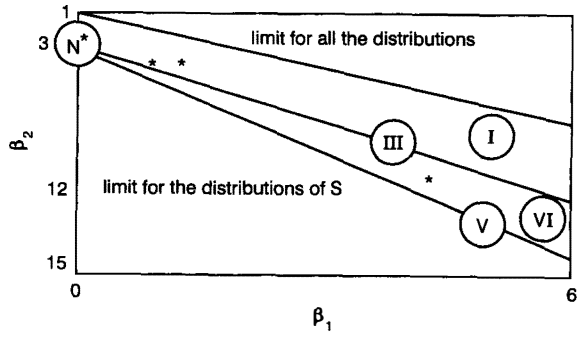

Fig. 10 *S distributions of images in (β_1,β_2) plane*

Also note that our method is more robust, because the Kolmogorov–Smirnov threshold varies from 0.8 to 1.31 and the K law fitting results vary from 2.45 to 32. This is a second advantage of this statistical study.

Finally, we propose to compare our results with those obtained by considering the parametric distributions, without making any hypotheses about backscattering. The estimation method is more traditional because it does not take into account the specific nature of the radar images. The usual distributions are the lognormal, beta of the first and second kind distributions, and also the Weibull one. The results are shown in Table 3. The Kolmogorov–Smirnov test demonstrates the superiority of the lognormal and beta distributions for the intensity. These results can be explained by considering that the random-walk theory is based on hypotheses which are not always fully justified. Nevertheless, these advantages should be balanced by the fact that this method does not give us the distribution of *S*.

Table 3: Results of the estimation of beta, lognormal and Weibull distributions of the Seasat images

	OLERON	ATLANTIQUE	JASIN1	JASIN4
Beta	0.55	0.91	0.71	0.73
Lognormal	0.93	0.76	1.38	0.68
Weibull	3.08	5,11	3.54	7.06

6 Conclusion

In this paper, we have introduced a method for modelling the variability of the scene texture of ocean SAR images. To avoid making a choice of *S* distribution, we have proposed a method to automatically select a well suited distribution of the histogram in a system of parametric distributions. This has been chosen so that it contains distributions of varied shapes and most of the well known parametric distributions. For a given sample, a distribution is selected in accordance with the skewness and the flatness of its histogram. The corresponding intensity distribution set has been processed and called KUBW, with reference to the K, U, B and W special functions. We can show definite advantages for this estimation method.

The automatic selection of the *S* distribution ensures good fitting results to the histogram and also a more robust estimation than the one produced by the classical gamma distribution. The same remark can be made about the intensity variable. We have introduced new distributions for *S* and the intensity variables. These are the beta of the first and second kind distributions for *S* and the U and W distributions for the intensity. The former distributions depend on one more shape parameter than the gamma probability-density function. Note also that the latter is a limit form of the beta of the first and second kind density. Consequently, these new distributions are more flexible than the classical gamma distribution and offer better adjustment to the histogram. Hence, we retain the beta of the first and second kind distributions for the reflectivity variable and the U and W distributions for the intensity.

Nevertheless, the K law results are not bad, its location in the (β_1, β_2) graph shows that the gamma line location is a good compromise between the two beta laws. This statistical modelling, therefore, provides an adequate parametric distribution of the intensity which can be used for the design of segmentation, texture analysis or filtering algorithms [18–20].

7 References

1 FRANKOT, R.T., and CHELLAPPA, R.: 'Lognormal random field models and their applications to radar image synthesis', *IEEE Trans. Geosci. Electron.*, 1987, **GE—25**, (2), pp. 195–206

2 TRUNK, G.V., and GEORGE, S.F.: 'Detection of targets in non-Gaussian sea-clutter', *IEEE Trans. Aerosp. Electron. Syst.*, 1970, **AES—6**, (5), pp. 620–628

3 GARELLO, R., and CHAPRON, B.: 'Use of statistical analysis for Ice-Ocean discrimination'. URSI Commission-F Microwave Specialist symposium on *Microwave remote sensing of the Earth, oceans, ice and atmosphere*, Lawrence, Kansas, USA, 1994, pp. 52

4 CHAPRON, B., GARELLO, R., KERBAOL, V., and LEFEVRE, J.M.: 'Non linear theory of ocean-SAR transformation and statistical analysis of ERS1-1 SAR wave mode imagettes'. Proceedings of the second ERS1 symposium, Hambourg, Germany, 1993, pp. 247–250

5 MAFFETT, A.L., and WACKERMAN, C.C.: 'The modified beta density function as a model for synthetic aperture radar clutter statistics', *IEEE Trans. Geosci. Electron.*, 1991, **GE—29**, (2), pp. 277–283

6 DELIGNON, Y.: 'Statistical study of sea surface radar images'. PhD dissertation, University of Rennes I, France, 1993

7 JOHNSON, N.L., and KOTZ, S.: 'Distribution in statistics: Continuous univariate distributions' (Wiley, 1969)

8 GOODMAN, J.W.: 'Laser speckle and related phenomena' (Ed. J.C. Dainty, 1975)

9 ULABY, F.T., MOORE, R.K., and FUNG, A.K.: 'Active microwave sensing of the ocean' *in* 'Microwave remote sensing active and passive 3' (Artech, MA, 1982)

10 WARD, K.D.: 'Compound representaton of high resolution sea clutter', *Electron. Lett.*, 1981, **17**, pp. 561–563

11 JAKEMAN, E., and PUSEY, P.N.: 'A model for non-Rayleigh sea echo', *IEEE Trans. Antennas Propag.*, 1976, **AP—24**, (6), pp. 806–814

12 JAKEMAN, E., and PUSEY, P.N.: 'Significance of K distribution in scattering experiments', *Phys. Rev. Lett.*, 1978, **40**, (9), pp. 546–550

13 JAO, J.K.: 'Amplitude of composite terrain radar clutter and the K distribution', *IEEE Trans.*, 1984, **AP—32**, (10), pp. 1049–1062

14 LAUR, H.: 'Radar image analysis in remote sensing: textural nd radiometric discriminators'. PhD dissertation, University of Paul Sabatier, Toulouse, 1989

15 BARAKAT, R.: 'Weak scattering generalisation of the K density function with application to laser scattering in atmospheric turbulence', *J. Opt. Soc. Am. A,* 1986, **3**, (4), pp. 401–409
16 WACKERMAN, C.C.: 'Calculation of the spatial distribution of the scatterers in a diffuse scene from SAR data'. Proceedings of IGARSS, 1989, pp. 1012–1015
17 LOPES, A., LAUR, H., and NEZRY, E.: 'Statistical distribution and texture in multilook and complex SAR images'. Proceedings of IGARSS symposium, Washington, USA, 1990, pp. 2427–2430
18 HILLION, A., and BOUCHER, J.M.: 'SAR images filtering', *Stat. Anal. de Données,* 1991, **16**, (2)
19 QUELLE, H.C., DELIGNON, Y., and MARZOUKI, A.: 'Unsupervised bayesian segmentation of SAR images using the Pearson system distributions'. Proceedings of IGARSS'93 symposium, Tokyo, Japan, 1993, pp. 1538–1540
20 DELIGNON, Y., MARZOUKI, A., and PIECZYNSKI, W.: 'Estimation of generalised mixture and unsupervised statistical radar image segmentation', *IEEE Trans. Image Process.*, 1997, **6**, (10), pp. 1364–1375