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# Turning up and down strong magnetic fields in relativistic nuclear collisions 

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I show that the average transverse momentum, $\left\langle p_{t}\right\rangle$, of the hadrons emitted in relativistic nuclear collisions can be used as a "knob" to control the strength of the magnetic field induced by the spectator and the participant protons over the overlap region. I thus argue that any observable sensitive to this magnetic field is nontrivially correlated with $\left\langle p_{t}\right\rangle$ at a given collision centrality.

Heavy atomic nuclei are smashed at relativistic energy at the BNL Relativistic Heavy Ion Collider (RHIC) and at the CERN Large Hadron Collider (LHC) to produce and characterize the quark-gluon plasma, the hot fluidlike state of strong-interaction matter. These processes involve the interaction of highly charged objects, i.e., ions with $Z \sim 80$, moving in opposite directions at nearly the speed of light, and are therefore associated with the emergence of magnetic fields of gigantic strength $[1-5], B \sim 10^{15} \mathrm{~T}$, the strongest ever created in a laboratory. Experimental searches for signatures of the magnetic field in relativistic nuclear collisions are actively pursued at both RHIC [6-13] and LHC [14-18], and theoretical studies aimed at establishing a quantitative phenomenology of $B$ field-related effects have recently appeared in the literature [19-30]. This effort is driven by the fact that a strong $B$ field acting on the hot quark-gluon medium may lead to the emergence of so-called chiral anomalous effects [31-37], whose experimental observation would have far-reaching implications, bringing evidence of local strong parity violation in highenergy nuclear experiments.

However, this is an outstanding challenge. The observable effects driven by the $B$ field are typically of the same kind as the observable effects driven by the strong interaction governing the quark-gluon plasma [38], and it is difficult to separate these two contributions in the data. The strength of the $B$ field in a given collision depends on the number of participant protons, $N_{\text {part }}$, and on the number of spectator protons, $N_{s}$. In this paper, I introduce a new method that allows one to have an experimental handle on these numbers, and thus on the manifestations of the strong $B$ field.

The idea is to look at events that yield the same number of particles in the final state (i.e., same multiplicity), and then sort these events according the mean transverse momentum, $\left\langle p_{t}\right\rangle$, of their final-state hadrons. In the hydrodynamic framework of high-energy nuclear collisions, the mean transverse momentum is a measure of the energy of the fluid from which the particles are emitted [3941]. In one event, and assuming that the quark-gluon plasma is invariant under longitudinal boosts:

$$
\begin{equation*}
\left\langle p_{t}\right\rangle=\frac{1}{N} \int_{\mathbf{p}_{t}} \frac{d N}{d^{2} \mathbf{p}_{t}} p_{t} \tag{1}
\end{equation*}
$$

where $N$ is the total number of particles detected in one
event, and $\frac{d N}{d^{2} \mathbf{p}_{t}}$ is the spectrum of all charged hadrons observed at a given rapidity. Now, collisions with fixed final-state multiplicity correspond to a good approximation to events where the entropy of the medium is fixed. As a consequence, at fixed multiplicity there exists a tight correlation between $\left\langle p_{t}\right\rangle$ and the size of the system, because if two events have the same entropy, but different volumes, then the event contained within a smaller volume corresponds to a medium with larger energy, and in turn a larger $\left\langle p_{t}\right\rangle$. This well-known feature of hydrodynamics [41-45] implies that a selection of events based on $\left\langle p_{t}\right\rangle$ at fixed multiplicity corresponds to a selection based on their size, where a large system size corresponds to a small value of $\left\langle p_{t}\right\rangle$, and vice versa.

The argument of the present paper is that the variation of system size induced by a variation of $\left\langle p_{t}\right\rangle$ corresponds in turn to a significant variation of the collision impact parameter, and consequently of the number of nucleons that participate, or do not participate, in the collision. The value of $\left\langle p_{t}\right\rangle$ provides thus an experimental handle on $N_{\text {part }}$ and $N_{\mathrm{s}}$, with nontrivial implications for the manifestation of the $B$ field at a given collision centrality.

To show that this works in practice, I perform simulations of the collision process using a phenomenological model. I use the $\mathrm{T}_{\mathrm{R}}$ ENTo model of initial conditions [46], tuned as in Ref. [47] to simulate ${ }^{208} \mathrm{~Pb}+{ }^{208} \mathrm{~Pb}$ collisions at LHC. This model provides a prescription for the entropy density, $s\left(\mathbf{x}, \tau_{0}\right)$, created in the interaction of two nuclei $A$ and $B: s\left(\mathbf{x}, \tau_{0}\right) \propto \sqrt{T_{A}(\mathbf{x}+\mathbf{b} / 2) T_{B}(\mathbf{x}-\mathbf{b} / 2)}$, where $\tau_{0}$ is the time at which the hydrodynamics description of the system becomes applicable, $\mathbf{b}$ is the impact parameter of the collision, and $T_{A(B)}$ is a Lorentz-boosted density of participant matter. The $\mathrm{T}_{\mathrm{R}}$ ENTo model does not allow to evaluate $\left\langle p_{t}\right\rangle$, nevertheless, following recent studies [39, 40], a good approximation of the relative variation of this quantity can be obtained as follows. I denote the initial energy per rapidity and the initial entropy per rapidity in the quark-gluon plasma respectively by:

$$
\begin{equation*}
E_{0}=\tau_{0} \int_{\mathbf{x}} e\left(\mathbf{x}, \tau_{0}\right), \quad S=\tau_{0} \int_{\mathbf{x}} s\left(\mathbf{x}, \tau_{0}\right) \tag{2}
\end{equation*}
$$

where $e\left(\mathbf{x}, \tau_{0}\right)$ is the energy density of the system, $e \propto$ $s^{4 / 3}$, at the beginning of hydrodynamics. Dubbing $\delta\left\langle p_{t}\right\rangle=\left\langle p_{t}\right\rangle-\left\langle\left\langle p_{t}\right\rangle\right\rangle$, where $\left\langle\left\langle p_{t}\right\rangle\right\rangle$ is the average value of $\left\langle p_{t}\right\rangle$ at a given centrality, the relative variation of the


FIG. 1. Top: Average impact parameter in ${ }^{208} \mathrm{~Pb}+{ }^{208} \mathrm{~Pb}$ (squares) collisions at top LHC energy as a function of $\left\langle p_{t}\right\rangle$. Bottom: Average number of spectator nucleons. Left: central collisions, 1-2\%. Right: Peripheral collisions, 39-40\%.
average transverse momentum is then provided by:

$$
\begin{equation*}
\frac{\delta\left\langle p_{t}\right\rangle}{\left\langle\left\langle p_{t}\right\rangle\right\rangle}=\kappa_{0} \frac{E_{0} / S-\left\langle E_{0} / S\right\rangle}{\left\langle E_{0} / S\right\rangle} \tag{3}
\end{equation*}
$$

where $\kappa_{0}$ is a constant which depends on the thermodynamic and viscous properties of the system [45, 48], and has to be chosen to reproduce the relative dynamical fluctuation of $\left\langle p_{t}\right\rangle$ measured in experimental data [49, 50]. Doing so, one can study observables as a function of the relative variation of the final-state $\left\langle p_{t}\right\rangle$. I calculate observables at fixed centrality. Following the experimental procedure, where the centrality of a collision is defined by the multiplicity [51, 52], I define centrality classes from the amount of produced entropy, $S$.

The results of the model can be found in the leftmost panels of Fig. 1, where I analyze ${ }^{208} \mathrm{~Pb}+{ }^{208} \mathrm{~Pb}$ collisions with a central cut, $1-2 \%$. The upper panel shows the collision impact parameter, $b$, as a function of $\left\langle p_{t}\right\rangle$. I remark that $b$ and $\left\langle p_{t}\right\rangle$ are positively correlated, and that the impact parameter increases by a significant factor from low to high values of $\left\langle p_{t}\right\rangle$. The lower panel shows instead the average number of spectator nucleons, $\left\langle N_{s}\right\rangle$, as a function of $\left\langle p_{t}\right\rangle$. One observes a strong correlation between $\left\langle p_{t}\right\rangle$ and $N_{s}$, as the number of spectators increases by as much as a factor 4 moving towards the high- $\left\langle p_{t}\right\rangle$ tail.

I assess now the dependence of the previous result on the collision centrality, on the type of colliding species, and on the beam energy.

The rightmost panels of Fig. 1 show the same results discussed previously, but in peripheral collisions, corresponding to $39-40 \%$ centrality. One notes that the curves preserve their positive slope, however, the correlation between $\left\langle N_{s}\right\rangle$ and $\left\langle p_{t}\right\rangle$ is depleted. The overall relative increase of $\left\langle N_{s}\right\rangle$ is indeed less than $20 \%$.


FIG. 2. Average number of spectator nucleons in ${ }^{197} \mathrm{Au}+{ }^{197} \mathrm{Au},{ }^{238} \mathrm{U}+{ }^{238} \mathrm{U}$, and ${ }^{96} \mathrm{Ru}+{ }^{96} \mathrm{Ru}$ collisions at top RHIC energy. Left: central collisions, $1-2 \%$. Right: peripheral collisions, 39-40\%.

I repeat now the same calculation for systems collided at RHIC. I use the $\mathrm{T}_{\mathrm{R}}$ ENTo model tuned as in Ref. [53] to simulate ${ }^{197} \mathrm{Au}+{ }^{197} \mathrm{Au},{ }^{238} \mathrm{U}+{ }^{238} \mathrm{U}$ collisions, and also ${ }^{96} \mathrm{Ru}+{ }^{96} \mathrm{Ru}$ collisions, recently performed at RHIC, whose value of $\kappa_{0}$ in Eq. (3) is chosen by assuming that relative $\left\langle p_{t}\right\rangle$ fluctuations scale like $A^{-1 / 2}$. The left panel of Fig. 2 shows a very strong correlation between $\left\langle p_{t}\right\rangle$ and $\left\langle N_{s}\right\rangle$ in central collisions at RHIC. This correlation is in fact stronger in this figure than in the previous one. The reason is that RHIC systems fluctuate more [54], so that to a given collision centrality corresponds a broader range of impact parameters. In central ${ }^{197} \mathrm{Au}+{ }^{197} \mathrm{Au}$, for instance, the number of spectators increases by roughly a factor 7 . I further note that the results shown for ${ }^{238} \mathrm{U}+{ }^{238} \mathrm{U}$ collisions are obtained by implementing deformed nuclei $(\beta=0.3)$. However, while it has been established that a selection of central events based on $\left\langle p_{t}\right\rangle$ allows to discern body-body and tip-tip geometries $[55,56]$, this appears to have a negligible impact on the average spectator number, shown in Fig. 2. Results for more peripheral collisions are finally shown in the right panel of Fig. 2. I note that $\left\langle N_{s}\right\rangle$ increases by about 60 units in all systems, an effect which is quite significant in ${ }^{96} \mathrm{Ru}+{ }^{96} \mathrm{Ru}$ collisions. One should nevertheless keep in mind that, for peripheral collisions at RHIC beam energy, the physics of $\left\langle p_{t}\right\rangle$ is nontrivially influenced by the presence of pre-hydrodynamic flow [57], which will have to be properly addressed in future quantitative evaluations.

I summarize these findings in Fig. 3, which provides an illustration of my argument for central ${ }^{208} \mathrm{~Pb}+{ }^{208} \mathrm{~Pb}$ collisions. The $B$ field indicated in the figure is the $B$ field induced by the spectators. Collisions at low $\left\langle p_{t}\right\rangle$ (left panel in Fig. 3) correspond to events at small impact parameter, small number of spectators, and thus a small $B$ field. Moving to high $\left\langle p_{t}\right\rangle$ (right panel in the figure), the impact parameter increases, and this triggers an enhancement in the number of spectator nucleons, which does turn the $B$ field up. The mean transverse


FIG. 3. Transverse plane projection of the average geometry of central (1-2\%) collisions of ${ }^{208} \mathrm{~Pb}$ nuclei, and its implication for the manifestation of the $B$ field induced by the spectator protons. Left: low $\left\langle p_{t}\right\rangle$, corresponding to $b \sim 1 \mathrm{fm}$, and spectator number $N_{s}=10$. Right: high $\left\langle p_{t}\right\rangle$, corresponding to $b \sim 2.8 \mathrm{fm}$, and $N_{s}=40$. The spectator nucleons are displayed as small circles.
momentum, hence, serves as a sort of knob to turn up and down the strong $B$ field created in high-energy nuclear collisions at a given collision centrality. An analogous picture could be drawn for the $B$ field produced by the participant nucleons.

A couple of comments are in order. The proposed method amounts to an event-shape selection in which one uses $\left\langle p_{t}\right\rangle$ to sort events according to their size. Another event-shape engineering method commonly used in heavy-ion analyses uses instead the elliptic flow, $v_{2}$, to sort events according to their ellipticity, $\varepsilon_{2}$. In principle, the ellipticity is correlated with the impact parameter, i.e., with the system size, at a given centrality, however, this correlation is small [41]. Within the $\mathrm{T}_{\mathrm{R}}$ ENTo model, I have indeed checked that the correlation between $\left\langle p_{t}\right\rangle$ $\operatorname{ad}\left\langle N_{s}\right\rangle$ is in fact twice as strong as the correlation between $\varepsilon_{2}$ and $\left\langle N_{s}\right\rangle$. The average transverse momentum stands out, then, as the final-state observable presenting the strongest correlation with the number of spectators (or participants) at a given collision centrality. It is finally important to appreciate that, while the fine details of the results shown in Fig. 1 and Fig. 2 depend on the specific $\mathrm{T}_{\mathrm{R}}$ ENTo setup, the fact that the curves have positive slope is fully generic. Hydrodynamics implies only that $\left\langle p_{t}\right\rangle$ is proportional to the system size at fixed multiplicity. The larger $B$ field produced at high $\left\langle p_{t}\right\rangle$ appears to be, then, a built-in feature of the Glauber modeling [58] of nuclear collisions.

This result is remarkably simple, but its phenomenological consequences are vast. The selection of events based on $\left\langle p_{t}\right\rangle$ gives a new experimental handle on $N_{s}$ and $N_{\text {part }}$. Therefore, observables and phenomena which are driven by the magnetic should present a nontrivial correlation with the average transverse momentum, a feature which should be investigated in theoretical calculations. One is given an observable, $\mathcal{O}$, and wants to study its cor-
relation with $\left\langle p_{t}\right\rangle$. A method to do this, and which allows one to obtain results directly comparable to experimental data, consists in the evaluation a Pearson correlation coefficient, as done, e.g., by Bożek in Ref. [59]. Dubbing $\delta\left\langle p_{t}\right\rangle=\left\langle p_{t}\right\rangle-\left\langle\left\langle p_{t}\right\rangle\right\rangle$, and $\delta \mathcal{O}=\mathcal{O}-\langle\mathcal{O}\rangle$, their correlation is defined by:

$$
\begin{equation*}
\rho\left(\left\langle p_{t}\right\rangle, \mathcal{O}\right)=\frac{\left\langle\delta\left\langle p_{t}\right\rangle \delta \mathcal{O}\right\rangle}{\sqrt{\left\langle\left(\delta\left\langle p_{t}\right\rangle\right)^{2}\right\rangle\left\langle(\delta \mathcal{O})^{2}\right\rangle}} \tag{4}
\end{equation*}
$$

This quantity isolates the genuine correlation between $\left\langle p_{t}\right\rangle$ and $\mathcal{O}$ originating from collective effects. Note that this correlation should be evaluated in a narrow class of multiplicity, e.g., centrality bins of size $1 \%$ or smaller. This is typically doable with experimental data, where millions or billions of collisions are recorded, but can be problematic in full hydrodynamic calculations, due to the limited statistics of events. However, methods to address this issue exist [60], and are currently in use in studies of heavy-ion collisions [45, 61].

I conclude with an example of application of the idea introduced in this manuscript. I explain in particular how it can be applied to the observable used to infer signatures of the chiral magnetic effect (CME) in high-energy nuclear experiments. The CME is a manifestation of local strong parity violation which is expected to occur in relativistic nuclear collisions [62,63]. At the high temperatures achieved in the early stages of the quark-gluon plasma, one expects the emergence of local domains of chirally-imbalanced matter with a nonzero axial chemical potential, $\mu_{5}$. In presence of an external magnetic field, such as that produced by the spectator protons, an electric current is thus induced, $\vec{J} \propto \mu_{5} \vec{B}$. Positivelyand negatively-charged particles get pushed (in opposite directions) along this current, i.e., along the direction of the $B$ field [64].

The CME is thus a dipole-like charge-dependent deformation of the system in momentum space, and as such it contributes to the variance of the charge-dependent hadron dipolar flow, $v_{1}^{ \pm} \equiv\left\langle\cos \left(\phi_{1}^{ \pm}-\phi_{2}^{ \pm}\right)\right\rangle$, in the final state. Since in off-central collisions the direction of the $B$ field and the direction of the impact parameter are strongly correlated $[37,65]$, the signal of the CME is typically measured [66] as a correlation between the plane of $v_{1}^{ \pm}$and the reaction plane, which, up to fluctuations, is the same as the plane of elliptic flow, $v_{2}$. This corresponds to the following 3-particle correlation:

$$
\begin{equation*}
g^{ \pm}=\left\langle\cos \left(\phi_{1}^{ \pm}+\phi_{2}^{ \pm}-2 \phi_{3}\right)\right\rangle=\left(v_{1}^{ \pm}\right)^{2} v_{2} \tag{5}
\end{equation*}
$$

where $v_{2} \equiv\left\langle\cos 2\left(\phi_{1}-\phi_{2}\right)\right\rangle$ is the elliptic flow of all hadrons. Now, the strength of the CME signal grows with the strength of the $B$ field, and thus, according to my results, it should increase with $\left\langle p_{t}\right\rangle$ at a given collision centrality. The relevant measure of the correlation
between the CME signal and $\left\langle p_{t}\right\rangle$ is hence given by the following 3-particle correlator:

$$
\begin{equation*}
\rho^{ \pm}\left(\left\langle p_{t}\right\rangle, v_{1}^{ \pm}\right)=\frac{\left\langle\delta\left\langle p_{t}\right\rangle \cos \left(\phi_{1}^{ \pm}-\phi_{2}^{ \pm}\right)\right\rangle}{\sqrt{\left\langle\left(\delta\left\langle p_{t}\right\rangle\right)^{2}\right\rangle\left(g^{ \pm} / v_{2}\right)}} \tag{6}
\end{equation*}
$$

This gives the statistical correlation between $v_{1}^{ \pm}$and $\left\langle p_{t}\right\rangle$. In presence of CME signal, this quantity is positive, whereas a baseline for its value in absence of CME could be estimated following the calculations of Ref. [67].

Indeed, like in the case of $g^{ \pm}$, background effects [6871] contribute to $\rho^{ \pm}$, although this observable presents novel qualitative features. First of all, major background contributions to the CME signal, such as global momentum conservation [70], scale like $1 / N$, where $N$ is the multiplicity. The positive slope of the curves shown in Fig. 1 and in Fig. 2 occurs, however, at fixed $N$. Hence, the selection of events based on $\left\langle p_{t}\right\rangle$ allows one to enhance the CME signal, and to yield a positive value for $\rho^{ \pm}$, while keeping the background fixed. Additionally, background phenomena get themselves correlated with $\left\langle p_{t}\right\rangle$ in the evaluation of $\rho^{ \pm}$. Suppose that a given background effect is responsible for a large fraction of the CME signal, $g^{ \pm}$, measured at a given collision centrality. However, if such a background effect is uncorrelated with $\left\langle p_{t}\right\rangle$, then it does not contribute to $\rho^{ \pm}$. I thus strongly recommend experimental investigations of $\rho^{ \pm}$, which should pave the way for new studies of CME-related effects, especially in collisions at small impact parameter, that will nicely complement the ongoing searches.

I reiterate that this application concerns only the CME signal. A correlation such as that given by Eq. (6), with its nontrivial implications, should be indeed constructed for all observables that present a sensitivity to the strong $B$ field produced in high-energy nuclear collisions.

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