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Intricacies of classical de Sitter string backgrounds

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Up-to-date, there is no known example of a classical de Sitter solution of string theory, despite several good candidates. We consider here two newly discovered 10d supergravity de Sitter solutions, and analyse in great detail whether they can be promoted to classical string backgrounds. To that end, we identify five requirements to be met, and develop the necessary 10d tools to test the solutions. Eventually, they both fail to verify simultaneously all requirements, in spite of positive partial results. The explicit values obtained offer a clear illustration of the situation, and the analysis highlights various subtleties. We finally discuss the relation to the problem of scale separation.

I. INTRODUCTION

Obtaining a background of string theory with a four-dimensional (4d) de Sitter space-time in a well-controlled framework is challenging [1]. This becomes an issue if one tries to reproduce from string theory – as a candidate for a fundamental theory of nature – standard models of cosmology, which describe the early or the future universe with (quasi) de Sitter space-times. This problem has been recently revived, not only by cosmological observations becoming more and more accurate, but also on the theory side with the swampland program. In the latter, conjectures were proposed asserting that any theory of quantum gravity, like string theory, and its 4d low energy effective theories, cannot admit solutions with a (quasi) de Sitter space-time [2], at least in some limit [3, 4] that could correspond to a classical perturbative regime. In this paper we revisit this situation and claims in great detail on interesting examples. We focus on 10d supergravity solutions admitting a 4d de Sitter space-time, and we test to a rare extent all requirements for them to be a classical string background. Even though the solutions do not pass all tests, this detailed analysis should provide useful tools for such studies, as well as a better understanding of the mechanisms behind this apparent obstruction.

Finding classical de Sitter solutions of string theory is usually done in two steps. First, one typically looks for solutions of 10d type II supergravities with a 4d de Sitter space-time and a compact 6d manifold \mathcal{M} . Such solutions with intersecting D_p -branes and orientifold O_p -planes were found on \mathcal{M} being a group manifold in [5–11]. The second step is to verify that these supergravity solutions are classical string backgrounds, by satisfying a list of requirements, including a small string coupling g_s and a large 6d volume in units of the string length l_s . Studies on this topic were so far rather negative [10, 12–15] de-

spite possible loopholes, leading to the situation that no stringy classical de Sitter solution has been found, in line with the swampland conjectures. In the companion work [11], we have obtained new de Sitter solutions of 10d type IIB supergravity with intersecting O_5/D_5 sources, thus fulfilling the above first step. In this work, we provide a detailed 10d analysis of the second step, on two of these new solutions.

Group manifolds are characterised by an underlying Lie algebra with structure constants $f^a{}_{bc}$. The latter can be related to the spin connection coefficients of the 6d metric, as can be seen e.g. in the Maurer-Cartan equations $de^a = -\frac{1}{2}f^a{}_{bc}e^b \wedge e^c$. The $f^a{}_{bc}$ then enter our 10d equations as variables. When looking for solutions in [11], we allowed for a maximal freedom in the $f^a{}_{bc}$, in a basis of one-forms e^a where the metric was δ_{ab} . This simplified the search of solutions, but had the drawback of making the identification of the algebra and group manifold cumbersome. We typically got many non-zero structure constants, while Lie algebra representatives in classification tables only have a few. Algebras of some solutions could still be identified, with an isomorphism to our sets of $f^a{}_{bc}$. We could then verify the existence of a lattice, a discrete action on the 6d group manifold that provides its compactness. Algebras and lattices are particularly simple for the de Sitter solutions 14 and 15 of [11], providing us, up to a change of basis, with a complete knowledge of their 6d geometry. So we focus here on solution 14, and treat solution 15 in the appendix. We briefly comment on algebras of other solutions at the end of the appendix.

To ensure that a 10d supergravity solution is a classical string background, we identify in practice five requirements to be met: a small g_s , large 6d radii $r^a=1, \dots, 6$ in units of l_s , quantization of fluxes, a fixed number of orientifolds $N_{O_5}^I$, and lattice quantization conditions. We believe these requirements are sufficient; whether they are necessary is discussed at the end of section V. The

last three requirements need the detailed 6d geometry, so they were only partially checked on the solutions in [11]. We now complete the study for solutions 14 and 15. We summarize the requirements as follows

$$\begin{aligned} g_s \ll 1, \quad r^a \gg 1, \quad N_{q a_1 \dots a_q} \in \mathbb{Z}, \\ N_s^I \in \mathbb{Z}, \quad N_s^I \leq N_{O_5}^I, \quad N_a \text{ quantized}, \end{aligned} \quad (1)$$

where for simplicity, we will choose a hierarchy factor of 10, i.e. $0 < g_s \leq 10^{-1}, r^a \geq 10$. The index $I = 1, 2, 3$ refers to the sets of internal dimensions (ab) wrapped by the O_5/D_5 sources: $I = 1$ along (12), $I = 2$ along (34), $I = 3$ along (56). The number of sources along each set is defined as $N_s^I = N_{O_5}^I - N_{D_5}^I$. It enters the source contributions T_{10}^I to the equations of motion and Bianchi identities. Our solutions have non-zero fluxes F_1, F_3, H , giving the flux integers $N_{q a_1 \dots a_q}$ along some of their components. Finally, the N_a are numbers entering the f^a_{bc} , quantized because of the lattice. The relations to the supergravity solution data (in the left-hand side below) go as follows

$$\begin{aligned} g_s F_{q a_1 \dots a_q} &= \frac{g_s \lambda N_{q a_1 \dots a_q}}{r^{a_1} \dots r^{a_q}}, \\ g_s T_{10}^I &= \frac{6 g_s \lambda^2 N_s^I}{r^{a_{1 \perp I}} \dots r^{a_{4 \perp I}}}, \quad f^a_{bc} = \frac{r^a \lambda N_a}{r^b r^c}, \end{aligned} \quad (2)$$

without sum on the indices, and the indices $a_{\perp I}$ denote the directions transverse to the set I . The three types of supergravity variables (fluxes, source contributions, structure constants) are here expressed in units of $2\pi l_s$. The radii r^a are introduced through a normalisation convention of the one-forms e^a that we will come back to. The parameter $\lambda > 0$ is an overall rescaling parameter of the solution that we are free to introduce. We refer to section 4 of [11] for more details [44].

In the following, we test solutions 14 and 15, expressed in the appropriate basis, upon the requirements (1). To that end, we provide in section II the material needed by first discussing the 6d geometry of the group manifold for solution 14. We give a basis of globally defined one-forms, determine lattice quantization conditions, and discuss the number of orientifolds. We determine the needed harmonic forms and related flux components. We then test in section III the solution 14 upon the different constraints (1). As announced, it does not succeed in satisfying all of them, but we give explicitly various values obtained, allowing to evaluate how far the solution is from a classical string background. The same procedure is followed in the appendix for solution 15, for which the results are worse. We finally relate the problem of classical de Sitter solutions to that of scale separation in section IV. We argue there that one should at best expect a bounded region in parameter space for both problems. We end with an outlook in section V.

II. 6D GEOMETRY OF SOLUTION 14

In this section, we present in detail the geometry of the 6d group manifold for our de Sitter solution 14. We also obtain the material needed to test, in the next section, the requirements (1) that would allow this solution to be a classical string background.

A. Foreword on the change of basis

As explained in the introduction, solutions 14 and 15 of [11] were found in a basis of one-forms associated to a 6d metric δ_{ab} , $a = 1, \dots, 6$, giving respectively 8 or 7 non-zero structure constants. As detailed in section 2.3 and appendix C of [11], a change of basis can be performed to new one-forms $e^a = e^a_m dy^m$, associated to a new “metric” denoted g_{ab}

$$ds_6^2 = g_{mn} dy^m dy^n = g_{ab} e^a e^b. \quad (3)$$

The coordinates $y^{m=1, \dots, 6}$ parameterize circles: $y^m \in [0, 2\pi[$ and we require the identifications $y^m \sim y^m + 2\pi$. As we will see, the radii $r^a > 0$ are inside the e^a_m . The metric g_{ab} has the interesting properties of being block diagonal along the pairs of a -indices (12), (34), (56), along which are the sources, and the determinant of these blocks is equal to 1. For both solutions, the new basis gives only 4 structure constants. The corresponding algebras are then easy to identify: solution 14 is on $\mathfrak{g}_{3.5}^0 \oplus \mathfrak{g}_{3.5}^0$, and solution 15 is on $\mathfrak{g}_{3.4}^{-1} \oplus \mathfrak{g}_{3.4}^{-1}$, using notations of [16]. Both admit lattices, allowing the group manifolds to be compact. We come back to those in detail below. The new one-forms were denoted with a prime in [11], and the solution data (flux components, structure constants, etc.) was given explicitly in appendix A of [11] under the names solution 14' and 15'. We refer to section 2.3 and appendices A and C for more details on these solutions; in this paper, we work in the new basis and drop the prime. We now focus on solution 14, while solution 15 is treated in the appendix.

B. Lattice, orientifolds

For solution 14, the 4 structure constants are given by

$$\begin{aligned} f^2_{35} &= -0.28930, \quad f^3_{25} = 0.013433, \\ f^1_{64} &= -0.67154, \quad f^6_{14} = 0.41310. \end{aligned} \quad (4)$$

This corresponds to two copies of the three-dimensional solvable algebra $\mathfrak{g}_{3.5}^0$. We rewrite them in terms of real positive numbers $N_{1,2,3,6} > 0$ and the radii r^a , as in the

following Maurer-Cartan equations

$$\begin{aligned} de^2 &= \frac{N_2 r^2}{r^3 r^5} e^3 \wedge e^5, \quad de^3 = -\frac{N_3 r^3}{r^2 r^5} e^2 \wedge e^5, \quad de^5 = 0 \quad (5) \\ de^1 &= \frac{N_1 r^1}{r^4 r^6} e^6 \wedge e^4, \quad de^6 = -\frac{N_6 r^6}{r^1 r^4} e^1 \wedge e^4, \quad de^4 = 0. \end{aligned}$$

Let us focus on one of the two copies. As discussed below, an expression for globally defined one-forms is given by

$$\begin{aligned} e^2 &= r^2 \left(\frac{N_2}{N_3} \right)^{\frac{1}{4}} \left(\cos(\sqrt{N_2 N_3} y^5) dy^2 - \sin(\sqrt{N_2 N_3} y^5) dy^3 \right) \\ e^3 &= r^3 \left(\frac{N_3}{N_2} \right)^{\frac{1}{4}} \left(\sin(\sqrt{N_2 N_3} y^5) dy^2 + \cos(\sqrt{N_2 N_3} y^5) dy^3 \right) \\ e^5 &= r^5 dy^5. \end{aligned} \quad (6)$$

Their normalisation is such that $e^2 \wedge e^3 = r^2 r^3 dy^2 \wedge dy^3$. The forms $e^{2,3}$ can be written in terms of the rotation matrix

$$A(y^5) = \begin{pmatrix} \cos(\sqrt{N_2 N_3} y^5) & -\sin(\sqrt{N_2 N_3} y^5) \\ \sin(\sqrt{N_2 N_3} y^5) & \cos(\sqrt{N_2 N_3} y^5) \end{pmatrix}, \quad (7)$$

which plays an important role. One verifies that the one-forms e^α are globally defined, meaning invariant under $y^5 \sim y^5 + 2\pi$, thanks to a coordinate identification

$$(y)_{y^5+2\pi} = A(-2\pi) (y)_{y^5}, \quad (8)$$

where the 2-vector (y) stands for y^2, y^3 . Such an identification is admissible if the entries of $A(-2\pi)$ are integer (more generally, one may also allow for shifts of coordinates by multiples of 2π). This gives the lattice quantization conditions, and ensures at the same time that we have globally defined one-forms. We refer to [17] for more details. So here, a first possibility is to have $\sqrt{N_2 N_3} \in \mathbb{N}^*$. In that case, the coordinates are simply mapped to themselves, i.e. they are globally defined: one is back topologically to a torus T^6 , with a non-Ricci flat metric. A second possibility is $\sqrt{N_2 N_3} \in \mathbb{N} + \frac{1}{2}$, where the rotation gluing acts as a \mathbb{Z}_2 on the coordinates. Another possibility is $\sqrt{N_2 N_3} \in \mathbb{N} + \frac{1}{4}$ that mixes y^2 and y^3 . These different lattices give rise to different topologies. We refer to [18] or [19] (sections 2.3 and 5) for more details on these geometries. Further discussions and Betti numbers can also be found in [18], as well as two more lattices involving constant shifts of coordinates, not included here. Ricci flat versions were recently considered in [20, 21].

In the case of a torus, with $\sqrt{N_2 N_3} \in \mathbb{N}^*$, each coordinate is that of a circle without further identifications. Our orientifold involution conditions can then be mapped to conditions on coordinates: $\sigma(e^2) = e^2, \sigma(e^{3,5}) = -e^{3,5}$ are equivalent to $\sigma(y^2) = y^2, \sigma(y^{3,5}) = -y^{3,5}$, as can be seen through the forms (6). This involution action on coordinates is the standard one on circles, so the counting of fixed points is the usual one, i.e. 2 per transverse circle. This gives $N_{O_5}^I = 2^4 = 16$. In the following,

having this lattice in mind, we will impose the bound $N_s^I \leq 16$. For the other, more complicated lattices, this bound could be lowered, but smaller values for N_s^I will as well be obtained.

C. Harmonic forms

With the above basis $\{e^\alpha\}$ of globally defined one-forms, we can now determine the harmonic 1- and 3-forms with constant coefficients [45]: those will be needed for flux quantization. To that end, we look for all closed and co-closed 1- and 3-forms [46]. This is complicated due to the Hodge star, involving here the inverse metric (3) which has off-diagonal components g^{ab} . As a warm-up, let us consider the metric to be δ_{ab} . In that case, one obtains e^4, e^5 and $e^1 \wedge e^4 \wedge e^6, e^1 \wedge e^5 \wedge e^6, e^2 \wedge e^3 \wedge e^4, e^2 \wedge e^3 \wedge e^5$. These forms are representative of cohomology equivalence classes. Changing the metric to g_{ab} should not change these classes, since the latter are topological, so we should get the same number of forms. In addition, the representatives may only differ by an exact piece. This is what we obtain by an explicit computation: the 1-forms remain e^4, e^5 , while the harmonic 3-forms with constant coefficients are now

$$\begin{aligned} \omega_1 &= e^1 \wedge e^4 \wedge e^6 + d\omega_1, \quad \omega_2 = e^2 \wedge e^3 \wedge e^5 + d\omega_2 \quad (9) \\ \omega_3 &= e^1 \wedge e^5 \wedge e^6 + d\omega_3, \quad \omega_4 = e^2 \wedge e^3 \wedge e^4 + d\omega_4, \end{aligned}$$

with the following exact pieces

$$\begin{aligned} \omega_1 &= \alpha f^1_{46} \left(e^1 \wedge e^2 + \frac{f^1_{46}}{f^3_{25}} g^{56} e^3 \wedge e^4 \right) \\ &\quad - \frac{\alpha f^1_{46} f^2_{35} g^{34} + g^{56}}{f^6_{14}} e^5 \wedge e^6 \\ \omega_2 &= \alpha f^2_{35} \left(-e^1 \wedge e^2 + \frac{f^2_{35}}{f^6_{14}} g^{34} e^5 \wedge e^6 \right) \\ &\quad - \frac{\alpha f^2_{35} f^1_{46} g^{56} + g^{34}}{f^3_{25}} e^3 \wedge e^4 \\ \omega_3 &= \frac{g^{12} f^3_{25}}{(f^6_{14})^2 + (f^3_{25})^2 (1 + (g^{12})^2)} e^3 \wedge e^6 \\ \omega_4 &= \frac{g^{12} f^6_{14}}{(f^6_{14})^2 + (f^3_{25})^2 (1 + (g^{12})^2)} e^3 \wedge e^6 \\ \text{where } \alpha &= \frac{g^{12}}{(f^2_{35})^2 + (f^1_{46})^2 (1 + (g^{12})^2)}. \end{aligned} \quad (10)$$

We verify that those are non-zero only because of the (inverse) metric off-diagonal components.

To perform the quantization, we will need to normalise these harmonic forms. Using the duality to the homology, and changing representatives of a same (co)homology class, we have the following equalities

$$\begin{aligned} \int_{\Sigma_3} \omega_1 &= \int_{\tilde{\Sigma}_3} e^1 \wedge e^4 \wedge e^6 = r^1 r^4 r^6 \int \int \int_0^{2\pi} dy^1 \wedge dy^4 \wedge dy^6 \\ &= (2\pi)^3 r^1 r^4 r^6, \end{aligned} \quad (11)$$

and the same holds for all four ω_i . Indeed, the $e^a \wedge e^b \wedge e^c$ involved are precisely those that give $dy^m \wedge dy^n \wedge dy^p$ times an appropriate (constant) normalisation. This is consistent with the formulas used so far for flux quantization, given that the only flux components that need to be quantized are those along harmonic forms. We have just verified here that these forms can be integrated, and the result is in agreement with the normalisation ansatz used so far, that relates the radii r^a to the integral of e^a . The formula (2) for the quantized flux can thus be consistently used on the harmonic components.

D. Flux quantization

Fluxes should be quantized if and only if they are harmonic forms. This condition comes from having a flux $F = dA$, where A is locally but not globally defined, i.e. F is closed but not exact, in other words harmonic. It is then the transition function or patching of the gauge potential A that gives the quantization condition. Let us consider the equations solved by the fluxes in our solutions

$$\begin{aligned} dF_1 = 0, \quad d *_6 F_1 = 0, \quad dF_3 \neq 0, \quad d *_6 F_3 = 0, \quad (12) \\ dH = 0, \quad d *_6 H \neq 0. \end{aligned}$$

F_1 is thus harmonic, while F_3 and H are not but may contain pieces which are. Only the harmonic parts of these fluxes should be quantized. Doing such a proper flux quantization requires a good knowledge of the geometry, and in particular of the harmonic forms. The lack of such a knowledge for most solutions of [11] led us to quantize all flux components, as also done in the literature, but this is overconstraining. Here, using the harmonic 1- and 3-forms identified above for solution 14, we are able to express our fluxes within the general Hodge decomposition

$$F_q = F_{q \text{ harmo}} + dA + *_6 dB, \quad (13)$$

where $F_{q \text{ harmo}}, A, B$ are globally defined forms. We obtain more explicitly for solution 14

$$\begin{aligned} H = d(b_{13} e^1 \wedge e^3 + b_{24} e^2 \wedge e^4), \quad F_1 = F_{15} e^5, \quad (14) \\ F_3 = F_{3\omega_1} \omega_1 + F_{3\omega_2} \omega_2 \\ + g_s^{-1} *_6 d(a_{12} e^1 \wedge e^2 + a_{34} e^3 \wedge e^4 + a_{56} e^5 \wedge e^6), \end{aligned}$$

where g_s is introduced for notation convenience, and

$$\begin{aligned} b_{13} = -0.34083, \quad b_{24} = 0.99383, \quad g_s F_{15} = -0.27398, \\ g_s F_{3\omega_1} = 0.12430, \quad g_s F_{3\omega_2} = 0.012539, \quad (15) \\ a_{12} = 0.82025, \quad a_{34} = -2.0877, \quad a_{56} = -0.55448. \end{aligned}$$

We deduce that only three flux components need to be quantized: $F_{15}, F_{3\omega_1}, F_{3\omega_2}$. To that end, we use the normalisation (11) for the harmonic forms, which justifies the use of the initial formulas (2).

III. CHECKING REQUIREMENTS FOR A CLASSICAL SOLUTION

With all the material obtained in the previous section, we are now ready to test solution 14 against the requirements (1). Given the supergravity solution data, namely the left-hand sides of (2), we need to find 8 real parameters (g_s, r^a, λ) and 8 integers ($N_{15}, N_{3\omega_1}, N_{3\omega_2}, N_s^I, N_2 N_3, N_1 N_6$), that satisfy the constraints (1). The source contributions are given by

$$g_s T_{10}^1 = 10, \quad g_s T_{10}^2 = -0.088507, \quad g_s T_{10}^3 = -0.77652. \quad (16)$$

Solution 14 has the particularity of having $T_{10}^2 < 0$: this implies that among the N_s^I , an upper bound should only be imposed for $I = 1$. We recall from [11] that O_5 may still be present in the set $I = 2$.

For a better comparison to the results of [11], we start by testing our solution without imposing the lattice conditions. We then get the following solution to the other constraints

No lattice condition: (17)

$$\begin{aligned} r^1 = 86.658, \quad r^2 = 272.28, \quad r^3 = 10.834, \quad r^4 = 18.142, \\ r^5 = 198.25, \quad r^6 = 10.562, \quad \lambda = 789.30, \quad g_s = 0.068818, \\ N_{15} = -1, \quad N_{3\omega_1} = 38, \quad N_{3\omega_2} = 135, \\ N_s^1 = 16, \quad N_s^2 = -17, \quad N_s^3 = -14, \\ N_2 = 0.0028913, \quad N_3 = 0.084801 = (0.015659)^2 / N_2, \\ N_1 = 0.0018814, \quad N_6 = 0.077905 = (0.012107)^2 / N_1, \end{aligned}$$

where we give the highest products $N_1 N_6$ and $N_2 N_3$ obtained. Those remain far from 1^2 , but are less far from the other lattice allowing for $(1/4)^2$. We also obtain a solution to these constraints with $N_s^1 = 14$, $N_2 N_3 = 1$ and $N_1 N_6 \approx 10^{-8}$. Solutions with lower N_s^1 , down to $N_s^1 = 1$ can also be found. On the contrary, imposing lattice conditions (and the other requirements) without the bound on N_s^1 leads us at best to $N_s^1 = 50960$, namely

No orientifold bound: (18)

$$\begin{aligned} r^1 = 57.907, \quad r^2 = 162.65, \quad r^3 = 10.014, \quad r^4 = 354.99, \\ r^5 = 2999.3, \quad r^6 = 10.014, \quad \lambda = 186.97, \quad g_s = 0.099885, \\ N_{15} = -44, \quad N_{3\omega_1} = 1370, \quad N_{3\omega_2} = 3280, \\ N_s^1 = 50960, \quad N_s^2 = -1195, \quad N_s^3 = -1241, \\ N_2 = 0.28572, \quad N_3 = 3.5000 = 1^2 / N_2, \\ N_1 = 0.22048, \quad N_6 = 4.5356 = 1^2 / N_1. \end{aligned}$$

As expected, we can lower this N_s^1 when rather verifying

the other lattice quantization conditions

$$N_2 = 0.20794, N_3 = 1.2023 = (\frac{1}{2})^2 / N_2, \quad (19)$$

$$N_1 = 0.10602, N_6 = 2.3581 = (\frac{1}{2})^2 / N_1,$$

and $N_s^1 = 13000$,

$$N_2 = 0.56756, N_3 = 0.11012 = (\frac{1}{4})^2 / N_2, \quad (20)$$

$$N_1 = 0.18581, N_6 = 0.33636 = (\frac{1}{4})^2 / N_1,$$

and $N_s^1 = 3219$,

and the other quantities verifying their constraints. These values of N_s^1 remain far too high.

Lattice conditions and the bound on N_s^I have however to be respected in any case for the compactification to make sense, and we should rather test the string regime through the values of g_s and the radii. Imposing all requirements but those on the radii, we find the solution

No large radius condition: (21)

$$r^1 = 7.4234, r^2 = 4.0198, r^3 = 0.24159, r^4 = 1.9131, \\ r^5 = 16.164, r^6 = 0.11150, \lambda = 1.0076, g_s = 0.097666,$$

$$N_{15} = -45, N_{3\omega_1} = 2, N_{3\omega_2} = 2,$$

$$N_s^1 = 14, N_s^2 = -8, N_s^3 = -18,$$

$$N_2 = 0.27890, N_3 = 3.5855 = 1^2 / N_2,$$

$$N_1 = 0.019150, N_6 = 52.218 = 1^2 / N_1.$$

Two radii are substringy, i.e. smaller than 1. We can also bring all of them to be greater than 1, except r^6 which gets lowered to 0.025984. We get as well $r^{1,2,4,5} > 10$ at the cost of having $r^3 = 0.032184$ and $r^6 = 0.93577$. Using the lattice with $(1/4)^2$ does not change this situation. If rather we require large radii but relax the bound on g_s , we do not find any solution to the constraints (1). Note that a similar analysis and result has been obtained for a known de Sitter solution of type IIA supergravity in section 6 of [9]. As there, we conclude that our de Sitter supergravity solution 14 cannot be made a classical string background.

IV. SCALE SEPARATION FOR DE SITTER

In this section, we discuss the matter of scale separation, and its relation to the requirements for a string classical regime. Consider a 10d (anti-)de Sitter solution with 4d cosmological constant Λ , and a tower of Kaluza–Klein states of mass scale m_{KK} coming from the compactification on the 6d manifold. Having a 4d low energy effective theory, involving a finite number of degrees of freedom, is only possible if one truncates the tower of states by the 4d energy cutoff. This requires what is called scale separation, namely

$$\sqrt{|\Lambda|} \ll m_{KK}. \quad (22)$$

It means that the typical 4d energy scale can be decoupled from the 6d one. In view of phenomenology, it is important to know whether scale separation can be achieved.

In many anti-de Sitter solutions, scale separation can actually not be reached: see e.g. [22–26] for recent papers on this point, and [27–30] and references therein for older related works. This has led to recent swampland conjectures [31, 32] forbidding scale separation (see also [33–35]). Interestingly, one classical counter-example exists, the so-called DGKT anti-de Sitter solution [36] (extended recently in [37–39]). A criticism of this solution is the problem of “smeared” sources, and we refer to [11] for more references and discussion on this point. This example remains interesting here for two reasons. First, its framework is very analogous to ours: it is a classical 10d supergravity solution, on a torus, and its intersecting sources configuration is analogous to ours. Another similarity is the presence of flux integers not constrained through the tadpole or Bianchi identity: here these are the harmonic components of F_3 , giving the $N_{3\omega_i}$. Secondly, the DGKT solution admits a parametric control (with asymptotic limit) on the scale separation, which is the same that governs the validity of the solution as a classical string background: the more classical (small g_s , large volume, etc.), the better the scale separation. This provides an example of a relation between these two important problems.

For de Sitter solutions, a similar relation was sketched in [14]. First, it was first shown there that a classical de Sitter solution requires a small 6d curvature scale $|\mathcal{R}_6|$ compared to that of the average internal length $2\pi r$: $|\mathcal{R}_6| \times (2\pi r)^2 \ll 1$. This is a priori possible on some manifolds thanks to internal hierarchies and/or fine-tunings. In addition, such a hierarchy in a de Sitter solution was shown to automatically imply scale separation, in the sense of

$$\mathcal{R}_4 \ll \frac{1}{(2\pi r)^2}. \quad (23)$$

Let us test here scale separation with the de Sitter solution 14 in the case where we ignore lattice conditions (17): all other requirements for a classical solution are then satisfied. With these values and the λ -rescaling, we first compute (in units of $2\pi l_s$)

$$(\mathcal{R}_4 = 3.6370 \cdot 10^{-8}) < (1/(2\pi r)^2 = 1.1873 \cdot 10^{-5}), \quad (24)$$

where $r = (\prod r^a)^{\frac{1}{6}}$ [48]. The scale separation condition (23) is verified, since we consider here hierarchies of order 10 per length. This result is however only preliminary, since the largest radius r^2 gives a smaller value, as well as the 6d curvature

$$1/(2\pi r^2)^2 = 3.4167 \cdot 10^{-7}, |\mathcal{R}_6| = 1.2162 \cdot 10^{-6}. \quad (25)$$

A tower of modes associated to the radius r^2 would then be difficult to separate or truncate [49]. The internal

hierarchy $|\mathcal{R}_6| \times (2\pi r)^2 \ll 1$ is also not verified, due to the unsatisfied lattice conditions. The lack of complete classicality (in the sense of satisfying requirements (1)) seems here again related to difficulties in having scale separation.

Even though classical de Sitter solutions may admit scale separation, we do not expect the same parametric control as in DGKT. As expressed in [11], classical de Sitter solutions may not exist at parametric control in an asymptotic limit, but they could still be present in a bounded region of parameter space. In such a “grey zone”, parameters are large/small enough to accommodate a classical regime as in the requirements (1), but they remain bounded by these same constraints and cannot be taken asymptotically. At best, one may then have scale separation in this bounded region, controlled by a bounded parameter. In the following, and later in [50], we illustrate this idea with a simple transformation that allows us to move in parameter space. We use a scaling parameter $\gamma > 1$ to act as follows on the entries in the right-hand sides of (2)

$$r^a \rightarrow \gamma^{x_a} r^a, \quad g_s \rightarrow \gamma^g g_s, \quad N_K \rightarrow \gamma^{n_K} N_K, \quad \lambda \rightarrow \lambda \quad (26)$$

and we do not consider any quantized H -flux nor F_5 above because there are none in solutions 14 and 15. The powers x_a, g, n_K and their signs are not fixed. However, we relate them in such a way that the left-hand sides of (2) remain invariant. Taking for simplicity $\forall a, x_a = x$, we get

$$n_{q=1,3} = qx - g, \quad n_s^I = 4x - g, \quad n_a = x. \quad (27)$$

By not affecting the quantities in the left-hand sides of (2), we are guaranteed to still have a solution to the 10d equations. \mathcal{R}_4 is also invariant under this simple transformation. The 6d radii are however changed, so the ratio of the cosmological constant to a Kaluza–Klein mass goes as the latter, and the scale separation is improved with smaller r^a , i.e. $x < 0$. This specific γ -scaling then does not help in getting classical solutions, which rather require large radii. Such a situation where two requirements go in different parametric directions is common for classical de Sitter solutions, due to the many bounds to satisfy. This leads to the bounded region in parameter space, as we illustrate in table I with the different possible signs of x, g , corresponding to various parametric directions.

We see from table I that in any direction of the γ -scaling, a parameter bound is met. Some cases in this table were also observed in the explicit example in section III. The simple γ -scaling was considered here for the purpose of illustration. Other transformations may do better on some of the requirements, e.g. for the lattice conditions or the scale separation: we introduce one at the end of this paper in [50], the β -scaling. Specificities of given solutions may also help. Nevertheless, it remains

unlikely that an asymptotic limit would be opened in some parametric direction.

Let us also stress two differences with an anti-de Sitter solution, that are made more obvious with table I. An anti-de Sitter solution would allow for all $N_s^I \leq 0$, in which case there is no upper bound on $|N_s^I|$, contrary to the above $N_s^1 \leq N_{O_p}^1$. In addition, in the DGKT anti-de Sitter solution, one has $f^a{}_{bc} = 0$, which is not possible for de Sitter solutions on group manifolds that require $\mathcal{R}_6 < 0$. The former thus does not face the lattice conditions. Removing these two constraints relieves bounds in parameter space.

	$x > 0$	$x < 0$
$g < 0$	classical \checkmark , scale sep. \times , fluxes, lattice \checkmark , sources \times ($N_s^1 \leq N_{O_p}^1$)	With $4x = g$: classical \times (r^a bound), scale sep. \checkmark , fluxes, sources \checkmark , lattice \times (integer cond.)
$g > 0$	With $4x = g$: classical \times (g_s bound), scale sep. \times , lattice, sources \checkmark , fluxes \times (integer cond.)	classical \times (r^a, g_s bounds), scale sep. \checkmark , fluxes, sources, lattice \times (integer cond.)

TABLE I: Starting with a de Sitter solution, we act with the γ -scaling (26), (27) on the parameters. We indicate with a \checkmark or \times whether or not it helps in meeting the various requirements. When it does not, the limiting bound is written in parentheses. The “integer condition” bound refers to integer numbers having to be greater than 1. Since N_s^1 is very constrained, $1 \leq N_s^1 \leq N_{O_p}^1$ in a de Sitter solution, we restrict when possible to the scaling leaving this number invariant, namely $4x = g$.

To conclude, if a region of classical de Sitter solutions exists, we do not exclude that scale separation also takes place there. We believe however that it would be at best controlled by a bounded parameter, contrary to the DGKT solution and despite several similarities with that framework. Such a bounded region or grey zone with classical and scale separated de Sitter solutions remains to be found [51], and we hope to come back to these questions in future work.

V. OUTLOOK

In this work we have tested whether two 10d supergravity de Sitter solutions of [11] could be promoted to classical string backgrounds. Having identified the requirements (1) to be met, we have derived all the necessary material to perform these checks, to a rare extent in the literature. Eventually, our solutions do not pass these

tests, as they fail to satisfy simultaneously all requirements, despite partial positive results. We also discussed the relation of this problem to that of scale separation, and what one should expect on the latter for classical de Sitter solutions. We believe that the tools developed here and the explicit results obtained should be useful in future studies.

Whether supergravity solutions could be classical string backgrounds has already been partially analysed on de Sitter solutions of [11]. Solution 14 did not appear as the most favored one: four other solutions were doing better regarding the partial requirements imposed there. But for those, a complete knowledge of the 6d geometry was missing, preventing us to go further (see however the appendix here). It would be interesting to focus more on them, or more generally, to try with other solutions, possibly obtained from ours by small deformations or transformations, that could help in satisfying the requirements. A family of de Sitter solutions obtained by varying one parameter is depicted in figure 1 of [8]. In such a family, one may eventually find a bounded region of classical solutions, as discussed in section IV.

As it appears from this work, the question of whether one can obtain classical de Sitter solutions of string theory is not settled. Even though we have not found an example for now, our study highlighted how close one can get, and how involved these tests actually are. A related matter is the definition of the requirements for a classical solution: one may consider our requirements on g_s and the radii as conservative. Can one allow for slightly higher values of g_s ? What length or volume should actually be bigger than l_s on our group manifolds? Precise answers to these questions depend on the corrections to effective theories, and those are typically difficult to determine, especially away from a more standard Ricci flat compactification. If any room could be gained from this side, it would certainly be interesting. We hope to come back to all these important problems in future work.

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Appendix: Solution 15

The 4 structure constants of solution 15 are given by

$$\begin{aligned} f^2_{35} &= -0.60208, & f^3_{25} &= -0.058853, \\ f^4_{61} &= -0.10206, & f^6_{41} &= -0.015345, \end{aligned} \quad (\text{A.1})$$

giving two copies of the solvable algebra $\mathfrak{g}_{3,4}^{-1}$. We focus here on one. We rewrite it as

$$de^2 = \frac{N_2 r^2}{r^3 r^5} e^3 \wedge e^5, \quad de^3 = \frac{N_3 r^3}{r^2 r^5} e^2 \wedge e^5, \quad de^5 = 0, \quad (\text{A.2})$$

with real positive numbers $N_{2,3} > 0$. Globally defined one-forms can be written as in (6) where one replaces the rotation matrix (7) by the following ‘‘weighted hyperbolic rotation matrix’’

$$\begin{pmatrix} \cosh(\sqrt{N_2 N_3} y^5) & -\left(\frac{N_3}{N_2}\right)^{\frac{1}{2}} \sinh(\sqrt{N_2 N_3} y^5) \\ -\left(\frac{N_2}{N_3}\right)^{\frac{1}{2}} \sinh(\sqrt{N_2 N_3} y^5) & \cosh(\sqrt{N_2 N_3} y^5) \end{pmatrix}$$

This gives the normalisation $e^2 \wedge e^3 = r^2 r^3 dy^2 \wedge dy^3$ [47]. The above matrix evaluated at -2π provides again the lattice quantization conditions, by requiring the entries to be integers. We obtain [17]

$$\cosh(\sqrt{N_2 N_3} 2\pi) = n_1, \quad n_1^2 - n_2 n_3 = 1, \quad \frac{N_3}{N_2} = \frac{n_3}{n_2}, \quad (\text{A.3})$$

for $n_{1,2,3} \in \mathbb{N}^*$. Note that $n_2 = n_3$ is not a solution to these conditions, hence the need of the ‘‘weight’’ in the hyperbolic rotation.

The harmonic 3-forms with constant coefficients in $\{e^a\}$ basis are formally the same as (9) and (10), up to the relabeling $1 \leftrightarrow 4$, $2 \leftrightarrow 3$, and setting $g^{56} = 0$; this is how we define $\omega_{1,2}$ here below. This can be understood by looking at the algebras and the metrics. Similarly, the harmonic 1-forms are now e^1, e^5 . We then obtain for this solution the following Hodge decomposition of the fluxes

$$\begin{aligned} H &= d(b_{24} e^2 \wedge e^4), \quad F_1 = F_{15} e^5, \\ F_3 &= F_{3\omega_1} \omega_1 + F_{3\omega_2} \omega_2 \\ &\quad + g_s^{-1} *6 d(a_{12} e^1 \wedge e^2 + a_{34} e^3 \wedge e^4 + a_{56} e^5 \wedge e^6), \end{aligned} \quad (\text{A.4})$$

where

$$\begin{aligned} b_{24} &= 0.0018085, \quad g_s F_{15} = 0.13944, \\ g_s F_{3\omega_1} &= 0.0014004, \quad g_s F_{3\omega_2} = 0.00013677, \\ a_{12} &= 1.0117, \quad a_{34} = 0.048104, \quad a_{56} = -0.15985. \end{aligned} \quad (\text{A.5})$$

This gives again only three flux components to quantize. For completeness, we finally give the source contributions

$$g_s T_{10}^1 = 10, \quad g_s T_{10}^2 = 0.49663, \quad g_s T_{10}^3 = -0.10585. \quad (\text{A.6})$$

We now test this solution as done above for solution 14. Without imposing the lattice conditions we obtain a first solution to the other requirements (1)

$$\text{No lattice condition:} \quad (\text{A.7})$$

$$\begin{aligned} r^1 &= 108.00, \quad r^2 = 47.861, \quad r^3 = 21.302, \quad r^4 = 12.912, \\ r^5 &= 25.270, \quad r^6 = 10.826, \quad \lambda = 2372.7, \quad g_s = 0.0014851, \\ N_{15} &= 1, \quad N_{3\omega_1} = 6, \quad N_{3\omega_2} = 1, \\ N_s^1 &= 15, \quad N_s^2 = 14, \quad N_s^3 = -3, \\ N_2 &= 0.0028540, \quad N_3 = 0.0014083, \\ N_4 &= 0.0038956, \quad N_6 = 0.00083305. \end{aligned}$$

We find other solutions down to $N_s^1 = N_s^2 = 1$. On the contrary, imposing the lattice quantization conditions is more difficult. Imposing no bound on the number of sources $N_s^{1,2}$, we can satisfy the other constraints if we trade the integer conditions for numbers greater than 1. We then get a solution with $N_s^1 \approx 755.40$, $N_s^2 \approx 1746.1$, which remains too high. Finally, imposing only the lattice conditions, the bound on N_s^I and the flux quantization, to test the string regime on g_s and r^a , we could not find any solution. These results are worse than for solution 14. The de Sitter supergravity solution 15 cannot be made a classical string background.

Let us finally comment on solutions 10 and 12 of [11]: those were found in that paper to satisfy partial requirements for classical string backgrounds. The algebras of these two solutions were not identified in [11], but we achieve here this identification. Indeed, we have found a change of basis transforming their structure constants to the following sets of non-zero $f^{a_{bc}}$

$$\text{Solution 10: } f^4_{26} = f^6_{42} = -f^2_{64} = 1, \quad (\text{A.8})$$

$$f^3_{15} = f^1_{35} = -1,$$

$$\text{Solution 12: } f^4_{61} = f^6_{14} = -f^1_{46} = 1, \quad (\text{A.9})$$

$$f^3_{25} = -f^2_{35} = 1,$$

allowing to identify the corresponding algebras: $\mathfrak{so}(2,1) \oplus \mathfrak{g}_{3.4}^{-1}$ for solution 10, and $\mathfrak{so}(2,1) \oplus \mathfrak{g}_{3.5}^0$ for solution 12. The $\mathfrak{so}(2,1)$ part leads to a non-compact group manifold, so these solutions are actually not valid as compactifications. This is reminiscent of the analysis of [40] and references therein, where de Sitter solutions can be found in 4d gauged supergravities with semi-simple gaugings corresponding to non-compact spaces.

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- [44] Note: when using conventions of [14] in [11], the sign ε_5 appearing on the right-hand side of the sourced Bianchi identity was wrongly computed to be -1 instead of $+1$. This is only a matter of conventions and has no physical impact, since this sign can be absorbed by $F_q \rightarrow -F_q$ that leaves all other equations invariant. This has no consequence either in our stability analysis nor in that of the classical regime; in the latter we can equally well consider the opposite of the flux integers $N_{q a_1 \dots a_q}$.
- [45] Further harmonic forms can be obtained in the $\{e^a\}$ basis with globally defined functions as coefficients, such as $\cos(\sqrt{N_2 N_3} y^5)$, etc.; we will not need those.
- [46] For a lattice that does not give a torus, some of the closed and co-closed forms may turn out not to be harmonic, because they are actually not globally defined; we refer to [18] on that point. This subtlety would only remove some flux components to quantize, i.e. lead to less constraints, so we do not consider this point any further.
- [47] Slightly different expressions of the one-forms would allow for the same normalisation. In particular another one more analogous to (6) is possible, but it would not lead to an appropriate lattice quantization.
- [48] For completeness and illustration, we compute for this set of values the square of the Planck mass in units of $2\pi l_s$
- $$(2\pi l_s)^2 \times M_p^2 = \frac{(2\pi l_s)^2}{\kappa_{10}^2} \int_6 d^6 y \sqrt{|g_6|} g_s^{-2} = \frac{4\pi}{g_s^2} \prod_{a=1}^6 \frac{r^a}{l_s} = 2.5767 \cdot 10^{13} = (5.0761 \cdot 10^6)^2 . \quad (\text{A.10})$$
- [49] The Kaluza–Klein spectrum on a group manifold can actually be more involved than a simple inverse radius dependence: see e.g. [41] and references therein for explicit spectra.
- [50] Another transformation worth being mentioned is the following β -scaling: it results in having $f^a{}_{bc}$, $g_s F_{q=1,3,5}$, H rescaled by $1/\beta$ and $g_s T_{10}^I$ by $1/\beta^2$, with $\beta > 1$. This mimicks the overall λ -rescaling introduced in [11] and leaves 10d equations invariant, thus guaranteeing a solution. It can be generated as follows, with $\beta^{\frac{2}{3}} \in \mathbb{N}^*$
- $$r^a \rightarrow \beta^{\frac{1}{3}} r^a , \quad g_s \rightarrow \beta^{-\frac{2}{3}} g_s , \quad \lambda \rightarrow \lambda , \quad N_s^I \rightarrow N_s^I , \\ N_{1a} \rightarrow N_{1a} , \quad N_{3abc} \rightarrow \beta^{\frac{2}{3}} N_{3abc} , \quad N_{Habc} \rightarrow N_{Habc} , \\ N_{5abcde} \rightarrow \beta^{\frac{4}{3}} N_{5abcde} , \quad N_a \rightarrow \beta^{-\frac{2}{3}} N_a . \quad (\text{A.11})$$
- Since \mathcal{R}_4 scales as β^{-2} , the quantity $\mathcal{R}_4 \times (2\pi r^a)^2$ scales as $\beta^{-\frac{4}{3}}$, meaning that the scale separation gets enhanced when the classical regime is better verified. This is an improvement with respect to the γ -scaling discussed in section IV. Constraints on fluxes and sources remain satisfied, but this scaling will hit a bound due to the lattice conditions, as in the top right corner of table I. Our conclusion of a bounded region for classical de Sitter is thus not altered, but this β -scaling could be interesting for anti-de Sitter solutions on a flat torus, if any. In particular, the exponent in the conjecture of [32] would be given by $\alpha = \frac{1}{6}$.
- [51] As recalled in [11], an asymptotic limit for the 6d radii is not appropriate from a phenomenological point of view, since those should get bounded from above by observations. A conservative lower bound on the average mass $1/(2\pi r)$ in (24) would be the LHC energy scale, namely 14 TeV. We now rewrite the formula (A.10) for the (reduced) 4d Planck mass as follows
- $$\frac{l_s}{r} = \left(\frac{2\sqrt{\pi} l_p}{g_s} \right)^{\frac{1}{4}} \geq 6.7188 \cdot 10^{-4} , \quad (\text{A.12})$$
- with the length $2\pi l_p = 1/M_p$. The lower bound in (A.12) is then obtained using the above LHC value and $g_s \leq 0.1$. From this perspective, the room for classicality is actually restricted. If one asks in addition for scale separation, and if \mathcal{R}_4 should then match the scale of early universe inflation, most likely higher than the LHC scale, the range (A.12) gets even more restricted [42]. This is where having a warp factor and a varying dilaton could help with hierarchies, even though in the concrete setting of [43], the values were not drastically altered by such effect.