

# Simple method to reduce computation time in planar airgap 3D FEM non-linear problems.

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In the present work, an easily implementable method to reduce CPU time for some 3D FEM problems is presented. A translational displacement between two parts on both sides of a planar airgap is taken into account with a change of coordinates in the airgap region along the z-axis. To achieve this, formulation and post processing are slightly adapted. The proposed method is applied to an academic problem and to an axial electric rotating machine. Results are compared to a usual method. Time saving, ease of implementation and precision on results are really encouraging with a strong reduction of CPU time keeping a low relative deviation. The method could be easily applied to any configurations with planar airgap as axial motor.

**Index Terms**—Air gaps, Electromagnetic forces, Finite element analysis, Iterative methods, Nonlinear magnetics

## I. INTRODUCTION

In 3D finite element analysis, CPU time is a crucial issue. For multiple static problems with displacement, there are three main difficulties. The mesh has to be adapted for each position, either by fully or partially remeshing, or by applying a deformation to each element [1]. Moreover, in case of non linear material, re-using the previous solution to initialize the next iterative process is sometimes difficult. Finally, the thickness of the airgap can cause a poor quality mesh. Therefore, some methods (as shell elements [2] for instance) are used in FEM problems including thin region.

For FEM problems with translational displacement, the proposed method solves these difficulties by slightly adapting the weak formulation without changing the mesh. This paper is an extension of 2D dilation method ([3] and [4]) to 3D electric motors. The main benefit of the proposed method is its ease of implementation. To move away the two parts on both sides of a planar airgap, a variable substitution is made in the airgap region along the Z-axis. Keeping the mesh unchanged allows the initialization of the non-linear resolution with the previous solution. First, an academic problem is proposed to highlight the advantages of this method, already shown in [4] for 2D cases, using free software Gmsh/GetDP [5][6]. Then, the method is applied to an axial claw pole motor.

## II. DILATION FACTOR METHOD

### A. Formulation

The proposed method can be applied to a magnetic device with two parts ( $\Omega_1$  and  $\Omega_2$ ) separated by a planar airgap ( $\Omega_e$ ) as in Fig 1.  $\Omega_1$  has a translational displacement along the Z-Axis. With this method, the model is meshed for a constant airgap value  $e_d$  and the problem is solved for all the airgap values  $e_i$  without changing the geometry. A dilation factor  $\alpha$  is used for each position. The constant airgap  $\Omega_d(x_d, y_d, z_d)$

is the result of the dilation along the Z-Axis of the real airgap  $\Omega_e(x, y, z)$  as:  $x_d = x$ ,  $y_d = y$ ,  $z_d = \alpha z$  (see Fig. 1).

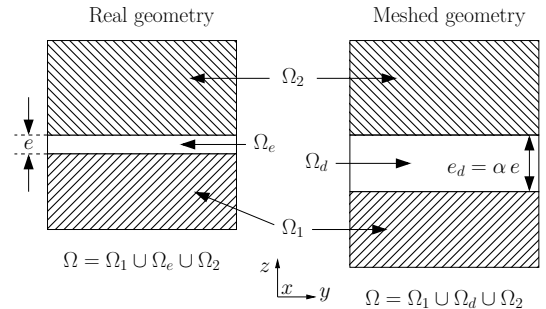


Fig. 1. Dilation along the Z-Axis.

The jacobian  $J$  corresponding to the change of coordinates and its determinant are simply:

$$J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\alpha} \end{pmatrix}, \quad |J| = \frac{1}{\alpha} \quad (1)$$

Magnetic scalar potential ( $\varphi$ ) formulation is used in this method. A dummy magnetic field  $\mathbf{h}_s$  is calculated to replace the electric current  $\mathbf{j}$  in the sources domain  $\Omega_s$ . Without dilation, the weak formulation in the total domain  $\Omega$  is:

$$\int_{\Omega_s} \mu_0 \mathbf{h}_s \cdot \nabla \varphi' d\Omega - \int_{\Omega} \mu (|\nabla \varphi|) \nabla \varphi \cdot \nabla \varphi' d\Omega = 0 \quad (2)$$

To solve another position ( $\Omega_e$ ) keeping the geometry ( $\Omega_d$ ), the gradient of  $\varphi$  in  $\Omega_e$  has to be express as a function of the

gradient of  $\varphi$  in  $\Omega_d$ . With a symmetric second order tensor  $\bar{\alpha}$ :

$$\begin{aligned} \nabla\varphi|_{\Omega_e} &= \begin{pmatrix} \frac{\partial\varphi}{\partial x} \\ \frac{\partial\varphi}{\partial y} \\ \frac{\partial\varphi}{\partial z} \end{pmatrix} = \begin{pmatrix} \frac{\partial\varphi}{\partial x_d} \frac{\partial x_d}{\partial x} \\ \frac{\partial\varphi}{\partial y_d} \frac{\partial y_d}{\partial y} \\ \frac{\partial\varphi}{\partial z_d} \frac{\partial z_d}{\partial z} \end{pmatrix} \\ &= \begin{pmatrix} \frac{\partial\varphi}{\partial x_d} \\ \frac{\partial\varphi}{\partial y_d} \\ \frac{\partial\varphi}{\partial z_d} \alpha \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \alpha \end{pmatrix}}_{\bar{\alpha}} \nabla\varphi|_{\Omega_d} \end{aligned} \quad (3)$$

Only the term corresponding to the dilated subregion is modified in the weak formulation. Considering (3) in (2) it becomes:

$$\begin{aligned} \int_{\Omega_s} \mu_0 \mathbf{h}_s \cdot \nabla\varphi' d\Omega - \int_{\Omega_1 \cup \Omega_2} \mu (|\nabla\varphi|) \nabla\varphi \cdot \nabla\varphi' d\Omega \\ - \int_{\Omega_d} \mu_0 |J| \bar{\alpha} \bar{\alpha}^T \nabla\varphi|_{\Omega_d} \cdot \nabla\varphi'|_{\Omega_d} d\Omega = 0 \end{aligned} \quad (4)$$

After solving the problem, Post processing quantities have to be adapted in the dilated region by means of the tensor  $\bar{\alpha}$ . This approach has already been proven and implemented for a 2D case in [4].

#### B. Magnetic flux density: $\mathbf{b}$

- Usual method:  $\mathbf{b} = -\mu_0 \nabla\varphi$
- Dilation method:  $\mathbf{b} = -\mu_0 \bar{\alpha} \nabla\varphi|_{\Omega_d}$

Keep in mind that  $\varphi|_{\Omega_d}$  is not the magnetic scalar potential but a modified scalar potential which assures that the flux density  $\mathbf{b}$  is div-free.

#### C. Magnetic Coenergy: $W_{co}$

Magnetic Coenergy is a global quantity and has to be integrated. The integration volume in dilated region has to be adjusted with  $|J| = \frac{1}{\alpha}$ .

- Usual method:  $W_{co} = \int_{\Omega_e} \frac{1}{2} \mu_0 |\nabla\varphi|^2 d\Omega_e$
- Dilation method:  $W_{co} = \frac{1}{\alpha} \int_{\Omega_d} \frac{1}{2} \mu_0 |\bar{\alpha} \nabla\varphi|^2 d\Omega_d$

#### D. Maxwell stress tensor: $\bar{T}_m$

With the Maxwell stress tensor used on a surface  $\partial\Omega_f$ , **Force** =  $\oint_{\partial\Omega_f} \bar{T}_m d\mathbf{S}$  with:

$$\bar{T}_m = \frac{1}{\mu_0} \begin{bmatrix} b_x^2 - \frac{|\mathbf{b}|^2}{2} & b_x b_y & b_x b_z \\ b_x b_y & b_y^2 - \frac{|\mathbf{b}|^2}{2} & b_y b_z \\ b_x b_z & b_y b_z & b_z^2 - \frac{|\mathbf{b}|^2}{2} \end{bmatrix} \quad (5)$$

The surface  $\partial\Omega_f$  goes to the domain frontier where  $\mathbf{b} \cdot \mathbf{n} = 0$ . The part of the surface with  $b \neq 0$  has a normal  $\mathbf{n}$  along the dilation axis as shown Fig. 2.

By using an average on a skin volume with a thickness  $e_d$ : **Force** =  $\frac{1}{e_d} \int_{\Omega_f} T_m \mathbf{n} dV$  The dilation method also uses the equation 3. Only the calculation of flux density is different.

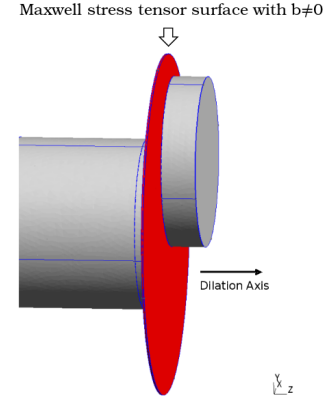


Fig. 2. Maxwell stress tensor surface

### III. TRANSLATIONAL DISPLACEMENT EXAMPLE

#### A. FEM problem

The method is illustrated with an academic problem where structured mesh is strongly difficult and is compared to a usual method meshing all positions. An axial electromagnet with an off-centre iron plate (see Fig. 3) is chosen. The aim is to calculate the force in any directions on the iron plate for several airgap lengths.

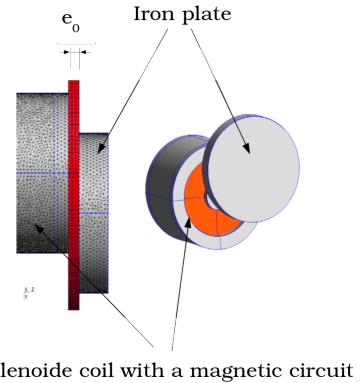


Fig. 3. Electromagnet geometry

Using the dilated subregion method, only one meshing with a given  $e_0$  value is necessary.  $e_0$  is chosen to have a good quality of mesh elements without expending the element number. Then the dilation factor  $\alpha = \frac{e_0}{e}$  is calculated,  $e$  being the real airgap length.  $\alpha$  is used in the weak formulation as explained in section II.

For three calculations corresponding to three airgap values (see Fig. 4) using the usual method, 3 meshes and 3 FEM resolutions are needed. Then,  $\text{CPU}_{\text{Time}} = 3 \text{ Meshing}_{\text{Time}} + 3 \text{ Resolution}_{\text{Time}}$ .

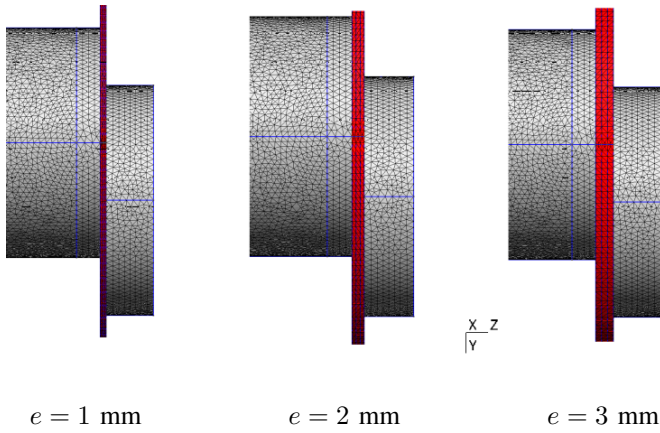


Fig. 4. Three positions with usual method

To solve the same three FEM problems using the proposed method,  $e_0 = 3\text{mm}$  is chosen (as in Fig. 3) and the three factors  $\alpha_i = \frac{e_0}{e_i}$  are calculated as:

- $\alpha_1 = 3/1 = 3$  to solve  $e = 1\text{ mm}$ .
- $\alpha_2 = 3/2 = 1.5$  to solve  $e = 2\text{ mm}$ .
- $\alpha_3 = 3/3 = 1$  to solve  $e = 3\text{ mm}$ .

In this case, as only one position is meshed,  $\text{CPU}_{\text{Time}} = \text{Meshing}_{\text{Time}} + 3 \text{ Resolution}_{\text{Time}}$ .

### B. Result

The problem has been solved for 100 airgap lengths with a linear and then a non-linear permeability  $\mu(b)$  in iron. Comparing the two methods, with and without remeshing, the maximum relative deviation on the three force components and the Coenergy is about 6% and lower than 2% in most cases as shown in Fig. 5, Fig. 6 and Fig. 7.

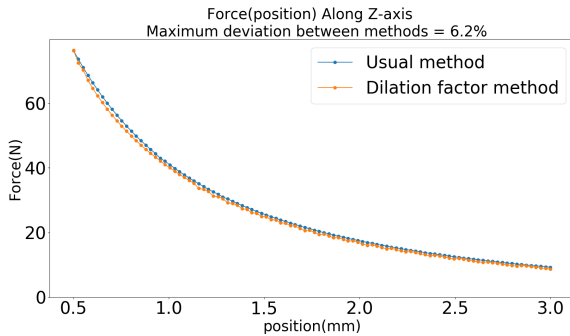


Fig. 5. Electromagnet: Force along Z-Axis

With the proposed method, only one geometry is meshed instead of 100 with the usual method both for linear and non-linear resolution. For linear problem, resolution time is the same using the two method. With the non-linear permeability  $\mu(b)$ , resolution time decrease using the dilation factor method. This is due to a better initialization of the fixed-point iterative process. Indeed, the proposed method keeps the mesh unchanged for all positions and the non-linear resolution is initialized with the solution of the previous position. Convergence criterion is reached in 2 or 3 iterations instead of 8

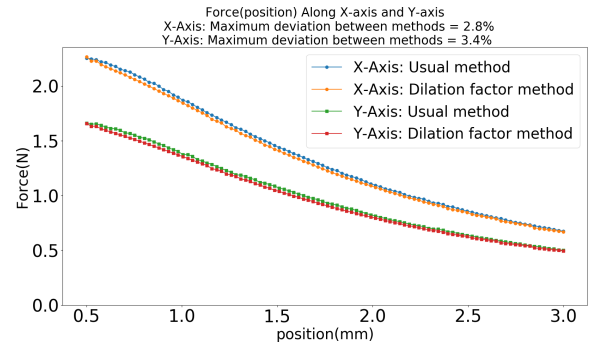


Fig. 6. Electromagnet: Force along X-Axis and Y-Axis

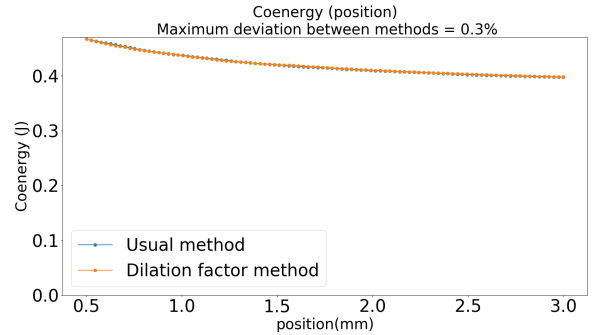


Fig. 7. Electromagnet: Co-energy

and resolution time divided by 7 in this problem (time saving depends of magnetic saturation, number of mesh elements, number of positions). Time results are summarized in TABLE I.

TABLE I  
ELECTROMAGNET: TIME RESULTS

For Nb = 100 positions	Usual method	Dilation factor method
<b>Linear Problem:</b>		
Meshing Time (per unit)	1	1/Nb
Resolution Time (per unit)	1	1
<b>Non-Linear Problem:</b>		
Meshing Time (per unit)	1	1/Nb
Resolution Time (per unit)	1	1/7

## IV. AXIAL CLAW POLE MOTOR

### A. FEM problem

The method is applied to an axial claw pole motor with double airgap shown in Fig. 8 (the winding head are taken into account in calculations but are not represented)

The torque is calculated for several airgap values. As previously, with the dilation method, only one meshing is made for an angular position with a constant airgap  $e_d$ . The torque is calculated for all the airgap values by applying several dilations factors. The non-linear resolution is initialized with the solution of the previous airgap value. The method is compared to an usual method meshing all positions and all airgap values.

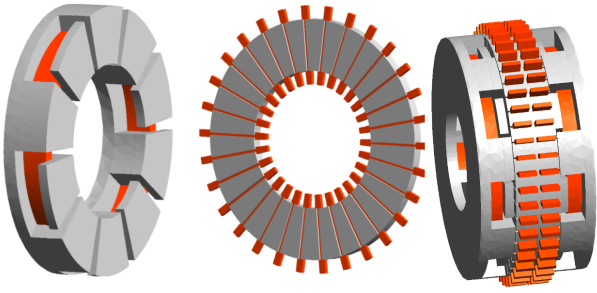


Fig. 8. Synchronous axial claw pole motor

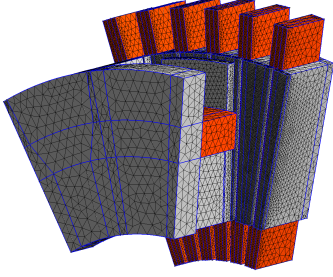


Fig. 9. Periodic model

### B. Results

The problem has been solved for 10 angular positions and 10 airgap values from 0.6mm to 1.5mm with a periodic model using a non linear permeability (see Fig. 9). The geometry is meshed for  $e_d = 1.5mm$  and dilation factor is between 1 and 2.5. All curves are plot with torque in per unit as:

$$Torque_{(pu)} = \frac{Torque_{actual}}{Torque_{Base}} \quad (6)$$

$Torque_{Base}$  being the mean value of torque for the finest airgap. Comparing the two methods, the maximum relative deviation on the mean of torque is lower than 1% as shown in Fig. 10.

The meshing time is divided by 10 (number of airgap value)

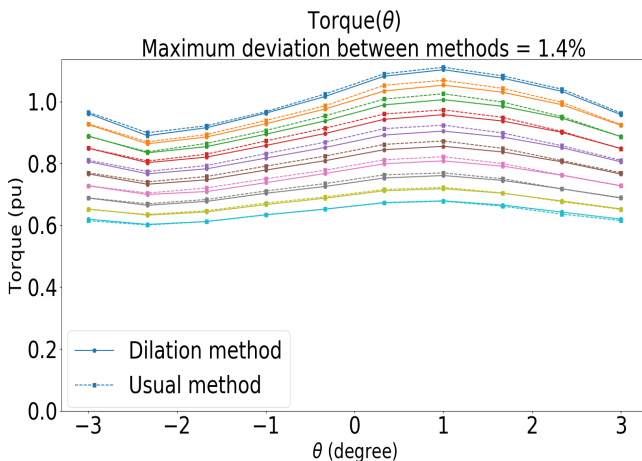


Fig. 10. Axial claw pole motor: Torque

and time resolution by 1.8 due to a better initialization of the

TABLE II  
AXIAL CLAW POLE MOTOR: TIME RESULTS

$N_{bp} = 10$ positions $N_{ba} = 10$ airgap	Usual method	Dilation factor method
<b>Meshing Time:</b>	$N_{bp}N_{ba} \cdot 17s = 28min$	$N_{bp} \cdot 17s = 2min50s$
<b>Resolution Time</b>	1h37min	54min
<b>CPU Time</b>	2h 5min	57min

iterative process. Moreover, convergence criterion is reached in 4 iterations instead of 10, because of a better starting point. Time results are summarized in TABLE II.

With too large airgap  $e_d$ , dofs number and CPU time drastically increase, which makes this method irrelevant. Moreover, dilation factor can not be too high. Indeed, a high dilation factor corresponds to a very thin airgap. In this case, meshing with usual method is impossible and comparison can not be performed. In practical, for the two applications of this paper, choosing  $1 < \alpha < 6$  (and then  $e_d = 6 \min(e)$ ) gives satisfactory results. For other devices, this criterion has to be validated case by case comparing dilation and usual method with one computation using  $\alpha = 6$ .

### V. CONCLUSION

Displacement or different airgap values has been taken into account by slightly adapting the weak formulation. The relative deviation between coenergy, force or torque is only about 6% with a reduction of CPU time. Time saving depends on magnetic saturation, on number of mesh elements and mainly on the number of calculations/positions. It could be easily applied to any configurations with planar airgap such as axial motor or maybe with thin conductors.

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