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Modeling soil moisture redistribution and infiltration dynamics in urban drainage systems

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ABSTRACT

Hydrological modeling has become a common approach for the design of stormwater management strategies which, at the same time, increasingly rely on permeable and decentralized systems to mitigate the impacts of urbanization. In most applications, little attention is however paid to the temporal variability of infiltration fluxes resulting from soil moisture redistribution between rain events. In this study, a conceptual infiltration-redistribution model is introduced to investigate the importance of the description of infiltration fluxes for the modeling of sustainable urban drainage systems (SUDS). Model verification against numerical solution of Richards equation is first conducted. Performance indicators simulated with the model for a large range of infiltration scenarios are later compared to those obtained with simpler approaches, commonly used in urban hydrology. The model is shown to replicate at a low computational cost numerical solutions of Richards equation. Regarding SUDS modeling, results indicate that a correct description of the temporal variability of infiltration fluxes (1) may be needed for some configurations to assess long-term volume-reduction efficiencies and (2) is more generally required when examining frequency-based performance indicators.

KEYWORDS: Green-Ampt; Soil; Stormwater; Water content; Redistribution; Temporal

INTRODUCTION

On-site stormwater management, in relatively simple facilities such as ponds or swales, has been widely encouraged to minimize the adverse effects of urban discharges on surface waters. In this context, hydrological modeling has received significant attention and appears to be a relevant tool for the design and performance assessment of stormwater management practices.

The efficiency of on-site stormwater management facilities largely originates from the runoff volume reduction associated with infiltration and evapotranspiration (Bressy et al. 2014). Significant pollution control may thus be achieved through the management of small events in simple permeable systems, using native soils. While an assessment of these processes is presumably needed for the development of efficient stormwater management strategies, the level of accuracy required for their description in modeling applications yet remains unclear.

The importance of continuous simulation to evaluate the performance of sustainable urban drainage systems (SUDS) is now well accepted (Fletcher et al. 2013; Salvadore et al. 2015). However, if the variability of rainfall events and initial storage conditions in such facilities is generally accounted for, the description of the infiltration process is yet seldom questioned and often relies on very simple conceptual models (Browne et al. 2008). Particularly, simulation of soil moisture variations from a rain event to another remains unusual, although changes in water content affect the infiltration process. Ignoring soil moisture redistribution generally avoids the use of complex unsaturated flow equations or the introduction of a detailed soil description and could possibly be relevant for such applications. Nonetheless, there is no evidence that this simplification does not lead to erroneous evaluation of the performance of stormwater management systems.

The Green and Ampt (1911) (GA) equation is a conceptual and physically-based model which has been widely adopted in both urban and general hydrology because of its simplicity and its ability to describe typical soil infiltration patterns, with high initial infiltration rates followed by a decrease to a steady-state value. In this model, infiltration rates depend upon the initial soil water content at the beginning of rain events which is often (especially in urban hydrology) assumed to remain unchanged from an event to another and set to a constant value such as field capacity (Abi Aad et al. 2010; Akan 2013; Heasom et al. 2006; Lee et al. 2013; Rosa et al. 2015).

Direct resolution of the unsaturated flow problem to evaluate the changes in soil moisture content (Richards' equation, 1931) would generally be cumbersome for simple applications such as the evaluation of stormwater management practices. It might as well precludes (or at least complicate) the use of optimization or sensitivity analysis techniques involving a large number of simulations (Brunetti et al. 2019), especially when a variety of conditions are considered in terms of surface boundary conditions and material properties (as discretization has to be adapted on a case by case basis). This approach can also become computationally demanding in the context of distributed models (Fatichi et al. 2016).

While relatively sophisticated alternative to Richards' equation have successfully been applied in general hydrology to simulate soil moisture redistribution, the models adopted in urban hydrology generally remain very simple. Most common approaches indeed rely on empirical equations (Lynn et al. 2018; Rossman 2010a) or assume gravity driven flow over a storage zone with homogeneous water content (Dussaillant et al. 2003; Her et al. 2017; Rossman 2010b) to account for the variability of initial moisture conditions in the GA model. Although the two aforementioned solutions have obvious limitations, their implications have rarely been

discussed. The empirical redistribution equation implemented in the US-EPA SWMM model (Rossman 2010a) for instance does not account for evapotranspiration and presumably requires calibration. Similarly, the second methods involve unrealistic averaging of the soil moisture profile and neglect the effect of capillary forces on the redistribution process (Rossman 2010b).

Among the more detailed approaches adopted in watershed hydrology, extensions of the GA infiltration equation, that simulate the redistribution of the saturation profile during non-rainfall periods (Gowdish and Muñoz-Carpena 2009; Lai et al. 2015), have been shown to perform relatively well and appear as a promising alternative for the modeling of SUDS. These models were however not tested for such applications and exhibit some limitations due to their inability to replicate near-surface water content over long simulation periods (especially for coarse soils or under the effect of evapotranspiration, as shown in Ogden and Saghafian (1997).

In this paper, an improved infiltration-redistribution scheme (I2RS) is introduced to simulate the evolution of soil moisture content and infiltration fluxes within SUDS over a wide range of surface boundary conditions and for long period applications. The I2RS model was specifically developed to overcome the difficulties associated with the resolution of Richards' equation, for which convergence is generally not granted. The objectives of the study are: (1) to evaluate the applicability of the I2RS model and (2) to perform simulations with the I2RS model so as to assess the importance of a correct description of the temporal variability of infiltration fluxes in the context of SUDS modeling. The ability of I2RS to replicate infiltration rates as well as variations of water content near soil surface, for simple case studies and over longer simulation periods, is first checked against the numerical solution of Richards' equations for different soil types and magnitude of surface fluxes. The I2RS model is later adopted to simulate a large number of stormwater infiltration scenarios and to derive several volume-reduction efficiency indicators. The consequences of using as less accurate description of the temporal variability of infiltration are then examined by comparing the results associated with the I2RS model to those computed for a simpler redistribution scheme and under the "traditional" GA assumption of constant initial moisture content.

REDISTRIBUTION OF THE GREEN-AMPT SATURATED PROFILE

THE GREEN-AMPT MODEL

The Green-Ampt (GA) model can be used to compute infiltration-rates under ponded conditions into a homogeneous soil, assuming a piston displacement of the saturated profile (e.g. saturated zone delimited by a sharp wetting front. Under ponded conditions, the infiltration rate f_{sup} at the soil surface can be computed as:

$$f_{\text{sup}} = K\left[1 + \frac{H_f + z_p}{Z_f}\right] = K\left[1 + \frac{\Delta\theta\left(H_f + z_p\right)}{F}\right] \tag{1}$$

Where: K = effective hydraulic conductivity often taken equal to saturated hydraulic conductivity K_s ([L.T^{-1}]), z_p = ponding depth ([L]), Z_f = vertical extension of the saturated profile ([L]), $\Delta\theta = \theta_s - \theta_0$, water deficit where θ_s = saturated water content and θ_0 = initial water content, $F = \Delta\theta \times Z_f$ cumulative infiltration depth in the saturated profile ([L]), and H_f = capillary pressure or suction head at the wetting front ([L]), which theoretically depends on θ_0 (Stewart et al. 2013).

WETTING PROFILE REDISTRIBUTION MODELS: STATE OF THE ART

The existence of a wetting front, as described by the GA equation (although diffuse rather than sharp), during infiltration is supported by experimental and theoretical evidences. Once saturation of the soil surface stops, this wetting front continues to move downward, while the water content in the wetting profile decreases (Gardner et al. 1970) (cf. figure 1). Several extensions of the GA equation were thus developed, as alternatives to the resolution of Richards equation, to describe this movement of water in the soil during redistribution.
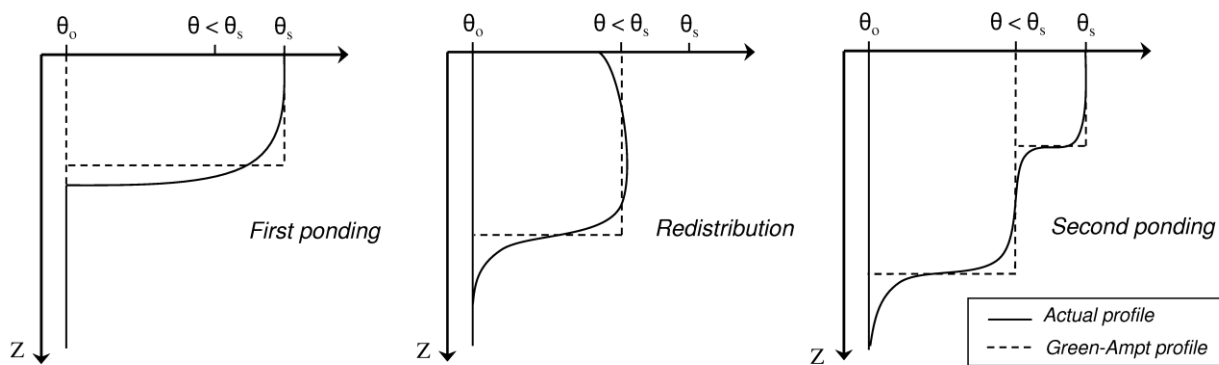

Figure 1. Redistribution of the saturated profile and creation of second saturation profile after redistribution (Ogden and Saghafian 1997)

Most of these "wetting profile redistribution models" result from vertical averaging of Darcy's law and derive from stochastic upscaling studies (Clapp et al. 1983; Dagan and Bresler 1983) or climate model applications (Milly 1986). Numerous variants of these simulation schemes have since then been proposed to simulate the movement of the wetting front during redistribution, accounting for distortion of the wetting profile (Corradini et al. 1997; Smith et al. 1993), or under the simpler piston-flow assumption (Govindaraju and Levent Kavvas 1993; Gowdish and Muñoz-Carpena 2009; Markar and Mein 1987; Ogden and Saghafian 1997).

Recent applications of the wetting profile redistribution formalism (Gowdish and Muñoz-Carpena 2009; Lai et al. 2015) consisted in extensions of the GAR (Green-Ampt with Redistribution) model developed by Ogden and Saghafian (1997). These methods were generally found to satisfactorily approximate the solutions of Richards' equation on an event basis, with relatively accurate replication of both infiltration fluxes and average water contents over the soil profile.

Systematic deviations were however observed after few redistribution cycles, requiring the introduction of corrective terms for multi-storm applications (Gowdish and Muñoz-Carpena 2009; Ogden and Saghafian 1997). Another limitation of these models is the assumption of a monotonic moisture profile (systematic decrease of water content with depth) that precludes the representation of a faster drying of superficial soil layers under the action of redistribution or evapotranspiration.

The I2RS model presented in this article relies on a more general formulation of the redistribution process to simulate the variations of soil water content under the effect of gravity, capillary forces and evapotranspiration. The redistribution procedure is based on a specific component of the Event Based simulation Model (EBM) of Milly (1986) and coupled with the GA equation. Although adapted from the work of Milly (1986), the I2RS model differs in several respects from the original contribution:

- The influence of soil moisture conditions on infiltration fluxes was not accounted for in Milly's EBM, which relied on a simple Philips equation with constant sorptivity parameters (thus exhibiting a similar behavior for each rain event). Here, a complete and consistent coupling between the GA equation and Milly's redistribution scheme is introduced to account for the effect of redistribution.

- Because the purpose of the EBM model was the quantification of water and energy fluxes at the earth-atmosphere interface, a relatively complex description of evaporation based on the desorptivity concept was adopted. The effect of this process on the moisture profile was however very roughly accounted for (setting near-surface water content to a constant negligible value), as the model was not intended to provide realistic estimates of near-surface water contents. Conversely, the infiltration-redistribution scheme introduced afterwards incorporates a rather simple representation of evapotranspiration that nonetheless affects soil moisture content A further modification is also implemented to provide a more accurate description of near surface water content

The I2RS model shares similarities with previous GAR approaches (Gowdish and Muñoz-Carpena 2009; Ogden and Saghafian 1997), namely the coupling of the GA equation with a simplified method for the redistribution of the saturated profile. Two improvements are however introduced:

- I2RS relies on a specific procedure to simulate non-monotonic moisture profile associated with the faster drying of superficial soil layers and thus provides better water content estimates for the application of the GA equation

- In connection with the aforementioned procedure, I2RS incorporates a simple parameterization of evapotranspiration that interacts with near surface water content

(It is worth mentioning that I2RS exhibits two features of the more sophisticated and conceptually different "finite water content approaches" recently developed by Ogden et al. (2015) and Talbot and Ogden (2008), although it retains the relative simplicity of the GAR approach).

THE IMPROVED INFILTRATION-REDISTRIBUTION SCHEME (I2RS)

The soil profile is described as a homogeneous domain, consisting of n fronts delimitating $n+1$ soil layers (subscript n being associated with the uppermost front). Each front $i_{\in[1,n]}$ is characterized by an uniform water content θ_i and a depth of penetration denoted as Z_i (cf. figure 2). The soil layer located beneath the lowermost front is assumed to have a constant water content θ_0 and an infinite extension.
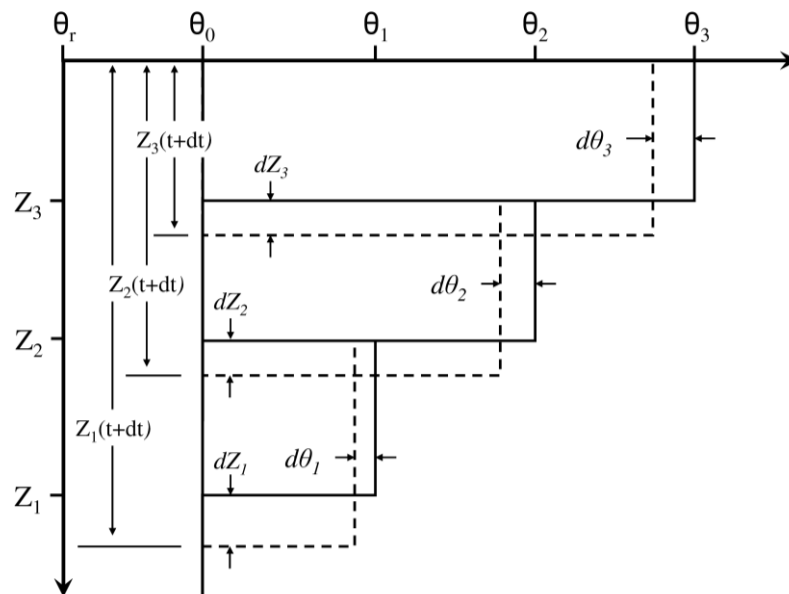

Figure 2. Water profile redistribution as represented in the conceptual multi-front redistribution model

The movement of fronts n to 1 is governed by a set of ordinary differential equations for θ_i and Z_i. For $\theta_n = \theta_s$, these equations revert to the GA infiltration equation.

The approach adopted to carry out the coupling between the redistribution model and the GA equation is adapted from previous studies (Gowdish and Muñoz-Carpena 2009; Ogden and Saghafian 1997; Smith et al. 1993). A separation between redistribution (or "hiatus") periods and potential surface saturation periods is introduced. A "hiatus" period starts once the water inflow in the system q_e becomes inferior to K_s and ponded water is completely infiltrated. It stops when water inflow q_e becomes greater than K_s (e.g. when saturation can potentially occur). A new front $n+1$ is generated at the end of each hiatus period. While the other fronts are allowed to keep on redistributing, this new front is treated as the saturated profile of the GA equation. Once saturation stops, the saturated profile redistributes along with underlying fronts.

The sole redistribution of the profiles generated with the GA equation (often exhibiting a large extension and thus providing a very rough description of the moisture domain at the end of saturation period) cannot account for a faster water content decrease near soil surface. A specific procedure is thus adopted to allow for a more detailed representation of the uppermost layers of soils and to enable moisture profile inversion under the effect of redistribution and evapotranspiration.

DERIVATION OF REDISTRIBUTION EQUATIONS

In the original EBM model an expression of the downward flux q_i in each uniform θ_i region (e.g. between Z_{i+1} and Z_i) is derived from the continuity equation (Milly 1986):

$$\frac{\partial \theta_i}{\partial t} = -\frac{\partial q_i}{\partial z} \Rightarrow \frac{\partial}{\partial z}\left(\frac{\partial \theta_i}{\partial t}\right) = -\frac{\partial}{\partial z}\left(\frac{\partial q_i}{\partial z}\right) = 0 \tag{2}$$

Because the water content θ_i is assumed to be uniform between Z_{i+1} and Z_i, the second derivative of q_i with respect to z is zero, and the following expression is thus adopted for $q_i(z)$:

$$q_i(z) = q_{in,i} + \left(\frac{z - Z_{i+1}}{L_i}\right) \times \Delta q_i \tag{3}$$

Where: $L_i = Z_i - Z_{i+1}$, Δq_i = a function to be determined and $q_{in,i}$ = water flux entering the uniform θ_i region = $q(Z_{i+1})$ with $q_{in,n}$ = water inflow q_e (= surface infiltration f_{sup} during redistribution) for the uppermost front ($i = n$) and $q_{in,i\in[1,n-1]} = K(\theta_i)$ otherwise. It may immediately be noted that, unless $\Delta q_i = 0$, a contradiction arises between equations (3) and Darcy's law which predicts a uniform flux over a uniform θ region. As stated by Milly (1986), this inconsistency however originates from our representation of the moisture profile, which is necessarily unrealistic and should only be interpreted in terms of averaged water contents.

The expression for the remaining term Δq_i may be obtained from a depth integration of Darcy's law and equation (3) between Z_{i+1} and Z_i:

$$\frac{1}{L_i}\int_{Z_{i+1}}^{Z_i} q_i(z)\,dz = \frac{1}{L_i}\int_{Z_{i+1}}^{Z_i}\left(-D(\theta)\frac{\partial \theta}{\partial z} + K(\theta)\right)dz = K(\theta_i) - \frac{1}{L_i}\int_{Z_{i+1}}^{Z_i} D(\theta)\frac{\partial \theta}{\partial z}\,dz \tag{4}$$

Where: $D(\theta)$ is the soil water diffusivity = $K(\theta) \times d\Psi/d\theta$ ($[L^2.T^{-1}]$) with Ψ the soil matric suction ($[L]$). Again, because the model introduces discontinuities in the moisture profile $\theta(z)$, hypotheses regarding $D(\theta)$ are needed to carry out the integration of the diffusive term in equation 4. In the case of the GA model ($i = n$ and $\theta_n = \theta_s$ in equation 4), $D(\theta)$ follows a delta-Dirac distribution and the diffusive term in equation (4) can be expressed as a function of the mean diffusivity $\overline{D}$ estimated from sorptivity measurements. $\overline{D}$ may be related to the integral of $D(\theta)$ from θ_s to θ_{n-1} through a shape factor $\beta \geq 1$ which accounts for the deviation from a sharp wetting front (Stewart et al. 2013; Triadis and Broadbridge 2012; White and Sully 1987). Following a similar approach, the contribution of capillary forces in equation (4) becomes:

$$-\frac{1}{L_i}\int_{Z_{i+1}}^{Z_i} D(\theta)\frac{\partial \theta}{\partial z}\,dz = \frac{\overline{D}}{L_i} = \frac{1}{L_i \beta}\int_{\theta_i}^{\theta_{i+1}} D(\theta)\,d\theta \tag{5}$$

Equation (4) may thus be rewritten as follow:

$$\frac{1}{L_i} \int_{Z_{i+1}}^{Z_i} q_i(z)\,dz = K(\theta_i) + K_s \frac{G(\theta_i,\theta_{i-1})}{\beta L_i} \tag{6}$$

Where $G(\theta_i,\theta_{i-1})$ represents the capillary drive between two fronts (Morel-Seytoux et al. 1996; Ogden and Saghafian 1997) ([L]), which can be calculated from soil water retention curve $\Psi(\theta)$ and hydraulic conductivity function $K(\theta)$:

$$G(\theta_i,\theta_{i-1}) = \frac{1}{K_s} \int_{\theta_i}^{\theta_{i+1}} D(\theta)\,d\theta = \frac{1}{K_s} \int_{\theta_i}^{\theta_{i+1}} K(\theta)\frac{\partial \psi}{\partial \theta}\,d\theta \tag{7}$$

Combination of equation (6) and (7) with an integrate form of (3) therefore yields the following expression for Δq_i:

$$\frac{\Delta q_i}{2} = -q_{in,i} + K(\theta_i) + K_s \frac{G(\theta_i,\theta_{i-1})}{\beta L_i} \tag{8}$$

The net downward flux at the ith front is partitioned between the elongation dZ_i/dt of the wetted profile and the flux $q_{in,i-1}$ across the wetting front supplying the underlying uniform θ_{i-1} region. An expression of this term can be obtained from equation (3) for $z = Z_i$ (Milly 1986):

$$q_i(Z_i) = q_{in,i} + \Delta q_i = (\theta_i - \theta_{i-1})\frac{dZ_i}{dt} + q_{in,i-1} \tag{9}$$

The differential equation for the water content θ_i associated with each front can then be expressed from a depth integration of the continuity equation using equation (3):

$$\frac{1}{L_i} \int_{Z_{i+1}}^{Z_i} \frac{\partial \theta_i}{\partial t}\,dz = -\frac{1}{L_i} \int_{Z_{i+1}}^{Z_i} \frac{\partial q_i}{\partial z}\,dz \Rightarrow \frac{d\theta_i}{dt} = -\frac{\Delta q_i}{L_i} \tag{10}$$

Differential equations associated with θ_i and Z_i can respectively be obtained substituting equation (8) in (10) and (9). Given that $q_{in,i} = K(\theta_i)$ for $i<n$, redistribution equations associated with soil fronts n-1 to 2 are thus:

$$\frac{d\theta_i}{dt} = -2K_s \frac{G(\theta_i,\theta_{i-1})}{\beta L_i^2} \bigg|_{i<n} \tag{11}$$

$$(\theta_i - \theta_{i-1})\frac{dZ_i}{dt} = K(\theta_i) - K(\theta_{i-1}) + 2K_s \frac{G(\theta_i,\theta_{i-1})}{\beta L_i} \bigg|_{i<n} \tag{12}$$

Despite differences related to the introduction of the capillary drive term, these expressions are equivalent to those of Milly (1986). Similarly, equations associated with the uppermost front ($i = n$) can be obtained from equation (8) to (10) noting that $q_{in,n} = f_{sup} \neq K(\theta_n)$ and $q_{in,n-1} = K(\theta_{n-1})$. A modification is introduced to account for the effect of evapotranspiration on the uppermost uniform θ region. Following the approach of Markar and Mein (1987) redistribution equations can be rewritten as follow for $i = n$:

$$Z_n \frac{d\theta_n}{dt} = 2\left[f_{\text{sup}} - K(\theta_n) - K_s \frac{G(\theta_n,\theta_{n-1})}{\beta Z_n} \right] - ET(\theta_n)_{(i=n)} \tag{13}$$

$$(\theta_n - \theta_{n-1})\frac{dZ_n}{dt} = 2K(\theta_n) + 2K_s \frac{G(\theta_n,\theta_{n-1})}{\beta Z_n} - f_{\text{sup}} - K(\theta_{n-1})_{(i=n)} \tag{14}$$

Here the assumption that evapotranspiration preferentially affects the uppermost constant θ region allows for a relatively simple parameterization, avoiding hypothesis regarding roots distribution.

Here, $ET(\theta_n)$ is computed from the Penman-Monteith potential evapotranspiration ET_p using the soil moisture index (Morbidelli et al. 2011):

$$ET(\theta_n) = \begin{cases} ET_p & if & \theta_n > \theta_{fc} \\ SMI \times ET_p & if & \theta_{fc} \geq \theta_n \geq \theta_{wp} \\ 0 & if & \theta_{wp} > \theta_n \end{cases} \quad with \quad SMI = \frac{\theta_n - \theta_{wp}}{\theta_{fc} - \theta_{wp}} \tag{15}$$

Where SMI = soil moisture index, θ_n = water content associated with the uppermost front, θ_{wp} = wilting point (bulk water content at -15 bar pressure) and θ_{fc} = field capacity estimated from the hydraulic conductivity function with $K(\theta_{fc}) = 0.01$ cm/d.

Finally, redistribution equation associated with the lowermost front ($i = 1$) depends on the boundary condition specified at the end the soil domain. In the original EBM model, a maximum wetting front penetration depth Z_{max} is introduced to limit the extension of the simulation domain (Milly 1986). Corresponding redistribution equation is however not defined. Here, a "free drainage" boundary condition is adopted at the bottom of the soil profile (the groundwater is assumed to lie far below the domain of interest). When the extension of the lowermost front Z_1 reaches Z_{max}, equations (12) or (14) no longer holds and $dZ_1/dt = 0$. The flux across Z_{max} is therefore set to $q_{in,0} = K(\theta_1)$ and equations (11) and (13) become:

$$L_1 \frac{d\theta_1}{dt}\Big|_{n>1} = 0 \tag{16}$$

$$Z_1 \frac{d\theta_1}{dt}\Big|_{n=1} = f_{\text{sup}} - K(\theta_1) - ET(\theta_1) \tag{17}$$

(Note that $d\theta_1/dt = 0$ for $n>1$ does not imply that water content remains constant at the bottom of soil domain since overlying fronts move down and progressively reach $Z = Z_{max}$. In other words, $\theta_1(t)$ is a simple step function providing a rough description of moisture content at the bottom of the soil profile)

INFILTRATION EQUATIONS

As long as $q_e \leq K_S$, infiltration is not limited by soil moisture conditions and $f_{sup} = q_e$. The GA-based infiltration method is implemented as soon as q_e becomes greater than K_S. Before saturation occurs $f_{sup} = q_e$ ($>K_S$) and both the penetration depth Z_{n+1} and the water content θ_{n+1} of the newly formed profile are "undefined" (Ogden and

Saghafian 1997); I2RS hence simply tracks the cumulative infiltration volume F_{n+1} associated with this fictitious front without affecting a value to θ_{n+1} and Z_{n+1} until saturation occurs.

Once the soil is saturated, $\theta_{n+1} = \theta_s$ and Z_{n+1} may be calculated from the cumulative infiltration volume in the saturated profile F_{n+1}. Given the condition $q_{in,n} = K(\theta_n)$ at the lower end of the saturated profile:

$$F_{n+1} = \int_{t_{start}}^{t} \left[q_e - K(\theta_n) - ET(\theta_n) \right] dt = Z_{n+1} \times (\theta_s - \theta_n) \qquad (18)$$

Where t_{start} is the time associated with the end of previous hiatus period (i.e. when q_e becomes larger than K_s).

The procedure of Corradini et al. (1997) can then be adopted to compute time to ponding t_p and cumulative infiltration volume $F_{n+1}(t_p)$, with the following conditions in equation (13): $d\theta_{n+1}/dt = 0$ (as $\theta_{n+1} = \theta_s$) and $ET(\theta_{n+1}) = 0$ (evaporation no longer applies to the soil profile under ponded conditions). Corresponding equation (18) may then be solved for unsteady rainfall intensities by verifying at each simulation time step whether $F_{n+1}(t_p)$ has been reached.

$$\int_{t_{start}}^{t_p} \left[q_e - K(\theta_n) - ET(\theta_n) \right] dt = Z_{n+1} \times (\theta_s - \theta_n) = \frac{K_s}{\beta} \frac{G(\theta_s, \theta_n)}{q_e - K_s} \times (\theta_s - \theta_n) \qquad (19)$$

Once saturation has occurred, equation (13) becomes similar to equation (1) and precisely reverts to the GA formula for $\beta = 1$.

$$f_{sup} = K_s \left[1 + \frac{(\theta_s - \theta_n) G(\theta_s, \theta_n)}{\beta F_{n+1}} \right] \qquad (20)$$

In this study, β appears in both redistribution and infiltration equations. This coefficient reflects the deviation from the GA assumption of sharp rectangular moisture profile and theoretically depends on soil type, the magnitude of water fluxes at soil surface and moisture content below the wetting front (Corradini et al. 1997; White and Sully 1987). In the case of the saturated profile, β may be expected to range between 1 and $\pi/2$ but literature results suggest that using $\beta = 1.1$ would be a reasonable approximation for most soils (Morel-Seytoux et al. 1996; Warrick and Broadbridge 1992; White and Sully 1987). In this study, β is thus assumed to be invariant and is set to 1.1. (The relevance of this correction and its implications as compared to the traditional GA assumption $\beta = 1$ are discussed later on).

Under ponded conditions surface water elevation z_p is added to the capillary term $G(\theta_s, \theta_n)/\beta$ in coherence with equation (1). Equation (20) hence becomes:

$$f_{sup} = K_s \left[1 + (\theta_s - \theta_n) \left(\frac{G(\theta_s, \theta_n)/\beta + z_P}{F_{n+1}} \right) \right] \qquad (21)$$

An analytical expression of the integral term in (7) can be derived from the Brooks and Corey (1964) (BC) equations for $\Psi(\theta)$ and $K(\theta)$. Following the approach of Morel-Seytoux et al. (1996) who formulated an expression of G over the whole capillary pressure range, and introducing the relative water content $\Theta = (\theta - \theta_r)/(\theta_s - \theta_r)$, equation (7) may be rewritten as:

$$G\left(\Theta_i, \Theta_{i-1}\right) = \Psi_b \times \delta_1 + \frac{\Psi_b}{\lambda}\left(\frac{\Theta_i^{3+1/\lambda} - \Theta_{i-1}^{3+1/\lambda}}{3+1/\lambda}\right) \qquad (22)$$

Where Ψ_b = air entry-pressure ([L]) and λ = dimensionless pore size distribution index (parameters of the BC relationship). The use of a Kronecker symbol δ_1 in (22) is related to the discontinuity in the water retention curve for $\Theta = 1$ (e.g. $\theta = \theta_s$). Here, the limit of $G(\theta_s, \theta_{n-1})$ as θ_{n-1} approaches θ_s is Ψ_b instead of 0 in the original GAR model (Ogden and Saghafian 1997). Equation (22) also exactly reverts to the maximum capillary drive as defined by Morel-Seytoux et al. (1996) for $\theta_{n-1} = \theta_r$ (while accounting for the effect of soil moisture conditions on the wetting front potential when $\theta_{n-1} > \theta_r$). If water inflow q_e falls below K_s before surface saturation occurs, water content and penetration depth associated with the uppermost front (which has no physical significance) cannot be evaluated Ogden and Saghafian (1997). In this study, the water volume associated F_{n+1} with this "undefined" front is redistributed over the soil profile assuming a water content increase in underlying θ regions. The latter is computed as $(\theta_s - \theta_n) \times F_{n+1}/F_{n+1}(t_P)$ where $F_{n+1}(t_P)$ represents the theoretical time-to-ponding volume $F_{n+1}(t_P)$ (calculated with an average q_e value in the right hand term of equation (19)).

MODEL IMPLEMENTATION

Redistribution equations (11) to (17) are solved applying semi-implicit Euler method whereas a second order Runge-Kutta method is adopted to compute infiltration during saturation periods. An adaptive step-size control is implemented to solve both infiltration and redistribution equations. Similarly to the work of Struthers et al. (2006), a water content and a penetration depth "tolerance" are introduced to avoid computational errors during the redistribution process. Two profiles i and i-1 are thus merged when their water content becomes similar ($|\theta_i - \theta_{i-1}| < 10^{-4}$) or when a given front overlaps the underlying one ($|Z_i - Z_{i-1}| < 10^{-1}$ mm).

The procedure adopted to simulate moisture profile inversion (faster water content decrease near soil surface) period is described hereafter. During hiatus periods, a new fictitious front is generated at each time step near soil surface unless the total number of fronts in the first meter of soil exceeds a given value $n_{s,max}$. The thickness of the newly formed soil layer (denoted as n+1) is set to a small and arbitrary value $z_{s,ini}$ and its water content to θ_n. Depending on underlying moisture conditions and evapotranspiration fluxes, this thin layer of soil may either be merged at the end of the time step or result in a decrease of θ_{n+1} below θ_n. Here, $n_{s,max} = 4$ is found to be reasonable tradeoff between computational effort and model performance (as shown in Table S5). The effect of $z_{s,ini}$ on infiltration-rates (Table S6 and S15-16) tends to be negligible unless considering values greater than 15cm in which case mode moisture profile inversion can no longer occur. Here $z_{s,ini}$ is thus set to 10cm.

MODEL EVALUATION

GENERAL METHODOLOGY

Testing against the solutions of Richards' equation

Validation of redistribution models against numerical solutions of Richards' equation has generally been carried out for theoretical case studies consisting in successions of rain pulses (Lai et al. 2015; Ogden and Saghafian 1997). In this paper, a similar approach is first adopted to evaluate the I2RS model under simple hydrological forcing scenarios, for different soil types (cf. Table 1). A more complete evaluation is later conducted for long simulation periods, considering a variety of surface boundary conditions (water inflow and ponding depths) representative of those prevailing in SUDS. The characteristics of these tests are provided in corresponding subsections. (verification tests and long period applications).

Soil n°	Soil type	K_s (mm/h)	θ_r	θ_s	Ψ_b (mm)	λ	θ_{wp}	θ_{fc}
1	Sandy Clay	1.2	0.109	0.430	291.7	0.168	0.222	0.329
2	Silty Clay Loam	1.5	0.040	0.471	326.7	0.151	0.211	0.340
3	Clay Loam	2.3	0.075	0.464	258.9	0.194	0.188	0.317
4	Sandy Clay Loam	4.3	0.068	0.398	280.8	0.250	0.137	0.244
5	Silt Loam	6.8	0.015	0.501	207.6	0.211	0.136	0.284
6	Loam	13.2	0.027	0.463	111.5	0.220	0.116	0.251
7	Sandy Loam	25.9	0.041	0.453	146.6	0.322	0.085	0.201
8	Loamy Sand	61.1	0.035	0.437	86.9	0.474	0.047	0.141

Table 1. Hydraulic parameters for selected soil types (Brooks and Corey model from Rawls et al. (1982))

For each test, Richards equation is solved numerically using the HYDRUS-1D software (Šimůnek et al. 2009). The implementation of a free drainage boundary condition in HYDRUS-1D results in convergence difficulties for long-term applications when considering limited soil domain extensions (few meters) due to inconsistence in surface and bottom boundary conditions. The evaluation is thus conducted for a 40m deep soil profile with free drainage condition. Obviously, such a condition is very unlikely to be met in real world applications: this choice is solely justified by the need to avoid numerical problems as the study mostly focuses on infiltration rates and near surface water content. The soil column is discretized into 9001 nodes, with 2000 nodes for $z < 2$m. As opposed to the I2RS model, HYDRUSD requires root-depth Z_{ROOT} and root density to be specified to compute evapotranspiration. A 25cm value is first adopted assuming uniform root distribution (the importance of this choice is discussed later on, adding Z_{ROOT} to the parameters identified for sensitivity tests). Water stress reduction in the root zone is accounted for using the standard Feddes function (Šimůnek et al. 2009). The soil cover fraction used in HYDRUS to partition evaporation and transpiration is set to 85%.

The accuracy of infiltration-rates f_{sup} predictions is evaluated at a 1 min time step from two performance criteria: a benchmark-based index *BE* (Schaefli and Gupta 2007) derived from the Nash-Sutcliffe coefficient (NS) and a volumetric efficiency *VE* (Criss and Winston 2008).

While the original NS coefficient simply reflects the performance of the model as compared to the prediction associated with the use of mean observed data, BE allows for the introduction of more sophisticated "benchmark models". In this study, a constant rate infiltration model $f_{sup} = K_s$ (referred to as CR) is introduced to compute BE:

$$BE = 1 - \frac{\sum_{i=1}^{N}\left(Y_{ref,i} - Y_{sim,i}\right)^2}{\sum_{i=1}^{N}\left(Y_{ref,i} - Y_{ben,i}\right)^2} \tag{21}$$

Where $Y_{ben,i}$ = infiltration-rate value calculated with the CR model, $Y_{ref,i}$ = infiltration-rate calculated with HYDRUS-1D, $Y_{sim,i}$ = infiltration-rate predicted by the I2RS model, i = subscript associated with each time-step of the simulation series and N = length of the time series. As for the NS criterion, BE ranges from $-\infty$ to 1 and $BE > 0$ indicates the I2RS model has a larger explanatory power than a simpler constant infiltration model, which neglects the variability of f_{sup} ($BE = 1$ implies perfect fit between reference and simulated time series).

The advantage of BE lies in its ability to express the benefits associated with the representation of the temporal variability of infiltration rates.

This performance metrics is complemented by the more straightforward measure of the deviations in infiltrated volumes (differences between $Y_{ref,i} \times dt$ and $Y_{sim,i} \times dt$) provided by VE which represents "the fraction of water [infiltrated] at a proper time" (Criss and Winston 2008):

$$VE = 1 - \frac{\sum_{i=1}^{N}\left|Y_{ref,i} - Y_{sim,i}\right|}{\sum_{i=1}^{N}Y_{ref,i}} \tag{22}$$

The performance for water content predictions is evaluated from the mean absolute percentage error (MAPE) computed for both (1) the water content at the soil surface θ_{TOP} and (2) the average water content associated with the first 25cm of soil θ_{25cm}:

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{\theta_{ref,i} - \theta_{sim,i}}{\theta_{ref,i}}\right| \tag{23}$$

Where: $\theta_{ref,i}$ = water content value (θ_{TOP} or θ_{25cm}) computed with HYDRUS-1D; $\theta_{sim,i}$ = water content value simulated by the I2RS model.

For both surface and average water contents, coefficient of determination R^2 is additionally calculated so as to verify that the temporal trends simulated by the infiltration-redistribution model are consistent with the HYDRUS-1D solution of Richards' equation.

Comparison to the results produced by simpler models

Infiltration rates simulated with the I2RS model for long period applications are compared to those computed from two basic modeling schemes (in addition to the CR benchmark model). The first one consists in the association of the GA equation with a rudimentary redistribution model, where the soil domain is represented as a 2-m deep homogeneous water content reservoir drained at $K(\theta)$ and affected by evapotranspiration. In this case, water content variations can be described by equation (17). This approach, similar to the one adopted in EPA's SWMM for rain garden modeling (Rossman 2010b), is later denoted as GA-RR (Rudimentary Redistribution).

The second option, referred to as GA-θ_0, is based on the traditional GA scheme, assuming that surface water content immediately switches to a constant value θ_0 when saturation stops (neither redistribution nor evapotranspiration process are represented). Here, θ_0 is alternatively set to field capacity θ_{FC} and to the initial water content θ_{OPT} resulting in the best fit to HYDRUS-1D.

VERIFICATION TESTS

Tests description

The initial verification is performed for the 8 USDA soil classes for which four successive rainfall pulses (cf. table 2) are applied to soils initially at field capacity, assuming that rainfall excess immediately turns into runoff. Rainfall intensities are adjusted to cause saturation after 15min and thus deliberately set very high for some soil classes (such inflow-rates nevertheless remain plausible in the case of stormwater management practices which may receive very large amounts of water). It may be noted that saturated hydraulic conductivity values associated with some soil classes may be regarded as low for conventional stormwater infiltration practices; these configurations are nonetheless considered here as the infiltration of a limited fraction of the volume collected by a facility may still provide some runoff control.

Soil n°	Adjusted intensity	Pulse n°1		Pulse n°2		Pulse n°3		Pulse n°4	
		Date	Duration	Date	Duration	Date	Duration	Date	Duration
1	14.5 mm/h								
2	19.6 mm/h								
3	22.8 mm/h								
4	33.3 mm/h	T = 0h	1h	T = 6h	30min	T = 7h	30min	T = 9h	1h
5	43.9 mm/h								
6	47.8 mm/h								
7	83.3 mm/h								
8	121.4 mm/h								

Table 2. Theoretical rainfall sequences used for model verification

The effect of initial moisture conditions and rainfall intensities on model performance was addressed through simple sensitivity tests conducted for soil n°5. Both parameters were found to have a limited incidence on model performance. Detailed results are provided as supplementary data (Table S2 to S3).

Results

The accuracy of the I2RS model for the prediction of infiltration rates is generally very good (cf. figure 3 and S1-8), with *BE* and *VE* values systematically greater than 0.99 (detailed results can be found in Table S1). The introduction of β in equations (18) to account for the deviation from a rectangular moisture profile (GA assumption) is found to improve model performance (cf. Table S4): the error associated with infiltration volumes for the different rainfall pulses indeed ranges from -1.4% to 2.2% (with an average over the 4 rain pulses of 0.4%) for $\beta = 1.1$ vs. -2.4% to 4.8% (with an average of 3.1%) for $\beta = 1$. This overestimation of infiltration rates observed for $\beta = 1$ is consistent with several findings regarding the GA equation(Barry et al. 1993; Freyberg et al. 1980; Triadis and Broadbridge 2012). As opposed to previous GAR-models (Ogden and Saghafian 1997), deviations associated with cumulative infiltration volumes here do not significantly increase after successive redistribution cycles.
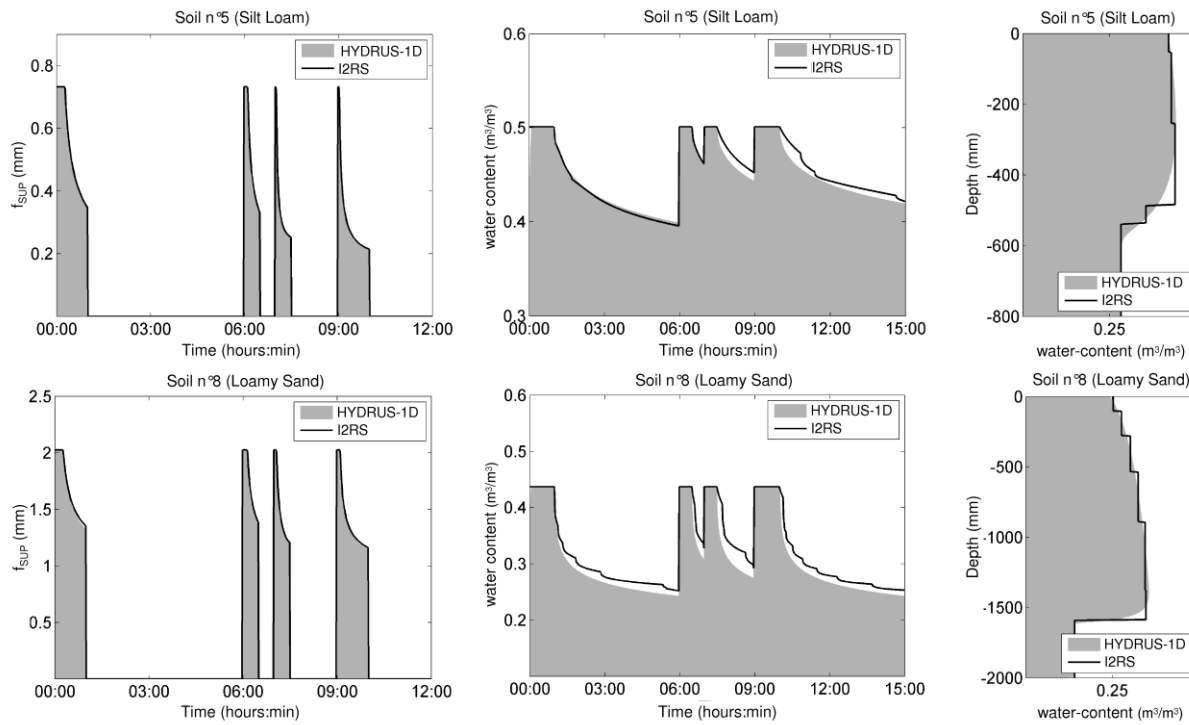

Figure 3. Simulation results for four successive rain pulses (infiltration-rates f, surface water content and moisture profile at the end of rainfall sequence) for two soil types

Although systematically overestimated for coarsest soils, surface water content θ_{TOP} predictions remain very satisfactory with MAPE value ranging from 0.6% for soil n°1 to 4.8% for soil n°8 whereas R^2 systematically exceeds 0.96. As for the GAR model (Ogden and Saghafian 1997), the aforementioned overestimation of θ_{TOP} is mainly related to the distortion of the actual wetting profile near soil surface during redistribution. These errors however remain very acceptable due to the ability of the I2RS model to simulate moisture profile inversion (as shown in figure 3). Water content predictions tend to become even more consistent when considering the average water content over the first 25cm of soil, with MAPE values systematically lower than

1.0% and R^2 greater than 0.99 for the 8 soil classes (deviations were additionally found to decrease for extensions larger than 25cm as shown in Table S19)

LONG PERIOD SIMULATIONS

Tests description

Long-term simulations are conducted for 7 soil types under realistic hydrologic conditions (soil n°8 could not be considered here due to convergence problems in the resolution of Richards' equation). For each scenario, surface boundary conditions are adjusted to simulate a simple storage unit collecting runoff originating from urban surfaces. Because SUDS with more pervious soils can accommodate larger runoff volumes, a ratio b between the surface area of the facility and the drainage area is introduced for the definition of stormwater infiltration scenarios. Maximum ponding depth over soil surface h_{MAX} is initially set to 20cm (a realistic value for SUDS as ponding depth is often limited for better site integration). The area ratio b associated with each soil type (table 3) is adjusted to result in an approximately 80% annual runoff volume reduction, which is a relatively usual target in the context of on-site stormwater management (Sage et al. 2015). These simulations clearly do not cover the diversity of possible SUDS design; they rather provide realistic surface boundary conditions so as to evaluate the ability of the I2RS model to simulate infiltration-rates and near surface water content under such conditions.

Simulations are conducted for a 1 year period (June 2007 to June 2008) using rainfall records (5-min time-step) collected on different stations of the Hauts-de-Seine department near Paris (additional details can be found in Sage (2016) p. 330). The cumulative rainfall volume for that period (613 mm) is close to the average annual value observed in Paris region. Runoff time-series are under the assumption that rainfall is immediately and totally converted into runoff (e.g. no losses and instantaneous transfer on drainage area). Evapotranspiration is computed from daily Penman-Monteith potential evapotranspiration values ET_{REF} calculated with the French meteorological service method (Choisnel 1988) in the Hauts-de-Seine department. Potential evapotranspiration ET_P rates are adjusted from ET_{REF} using a simple crop coefficient K_{CROP} ($ET_P = K_{CROP} \times ET_{REF}$) set to 0.75 (a value consistent with those reported in Romero and Dukes (2015) for turfgrass).

A simple "one-factor-at-a-time" sensitivity analysis was performed for soil n°1 and n°5 to assess the effect of K_{CROP} and Z_{ROOT} (additional root depth parameter introduced for the resolution of Richards' equation) and of the design parameters b and h_{MAX} on the performance of the I2RS model. Overall, the accuracy of infiltration rates prediction was not significantly affected by b (infiltration to drainage area ratio) and h_{MAX} (maximum ponding depth). Results associated with these two parameters are provided as supplementary data (Table S7 and S8). The effect of evapotranspiration related parameters is presented later on. Finally, the benefits of an accurate description of near surface water content (in relation with the introduction of an evapotranspiration term in redistribution equations and the procedure implemented to simulate moisture profile inversion) are briefly illustrated by disabling evapotranspiration and setting discretization parameter n_{smax} to 0 in the I2RS model.

Overall-model performance

As indicated in table 3, the I2RS model provides very satisfactory estimates of infiltration rates over the 1-year simulation period, resulting in high efficiency coefficient values for all soil types. Comparison to verification results evidences a minor decrease of the two efficiency coefficients BE and VE. This slightly lower model performance believably results from a less accurate estimation of surface and average water contents originating from the multiplication of redistribution cycles as well as the introduction of the evapotranspiration term. Water content predictions nonetheless remain very satisfactory for both θ_{TOP} and θ_{25cm}.

Soil n°	b (%)	f_{sup}		θ_{TOP}		$\theta_{25\text{-}cm}$	
		BE	VE	MAPE	R²	MAPE	R²
1	8.0	0.97	0.97	0.9 %	0.98	0.6 %	0.99
2	7.0	0.97	0.97	0.9 %	0.98	0.9 %	0.98
3	6.0	0.98	0.98	1.4 %	0.98	0.9 %	0.99
4	4.5	0.98	0.97	1.3 %	0.99	0.9 %	0.99
5	4.0	0.98	0.97	1.9 %	0.98	1.2 %	0.99
6	3.0	0.97	0.97	3.0 %	0.97	1.7 %	0.98
7	2.5	0.97	0.97	3.3 %	0.96	2.0 %	0.98

Table 3. Evaluation results for long-period applications

Detailed examination of the residuals between the outputs of I2RS and HYDRUS-1D (figure 4 and S9-15) indicates that infiltration-rates tend to be underestimated at early stages of infiltration (when f_{sup} largely exceeds K_s). As a consequence, the time required to infiltrate the water stored at soil surface is conversely slightly overestimated; negative residuals are therefore observed where HYDRUS-1D predicts that surface saturation has already stopped ($f_{sup} = 0$).
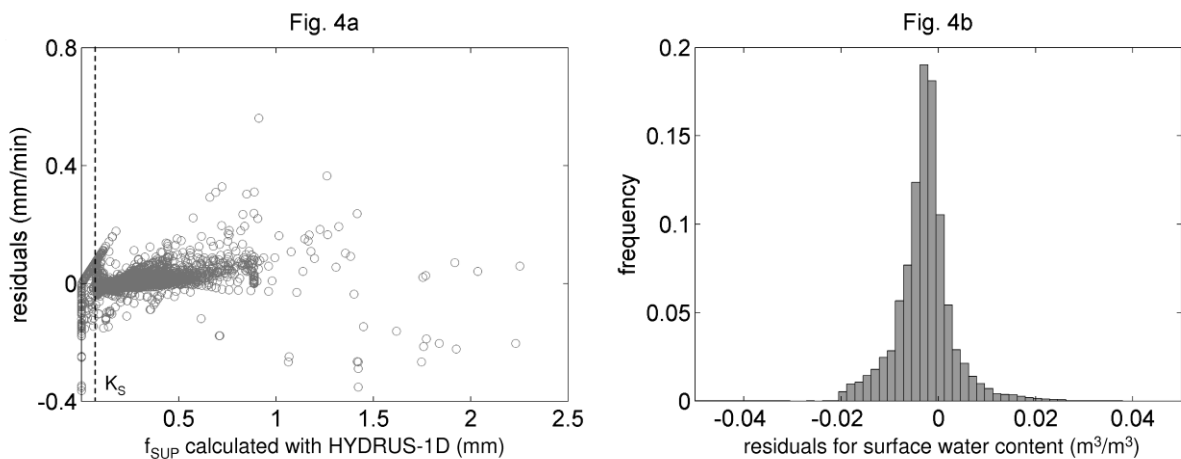

Figure 4. Analysis of residuals for soil n°4 (sandy clay loam) for the 1-year simulation (Difference between the values obtained with HYDRUS-1D and those simulated with I2RS) - 4a: Residuals (infiltration-rates) as a function of infiltration-rates as computed by HYDRUS-1D, 4b: distribution of residuals for surface water content predictions (during redistribution periods)

This observation is consistent with the slight overestimation of surface water content shown in figure 4b (negative residuals). The line "residuals = f_{sup}" in figure 4a however clearly suggest that this underestimation of infiltration rates is not systematic, as such pattern implies that HYDRUS-1D computes non-zero infiltration fluxes whereas ponding already stopped in the I2RS model.

These errors nevertheless remain very limited and the model generally provides very satisfactory predictions of both infiltration rate and water contents. The ability of the I2RS model to simulate the evolution of soil moisture profile is illustrated in figure 5:
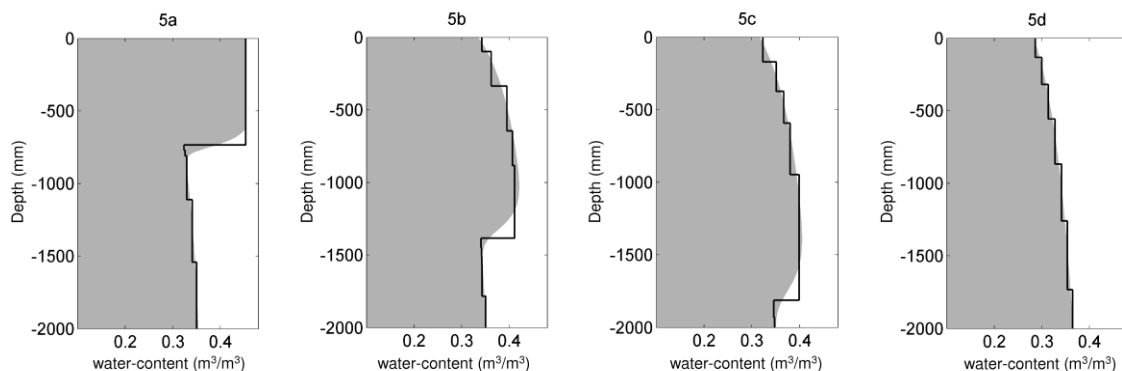

Figure 5. Water content profile simulated for an infiltration event (after 10 days of simulation) and during subsequent redistribution period for soil n°7 (sandy loam). Grey shaded area: Hydrus-1D, solid black line: I2RS model - 5a: end of the infiltration event (t = 10 days); 5b: after 3h; 5c: after 6h; 5d: end of the redistribution period, after approximately 1 day

Comparison with simpler modeling approaches

Comparison between the different modeling options (Table 4) primarily indicates that the I2RS model systematically outperforms simpler GA approaches for the prediction of infiltration fluxes. The benefits associated with a detailed representation of soil moisture content variations are particularly visible for moderate to low permeability soils (n°1 to 5). Although the adoption of a simpler method produces a less important decrease in *BE* and *VE* in the case of coarser soils, differences from the I2RS model remain substantial.

The association of the GA equation with a rudimentary redistribution model, assuming uniform water content over the soil profile and gravity drainage (GA-RR), seems to provide relatively solid estimates of f_{SUP}. Similarly, efficiency criteria computed for the simple GA approach reveals that neglecting the redistribution process may as well be acceptable, as long as the value of the fixed initial water content θ_0 is appropriately selected (GA-θ_{OPT} in Table 4). However, determination of this appropriate θ_0 value may be somewhat difficult (as the later obviously depends on the hydrological behavior of the facility) and a poor choice regarding the latter could strongly affect the accuracy of infiltration rate predictions (cf. Figure S16-17). (As shown in Figure S16, a 10% deviation from θ_{OPT} for instance results in an 80% and 30% decrease of BE and VE for soil n°2).

Model	I2RS		GA-RR		GA-θ_{OPT}		GA-θ_{FC}		CR	
Soil n°	BE	VE	BE	VE	BE	VE	BE	VE	BE	VE
1	0.97	0.97	0.79	0.88	0.76	0.89	-0.53	0.52	/	0.64
2	0.97	0.97	0.77	0.84	0.75	0.84	-0.33	0.44	/	0.51
3	0.98	0.98	0.78	0.84	0.80	0.86	-0.02	0.48	/	0.50
4	0.98	0.97	0.80	0.84	0.81	0.82	0.28	0.54	/	0.43
5	0.98	0.97	0.83	0.85	0.85	0.86	0.27	0.55	/	0.38
6	0.97	0.97	0.82	0.89	0.85	0.90	0.39	0.70	/	0.55
7	0.97	0.97	0.86	0.91	0.87	0.90	0.45	0.73	/	0.53

Table 4. Comparison of the I2RS model with simpler modeling approaches (GA-RR: GA equation and rudimentary redistribution scheme; GA-θ_{OPT} : GA with constant initial moisture content adjusted to produce the best fit to HYDRUS-1D, GA-θ_{FC} : GA with constant initial moisture set to field capacity; CR: constant rate model)

The distribution of the residuals associated with the different modeling approaches for soil n°3 and 6 is presented in figure 6. Comparison between the two soils first indicates that the frequency and the magnitude of model errors, relatively to the saturated hydraulic conductivity, tend to be more important in the case of soil n°6. Because the variability of infiltration rates is also larger in this case *($\sigma = 0.9 \times K_S$ vs. $\sigma = 0.67 \times K_S$)*, as a result of higher suction head in the GA equation value, the effect on *BE* and *VE* however remains moderate (cf. Table 4).
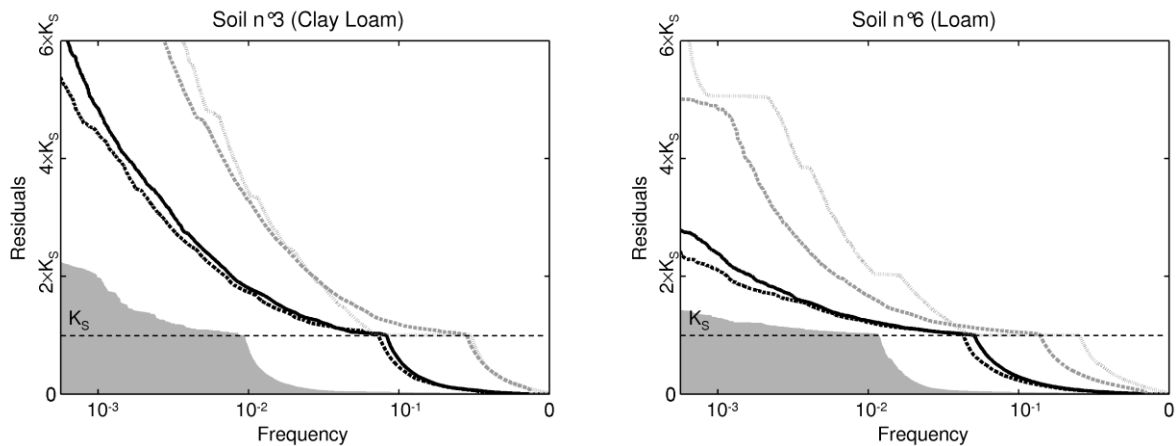

Figure 6. Distribution of absolute residuals (difference between the values obtained with HYDRUS-1D and those simulated with the different modeling options, excluding observations for which all models produce $f_{sup} = 0$) for the different modeling options for soil n°3 (clay loam) and soil n°6 (loam). Grey shaded area:I2RS model; Plain black line: GA-RR, Black dashed-line; GA-OPT; Grey dashed line: GA-FC; Light-grey dotted line: the constant rate model.

Both graphs confirm the poor performance of the constant rate infiltration model and the simple GA approach for $\theta_0 = \theta_{FC}$, with significantly higher deviations for a similar frequency. Although deviations from the I2RS model tend to be more limited in the case of the GA-RR and the GA-θ_{OPT} methods, distributions nevertheless corroborates the results presented in table 4. In the case of the GA-RR approach, the frequency associated with errors greater than $0.5 \times K_S$ is for instance more than 8 times as high as for the more sophisticated I2RS model in the case of soil n°3 and about 5 times as high for soil n°5. Although relatively

infrequent (about 10% of observations), these errors remain significant. These 10% represent approximately represent 16 days of simulation and a cumulative error of 2100 mm for soil n°3 and 6 days with 1100 mm error for soil n°5.

Effect of evapotranspiration related parameters

The variations of *BE* and *VE* associated with changes in evapotranspiration parameters K_{CROP} and Z_{ROOT} are presented in figure 7 (further details can be found in Table S9-12). Note that Z_{ROOT} is not a parameter of the I2RS model but rather an input to be specified in HYDRUS-1D that may affect reference infiltration and water-content time series. The sensitivity analysis primarily indicates that the two soils tested here do not exhibit the same behavior. While variations in model performance remain limited for the most pervious soil (n°5), the effect of K_{CROP} and Z_{ROOT} tends to be more important in the case of the less permeable one (n°1), for which evapotranspiration believably become substantial as compared to downward redistribution fluxes.
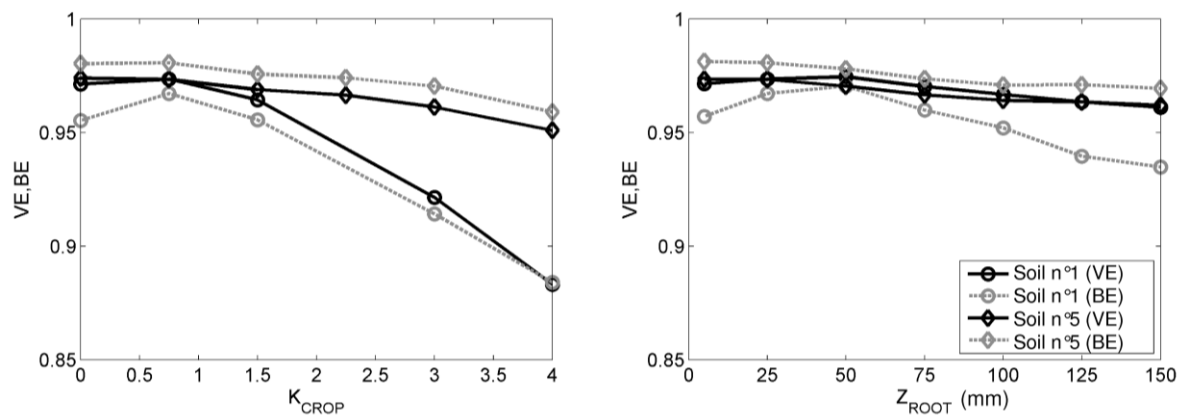

Figure 7. Incidence of evapotranspiration parameters K_{CROP} and Z_{ROOT} on the accuracy of infiltration rates predictions for soiln°5 (silt loam) and n°1 (sandy clay)

Expectedly, the accuracy of infiltration-rate predictions gradually deteriorates when large Z_{ROOT} values are adopted, as the assumption that evapotranspiration preferentially affects the uppermost layer of soil (adopted in the I2RS model) becomes less realistic. Although, model performance remains unaffected for usual root depth values (<50cm), the introduction of root depth parameter in the I2RS models may hence be considered for some applications.

Similarly, the augmentation of evapotranspiration fluxes causes a noticeable drop of *VE* and *BE* for soil n°1; deviations from the aforementioned hypothesis regarding the application of evapotranspiration in the IR2S model are here exacerbated when K_{CROP} increases. The application of higher K_{CROP} values as well results in a more pronounced inversion of the soil moisture profile that cannot completely be captured by the I2RS model (for soil n°1 and n°5, errors associated with θ_{TOP} hence reach 2.9% and 5.5% for $K_{CROP} = 4$ vs. 0.9% and 1.9% for the reference configuration). The upper range of K_{CROP} values shown in figure 7 are however not realistic, unless considering very specific plant species (or very high atmospheric water demand). Hence,

if the model may not be suitable for the simulation of low permeability soils under such conditions, its performance clearly remains satisfactory for SUDS modeling under temperate climates.

Importance of near surface water-content description

Neglecting evapotranspiration in the redistribution process produces contrasted effects depending on soil type. For $K_{amp} = 0.75$ in HYDRUS-1D, VE and BE respectively drop to 0.90 and 0.57 for soil n°1 whereas the effect is mostly negligible for soil n°5 (as shown in table S17, a more noticeable effect is observed for $K_{amp} = 3$). On the contrary, disabling the procedure implemented to simulate the fast decrease of near surface water content in the I2RS model systematically deteriorates model performance, with VE and BE value ranging from 0.78 to 0.88 and 0.48 to 0.82 (see table S18). Results therefore indicate that (1) accounting for evapotranspiration may often not be needed to accurately replicate infiltration-rates (although it remains necessary to properly describe water balance) whereas (2) simulating moisture profile inversions (mainly associated with gravity and capillary forces) can markedly improve model predictions.

COMPUTATIONAL EFFICIENCY

Conducting a proper evaluation of the numerical efficiency of the I2RS model as compared to HYDRUS-1D is a difficult exercise given the differences between the two codes. First, languages are not equivalent: HYDRUS-1D is a compiled FORTRAN program whereas I2RS has yet only been implemented in an interpreted and higher-level language (Matlab). Optimization of the I2RS model has not yet been conducted and the later still incorporates a relatively strict time-step control (for a given time step, simulations are systematically performed for at least dt and $dt/2$ and an exponential time-step refinement is used when tolerance regarding water content, water balance and infiltration rates are not satisfied). Secondly, the efficiency of HYDRUS-1D significantly depends on temporal and spatial discretization that should in principle be adapted on a case-by-case basis.

Here, the convergence issues encountered with HYDRUS-1D for the Brooks-Corey model resulted in very long simulation times (typically, several hours for a one-year period as a result of a particularly fine time and spatial discretization). Because these difficulties clearly originate from the large air-entry value of the Brooks-Corey model, a comparison is here performed using the more stable Van-Genuchten model in HYDRUS-1D. Simulations are conducted for a one year rainfall period (5-min time step in both HYDRUS-1D and I2RS) and a 2-m deep soil profile. The latter is described by 151 nodes mesh (1cm near soil surface to 3 cm at the bottom of the profile) in HYDRUS-1D. Iteration criteria are set to their default value (10^{-3} m^3/m^3 water content and 1 cm pressure head tolerance). Van Genuchten parameters are those provided in HYDRUS-1D except K_S which is set in accordance with table 1. The results of the comparison are presented in table 5 and S20.

		HYDRUS-1D				I2RS			
Soil n°	b (%)	[1]Time (s)	Iterations	[2]N_{nodes}	[3]ε_{BAL} (%)	[1]Time (s)	Iterations	[4]N_{front}	[1]ε_{BAL} (%)
1	8	43	3.6×10^5	151	0.3	23	2.6×10^5	6.0	0.0
2	7	91	8.8×10^5	151	0.1	25	3.3×10^5	6.7	0.0
3	6	129	1.2×10^6	151	0.1	28	3.2×10^5	7.4	0.2
4	4.5	75	6.1×10^5	151	0.1	33	3.7×10^5	8.4	0.1
5	4	253	2.6×10^6	151	0.6	33	3.3×10^5	8.9	0.5
6	3	391	4.8×10^6	151	0.1	33	3.3×10^5	8.5	0.4
7	2.5	217	2.2×10^6	151	0.3	41	4.4×10^5	8.9	0.5

Table 5. Computational efficiency of the I2RS model as compared to HYDRUS-1D ([1]for a 3.6 GHz Xeon processor; [2]number of nodes in HYDRUS-1D; [3]water balance error as defined in Šimůnek et al. (2009); [4]average number of front during the simulation)

Simulation times of the I2RS model are at least equivalent to those of HYDRUS-1D, and significantly lower for most test cases. Comparison between iteration requirements, up to one order of magnitude larger in the case of HYDRUS-1D, suggests that the performance of the I2RS model could probably be further improved. The relevance of the I2RS model is also illustrated by the average number of homogeneous water content profiles N_{front}, that can be interpreted in terms of vertical discretization (much coarser than the one typically used for the resolution of Richards equation).

APPLICATION TO MULTIPLE STORMWATER INFILTRATION SCENARIOS

Methodology

The I2RS model is implemented to simulate 5000 stormwater infiltration scenarios, generated randomly for the 8 soil classes presented in table 1 with ponding depth h_{MAX} ranging from 1 to 40 cm and area ratio b between 1 and 10%. As for I2RS evaluation, the facility is assumed to behave as a simple storage unit providing volume reduction through infiltration and evapotranspiration. Once again, the diversity of SUDS system is only addressed in terms of surface boundary conditions and soil properties. As for the evaluation the computational efficiency, the extension of the soil domain Z_{max} is limited to 2m.

A significantly longer simulation period (15 years, recorded at the same stations as for model evaluation) is here considered to derive a variety of metrics related to the performance of the different stormwater system designs:

- The overall volume reduction efficiency E_{TOT} (%) computed as $1 - Q_{TOT}/P_{TOT}$ where Q_{TOT} is the total overflow volume and P_{TOT} the cumulative rainfall volume entering the facility

- The return period T_{OUT} associated with the exceedance of the storage capacity (i.e. stormwater discharge)

- The volume reduction efficiency computed for different rainfall return periods ($T = 3$ months, 6 months and 1 year) as $E_T = 1 - Q_T/P_T$ where Q_T is the discharge volume (expressed as a runoff depth) and P_T the rainfall depth associated with T

These indicators are compared to those obtained for simpler modeling approaches to assess the importance of a detailed description of both the inter- and the intra-event variability of infiltration fluxes for the modeling of SUDS. Simplified models selected here include those presented in previous section (GA-RR and GA- θ_0 with $\theta_0 = \theta_{FC}$) as well as the constant-rate (CR) infiltration model $f_{sup} = K_s$. A detailed examination of the results is then performed to understand under which circumstances larger errors are likely to occur.

Results

Mean and median volume reduction efficiency obtained from this sample with the I2RS model are 75% and 82%. Percent deviations in the value of the different efficiency indicators (as compared to the I2RS model) are presented in figure 8.
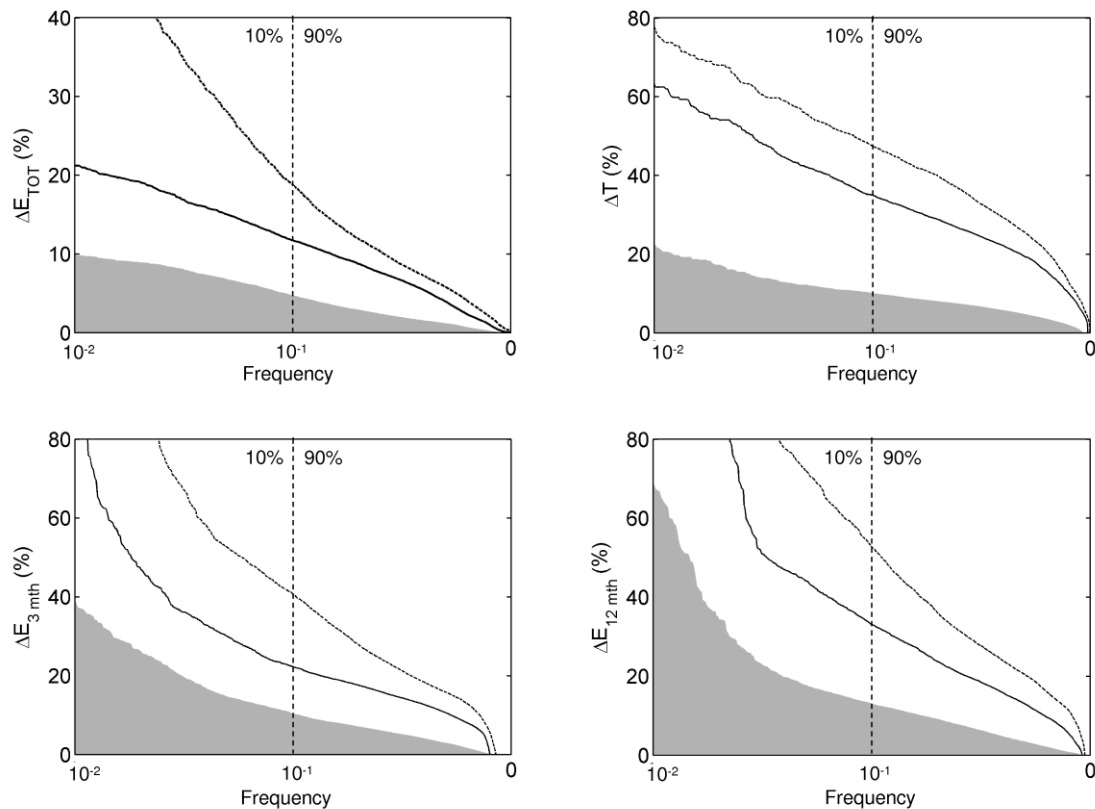

Figure 8. Distribution of absolute percentage deviations from the I2RS model for different model outputs (E_{TOT} is total volume reduction efficiency, T the return period associated with non-zero discharge and E-$_{3mths}$ and E-$_{12mths}$ volume reduction efficiencies computed for the 6 and 12 month retun period ;ΔE-$_{6mths}$ is not shown here). Grey shaded area: GA-RR model; Solid black line: GA-θ_{FC}; Black dashed-line: CR-model

Total volume reduction efficiencies E_{TOT} simulated with the rudimentary redistribution scheme (GA-RR) generally remain close to those computed with the I2RS model (with percentage deviations smaller than 5% for approximately 90% of the configurations tested here). Departures from these values tends to be much more important for the fixed initial water content approach (GA-θ_{FC}) and the constant-rate model

(CR), with relative errors larger than 35% and 45% for 10% of the simulations, respectively. Regarding the return period T_{OUT} associated with the exceedance of storage capacity, deviations from the I2RS model generally appear as significant, although more limited and less frequent in the case of the GA-RR model. The latter for instance produces differences larger than 10% for 10% of the simulations whereas these errors rise to 35 and 48% for GA-θ_{FC} and CR methods. Finally, frequency-based reduction efficiencies $E_{3\text{-}mths}$, $E_{6\text{-}mths}$ and $E_{12\text{-}mths}$ are also clearly affected by the adoption of a simpler modeling approach. Departures from I2RS are, once again, more important for the GA-θ_{FC} and the CR approaches, but may, nevertheless, become non-negligible in the case of the GA-RR model.
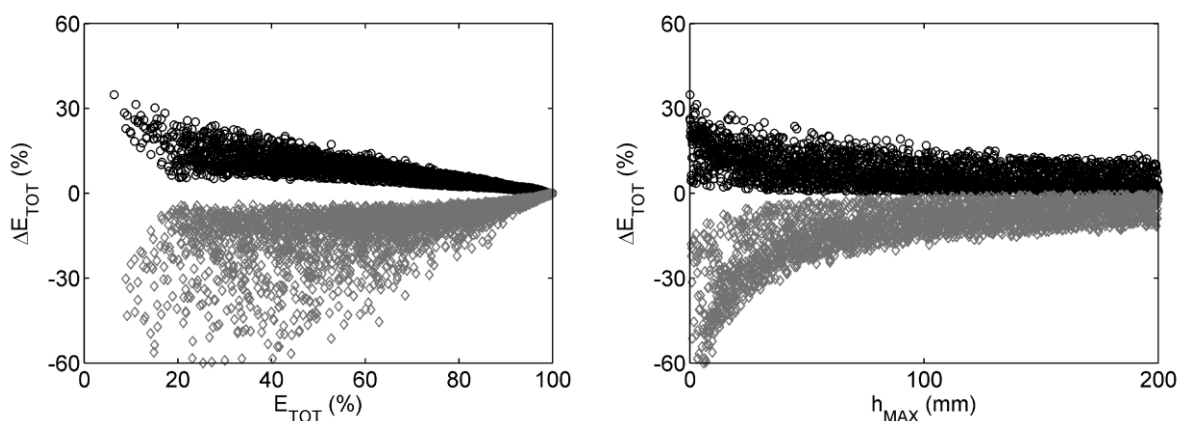

Figure 9. Percent deviations in E_{TOT} as function of E_{TOT} and h_{MAX} for the GA equation with fixed initial water content GA-θ_{FC} (black circles) and the constant rate CR infiltration model (grey diamonds) (the figure associated with GA-RR can be found in supplementary materials)

Further analysis of simulation results (see Tables S21-24 and figures S18-20) primarily indicates that relative deviations from the reference volume reduction efficiency (ΔE_{TOT}) tend to increase as E_{TOT} decreases (Kendall rank correlation coefficient $\tau = $ -0.69, -0.86 and -0.80 for GA-RR, GA-θ_{FC} and CR) and may become significant for SUDS with $E_{TOT}<75\%$ (5 to 10% for GA-RR and up to 60% for the CR model). Largest errors such as the 10% shown in figure 8 are therefore associated with moderate efficiency designs. For the simplest modeling approaches (GA-θ_{FC} and CR), ΔE_{TOT} is additionally found to increase as maximum ponding depth h_{MAX} decreases (cf. figure 9. Note that results associated with GA-RR can be found in figure S18 and S19). For these two models, relative differences in the estimation of T_{OUT} (ΔT_{OUT}) as well exhibit a dependence on E_{TOT} ($\tau > 0.45$). While extreme ΔT_{OUT} value are essentially associated with very efficient design scenarios (that presumably produce significant attenuation of runoff discharge), errors may nevertheless remain significant for medium to high efficiency designs ($50\%<E_{TOT}<80\%$) with maximum values ranging from 40% with GA-θ_{FC} to 60% for the CR model. Finally, deviations in $E_{3\text{-}mths}$ and $E_{6\text{-}mths}$ show a moderate dependence on E_{TOT} (with $-0.45<\tau<-0.56$) whereas $\Delta E_{12\text{-}mths}$ remains more erratic.

CONCLUSIONS

An improved infiltration-redistribution scheme (I2RS), based on the coupling of the Green-Ampt (GA) equation with a redistribution scheme adapted from previous studies, was introduced. The I2RS model was evaluated for both simple theoretical case studies and long-period applications under hydrologic conditions similar to those encountered in SUDS. The accuracy of infiltration rate and soil moisture predictions was checked against numerical solutions of Richards equation for seven USDA soil classes. The importance of a correct description of the temporal variability of infiltration fluxes was later assessed by comparing the results associated with the I2RS model to those computed for simpler and more usual modeling approaches, over a wide range of stormwater infiltration scenarios.

As opposed to previous rectangular profile redistribution models, that were either not tested over long rainfall periods or shown to produce increasing errors on infiltrated volumes, the I2RS model appears as a suitable option for long period applications. The redistribution scheme, originally developed by (Milly 1986), here incorporates a modification that allows for a relatively precise representation of near-surface water content and clearly helps to overcome the limitations of previous GA-based solutions. Although infiltration-rates computed with the I2RS model may slightly differ from the numerical solution of Richards equation, such deviations are believably expectable due to the approximate nature of the GA equation (Parlange et al. 2002) and nevertheless remain very limited. Given the influence evapotranspiration-related parameters on the reference simulations conducted with HYDRUS-1D, the description of this process in the I2RS model could be examined for further model improvement.

The computational efficiency of the I2RS model was found to be very satisfactory (and promising given the limitations of the current code). No convergence issues were encountered for the numerous tests associated with model evaluation. This numerical stability is a clear advantage as compared to the resolution of Richards' equation, especially when performing long period simulations, under a variety of surface boundary conditions (which is often the case in the context of urban drainage modeling).

The I2RS model also largely and systematically outperforms simpler modeling approaches for the prediction of infiltration rates. While the GA equation may be perceived as a more sophisticated method than constant rate infiltration model, the benefits associated with its adoption, under the common assumption that soil water content immediately drops to field capacity when infiltration stops, are found to be mostly negligible. Conversely, association of the GA equation with a rudimentary redistribution model, similar to the ones implemented in EPA's SWMM for rain garden modeling (Rossman 2010b), results in a noticeable improvement in the accuracy of infiltration rates predictions.

Application of the different modeling approaches over multiple stormwater infiltration scenarios primarily demonstrates that neglecting the inter- or intra-event variability of infiltration fluxes (assuming

constant rate infiltration or neglecting the redistribution process) may significantly bias the assessment of the performance of source control systems. Rudimentary representation of the redistribution process markedly improves the estimation of the different efficiency indicators, although deviations from the I2RS model remain noticeable for some configurations. Expectedly, long term volume reduction efficiencies are generally less affected by the adoption of a simpler modeling approach than distribution-based indicators (e.g. return period associated with non-zero discharge and efficiency associated with a given return period). Here, errors associated with instantaneous infiltration-rates prediction are presumably compensated by the ability of stormwater to accumulate over soil surface, resulting in small deviations in overall infiltration volumes. As a consequence, the influence of the infiltration model also tends to be more visible for small ponding depth and moderate efficiency designs (as the storage capacity is more frequently exceeded). These results highlight the importance of selecting infiltration models in accordance with the application considered and have important implications. First, more sophisticated approaches should probably be preferred (1) when short-duration performance indicators are investigated or (2) when moderate efficiency designs (such as those focusing on small storms) are considered. Second, although small ponding depth facilities yet remain unusual, the trend toward increasingly diffuse and integrated solutions (Fletcher et al. 2014) as well impervious cover reduction could possibly require an adaptation of modeling practices.

The I2RS model is currently implemented in a relatively accessible language that already enables adaptation to deal with some of the specificities of SUDS: discharge at controlled rate through a flow limiting device, relation between infiltration area and water storage, introduction of pollutants (Sage et al. 2016). Because the model incorporates a precise description of soil moisture content, it could also be a relevant tool to simulate water balance and evapotranspiration fluxes from SUDS or other vegetated surfaces.It is however important to acknowledge the limitation of this approach in the context of SUDS modeling. First, urban soils often exhibit heterogeneities and stormwater management increasingly relies on multi-layer systems such as bioretention devices; extensions of the I2RS model to heterogeneous soil domain should therefore be considered. Similarly, the implementation of a zero pressure or mixed boundary condition at the bottom of soil profile should presumably be carried out for facilities that. receive very large amounts of stormwater or when the water table does not lie far below soil surface (Locatelli et al. 2015).

More generally, the assumption of a strictly one-dimensional flow, adopted in both I2RS and HYDRUS-1D, may not always be appropriate(Freni et al. 2009). Besides, the ability of Richards' equation to describe infiltration fluxes is still the subject of debate in watershed hydrology due to the complex nature of soil (Beven 2004; McDonnell et al. 2007). The later nonetheless remain the most robust solution to simulate the effect of soil moisture redistribution on infiltration fluxes.

SUPPLEMENTAL DATA

Figures. S1–S20 and tables S1-S24 are available online in the ASCE Library (ascelibrary.org).

DATA AVAILABILITY STATEMENT

All data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

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