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The exact theory of the Stern-Gerlach experiment

and why it does not imply that a fermion can only have its spin up or down

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Abstract. The Stern-Gerlach experiment is notoriously counter-intuitive. The official explanation has it that the spin remains always aligned with the magnetic field such that the directions of space would be quantized: A fermion can only have its spin up or down. But that theory is based on several blatant mathematical errors in the way it (mis)treats spinors and group theory. We present here a mathematically rigorous theory for a fermion in a magnetic field, which is all but beyond human intuition. It is based on an understanding of spinors in SU(2) which as explained in [1] is only Euclidean geometry. Contrary to what everybody has been reading into the Stern-Gerlach experiment, the directions of space are not quantized. The energy $V = -\boldsymbol{\mu}\cdot\mathbf{B}$ is a statistical average. It is not a potential energy and it is not valid for individual fermions.

PACS. 02.20.-a, 03.65.Ta, 03.65.Ca Group theory, Quantum Mechanics

1 Confusion reigns

1.1 Preamble

In this section[1] we want to point out the total lack of intuition and the total lack of theory which prevail in the traditional presentation of the Stern-Gerlach experiment. We have shown in many examples on many occasions in the past [2] that the "geometry" that corresponds to the algebra of quantum mechanics, is the group theory of the rotation and Lorentz groups. We have in this respect often used the analogy of the correspondence between algebra and geometry in algebraic geometry to explain that the calculus of quantum mechanics, its algebra, is exact but that we do not know what its correct intuitive interpretation, its "geometry" should be. In this respect Villani uses the qualifiers "analytic" for what we call algebraic and "synthetic" for what we call "geometric" [3]. Perhaps this terminology is more accurate than ours.

When you know the algebraic part of the spinor formalism and you know that the corresponding synthetic part must be the group theory of the rotation and Lorentz groups, then you might expect that explaining the Stern-Gerlach experiment synthetically should not be too difficult. But lo and behold, this is certainly not the case. One reason for this is that the textbook algebra is very egregiously wrong.

1.2 Total absence of theory

We have pointed out many times before, especially in [1,2,4], that the terms $\mathbf{B}\cdot\boldsymbol{\sigma}$ or $\mathbf{B}\cdot\boldsymbol{\gamma}$ that occur in the equations are not the scalar product of the magnetic field with the spin vector $\mathbf{s}$. As a matter of fact $\mathbf{B}\cdot\boldsymbol{\sigma}$ or $\mathbf{B}\cdot\boldsymbol{\gamma}$ just express the magnetic field in the formalism of the Clifford algebra. There is absolutely no elbow room for eluding this death sentence. It is just an undeniable mathematical fact. Furthermore, the spin vector $\mathbf{s}$ is not represented by $\boldsymbol{\sigma}$ or $\boldsymbol{\gamma}$ but by $\mathbf{s}\cdot\boldsymbol{\sigma}$ or $\mathbf{s}\cdot\boldsymbol{\gamma}$, which often remains hidden inside the notation for the spinor ψ. When $\mathbf{s}\cdot\boldsymbol{\sigma}$ or $\mathbf{s}\cdot\boldsymbol{\gamma}$ do not explicitly occur in the equations, there cannot be any form of algebraic chemistry between $\mathbf{s}\cdot\boldsymbol{\sigma}$ and $\mathbf{B}\cdot\boldsymbol{\sigma}$ in those equations. Similar remarks apply for $\mathbf{s}\cdot\boldsymbol{\gamma}$ and $\mathbf{B}\cdot\boldsymbol{\gamma}$ in the Dirac formalism, but from now on we will only formulate things in the SU(2) formalism.

[1] In a Stern-Gerlach experiment neutral particles with spin 1/2 are used, e.g. Ag atoms. In our description we will all the time focus our attention on electrons, even if a Stern-Gerlach experiment on electrons might be extremely difficult to perform. The real problem we want to discuss is the case of an electron with spin 1/2 in a magnetic field, for which we have learned that the electron spin can be only up or down, and never tilted as we will assume in the attempt to describe precession, reported in Section 1.3.

The textbook theory exploits the mathematical errors mentioned to claim that the "spin vector" $\boldsymbol{\sigma}$, after multiplication by $\frac{\hbar q}{2m_0}$ defines the magnetic dipole $\boldsymbol{\mu} = \frac{\hbar q}{2m_0}\boldsymbol{\sigma}$. This slight of hand replaces the axial vector $\frac{\hbar q}{2m_0}\mathbf{B}\cdot\boldsymbol{\sigma}$ by a scalar $\mathbf{B}\cdot\boldsymbol{\mu}$, where $\boldsymbol{\mu}$ is now considered to be a magnetic dipole[2], and $V = -\mathbf{B}\cdot\boldsymbol{\mu}$ becomes a "potential energy". This balderdash of messing around with mathematical symbols is not a theory! However, the expression for this "potential energy" corresponds conveniently to our classical intuition, such that it can be accepted in blissful ignorance. It is then still difficult to understand within this picture why the spin should select two orientations in order to align with $\mathbf{B}$, rather than just one, *viz.* the one that would minimize its energy within the picture of a potential. Can the spin then also maximize its potential energy?

Despite its appeal, the ansatz $V = -\boldsymbol{\mu}\cdot\mathbf{B}$ is also problematic. We are talking here about the potential energy of a charged spinning object in a field $\mathbf{B}$, but this field $\mathbf{B}$ is not a force like the gravitational force $m\mathbf{g}$ exerted on a spinning top. Any analogy with the potential energy of a spinning top in a gravitational field is potentially misleading and conceptually wanting, as a magnetic field just cannot do any work on a charge. It can exert a force $\mathbf{F} = q(\mathbf{v}\wedge\mathbf{B})$, but this force is always perpendicular to the displacement $d\mathbf{r} = \mathbf{v}dt$ and therefore the work $-\mathbf{F}\cdot d\mathbf{r} = 0$.

1.3 Total absence of intuition

For a top which is precessing in a gravitational field, the energy of the top remains constant (if we assume that the dissipation of energy due to the friction is negligible, which of course can become wrong in the long run). But if you describe a precessing top within the spinor formalism of quantum mechanics, then the formalism says the energy is not constant and oscillates between two extreme values (see e.g. [2], p.307). We are referring here of course to the description of an electron in a magnetic field. That the energy could oscillate is really incomprehensible. We could imagine that the electron looses energy by e.g. radiation, but not how it could regain the energy lost, and what is more, exactly by the same amount.[3]

If we dare to be heretic and assume that there is something wrong with that calculation, and that the energy is constant like for a spinning top anyway, we may get a constant-energy term that has not the correct value, because it will contain an extra factor $\cos\theta$, where θ is the tilt of the spin axis with respect to the magnetic field, at least if you follow the common-sense arguments you have been taught. None of these speculations leads to a calculation that agrees with the startling experimental result, which seems to indicate that the spin of a fermion can only point up or down.

The cheap way out of these puzzling contradictions, is the textbook dogma that space would be quantized, and that this would be a quantum mystery. Whereas I fully agree that I do not understand the first world of it, such that calling this a mystery could be appropriate, I nevertheless think that this is logically and mathematically completely ramshackle. First of all we should refuse dogmatic mysteries. But there is something far worse at work than just a weird paradox. In fact, there is a fierce contradiction hidden within that statement.[4] The contradiction at stake here is that the formalism is completely based on SU(2), wherein the allowed axes of rotation explore all directions of $\mathbb{R}^3$ while it derives from this that the directions would be quantized in the sense that quantum mechanics would only allow for two directions, spin-up and spin-down! This brutal contradiction between the starting point of the reasoning and its final result shows that something must be badly wrong.

Gaslighting destroys the ability to think and in the end its effect is that nobody knows the truth anymore. This is what has happened here. We have been forcefed with so many wrong images, wrong maths and apparent internal contradictions that we do not know on which leg to stand anymore. We do no longer know how to winnow the chaff from the wheat. We even do not have the correct algebra.

The wrong images create even more puzzles in the light of the way we could derive the Dirac equation from the assumption that the electron spins in [2]. In developping the Dirac equation by expressing the rotational motion of a spinning electron with the aid of spinors, at a certain stage we must put $m_0c^2 = \hbar\omega_0/2$ in order to obtain the Dirac equation. Here the electron spins with angular frequency ω_0 around the spin axis $\mathbf{s}$, and m_0 is its rest mas. This is analogous to the equations for the angular momentum $L = I\omega$ and energy $E = I\omega^2/2$ of a top spinning at a frequency ω, which lead to $E = L\omega/2$. Here I is the moment of inertia. This means that the complete rest energy of the electron is rotational energy. Consider now the statement that in a magnetic field the spin axis aligns with the magnetic field, because the spin can only be up or down. We could e.g. imagine that the spin axis $\mathbf{s}$ is pointing in a given direction and that we turn on the magnetic field in a completely different direction.[5] There must then exist a really fast mechanism for the spin to align. This is puzzling, because the magnetic energy $\frac{\hbar qB}{2m_0}$ is much smaller than the energy m_0c^2. It is as though you would be able to align the rotation axis of a very heavy

[2] Mutiplying an axial vector $\mathbf{B}\cdot\boldsymbol{\sigma}$ by the constant $-\frac{\hbar q}{2m_0}$ can only yield another axial vector, such that identifying the result $-\frac{\hbar q}{2m_0}\mathbf{B}\cdot\boldsymbol{\sigma}$ with a scalar $-\mathbf{B}\cdot\boldsymbol{\mu}$ is a glaring error.

[3] The derivation of the Dirac equation in [2] relies on the assumption that the spin axis remains fixed. Therefore the motion of a precessing top cannot be studied with the traditional Dirac equation. One should first derive a generalized equation following the same methods as used for the Dirac equation in [2].

[4] What the current theory does is forcefully claiming in an authoritarian way that this contradiction would be correct. To sweeten the pill it is admitted that this is indeed weird but that quantum mechanics is beyond human undertanding. In reality, there is no theory at all, and all this is just gaslighting.

[5] Note that this implies also the temporary presence of an electric field.

fast-spinning top along a completely different direction with a very small force. Intuitively, this looks like utter nonsense. It does not comply with our daily-life experience and the conservation of angular momentum. What you expect on applying some external torque is precession, not alignement. If you tried to touch a fast-spinning heavy object you would find out that you cannot bring about such an alignment. With my apologies for the irony, you could rather become more or less aligned yourself. How is this then possible? Moreover, we do not understand how this alignment process is supposed to work. Is there some radiation emitted, and if so should this have been observed?

2 Tabula rasa approach based on spinors

In view of all this confusion, we must rebuild a theory from scratch. It will be based on a good understanding of spinors. This should not disheart the reader. Just remember, dear reader: SU(2) and spinors are only about rotations, i.e. Euclidean geometry. How could this possibly be difficult? We have shown that it is indeed not difficult in our account of spinors in [1]. Despite the fact that the author understands spinors quite well, the wrong pictorial arguments that are living on in the intuitive folk lore about the spin in a magnetic field amount to a formidable conceptual obstacle. They are a smoke screen of totally misleading hints that kept me in the dark for a very long time and rendered it extremely difficult to find the correct solution. I am confident that I am not the only one who has been running in circles for years in trying to make sense of this spin-up and spin-down narrative.[6]

We must thus warn the reader that he is in for a rough ride whereby a lot of what he has become used to take for granted will be ripped apart. We start from something we derived in [2] (see e.g. [2], p.142) , *viz.* that you can write a spinning motion in SU(2) in terms of two components. We will interpret this for the moment as a simultaneous description of left-handed and right-handed frames (see [1], p.33). E.g. if some spinning motion were to be described by $e^{-\imath\omega_0\tau}$ in the right-handed frame, then it would be described by $e^{+\imath\omega_0\tau}$ in the left-handed frame. It is just a matter of algebraic frequencies. The decomposition of a rotation into two components is as follows:

$$\mathbf{R}(\tau) = \frac{1}{2}\left[\,[\,\mathbb{1} + \mathbf{s}\cdot\boldsymbol{\sigma}\,]\,e^{-\imath\omega_0\tau/2} + [\,\mathbb{1} - \mathbf{s}\cdot\boldsymbol{\sigma}\,]\,e^{+\imath\omega_0\tau/2}\right]. \tag{1}$$

Eq. 1 is a direct consequence of the Rodrigues formula for a rotation over an angle φ around the axis $\mathbf{s}$:

$$\mathbf{R}(\mathbf{s},\varphi) = \cos(\varphi/2)\mathbb{1} - \imath\sin(\varphi/2)\,[\,\mathbf{s}\cdot\boldsymbol{\sigma}\,], \tag{2}$$

after putting $\varphi = \omega_0\tau$. Note that this is just Euclidean geometry. And here is then my question: What if these two components could correspond to a mixture of two beams? In fact, using Ehrenfest's interpretation of superposition states ([1], p.10), Eq. 1 means that a spinning frame can be left-handed or right-handed with equal probability.[7] Let us now write down Eq. 1 for a rotation with an axis $\mathbf{s}$ that is different from the z-axis:

$$\mathbf{R}(t) = \begin{bmatrix} \cos^2(\theta/2) & e^{-\imath\phi}\sin(\theta/2)\cos(\theta/2) \\ e^{\imath\phi}\sin(\theta/2)\cos(\theta/2) & \sin^2(\theta/2) \end{bmatrix} e^{-\imath\omega_0\tau/2} + \begin{bmatrix} \sin^2(\theta/2) & -e^{-\imath\phi}\sin(\theta/2)\cos(\theta/2) \\ -e^{\imath\phi}\sin(\theta/2)\cos(\theta/2) & \cos^2(\theta/2) \end{bmatrix} e^{\imath\omega_0\tau/2}. \tag{3}$$

Here (θ,ϕ) are the spherical coordinates of the spin axis $\mathbf{s}$. Note that we are using ϕ and φ as two different symbols in this document. Let us now inspect the two components. The $e^{-\imath\omega_0\tau/2}$ component is:

$$\begin{bmatrix} \cos^2(\theta/2) & e^{-\imath\phi}\sin(\theta/2)\cos(\theta/2) \\ e^{\imath\phi}\sin(\theta/2)\cos(\theta/2) & \sin^2(\theta/2) \end{bmatrix} = \begin{bmatrix} \cos(\theta/2)e^{-\imath\phi/2} \\ \sin(\theta/2)e^{+\imath\phi/2} \end{bmatrix} \otimes \begin{bmatrix} \cos(\theta/2)e^{\imath\phi/2} & \sin(\theta/2)e^{-\imath\phi/2} \end{bmatrix}. \tag{4}$$

We recover here the result $\mathbb{1} + \mathbf{s}\cdot\boldsymbol{\sigma} = 2\psi_1 \otimes \psi_1^\dagger$ from [2] (See Eqs. 3.28, 5.25). The $e^{+\imath\omega_0\tau/2}$ component is:

$$\begin{bmatrix} \sin^2(\theta/2) & -e^{-\imath\phi}\sin(\theta/2)\cos(\theta/2) \\ -e^{\imath\phi}\sin(\theta/2)\cos(\theta/2) & \cos^2(\theta/2) \end{bmatrix} = \begin{bmatrix} \sin(\theta/2)e^{-\imath\phi/2} \\ -\cos(\theta/2)e^{+\imath\phi/2} \end{bmatrix} \otimes \begin{bmatrix} \sin(\theta/2)e^{\imath\phi/2} & -\cos(\theta/2)e^{-\imath\phi/2} \end{bmatrix}. \tag{5}$$

[6] As we will see it is focusing the attention on the supposed aligning of the spin axis that sends us irrevokably down the rabbit hole. It is the unshakable belief that the experiment unmistakeably tells us that the spin must be aligned which keeps us in the total impossibility of breaking away from the conceptual death trap of space quantization.

[7] Due to the negative frequencies, it is costumary to interpret this rather by saying that an electron is a superposition of a particle and an anti-particle, but the possibility of negative frequencies is a trivial feature in SU(2) whose axioms do not accomodate for the existence of anti-particles. What you do not put in cannot come out by magic. Also the gauge symmetry from which the idea of anti-particles is derived absolutely does not play any rôle in the derivation of the Dirac equation. Hence, once again, what does not go in cannot come out. One can introduce *a posterio* anti-particles into the theory by adding the gauge symmetry, but in order to avoid ambiguity between the two types of negative frequencies one should then use the matrix γ_5 to define charge coordinates. Hence the hype of the antiparticle interpretation is just flawed within the present context.

This corresponds to $\mathbb{1} - \mathbf{s}\cdot\boldsymbol{\sigma} = 2\psi_2 \otimes \psi_2^\dagger$. Note that ψ_1 and ψ_2 are orthogonal. Now the idea is that a magnetic field would make the spin vector precess. We have explained this in earlier work (see [4]). For different radii of the circular motion within a magnetic field the cyclotron frequency remains the same in the non-relativistic limit. Every local co-traveling frame will spin at the same frequency, just like your horse on a merry-go-round does not only move along a circle but also spins around its own axis with repect to the frame of the observers on the ground.[8]. If you shrink the circular orbit in the magnetic field to a point the spinning motion with the cyclotron frequency around the axis will remain. Therefore a pointlike charged particle at rest in a magnetic field will be spinning even if it were initially spinless. But if it initially already spins and its spin axis is tilted, then this axis will be precessing, which corresponds to the intuitive narrative based on the analogy with a spinning top. The final test of this merry-go-round scenario will be whether it reproduces the experimental results. We have no *a priori* knowledge that would help us in deciding if this is correct or otherwise (except perhaps our remark in Footnote 8). We encounter this merry-go-round scenario also in Purcell's explanation of the Thomas precession [5]. We obtain then the motion:

$$\begin{bmatrix} e^{-\imath\Omega\tau/2} & \\ & e^{\imath\Omega\tau/2} \end{bmatrix} \mathbf{R}(\tau), \tag{6}$$

where $\Omega = \frac{qB}{m_0}$ is the cyclotron frequency. Let us write the effect of this precession on both components of $\mathbf{R}(\tau)$:

$$\begin{bmatrix} e^{-\imath\Omega\tau/2} & \\ & e^{+\imath\Omega\tau/2} \end{bmatrix} \begin{bmatrix} \sin^2(\theta/2) & -e^{-\imath\phi}\sin(\theta/2)\cos(\theta/2) \\ -e^{\imath\phi}\sin(\theta/2)\cos(\theta/2) & \cos^2(\theta/2) \end{bmatrix} e^{+\imath\omega_0\tau/2} =$$

$$\begin{bmatrix} \sin^2(\theta/2) & -e^{-\imath\phi}\sin(\theta/2)\cos(\theta/2) \\ 0 & 0 \end{bmatrix} e^{\imath(\omega_0-\Omega)\tau/2} + \begin{bmatrix} 0 & 0 \\ -e^{\imath\phi}\sin(\theta/2)\cos(\theta/2) & \cos^2(\theta/2) \end{bmatrix} e^{\imath(\omega_0+\Omega)\tau/2}. \tag{7}$$

The matrices are here again tensor products. But they are now of a novel type $\psi \otimes \chi^\dagger$, which no longer provides a familiar link with some rotation axis as in the equation $\mathbb{1} + \mathbf{s}\cdot\boldsymbol{\sigma} = 2\psi_1 \otimes \psi_1^\dagger$. This is quite normal because a precession has no fixed rotation axis. We could write actually the first term in the right-hand side of this equation as:

$$\begin{bmatrix} \sin^2(\theta/2) & -e^{-\imath\phi}\sin(\theta/2)\cos(\theta/2) \\ 0 & 0 \end{bmatrix} e^{\imath(\omega_0-\Omega)\tau/2} = e^{-\imath\Omega\tau/2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} \sin^2(\theta/2) & -e^{-\imath\phi}\sin(\theta/2)\cos(\theta/2) \end{bmatrix} e^{\imath\omega_0\tau/2}, \tag{8}$$

which interestingly contains traces of the history of what we have done. Actually we can write:

$$\begin{bmatrix} e^{-\imath\Omega\tau/2} & \\ & e^{\imath\Omega\tau/2} \end{bmatrix} = e^{-\imath\Omega\tau/2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \end{bmatrix} + e^{+\imath\Omega\tau/2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \end{bmatrix}. \tag{9}$$

Therefore the term that goes with $e^{\imath(\omega_0-\Omega)\tau/2}$ is:

$$e^{-\imath\Omega\tau} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \sin(\theta/2)e^{-\imath\phi/2} \\ -\cos(\theta/2)e^{+\imath\phi/2} \end{bmatrix} \otimes \begin{bmatrix} \sin(\theta/2)e^{\imath\phi/2} & -\cos(\theta/2)e^{-\imath\phi/2} \end{bmatrix}. \tag{10}$$

We can multiply the magenta matrices in the middle, which can be shown to be a correct procedure. We obtain then the scalar $\sin(\theta/2)e^{-\imath\phi/2}$. And this way we obtain again the same result as in Eq. 8. We are multiplying all the time with matrices that can be written as tensor products because they have determinant zero. This is a specifity of SU(2) as explained in [1]. The result of multiplying two such matrices will still have a zero determinant, such that it can be written as a tensor product, but it will no longer have the structure $\psi \otimes \psi^\dagger$. We should not worry that the expressions could be meaningless, because this is just Euclidean geometry. They are definitely not obvious to interpret, but they are exact. This calculus is very handsome because it reduces the calculations to a minimum. The other component yields:

$$\begin{bmatrix} e^{-\imath\Omega\tau/2} & \\ & e^{+\imath\Omega\tau/2} \end{bmatrix} \begin{bmatrix} \cos^2(\theta/2) & e^{-\imath\phi}\sin(\theta/2)\cos(\theta/2) \\ e^{\imath\phi}\sin(\theta/2)\cos(\theta/2) & \sin^2(\theta/2) \end{bmatrix} e^{-\imath\omega_0\tau/2} =$$

$$\begin{bmatrix} \cos^2(\theta/2) & e^{-\imath\phi}\sin(\theta/2)\cos(\theta/2) \\ 0 & 0 \end{bmatrix} e^{-\imath(\omega_0+\Omega)\tau/2} + \begin{bmatrix} 0 & 0\,, \\ e^{\imath\phi}\sin(\theta/2)\cos(\theta/2) & \sin^2(\theta/2) \end{bmatrix} e^{-\imath(\omega_0-\Omega)\tau/2}. \tag{11}$$

We can now rearrange the terms according to their energies:

[8] Note that this is different from what happens with a gyroscope in a space ship. There the gyroscope is not subject to gravitational forces because it is in free fall, such that it cannot precess. It is much harder to erase the electromagnetic field by an acceleration. One can erase the magnetic field by a rotating frame but then an electric field enters the scene. Note that in the rotating frame, the local co-moving frames are spinning like the horse in the merry-go-round. A magnetic field *is* a rotating frame.

$$\begin{bmatrix} \cos^2(\theta/2) & e^{-\imath\phi}\sin(\theta/2)\cos(\theta/2) \\ 0 & 0 \end{bmatrix} e^{-\imath(\omega_0+\Omega)\tau/2} + \begin{bmatrix} 0 & 0 \\ -e^{\imath\phi}\sin(\theta/2)\cos(\theta/2) & \cos^2(\theta/2) \end{bmatrix} e^{+\imath(\omega_0+\Omega)\tau/2}, \tag{12}$$

where we can factorize out the probability amplitude $\cos(\theta/2)$, and:

$$\begin{bmatrix} 0 & 0 \\ e^{\imath\phi}\sin(\theta/2)\cos(\theta/2) & \sin^2(\theta/2) \end{bmatrix} e^{-\imath(\omega_0-\Omega)\tau/2} + \begin{bmatrix} \sin^2(\theta/2) & -e^{-\imath\phi}\sin(\theta/2)\cos(\theta/2) \\ 0 & 0 \end{bmatrix} e^{+\imath(\omega_0-\Omega)\tau/2}, \tag{13}$$

where we can factorize out the probability amplitude $\sin(\theta/2)$. We see thus that there are two possible energies for the electron within the magnetic field. All the fuss of interpreting the formalism with the energy operator $-\frac{\hbar}{\imath}\frac{\partial}{\partial\tau}$ was thus self-defeating bogus.[9]. This is because we apply it to a mixed state with four different frequencies in all. This is the reason why we found a non-constant oscillating energy with the traditional operator. Now we have found an analysis that yields the correct energies.[10] It also explains the whole Stern-Gerlach experiment, provided we can still explain how these two energies lead to different trajectories (see below). Let us note that we have presented the effect of the magnetic field on the charge by Eq. 9. This is not something we find in textbooks, but we have explained this in [4] in terms of vorticity. The algebra does not contain a current loop. It is just contains a rotating point charge.[11] The whole puzzle why the magnetic moment would have to align with the field has now disappeared. We find the right energy without having to invoke alignments of axes with the magnetic field. Such alignments are just no longer part of the story. Furthermore, there is simply no longer a well-defined single fixed axis as transpires from the weird terms $\psi \otimes \chi^\dagger$ in the formalism. Eq. 12 describes a motion with energy $\hbar(\omega_0+\Omega)/2 = m_0c^2 + \frac{\hbar qB}{2m_0}$ and which occurs with probability $\cos^2(\theta/2)$, while Eq. 13 describes a motion with energy $\hbar(\omega_0-\Omega)/2 = m_0c^2 - \frac{\hbar qB}{2m_0}$ and which occurs with probability $\sin^2(\theta/2)$, in agreement with the experimental results. These are both complex motions that we cannot describe in simple terms like a rotation around some axis. We can safely assume that these two components just describe precession. The Stern-Gerlach filter separates these two energies into two different beams. It is one of those two rearranged combinations that in general will be fed into a next Stern-Gerlach apparatus. Note that the average energy is $\hbar(\omega_0+\Omega\cos\theta)/2$, such that $V = -\boldsymbol{\mu}\cdot\mathbf{B}$ is a macroscopic energy term, but not a potential energy.

Most textbooks calculate the force exerted on the electron starting from an equation for a potential energy $V = -\boldsymbol{\mu}\cdot\mathbf{B}$ and then using $\mathbf{F} = -\nabla V$. But the physical existence of such a potential energy is doubtful, because a magnetic field cannot do any work. The equation $V = -\boldsymbol{\mu}\cdot\mathbf{B}$ suggests that all directions of space are allowed which is actually what, according to the traditional theory, the experiment proves to be conceptually wrong. It is therefore better to base the analysis on the expression for the energy $E = \hbar\omega/2$ and then to use $\mathbf{F} = -\nabla E$. This will lead then to the same result as in the textbook analysis of the trajectories. Note that the traditional theory for the trajectories is classical because the aim is to show that our classical notions are wrong. To fully validate the theory one should also calculate the trajectories quantum mechanically. In the new theory, classical geometry still prevails and the mysterious quantum effects disappear. Let us finally note that it is not angular momentum which is quantized in physics but the representations of the three-dimensional rotation group SO(3). This is analogous to what transpires from Gell-Mann's theory of SU(3).

3 How to treat a concatenation of filters

How do we treat now two Stern-Gerlach filters in succession? First we assume that there is some air gap between the two filters where the field becomes zero. As soon as the electron enters this area, its precession will cease. The axis will thus remain oriented in the direction it had reached at that time τ_1. The axis $\mathbf{s}\cdot\boldsymbol{\sigma}$ transforms under rotations $\mathbf{R}(\mathbf{e},\Omega\tau)$ with axis $\mathbf{e}$ according to:

$$[\,\mathbf{s}(\tau)\cdot\boldsymbol{\sigma}\,] = [\,\mathbf{R}(\mathbf{e},\Omega\tau)\,]\,[\,\mathbf{s}(0)\cdot\boldsymbol{\sigma}\,]\,[\,\mathbf{R}(\mathbf{e},\Omega\tau)\,]^{-1}. \tag{14}$$

Hence at time τ_1 it will be:

$$[\,\mathbf{s}(\tau_1)\cdot\boldsymbol{\sigma}\,] = [\,\mathbf{R}(\mathbf{e},\varphi_1)\,]\,[\,\mathbf{s}(0)\cdot\boldsymbol{\sigma}\,]\,[\,\mathbf{R}(\mathbf{e},\varphi_1)\,]^{-1}, \quad \text{where:} \quad \varphi_1 = \Omega\tau_1, \tag{15}$$

[9] This operator has been derived by educated guesses from the de Broglie *ansatz*. It is obviously not universal and cannot be generalized to more complicated situations with non-scalar wave functions. See also Footnote 3.

[10] In the matrix for a spinning motion with frequency Ω around the z-axis in Eq. 9, the two frequencies $-\Omega$ and $+\Omega$ occur neatly in two different columns. When we work with spinors, i.e. columns of the rotation matrices, we avoid this way getting confronted with multiple frequencies such that we can use $-\frac{\hbar}{\imath}\frac{\partial}{\partial\tau}$ to obtain a meaningful result. But with four frequencies we can no longer avoid that there will be more than one frequency projected out of a spinor by $-\frac{\hbar}{\imath}\frac{\partial}{\partial\tau}$. We end up with more than one frequency within a spinor, which conjures op the image of a varying energy. But it is the operator $-\frac{\hbar}{\imath}\frac{\partial}{\partial\tau}$ which is then no longer correct.

[11] Let us for a while make the error of thinking that Eq. 9 represents the magnetic field. We see then that the spinning electron is expressed completely in the same way as the magnetic field. From this we could then conclude that the electron spin represents a magnetic field. See also our remark in Footnote 8.

and from τ_1 to τ_2, the moment where it enters the next filter, the motion will now be given by:

$$\mathbf{R}(\tau) = \frac{1}{2}\left[\,[\,\mathbb{1} + \mathbf{s}(\tau_1)\cdot\boldsymbol{\sigma}\,]\,e^{-\imath\omega_0(\tau-\tau_1)/2} + [\,\mathbb{1} - \mathbf{s}(\tau_1)\cdot\boldsymbol{\sigma}\,]\,e^{+\imath\omega_0(\tau-\tau_1)/2}\right]. \tag{16}$$

This is of course under the assumption that the changes are abrupt. In reality, the field will presumably be varying smoothly. This corresponds to the next level of difficulty where now the precession axis **e** is varying with time. When the motion of **e** is not a rotation, treating this correctly could be quite something.

References

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