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Highlights

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- Nonlinear locomotion equations of human arm are investigated.
- Linear frequencies vary with respect to the postures.
- The system is coupled to a nonlinear energy sink.
- Fast and slow dynamics are revealed.

Nonlinear passive tremor control of human arm

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Abstract

An academic model for passive vibration control of human arm suffering from essential tremors is proposed. The nonlinear uncontrolled system equations are linearised around arbitrary postural positions showing variations of linear natural frequencies as functions of postural positions. The coupled system, i.e. the model of the arm with the attached nonlinear absorber, is treated via a time multiple scale method leading to detection of fast and slow systems dynamics. These dynamics provide design tools for tuning parameters of the nonlinear absorber.

1. Introduction

The unwanted or large amplitude vibrations of systems seek for appropriate control devices to be able to reduce oscillation levels corresponding to comfort and undamaged states for systems. The control solutions cover large spectrum of categories expanding from active to passive solutions (Housner et al., 1997). The active category of absorbers demand injected (at least initial) energies for their activation while passive systems exploit direct linear or nonlinear interactions between oscillators for reducing energies of main systems. Roberson (1952) illustrated that via supplementing the linear restoring forcing function of an absorber by a cubic term, the suppression width of the absorber would increase. Since then different types of nonlinear absorbers are developed exploiting different sources of nonlinearities. One of such absorbers is named as nonlinear energy sink (NES) (Vakakis et al., 2008) which in its early developments possessed pure cubic nonlinearity, i.e. with no linear part (Vakakis, 2001; Vakakis and Gendelman, 2000). The control process by NES is accomplished via nonlinear energy exchanges between oscillators leading to periodic or non periodic regimes (Starosvetsky and Gendelman, 2008b; Ture Savadkoohi et al., 2016). The NES and its derived technologies are applied to different engineering systems such as acoustics (Cochelin et al., 2006), aero-elastic effects on wings (Hubbard et al., 2010), chatter instabilities (Gourc et al., 2015), buildings (Gourdon et al., 2007; Ture Savadkoohi et al., 2012; Wierschem et al., 2014), cables (Weiss et al., 2018), etc.

The idea of the current paper is "passive control" of tremors of human arm due to some disorders such as essential tremors, i.e. action and postural tremor, that is the most frequent movement problems and difficulties which affects daily life patients (Espay et al., 2017). There are several works and patents which aim at mitigation of the pathological tremors with "active" systems. From them we can name patents of Rosen which all of them incorporate velocity dependent forces in an active manner: for example, inducing linear velocity dependent forces (viscous damping) in two (Rosen, 1987) and four (three translation and one angular movement) degrees-of-freedom (dof) (Rosen, 1992). The device consists of a chamber filled with a viscous fluid and a positioned rotation-sensing actuators for sensing the positions and angles and generating electrical signals accordingly. Later on, a four-dof orthosis was invented which permits the patient to move the midpoint of the forearm in a horizontal plane and to rotate the forearm out of the plane via several rotational joints (Rosen and Baiges, 1993). The orthosis could apply linear (viscous) and square-velocity dependent forces (Arnold et al., 1993). Rocon et al. (2007) presented a robotic exoskeleton which exploits two control techniques based on biomechanical loading, namely tremor reduction by impedance control and notch filtering at tremor frequency (Rocon et al., 2012). Lora-Millán et al. (2019) studied effects of mechanical vibrations on tremors of the arm and forearm. They used some piezoelectric actuators in order to provide stimuli to the arm and forearm and monitoring tremors by sensors. They spotted that the mechanical vibration sometime could increase amplitudes of tremors. A literature review about different devices for tremor control is reported by Castrillo-Fraile et al. (2019). The linear passive control of tremors are already studied (see for example (Hashemi et al., 2004)). Our paper exploits

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the NES technology for passive control of tremors of a simplified model of human arm. The paper is organised as it follows: the simplified nonlinear mathematical model of the locomotion of human arm and detection of its linear natural frequencies are presented in Sect. 2. In Sect. 3 the system is coupled to a NES and its fast and slow dynamics are clarified. Application of the proposed system in passive control of human arm due to essential tremors is provided in Sect. 4. Finally, the paper is concluded in Sect. 5.

2. The mathematical model of the upper limb and linearisation about a postural position

The simplified academic model of the locomotion of human arm is illustrated in Fig. 1. In this model some biomechanical effects such as

- the elbow locking (Jackson et al., 1978), i.e. limitation of extension of human arm at elbow joint when arm and forearm are at the same line;
- the elastic activities of muscles;
- three-dimensional movement of the arm,

are ignored. Moreover, we neglect the movement of hand via assuming that the forearm and the hand constitute a single piece with the equivalent mass of both pieces which is situated at the centre of their masses. Let us define following parameters and variables:

- θ and ϕ : developed angles in arm and forearm, respectively;
- m_1 and m_2 : the mass of the arm and the equivalent mass of the forearm and the hand, respectively;
- l_1 : length of the arm;
- l_2 : distance of the equivalent mass m_2 from the elbow joint;
- d_1 : distance of the center of the mass of the arm to the glenohumeral joint;
- I : the moment of inertia of the upper arm with respect to the glenohumeral joint;
- H' and J' : applied torques about the glenohumeral and the elbow joints, respectively.

We use the developed model by Jackson et al. (1978) for representing locomotion of human upper limb; it reads:

$$\begin{cases} \rho_{11}\ddot{\theta} + \rho_{12}\ddot{\phi} + \rho_{13} - T_I + V_I = H' \\ \rho_{21}\ddot{\theta} + \rho_{22}\ddot{\phi} + \rho_{23} - T_{II} + V_{II} = J' \end{cases} \quad (1)$$

Different parameters of Eq. 1 are reported in Appendix A. It should be mentioned that other models exist which take into account more complete factors of the locomotion such as detailed bones and joints behaviours (Barhorst and Schovanec, 2009; Moody et al., 2009) or pennate (Wojnicz et al., 2017) and mono- and bi-articular muscles (Zagrodny et al., 2018).

Let us linearise system equations around a provided postural positions represented by θ_0 and ϕ_0 ; i.e.

$$\begin{aligned} \theta &= \theta_0 + \theta_s \\ \phi &= \phi_0 + \phi_s \end{aligned} \quad (2)$$

Following system is obtained:

$$\begin{cases} \mu_{11}\ddot{\theta}_s + \mu_{12}\ddot{\phi}_s + \kappa_{11}\theta_s + \kappa_{12}\phi_s + C_1 = H' \\ \mu_{21}\ddot{\theta}_s + \mu_{22}\ddot{\phi}_s + \kappa_{21}\theta_s + \kappa_{22}\phi_s + C_2 = J' \end{cases} \quad (3)$$

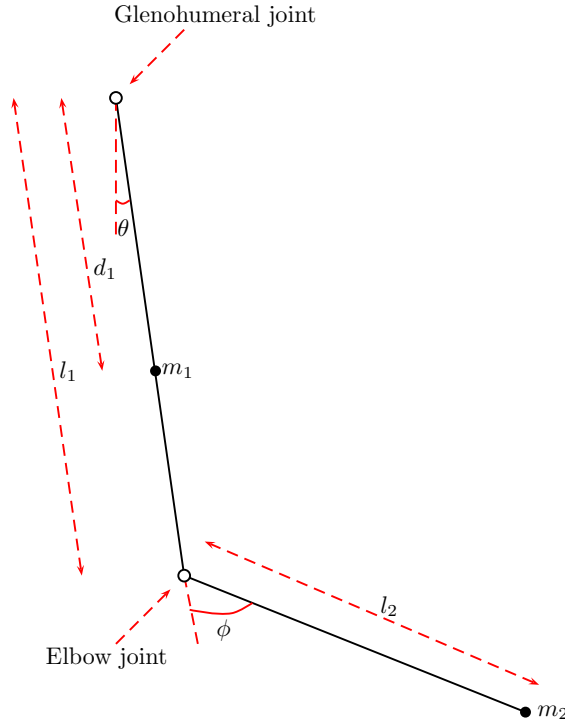


Figure 1: The simplified academic model of the locomotion of human arm.

All variables of the Eq. 3 are clarified in Appendix B.

The parameters C_1 and C_2 are fact the the resistive moments in order to maintain the postural position. Therefore, the total excitation due to tremor, F_{ext} , is summarised as:

$$F_{\text{ext}} = \begin{pmatrix} H' - C_1 \\ J' - C_2 \end{pmatrix} \quad (4)$$

We assume that F_{ext} can be decomposed in the form of Fourier series on the basis of the frequency Ω . To simplify the study, we take the first harmonics of the series and we consider a mono-frequency excitation; we can suppose:

$$F_{\text{ext}} = \begin{pmatrix} f_1 \sin(\Omega t) \\ f_2 \sin(\Omega t) \end{pmatrix} \quad (5)$$

Equation 3 can be written as:

$$\underbrace{\begin{pmatrix} \mu_{11} & \mu_{12} \\ \mu_{21} & \mu_{22} \end{pmatrix}}_M \begin{pmatrix} \ddot{\theta}_s \\ \ddot{\phi}_s \end{pmatrix} + \underbrace{\begin{pmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{pmatrix}}_K \begin{pmatrix} \theta_s \\ \phi_s \end{pmatrix} = \begin{pmatrix} f_1 \sin(\Omega t) \\ f_2 \sin(\Omega t) \end{pmatrix} \quad (6)$$

Variation of natural frequencies of the linear system of Eq. 6 with respect to different postures are illustrated in Fig. 2. The system parameters are reported in Table 1. These figures show that system frequencies are sensitive to initial postural positions. This means that classical linear absorbers, i.e. tuned mass dampers (Frahm, 1911), can not be efficient for passive control of tremors due to variations of natural frequencies according to different initial postural positions.

In the next section the two degrees of freedom pendulum will be coupled to a NES and different dynamics of the system will be clarified.

Table 1

System parameters of the arm (Drillis et al., 1964)

g (m.sec ⁻²)	l_1 (m)	l_2 (m)	d_1 (m)	m_1 (kg)	m_2 (kg)	I (kg.m ²)
9.81	0.364	0.293	0.155	2.07	1.7	0.076

3. The mathematical model of the upper limb coupled to a NES

Let us consider a simplified model of the upper limb which is coupled to a NES with the distance of l_3 from the elbow joint (see Fig. 3). We assign m_N and u as the mass and the generalised displacement of the NES which possesses a general nonlinear restoring forcing function as $f_{\text{NES}}(u)$, e.g. $f_{\text{NES}}(u) = k_N u^3$. The damping scenario of the NES is supposed to be linear and its coefficient is represented by c_N . Governing equations of the three-dof systems is written as:

$$\begin{cases} \rho_{11}\ddot{\theta} + \rho_{12}\ddot{\phi} + \rho_{13}\ddot{u} + \rho_{14} - T_1 + V_1 = H' \\ \rho_{21}\ddot{\theta} + \rho_{22}\ddot{\phi} + \rho_{23}\ddot{u} + \rho_{24} - T_2 + V_2 = J' \\ \rho_{31}\ddot{\theta} + \rho_{32}\ddot{\phi} + \rho_{33}\ddot{u} + \rho_{34} - T_3 + V_3 + c_N\dot{u} = 0 \end{cases} \quad (7)$$

Appendix C describes all variables of Eq. 7. Let us also assume that:

$$\begin{aligned} H' &= f_1 \sin(\Omega t) \\ J' &= f_2 \sin(\Omega t) \end{aligned} \quad (8)$$

We can perform linearisation of system equations around a postural position described in Eq. 2. Introducing the parameter $\epsilon = \frac{m_N}{m_2}$ and assuming $\epsilon \ll 1$, following system is obtained:

$$M' \begin{pmatrix} \ddot{\theta}_s \\ \ddot{\phi}_s \\ \ddot{u} \end{pmatrix} + K' \begin{pmatrix} \theta_s \\ \phi_s \\ u \end{pmatrix} + N' + A' + C' = \begin{pmatrix} F_1 \sin(\Omega t) \\ F_2 \sin(\Omega t) \\ 0 \end{pmatrix} \quad (9)$$

with variables which are defined in Appendix D.

Let us suppose that $p = \text{nint}\left(\frac{\omega_2(\theta_0, \phi_0)}{\omega_1(\theta_0, \phi_0)}\right)$, $p \in \mathbb{N}^*$, where $\text{nint}(\dots)$ stands for the nearest integer function. Moreover, we assume that:

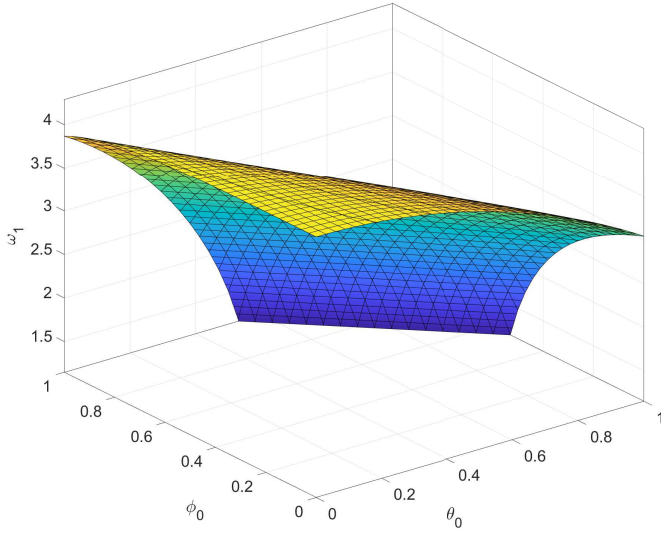
$$\begin{aligned} \omega_1 &= \Omega + \sigma_1 \epsilon \\ \omega_2 &= p\omega_1 + \tilde{\sigma}_2 \epsilon = p\Omega + \sigma_2 \epsilon \end{aligned} \quad (10)$$

In the modal coordinate, we apply following change of variables:

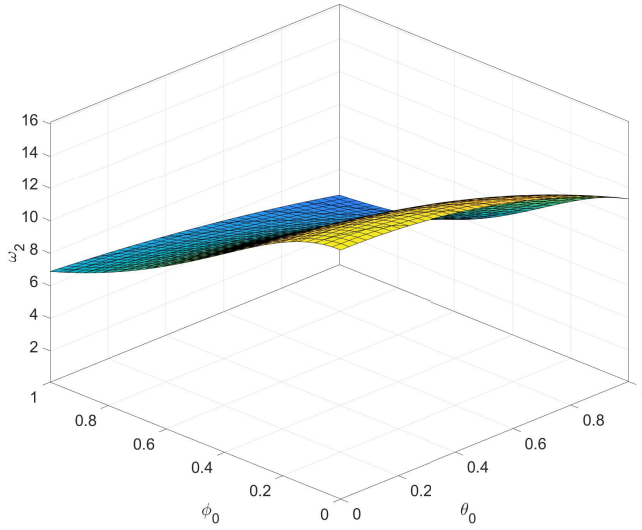
$$\begin{pmatrix} \theta_S \\ \phi_S \\ u \end{pmatrix} = P \begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix}, \text{ with } P = \begin{pmatrix} V_{11} & V_{12} & 0 \\ V_{21} & V_{22} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (11)$$

where P is a matrix of eigenvectors of $M'^{-1}K'$. To take into account dissipations of the main system, following damping matrix can be introduced to the system 9 as:

$$C'_d = \begin{pmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (12)$$



(a) ω_1



(b) ω_2

Figure 2: Variations of system linear frequencies with respect to initial postural positions θ_0 and ϕ_0 . a) ω_1 b) ω_2 .

Then, we set

$$P^{-1}M'^{-1}C'_dP = \epsilon C_D \quad (13)$$

with

$$C_D = \begin{pmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (14)$$

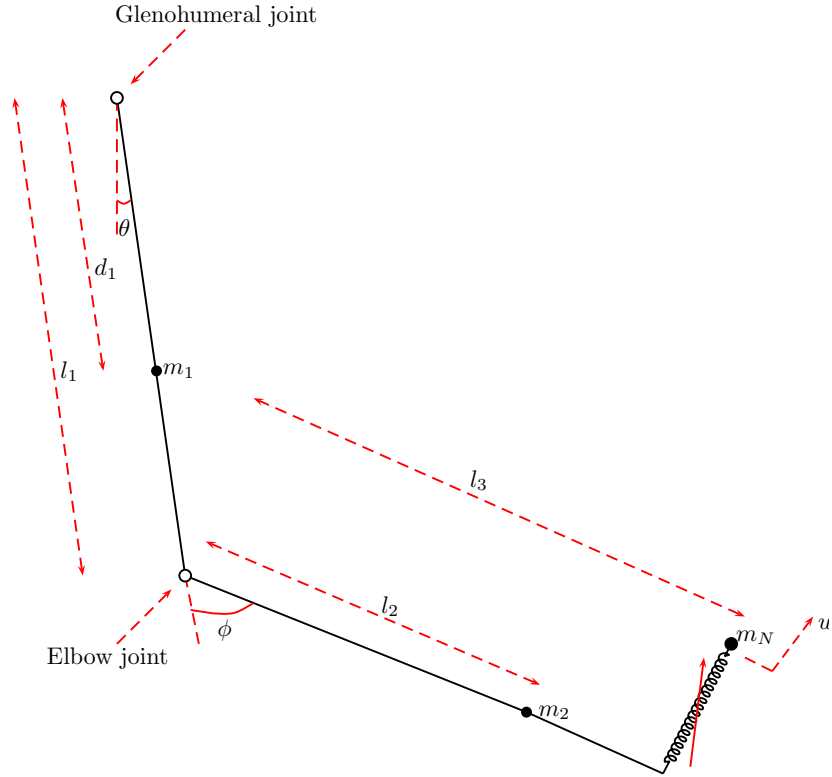


Figure 3: The academic model of the locomotion of human arm coupled with a NES with the mass m_N and the generalised displacement as u . The NES presents a nonlinear restoring forcing function and a linear damping scenario.

Considering explained change of variables in Eq. 9 following system is obtained:

$$\begin{pmatrix} \ddot{W}_1 \\ \ddot{W}_2 \\ \ddot{W}_3 \end{pmatrix} + (D + \epsilon\Lambda) \begin{pmatrix} \dot{W}_1 \\ \dot{W}_2 \\ \dot{W}_3 \end{pmatrix} + \epsilon C_D \begin{pmatrix} \dot{W}_1 \\ \dot{W}_2 \\ \dot{W}_3 \end{pmatrix} + \dot{W}_3(T + \epsilon U) + n_0(W_3)(X + \epsilon Y) + (A + \epsilon B) = \epsilon H \quad (15)$$

D and Λ are 3×3 matrices, where T, U, X, Y, A, B and H are 3×1 vectors. All of these, are clarified in Appendix E. In the next section the complexified form of system equations are treated by a time multiple scale method for clarifying its different dynamics.

3.1. Complexification of the system and keeping its first harmonics

Let us introduce complex variables of Manevitch (Manevitch, 2001):

$$\begin{cases} \Psi_1 \exp(i\Omega t) = \dot{W}_1 + i\Omega W_1 \\ \Psi_2 \exp(ip\Omega t) = \dot{W}_2 + ip\Omega W_2 \\ \Psi_3 \exp(i\Omega t) = \dot{W}_3 + i\Omega W_3 \end{cases} \quad (16)$$

We are interested to keep first harmonic of the system (Ture Savadkoohi et al., 2016). Following equation is obtained:

$$\begin{pmatrix} \dot{\Psi}_1 + \frac{i(\omega_1 - \sigma_1 \epsilon)}{2} \Psi_1 \\ \dot{\Psi}_2 + \frac{i(\omega_2 - \sigma_2 \epsilon)}{2} \Psi_2 \\ \dot{\Psi}_3 + \frac{i(\omega_1 - \sigma_1 \epsilon)}{2} \Psi_3 \end{pmatrix} + (D + \epsilon \Lambda) \begin{pmatrix} -\frac{i}{2(\omega_1 - \sigma_1 \epsilon)} \Psi_1 \\ -\frac{i}{2(\omega_2 - \sigma_2 \epsilon)} \Psi_2 \\ -\frac{i}{2(\omega_1 - \sigma_1 \epsilon)} \Psi_3 \end{pmatrix} + \epsilon C_D \begin{pmatrix} \dot{\Psi}_1 \\ \dot{\Psi}_2 \\ \dot{\Psi}_3 \end{pmatrix} + \frac{\Psi_3}{2} (T + \epsilon U) + (X + \epsilon Y) f_n(\Psi_3, \Psi_3^*) = \epsilon \mathbb{H} \quad (17)$$

where the function f_n is defined as (Gendelman, 2008):

$$f_n(\Psi_3, \Psi_3^*) = \frac{\Omega}{2\pi} \int_0^{\frac{2\pi}{\Omega}} n_0(W_3) \exp(-i\Omega t) dt = -\frac{i\Psi_3}{2} G(|\Psi_3|^2) \quad (18)$$

where $G(|\Psi_3|^2)$ is a function which depends on the nonlinearity type of the NES. The $*$ stands for the complex conjugate of the variable. Definition of \mathbb{H} is provided in Appendix F.

3.2. Fast/slow dynamics of the system

To treat system equations, we use a perturbation technique namely, the multiple scale method (Nayfeh, 2004). Different scales of the time t are defined as $\tau_0 = t$ (fast scale), and $\tau_j = \epsilon^j t$, $j \in \mathbb{N}^+$ (slow scales). These time scales are coupled to each other via the physical parameter ϵ . To clarify different systems dynamics, different orders of the ϵ should be considered in Eq. 17.

3.2.1. Fast dynamics: ϵ^0 order of system equations

The ϵ^0 order of the system 17 leads to:

$$\begin{cases} \frac{\partial \Psi_1}{\partial \tau_0} = 0 \\ \frac{\partial \Psi_2}{\partial \tau_0} = 0 \\ \frac{\partial \Psi_3}{\partial \tau_0} + \frac{i\omega_1}{2} \Psi_3 - \frac{i}{2} \left(\frac{D_{31}}{\omega_1} \Psi_1 + \frac{D_{32}}{\omega_2} \Psi_2 \right) + \frac{c_0 \Psi_3}{2} + f_n = 0 \end{cases} \quad (19)$$

For a NES with cubic nonlinearity, $G(|\Psi_3|^2) = \frac{3k_n}{4\omega_1^3} |\Psi_3|^2$, so $f_n = -\frac{i}{2} \Psi_3 \frac{3k_n}{4\omega_1^3} |\Psi_3|^2$. Fixed points of the system correspond to system behaviours when $\tau_0 \rightarrow \infty$, leading to $\frac{\partial \Psi_3}{\partial \tau_0} \rightarrow 0$ or

$$\mathcal{H}(\Psi_1, \Psi_1^*, \Psi_2, \Psi_2^*, \Psi_3, \Psi_3^*) = -\frac{i}{2\omega_1} (D_{31} \Psi_1 + \frac{D_{32}}{p} \Psi_2) + \left(\frac{i\omega_1}{2} + \frac{c_0}{2} \right) \Psi_3 + f_n = 0 \quad (20)$$

If we write system variables in polar forms as $\Psi_j = N_j \exp(i\delta_j)$, $j = 1, 2, 3$, then Eq. 20 reads:

$$\left(c_0^2 + \omega_1^2 + G^2(N_3^2) - 2\omega_1 G(N_3^2) \right) N_3^2 = \left(\frac{D_{31}}{\omega_1} \right)^2 N_1^2 + \left(\frac{D_{32}}{p\omega_1} \right)^2 N_2^2 + \frac{2D_{31}D_{32}}{p\omega_1^2} \cos(\delta_1 - \delta_2) N_1 N_2 \quad (21)$$

The complex function \mathcal{H} or the real equation of the system 21 are a geometrical surface corresponding to final destinations of the system which is named as slow invariant manifold (SIM). It is seen that the four-dimensional SIM is a function of amplitudes N_j , $j = 1, 2, 3$ and phase lag between Ψ_1 and Ψ_2 represented by $\delta = \delta_1 - \delta_2$. Different views of the four-dimensional SIM for some values of the δ are illustrated in Fig. 4. Some zones of the SIM are unstable. A stability analysis can be carried out via linear perturbation of variables of the Eq. 20 or 21 for tracing stable and unstable zones of the SIM (see for example Lamarque et al. (2017); Hurel et al. (2019)). When the system reaches to

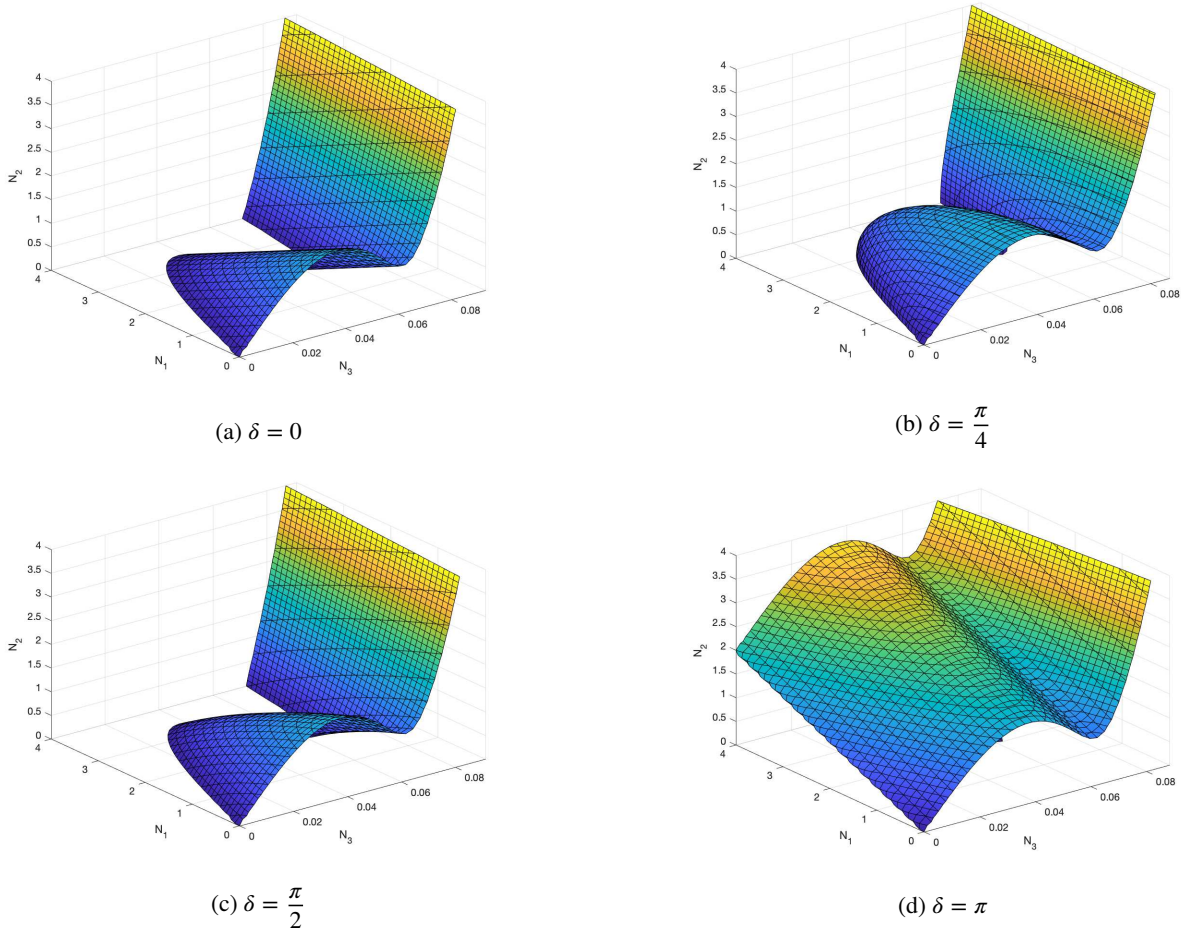


Figure 4: Three-dimensional views of the four-dimensional SIM (see Eq. 21) for some fixed phase lag ($\delta = \delta_1 - \delta_2$). System parameters are $k_n = 10$, $c_0 = 1$, $l_3 = 0.299$, $\theta_0 = 0$, $\phi_0 = 0$.

boundaries of the unstable zones, it bifurcates to its another stable zone. Depending on the characteristics of external forcing terms, the system can be attracted by periodic or non periodic regimes (Gendelman et al., 2008; Starosvetsky and Gendelman, 2008a). Different possible regimes can be revealed by studying slow dynamics of the system which is explained in the next section.

3.2.2. Slow dynamics: ϵ^1 order of system equations

The ϵ^1 order of first two equations of the system 17 reads:

$$\begin{cases} \frac{\partial \Psi_1}{\partial \tau_1} - \frac{i}{2\omega_1} \left((\Lambda_{11} + 2\sigma_1\omega_1)\Psi_1 + \frac{\Lambda_{12}}{p}\Psi_2 + \Lambda_{13}\Psi_3 \right) + \frac{c_1}{2}\Psi_1 + \frac{U_1}{2}\Psi_3 + f_n Y_1 = \mathbb{H}_1 \\ \frac{\partial \Psi_2}{\partial \tau_1} - \frac{ip}{2\omega_2} \left(\Lambda_{21}\Psi_1 + (\Lambda_{22} + 2\sigma_2\omega_2)\frac{\Psi_2}{p} + \Lambda_{23}\Psi_3 \right) + \frac{c_2}{2}\Psi_2 + \frac{U_2}{2}\Psi_3 + f_n Y_2 = \mathbb{H}_2 \end{cases} \quad (22)$$

Which can be written in a compact form as:

$$\begin{cases} \frac{\partial \Psi_1}{\partial \tau_1} = \mathcal{F}_1(\Psi_1, \Psi_1^*, \Psi_2, \Psi_2^*, \Psi_3, \Psi_3^*) \\ \frac{\partial \Psi_2}{\partial \tau_1} = \mathcal{F}_2(\Psi_1, \Psi_1^*, \Psi_2, \Psi_2^*, \Psi_3, \Psi_3^*) \end{cases} \quad (23)$$

The evolution of the SIM (see Eq. 20) at τ_1 time scale yields to:

$$\left(\begin{array}{cccccc} \frac{\partial \mathcal{H}}{\partial \Psi_1} & \frac{\partial \mathcal{H}}{\partial \Psi_1^*} & \frac{\partial \mathcal{H}}{\partial \Psi_2} & \frac{\partial \mathcal{H}}{\partial \Psi_2^*} & \frac{\partial \mathcal{H}}{\partial \Psi_3} & \frac{\partial \mathcal{H}}{\partial \Psi_3^*} \\ \frac{\partial \mathcal{H}^*}{\partial \Psi_1} & \frac{\partial \mathcal{H}^*}{\partial \Psi_1^*} & \frac{\partial \mathcal{H}^*}{\partial \Psi_2} & \frac{\partial \mathcal{H}^*}{\partial \Psi_2^*} & \frac{\partial \mathcal{H}^*}{\partial \Psi_3} & \frac{\partial \mathcal{H}^*}{\partial \Psi_3^*} \end{array} \right) \begin{pmatrix} \frac{\partial \Psi_1}{\partial \tau_1} \\ \frac{\partial \Psi_1^*}{\partial \tau_1} \\ \frac{\partial \Psi_2}{\partial \tau_1} \\ \frac{\partial \Psi_2^*}{\partial \tau_1} \\ \frac{\partial \Psi_3}{\partial \tau_1} \\ \frac{\partial \Psi_3^*}{\partial \tau_1} \end{pmatrix} = 0 \quad (24)$$

which can be reorganised as:

$$\underbrace{\left(\begin{array}{cc} \frac{\partial \mathcal{H}}{\partial \Psi_3} & \frac{\partial \mathcal{H}}{\partial \Psi_3^*} \\ \frac{\partial \mathcal{H}^*}{\partial \Psi_3} & \frac{\partial \mathcal{H}^*}{\partial \Psi_3^*} \end{array} \right)}_B \begin{pmatrix} \frac{\partial \Psi_3}{\partial \tau_1} \\ \frac{\partial \Psi_3^*}{\partial \tau_1} \end{pmatrix} = - \left(\begin{array}{cccc} \frac{\partial \mathcal{H}}{\partial \Psi_1} & \frac{\partial \mathcal{H}}{\partial \Psi_1^*} & \frac{\partial \mathcal{H}}{\partial \Psi_2} & \frac{\partial \mathcal{H}}{\partial \Psi_2^*} \\ \frac{\partial \mathcal{H}^*}{\partial \Psi_1} & \frac{\partial \mathcal{H}^*}{\partial \Psi_1^*} & \frac{\partial \mathcal{H}^*}{\partial \Psi_2} & \frac{\partial \mathcal{H}^*}{\partial \Psi_2^*} \end{array} \right) \underbrace{\begin{pmatrix} \frac{\partial \Psi_1}{\partial \tau_1} \\ \frac{\partial \Psi_1^*}{\partial \tau_1} \\ \frac{\partial \Psi_2}{\partial \tau_1} \\ \frac{\partial \Psi_2^*}{\partial \tau_1} \end{pmatrix}}_F \quad (25)$$

Looking at Eq. 23, we can write:

$$\mathcal{F}^T = (\mathcal{F}_1 \quad \mathcal{F}_1^* \quad \mathcal{F}_2 \quad \mathcal{F}_2^*) \quad (26)$$

Equilibrium points of the system (corresponding to periodic regimes of the initial system) are those which satisfy:

$$\left\{ \begin{array}{l} V = 0 \\ \mathcal{H} = 0 \\ \text{Det}(B) \neq 0 \end{array} \right. \quad (27)$$

while singularities of the system (corresponding to non periodic regimes of the initial system) verify:

$$\left\{ \begin{array}{l} V = 0 \\ \mathcal{H} = 0 \\ \text{Det}(B) = 0 \end{array} \right. \quad (28)$$

3.2.3. A numerical example

Let us consider the parameters of the arm which are presented in Table 1. The system is coupled to a NES which its supplementary parameters are reported in Table 2. All possible equilibrium points of the system for sweeping σ_1

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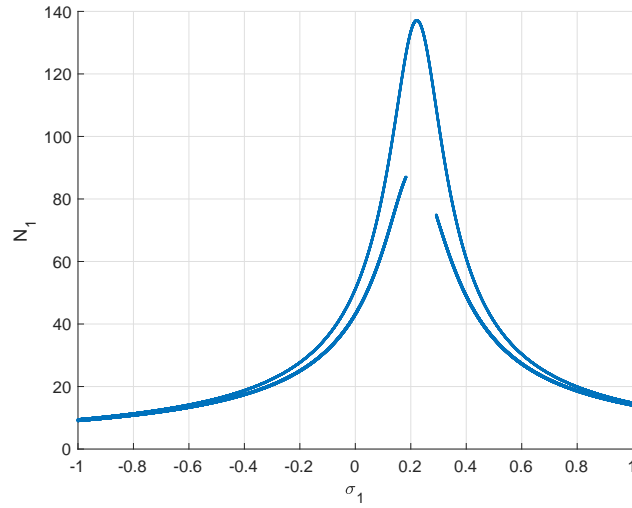


Figure 5: Equilibrium points of the system of Table 2 as a function of the detuning parameter: N_1 vs. σ_1

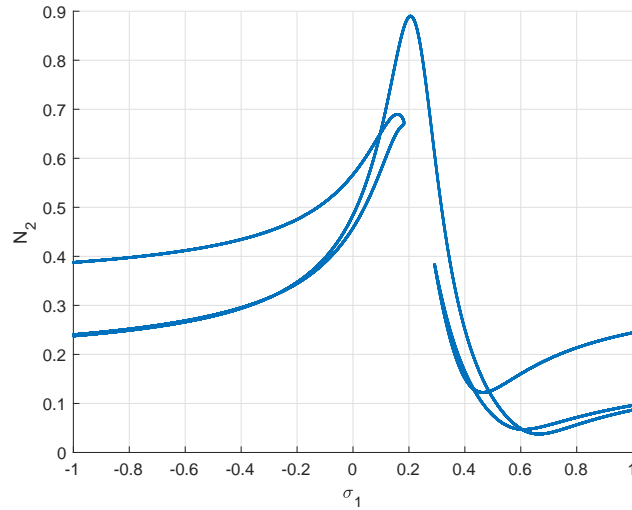


Figure 6: Equilibrium points of the system of Table 2 as a function of the detuning parameter: N_2 vs. σ_1

are presented in Figs. 5-7. As an example, let us take the case which correspond to $\sigma_1 = 0.2$. Equation 9 is integrated numerically by ode45 function of Matlab. Results for different system amplitudes are depicted in Figs. 8-10, showing that the system is being attracted by equilibrium points which are predicted in Figs. 5-7. Obtained numerical results are confronted with the three-dimensional view of the SIM in Fig. 11 for the section of $\delta = 0$. It is seen that (after starting from initial conditions), the system oscillates around the SIM and finally is attracted by the equilibrium point which is already clarified in Figs. 5-7. Different two-dimensional views of this confrontation are presented in Figs. 12.

Table 2
The coupled system parameters

k_n	c_0	l_3	c_1	c_2	θ_0	ϕ_0	ϵ	F_1	F_2
10^2	0.1	0.299	0.2	0.2	0	0	10^{-3}	10	10

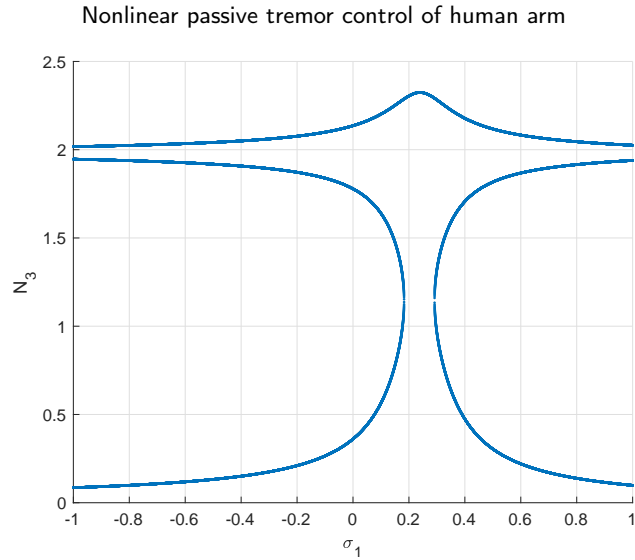


Figure 7: Equilibrium points of the system of Table 2 as a function of the detuning parameter: N_3 vs. σ_1

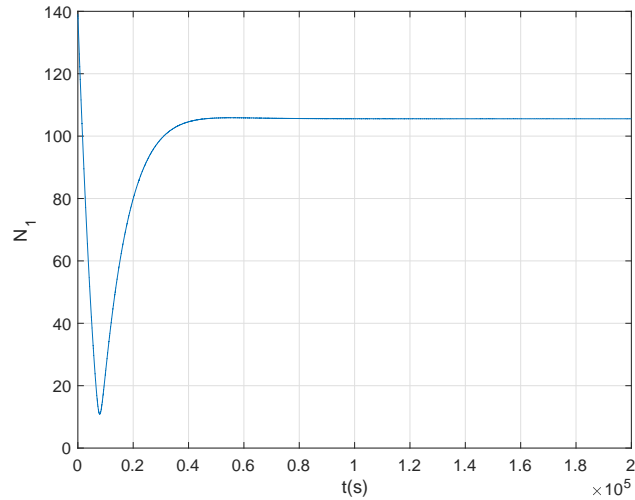


Figure 8: Time history of the system of Table 2 as a function of the detuning parameter: N_1 vs. t . Results are obtained by direct numerical integration of Eq. 9 for $\sigma_1 = 0.2$.

4. Application to passive control of the arm

In order to show the capability of the proposed device in tremor control of the arm, the undamped locomotion equations of human arm, i.e. Eq. 6, are integrated numerically (see Table 1 for parameters) and are compared with those which are equipped with the NES (see Table 3 for parameters). Results are shown in Figs. 13 and 14 showing the capability of the NES in controlling the diverging response of the main system.

Table 3
The coupled system parameters

k_n	c_0	l_3	c_1	c_2	θ_0	ϕ_0	ϵ	F_1	F_2
10^2	1	0.299	0	0	0	0	10^{-2}	0.1	0.1

Nonlinear passive tremor control of human arm

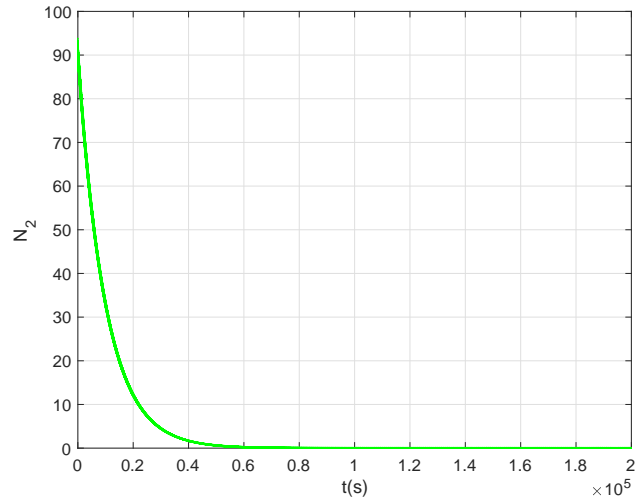


Figure 9: Time history of the system of Table 2 as a function of the detuning parameter: N_2 vs. t . Results are obtained by direct numerical integration of Eq. 9 for $\sigma_1 = 0.2$.

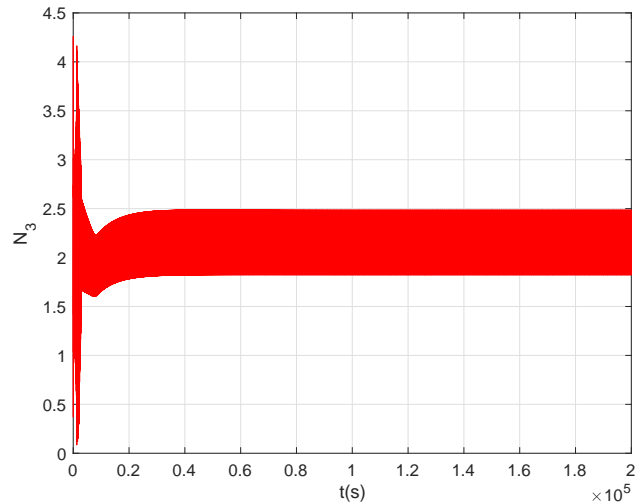


Figure 10: Time history of the system of Table 2 as a function of the detuning parameter: N_3 vs. t . Results are obtained by direct numerical integration of Eq. 9 for $\sigma_1 = 0.2$.

5. Conclusions

The mathematical model of the locomotion of human arm which has been developed by Jackson et al. (1978) and represented by a simplified two degrees of freedom nonlinear pendulum, is investigated:

- After linearisation of nonlinear system equation about an arbitrary posture, it is spotted that linear natural frequencies vary with the change of the posture. This means that classical linear passive absorbers, which are tuned to a spacial linear frequency, can not be efficient in general for the tremor control.
- The simplified locomotion model of human arm is coupled to a nonlinear energy sink. Detection of fast and slow system dynamics revealed its slow invariant manifold and characteristic points. Then, application of the proposed system in passive control of an academic diverging system is illustrated. It is seen that the proposed absorber is able to control (mathematically) diverging responses of the arm.

Perspective of this work will be a collaboration with the neurological hospital and medicines for having a real data of patients which suffer from essential tremor and to prepare design tools for such data.

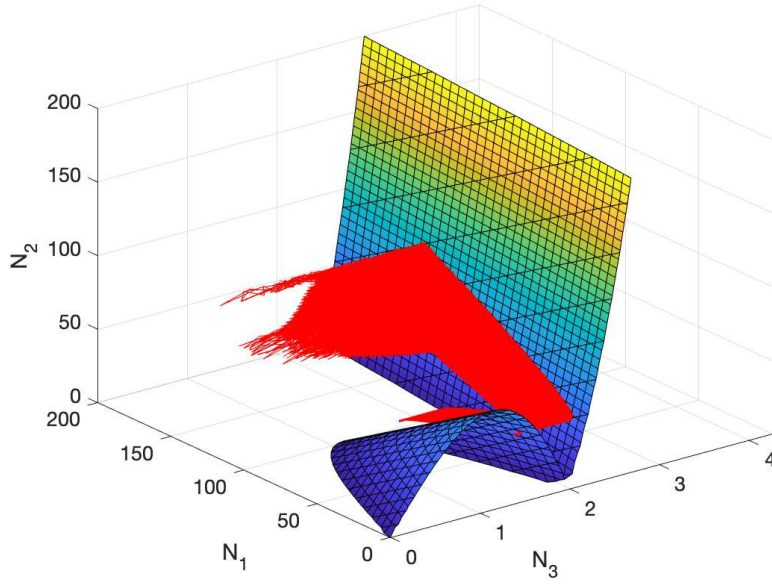


Figure 11: Three-dimensional view of the SIM of the system of Table 2 for $\delta = 0$. The SIM is accompanied by corresponding results.

Acknowledgement

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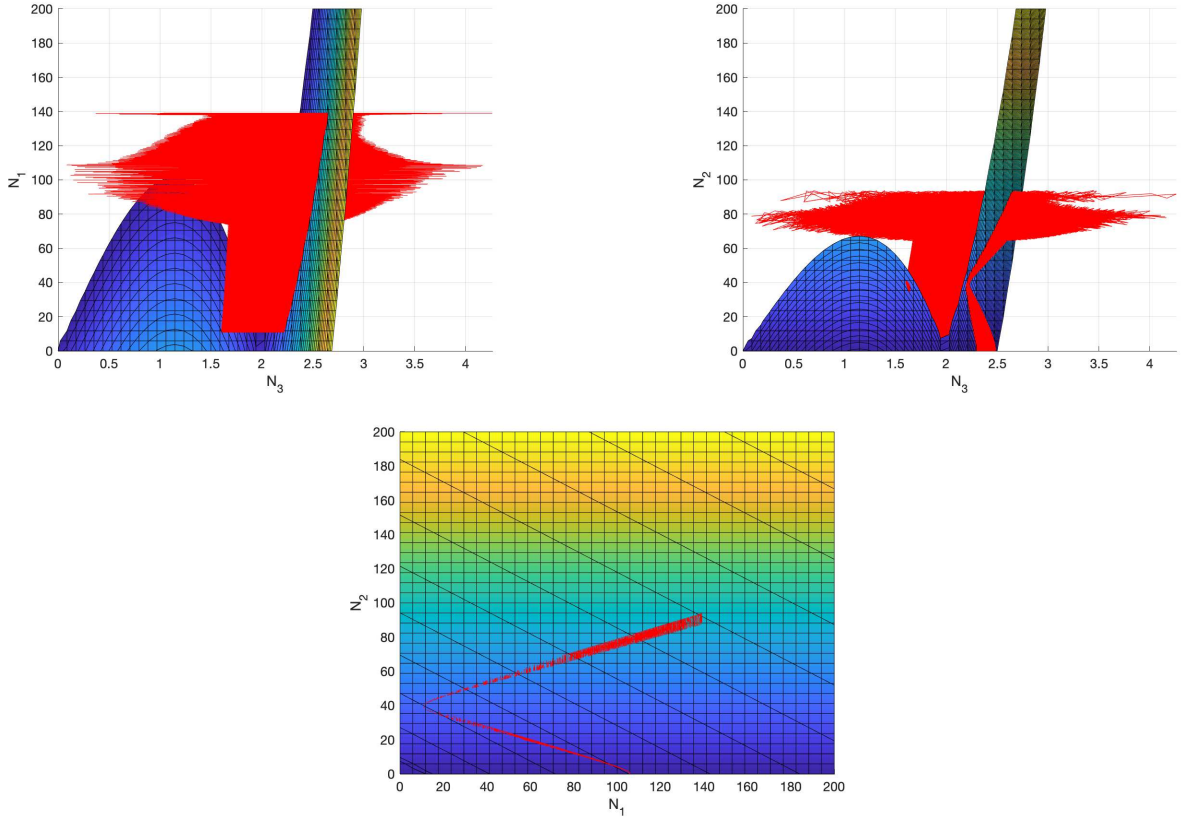


Figure 12: Different two-dimensional views of the SIM which is presented in Fig. 11

A. Definition of different variables of Eq. 1

$$\begin{aligned}
 \rho_{11} &= I + m_2(l_1^2 + l_2^2 + 2l_1l_2 \cos(\phi)) \\
 \rho_{12} &= m_2(l_2^2 + l_1l_2 \cos(\phi)) \\
 \rho_{13} &= -m_2l_1l_2(2\theta + \dot{\phi})\dot{\phi} \sin(\phi) \\
 \rho_{21} &= m_2(l_2^2 + l_1l_2 \cos(\phi)) \\
 \rho_{22} &= m_2l_2^2 \\
 \rho_{23} &= -m_2l_1l_2\theta\dot{\phi} \sin(\phi)
 \end{aligned} \tag{29}$$

B. Definition of different variables of Eq. 3

$$\begin{aligned}
 \mu_{11} &= I + m_2(l_1^2 + l_2^2 + 2l_1l_2 \cos(\phi_0)) \\
 \mu_{12} &= m_2(l_2^2 + l_1l_2 \cos(\phi_0)) \\
 \mu_{21} &= m_2(l_2^2 + l_1l_2 \cos(\phi_0)) \\
 \mu_{22} &= m_2l_2^2 \\
 \kappa_{11} &= g(m_1d_1 \cos(\theta_0) + m_2l_1 \cos(\theta_0) + m_2l_2 \cos(\theta_0 + \phi_0)) \\
 \kappa_{12} &= gm_2l_2 \cos(\theta_0 + \phi_0) \\
 \kappa_{21} &= m_2gl_2 \cos(\theta_0 + \phi_0) \\
 \kappa_{22} &= m_2gl_2 \cos(\theta_0 + \phi_0) \\
 C_1 &= gm_1d_1 \sin(\theta_0) + gm_2(l_1 \sin(\theta_0) + l_2 \sin(\theta_0 + \phi_0)) \\
 C_2 &= m_2gl_2 \sin(\theta_0 + \phi_0)
 \end{aligned} \tag{30}$$

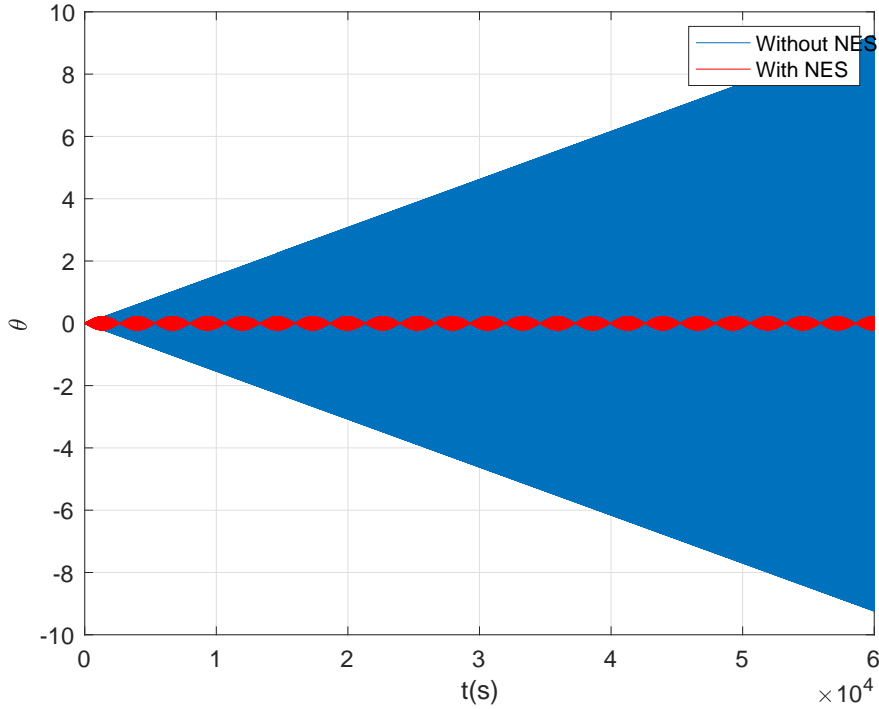


Figure 13: Time history of the θ for the system without and with the coupled NES ($\sigma_1 = 0$).

C. Definition of different variables of Eq. 7

$$\begin{aligned}
 \rho_{11} &= I + m_2(l_1^2 + l_2^2 + 2l_1l_2 \cos(\phi)) + m_N(l_1^2 + l_3^2 + u^2 + 2l_1l_3 \cos(\phi) - 2l_1u \sin(\phi)) \\
 \rho_{12} &= m_2[l_2^2 + l_1l_2 \cos(\phi)] + m_N[l_3^2 + u^2 + l_1l_3 \cos(\phi) - l_1u \sin(\phi)] \\
 \rho_{13} &= m_N[l_1 \cos(\phi) + l_3] \\
 \rho_{14} &= -m_2l_1l_2(2\dot{\theta} + \dot{\phi})\dot{\phi} \sin(\phi) \\
 &+ m_N[2\dot{u}u(\dot{\theta} + \dot{\phi}) - l_1l_3(2\dot{\theta} + \dot{\phi})\dot{\phi} \sin(\phi) - l_1\dot{u}\dot{\phi} \sin(\phi) - l_1\dot{u}(2\dot{\theta} + \dot{\phi}) \sin(\phi) - l_1u(2\dot{\theta} + \dot{\phi})\dot{\phi} \cos(\phi)] \\
 \rho_{21} &= m_2[l_2^2 + l_1l_2 \cos(\phi)] + m_N[l_3^2 + u^2 + l_1l_3 \cos(\phi) - l_1u \sin(\phi)] \\
 \rho_{22} &= m_2l_2^2 + m_N(l_3^2 + u^2) \\
 \rho_{23} &= m_Nl_3 \\
 \rho_{24} &= -m_2(l_1l_2\dot{\theta}\dot{\phi} \sin(\phi)) + m_N[2\dot{u}u(\dot{\theta} + \dot{\phi}) - l_1l_3\dot{\theta}\dot{\phi} \sin(\phi) - l_1\dot{u}\dot{\theta} \sin(\phi) - l_1u\dot{\theta}\dot{\phi} \cos(\phi)] \\
 \rho_{31} &= m_N[l_1 \cos(\phi) + l_3] \\
 \rho_{32} &= m_Nl_3 \\
 \rho_{33} &= m_N \\
 \rho_{34} &= -m_Nl_1\dot{\theta}\dot{\phi} \sin(\phi)
 \end{aligned} \tag{31}$$

and

$$\begin{aligned}
 T_1 &= 0 \\
 V_1 &= m_1gd_1 \sin(\theta) + m_2g(l_1 \sin(\theta) + l_2 \sin(\theta + \phi)) + m_Ng(l_1 \sin(\theta) + l_3 \sin(\theta + \phi) + u \cos(\theta + \phi)) \\
 T_2 &= -[m_2l_1l_2(\dot{\theta} + \dot{\phi})\dot{\theta} \sin(\phi) + m_N(l_1l_3(\dot{\theta} + \dot{\phi})\dot{\theta} \sin(\phi) + l_1u(\dot{\theta} + \dot{\phi})\dot{\theta} \cos(\phi) + l_1\dot{u}\dot{\theta} \sin(\phi))] \\
 V_2 &= m_2gl_2 \sin(\theta + \phi) + m_Ng(l_3 \sin(\theta + \phi) + u \cos(\theta + \phi)) \\
 T_3 &= m_N[(\dot{\theta} + \dot{\phi})^2u - l_1(\dot{\theta} + \dot{\phi})\dot{\theta} \sin(\phi)] \\
 V_3 &= m_Ng \sin(\theta + \phi) + f_{NES}(u)
 \end{aligned} \tag{32}$$

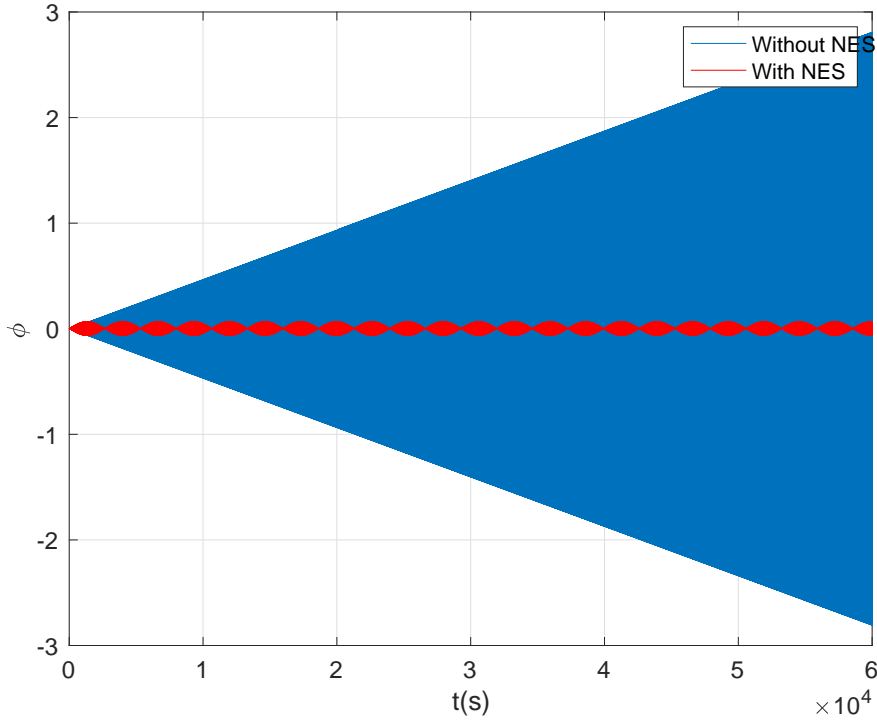


Figure 14: Time history of the ϕ for the system without and with the coupled NES ($\sigma_1 = 0$).

D. Definition of different variables of Eq. 9

Let us define $\frac{I}{m_2} = \alpha$ and $\frac{m_1}{m_2} = \beta$.

$$M' = \begin{pmatrix} \mu'_{11} & \mu'_{12} & \mu'_{13} \\ \mu'_{21} & \mu'_{22} & \mu'_{23} \\ \mu'_{31} & \mu'_{32} & \mu'_{33} \end{pmatrix} \quad (33)$$

$$\begin{aligned} \mu'_{11} &= \alpha + (l_1^2 + l_2^2 + 2l_1l_2 \cos(\phi_0)) + \epsilon(l_1^2 + l_3^2 + 2l_1l_3 \cos(\phi_0)) \\ \mu'_{12} &= l_2(l_2 + l_1 \cos(\phi_0)) + \epsilon l_3(\cos(\phi_0)l_1 + l_3) \\ \mu'_{13} &= \epsilon(\cos(\phi_0)l_1 + l_3) \\ \mu'_{21} &= l_2(l_2 + l_1 \cos(\phi_0)) + \epsilon l_3(\cos(\phi_0)l_1 + l_3) \\ \mu'_{22} &= l_2^2 + \epsilon l_3^2 \\ \mu'_{23} &= \epsilon l_3 \\ \mu'_{31} &= \epsilon(\cos(\phi_0)l_1 + l_3) \\ \mu'_{32} &= \epsilon l_3 \\ \mu'_{33} &= \epsilon \end{aligned} \quad (34)$$

$$K' = \begin{pmatrix} \kappa'_{11} & \kappa'_{12} & \kappa'_{13} \\ \kappa'_{21} & \kappa'_{22} & \kappa'_{23} \\ \kappa'_{31} & \kappa'_{32} & \kappa'_{33} \end{pmatrix} \quad (35)$$

$$\begin{aligned}
 \kappa'_{11} &= g (\cos (\phi_0 + \theta_0) l_2 + \cos (\phi_0 + \theta_0) l_3 \epsilon + \beta d_1 \cos (\theta_0) + \cos (\theta_0) l_1 + \cos (\theta_0) l_1 \epsilon) \\
 \kappa'_{12} &= \cos (\phi_0 + \theta_0) g (l_2 + \epsilon l_3) \\
 \kappa'_{13} &= \epsilon g \cos (\phi_0 + \theta_0) \\
 \kappa'_{21} &= \cos (\phi_0 + \theta_0) g (l_2 + \epsilon l_3) \\
 \kappa'_{22} &= \cos (\phi_0 + \theta_0) g (l_2 + \epsilon l_3) \\
 \kappa'_{23} &= \epsilon g \cos (\phi_0 + \theta_0) \\
 \kappa'_{31} &= \epsilon g \cos (\phi_0 + \theta_0) \\
 \kappa'_{32} &= \epsilon g \cos (\phi_0 + \theta_0) \\
 \kappa'_{33} &= 0
 \end{aligned} \tag{36}$$

$$N' = \begin{pmatrix} 0 \\ 0 \\ \epsilon n_0(u) \end{pmatrix} \tag{37}$$

with $\epsilon k_n = \frac{K_N}{m_2}$ and $\epsilon n_0(u) = \epsilon k_n u^3$.

$$A' = \begin{pmatrix} 0 \\ 0 \\ \epsilon c_0 \dot{u} \end{pmatrix} \tag{38}$$

with $\epsilon c_0 = \frac{c_N}{m_2}$.

$$C' = \begin{pmatrix} C'_1 \\ C'_2 \\ C'_3 \end{pmatrix} \tag{39}$$

$$\begin{aligned}
 C'_1 &= \beta g d_1 \sin(\theta_0) + g(l_1 \sin(\theta_0) + l_2 \sin(\theta_0 + \phi_0)) + \epsilon g(l_1 \sin(\theta_0) + l_3 \sin(\theta_0 + \phi_0)) \\
 C'_2 &= g l_2 \sin(\theta_0 + \phi_0) + \epsilon g l_3 \sin(\theta_0 + \phi_0) \\
 C'_3 &= \epsilon g \sin(\theta_0 + \phi_0)
 \end{aligned} \tag{40}$$

$$\begin{aligned}
 \epsilon F_1 &= \frac{f_1}{m_2} \\
 \epsilon F_2 &= \frac{f_2}{m_2}
 \end{aligned} \tag{41}$$

E. Definition of different variables of Eq. 15

The non zero arrays of Matrices $D_{3 \times 3}$ and $\Lambda_{3 \times 3}$ are:

$$\begin{aligned}
 D_{11} &= \omega_1^2 \\
 D_{22} &= \omega_2^2 \\
 D_{31} &= \frac{((V_{11} + V_{21})(l_1^2 + \alpha) \cos(\theta_0 + \phi_0) - \cos(\theta_0) \cos(\phi_0) l_1 V_{11} (\beta d_1 + l_1))(l_2 - l_3)g}{l_2 (-(\cos(\phi_0))^2 l_1^2 + l_1^2 + \alpha)} \\
 D_{32} &= \frac{((V_{12} + V_{22})(l_1^2 + \alpha) \cos(\theta_0 + \phi_0) - \cos(\theta_0) \cos(\phi_0) l_1 V_{12} (\beta d_1 + l_1))(l_2 - l_3)g}{l_2 (-(\cos(\phi_0))^2 l_1^2 + l_1^2 + \alpha)}
 \end{aligned} \tag{42}$$

$$\begin{aligned}
 \Lambda_{11} &= -\frac{(\cos(\phi_0)V_{12}l_1+l_2(V_{12}+V_{22}))g(\alpha\cos(\phi_0)(V_{11}+V_{21})\cos(\theta_0+\phi_0)-\cos(\theta_0)V_{11}(d_1l_1\beta(\cos(\phi_0))^2-d_1l_1\beta+\alpha))l_1}{(V_{11}V_{22}-V_{12}V_{21})l_2(-(\cos(\phi_0))^2l_1^2+l_1^2+\alpha)^2} \\
 \Lambda_{12} &= -\frac{(\cos(\phi_0)V_{12}l_1+l_2(V_{12}+V_{22}))(\alpha\cos(\phi_0)(V_{12}+V_{22})\cos(\theta_0+\phi_0)-\cos(\theta_0)V_{12}(d_1l_1\beta(\cos(\phi_0))^2-d_1l_1\beta+\alpha))gl_1}{(V_{11}V_{22}-V_{12}V_{21})l_2(-(\cos(\phi_0))^2l_1^2+l_1^2+\alpha)^2} \\
 \Lambda_{13} &= \frac{(l_1l_2(V_{12}+V_{22})\cos(\phi_0)+V_{12}(l_1^2+\alpha))g\cos(\theta_0+\phi_0)}{(V_{11}V_{22}-V_{12}V_{21})l_2^2((\cos(\phi_0))^2l_1^2-l_1^2-\alpha)} \\
 \Lambda_{21} &= \frac{(\cos(\phi_0)V_{11}l_1+l_2(V_{11}+V_{21}))g(\alpha\cos(\phi_0)(V_{11}+V_{21})\cos(\theta_0+\phi_0)-\cos(\theta_0)V_{11}(d_1l_1\beta(\cos(\phi_0))^2-d_1l_1\beta+\alpha))l_1}{(V_{11}V_{22}-V_{12}V_{21})l_2(-(\cos(\phi_0))^2l_1^2+l_1^2+\alpha)^2} \\
 \Lambda_{22} &= \frac{(\cos(\phi_0)V_{11}l_1+l_2(V_{11}+V_{21}))(\alpha\cos(\phi_0)(V_{12}+V_{22})\cos(\theta_0+\phi_0)-\cos(\theta_0)V_{12}(d_1l_1\beta(\cos(\phi_0))^2-d_1l_1\beta+\alpha))gl_1}{(V_{11}V_{22}-V_{12}V_{21})l_2(-(\cos(\phi_0))^2l_1^2+l_1^2+\alpha)^2} \\
 \Lambda_{23} &= -\frac{(l_1l_2(V_{11}+V_{21})\cos(\phi_0)+V_{11}(l_1^2+\alpha))g\cos(\theta_0+\phi_0)}{(V_{11}V_{22}-V_{12}V_{21})l_2^2((\cos(\phi_0))^2l_1^2-l_1^2-\alpha)}
 \end{aligned} \tag{43}$$

T, U, X, Y, A, B and H read:

$$T = \begin{pmatrix} 0 \\ 0 \\ c_0 \end{pmatrix} \tag{44}$$

$$U = \begin{pmatrix} \frac{c(-(\cos(\phi_0))^2V_{12}l_1^2l_2-l_1l_2(V_{12}+V_{22})(l_2-l_3)\cos(\phi_0)+l_3V_{12}(l_1^2+\alpha))}{(V_{11}V_{22}-V_{12}V_{21})l_2^2(-(\cos(\phi_0))^2l_1^2+l_1^2+\alpha)} \\ -\frac{(-(\cos(\phi_0))^2V_{11}l_1^2l_2-l_1l_2(V_{11}+V_{21})(l_2-l_3)\cos(\phi_0)+l_3V_{11}(l_1^2+\alpha))c}{(V_{11}V_{22}-V_{12}V_{21})l_2^2(-(\cos(\phi_0))^2l_1^2+l_1^2+\alpha)} \\ -\frac{(l_1^2l_2(l_2-2l_3)(\cos(\phi_0))^2+l_3^2(l_1^2+\alpha))c}{l_2^2((\cos(\phi_0))^2l_1^2-l_1^2-\alpha)} \end{pmatrix} \tag{45}$$

$$X = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{46}$$

$$Y = \begin{pmatrix} \frac{(-(\cos(\phi_0))^2V_{12}l_1^2l_2-l_1l_2(V_{12}+V_{22})(l_2-l_3)\cos(\phi_0)+l_3V_{12}(l_1^2+\alpha))}{(V_{11}V_{22}-V_{12}V_{21})l_2^2(-(\cos(\phi_0))^2l_1^2+l_1^2+\alpha)} \\ -\frac{(-(\cos(\phi_0))^2V_{11}l_1^2l_2-l_1l_2(V_{11}+V_{21})(l_2-l_3)\cos(\phi_0)+l_3V_{11}(l_1^2+\alpha))}{(V_{11}V_{22}-V_{12}V_{21})l_2^2(-(\cos(\phi_0))^2l_1^2+l_1^2+\alpha)} \\ -\frac{(l_1^2l_2(l_2-2l_3)(\cos(\phi_0))^2+l_3^2(l_1^2+\alpha))}{l_2^2((\cos(\phi_0))^2l_1^2-l_1^2-\alpha)} \end{pmatrix} \tag{47}$$

$$A = \begin{pmatrix} C'_1 \\ C'_2 \\ C'_3 \end{pmatrix} \tag{48}$$

$$H = P^{-1} M'^{-1} \begin{pmatrix} F_1 \sin(\Omega t) \\ F_2 \sin(\Omega t) \\ 0 \end{pmatrix} \quad (49)$$

F. Definition of \mathbb{H} in Eq. 17

$$\mathbb{H} = \begin{pmatrix} \frac{i/2(-I_1 I_2 ((F_1 - 2 F_2) V_{12} - F_2 V_{22}) \cos(\phi_0) - (V_{12} + V_{22})(F_1 - F_2) I_2^2 + F_2 V_{12} (I_1^2 + \alpha))}{(V_{11} V_{22} - V_{21} V_{12}) I_2^2 (-\cos(\phi_0))^2 I_1^2 + I_1^2 + \alpha} \\ \frac{-i/2(-I_1 ((F_1 - 2 F_2) V_{11} - F_2 V_{21}) I_2 \cos(\phi_0) - (V_{11} + V_{21})(F_1 - F_2) I_2^2 + F_2 V_{11} (I_1^2 + \alpha))}{(V_{11} V_{22} - V_{21} V_{12}) I_2^2 (-\cos(\phi_0))^2 I_1^2 + I_1^2 + \alpha} \\ 0 \end{pmatrix} \quad (50)$$

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