



HAL
open science

Towards a complete design of linear functional observers

Benoît Larroque, Farid Noureddine, Frédéric Rotella

► **To cite this version:**

Benoît Larroque, Farid Noureddine, Frédéric Rotella. Towards a complete design of linear functional observers. *International Review of Automatic Control*, 2008, 1 (2), pp.132-142. hal-02880892

HAL Id: hal-02880892

<https://hal.science/hal-02880892>

Submitted on 25 Jun 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Open Archive Toulouse Archive Ouverte

OATAO is an open access repository that collects the work of Toulouse researchers and makes it freely available over the web where possible

This is an author's version published in: <http://oatao.univ-toulouse.fr/20187>

To cite this version:

Larroque, Benoît  and Nouredine, Farid  and Rotella, Frédéric  *Towards a complete design of linear functional observers*. (2008) *International Review of Automatic Control - IREACO*, 1 (2). 132-142. ISSN 1974-6059

Any correspondence concerning this service should be sent to the repository administrator: tech-oatao@listes-diff.inp-toulouse.fr

Towards a Complete Design of Linear Functional Observers

B. Larroque, F. Noureddine, F. Rotella

Abstract – This paper provides a procedure for the design of a reduced order observer of a state linear functional for a linear time-invariant system. The case, defined in [1], where the observer order p is given by the number m of single independent linear functionals to be observed, is called in this paper the minimum case where $p = m$. The minimum case is revisited and numerically simplified. The aim of this paper is to extend the minimum case to the case where $m < p < n-l$, named minimal case. A constructive procedure is given to design the linear functional observer.

Keywords: Linear functional observers, linear time-invariant systems, generalized inverse

Nomenclature

Vectors and scalars

Name	Size (or definition)
State $x(t)$	n
Input $u(t)$	γ
Output $y(t)$	l
Observer state $z(t)$	p
Observer output $w(t)$	m
ρ	$l+m+\mu$
ε	$n-l-m$

System Matrices

Name	Size
A	$n \times n$
A_1 A_2 A_3	$n \times l$ $n \times m$ $n \times \varepsilon$
A_{11} A_{12} A_{13}	$l \times l$ $l \times m$ $l \times \varepsilon$
A_{21} A_{22} A_{23}	$m \times l$ $m \times m$ $m \times \varepsilon$
A_{31} A_{32} A_{33}	$\varepsilon \times l$ $\varepsilon \times m$ $\varepsilon \times \varepsilon$
B	$n \times \gamma$
C	$l \times n$
K	$m \times n$

Observer Matrices

Name	Size
D	$p \times p$
D_1 D_2	$(p-m) \times m$ $m \times m$
D_{11} D_{12}	$m \times m$ $m \times (p-m)$
D_{21} D_{22}	$(p-m) \times m$ $(p-m) \times (p-m)$
H	$p \times \gamma$
E	$p \times l$
E_2	$m \times l$ $(p-m) \times l$
P	$m \times p$
V	$m \times l$
T	$m \times n$
T_2	$m \times n$ $(p-m) \times n$
T_{21}	$(p-m) \times l$
T_{22}	$(p-m) \times m$
T_{23}	$(p-m) \times \varepsilon$

Other Matrices

Name	Size
Q	$n \times n$
Σ	$(m+2l) \times n$
Ψ	$p \times p$
Φ	$l+m$
J	$p \times l$
N_1 M_1	$(2l+m) \times l$ $n \times l$
N_2 M_2	$(2l+m) \times m$ $n \times m$
N_3 M_3	$(2l+m) \times l$ $n \times l$
L	$(l-\mu) \times \rho$
L_1 Z	$(l-\mu) \times l$ $m \times (2l+m)$
L_2 Z_1	$(l-\mu) \times m$ $m \times \rho$
L_3 Z_2	$(l-\mu) \times \mu$ $m \times (l-\mu)$
Γ_1 Γ_2	$m \times m$ $(p-m) \times m$

I. Introduction

From the seminal Luenberger's paper [2], several attempts have been proposed to design a reduced order observer of a linear functional of the state of a linear time-invariant system (see for instance [1], [3]-[16]). Thus:

- For the state space linear system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \quad (1)$$

- For a linear functional of the state, the aim is to design a system

$$\begin{aligned} \dot{z}(t) &= Dz(t) + Hu(t) + Ey(t) \\ w(t) &= Pz(t) + Vy(t) \end{aligned} \quad (2)$$

where for every t , the state $z(t)$, such that:

$$\lim_{t \rightarrow \infty} (w(t) - Kx(t)) = 0$$

If this last condition is fulfilled, then (2) is a linear functional observer (LFO). For instance, (2) can be used to implement a state feedback control with a minimal order controller.

From [2], it is known that such an observer exists if and only if the following well-known conditions are satisfied:

$$D \text{ is a Hurwitz matrix} \quad (3)$$

$$TA - DT = EC \quad (4)$$

$$\begin{aligned} K &= PT + VC \\ H &= TB \end{aligned} \quad (5)$$

where T is an unknown constant matrix such that $\lim_{t \rightarrow \infty} (z(t) - Tx(t)) = 0$. It can be seen that the last relation gives H . Thus, the design consists in determining the constant matrices of appropriate dimensions D , E , P , V and T and the order of the observer p such that conditions (3), (4) and (5) are verified, keeping in mind that we are looking for a minimal LFO.

Remark 1. In our study we have to define 2 different cases. In the first case, the minimum LFO is the LFO where $p = m$ and $P = I_m$. In the second case, the minimal LFO is the minimal order LFO, where $m < p < n - l$ which exists when existence conditions for minimum LFO are not fulfilled.

In the aforementioned literature we can underline interesting works. [1], [3], [4] propose a procedure to design a LFO in case $p = m$ [7] gives a procedure for a single LFO only. In [17], the authors use an LFO without feedthrough from input to output. Thus, the design problem of a minimal order LFO is not, to our standpoint, completely solved. Indeed, it can be seen that [15] uses the observable lower Hessenberg form of (A, C) which is numerically stable with respect to the observable canonical form. This form is also used in [7], [11], [16]. In the following, for numerical stability sake, we consider that C is full row rank only and the pair (A, C) is not supposed to be in a particular canonical form. Moreover, if a row of K is linearly dependent on the rows of C , the corresponding component of the linear functional (2) can be observed with the output alone. Hence it is supposed in the following that C and K are linearly independent. This property is a very important point of our development.

In [1], a necessary and sufficient condition has been proposed in the case when $p = m$. When the existence conditions are fulfilled, the observer design is based on the use of the Moore-Penrose pseudo-inverse of a matrix, and on an eigenvalue placement procedure to

verify the Hurwitz condition for D . Extending the previous work, [14] proposes conditions for the existence of a second order observer for a scalar linear functional of the state. It can be noted that it uses an observable form for the observer to obtain a procedure for the design of a minimal LFO.

The aim and the main motivation of our paper is twofold. Firstly, by considering the property between C and K mentioned before, the minimum case developed by [1] is revisited and some numerical reductions are introduced (minimum case section). Secondly, it is interesting to extend the standpoints proposed in [1]-[14] in case the necessary and sufficient conditions are not fulfilled for a minimum LFO ($p = m$). The case $p > m$ can therefore be called the minimal case and the observer is called minimal LFO (minimal case section).

These points of view induce the following organization for the paper. Firstly, the minimum case is revisited. Secondly, we develop the existence conditions for the minimal case. This leads us to propose a design procedure to get a minimal LFO. In our work, we propose to use the generalized inverse concept which offers more possibilities in the calculus than the Moore-Penrose pseudo-inverse generally used. Some useful results on generalized inverses are developed in [18].

II. The Minimum Case

II.1. The Minimum Case

We have $P = I_m$ and following [1], the condition (5) is written $T = K - VC$. Introducing $J = E - DV$, condition (4) can be written:

$$KA = [J \quad D \quad V]\Sigma, \text{ with } \Sigma = \begin{bmatrix} C \\ K \\ CA \end{bmatrix}$$

According to [2] this linear system has a solution if and only if:

$$\text{rank} \Sigma = \text{rank} \begin{bmatrix} \Sigma \\ KA \end{bmatrix} \quad (6)$$

When (6) is fulfilled, the solution is:

$$[J \quad D \quad V] = KA\Sigma^{\{1\}} + Z(I_{2l+m} - \Sigma\Sigma^{\{1\}}) \quad (7)$$

where $\Sigma^{\{1\}}$ is a generalized inverse of Σ (see [18]). That is to say, it verifies the relationship $\Sigma\Sigma^{\{1\}}\Sigma = \Sigma$ and Z is an arbitrary $(m \times (2l+m))$ matrix. With the following partitions:

$$\Sigma^{(1)} = \begin{bmatrix} M_1 & M_2 & M_3 \\ N_1 & N_2 & N_3 \end{bmatrix}$$

we obtain:

$$\begin{aligned} J &= KAM_1 + ZN_1 \\ D &= KAM_2 + ZN_2 \\ V &= KAM_3 + ZN_3 \end{aligned}$$

As seen in [1] the following theorem is recalled.

Theorem 1 [1] *The necessary and sufficient conditions for the existence and stability of the minimum ($p = m$) functional observer (2) for system (1) are condition (6) and the pair (KAM_2, N_2) is detectable.*

11.2. The Minimum Case Revisited

The aim of this section is to simplify the Theorem 1 established in [1]. We have to consider the properties mentioned before between C and K .

A full rank decomposition of Σ with $\text{rank}(\Sigma) = \rho$ leads to write:

$$FG = \Sigma \quad (8)$$

where F and G are such that $\text{rank}(F) = \text{rank}(G) = \rho$ with $F \in \mathfrak{R}^{(2l+m) \times \rho}$ and $G \in \mathfrak{R}^{\rho \times n}$.

Firstly, considering Σ , we order the row of C by making a suitable permutation in the output components in order to assure that the μ first rows of CA are linearly independent from each other and linearly independent of $\begin{bmatrix} C \\ K \end{bmatrix}$ too. We can deduce:

$$\rho = l + m + \mu \quad (9)$$

Let $(CA)^*$ be defined as these μ rows of CA . By choosing on the one hand $F = \begin{bmatrix} I_\rho \\ L_{l-\mu, \rho} \end{bmatrix}$ where $L_{l-\mu, \rho}$ will be identified later and on the other hand $G = \begin{bmatrix} C \\ K \\ (CA)^* \end{bmatrix}$, (8) is verified.

We consider a coordinates transformation Q which leads to define a similar matrix such that:

$$\bar{\Sigma}Q = \Sigma \quad (10)$$

with $\bar{\Sigma} \in \mathfrak{R}^{(n+2l) \times n}$. By comparing (8) with (10), Q and $\bar{\Sigma}$ are identified.

Due to the fact that Q must be nonsingular, transformations must be made. If $(l + m + \mu) < n$, $(n - \mu - l - m)$ arbitrary components noted Y are added to G it can be deduced:

$$Q = \begin{bmatrix} C \\ K \\ (CA)^* \\ Y \end{bmatrix} \quad (11)$$

As a consequence, by adding a column to F we finally obtain:

$$\bar{\Sigma} = \begin{bmatrix} I_\rho & 0_{\rho, n-\rho} \\ L_{l-\mu, \rho} & 0_{l-\mu, n-\rho} \end{bmatrix} \quad (12)$$

where $L_{l-\mu, \rho}$ is a known matrix obtained by identifying the sub-matrix of:

$$\bar{\Sigma} = \Sigma Q^{-1} = \begin{bmatrix} C \\ K \\ CA \end{bmatrix} \begin{bmatrix} C \\ K \\ (CA)^* \\ Y \end{bmatrix}^{-1}$$

By using this state variable change, we obtain $\bar{A} = QAQ^{-1}$ and:

$$\begin{aligned} \bar{C} &= CQ^{-1} = \begin{bmatrix} I_l & 0_{l, m} & 0_{l, n-l-m} \end{bmatrix} \\ \bar{K} &= KQ^{-1} = \begin{bmatrix} 0_{m, l} & I_m & 0_{m, n-l-m} \end{bmatrix} \end{aligned} \quad (13)$$

Remark 2. For notational simplicity, if one of the dimensions of the null matrix vanishes or is negative, this matrix must not be considered.

Let us partition L as:

$$L = \begin{bmatrix} L_1 & L_2 & L_3 \end{bmatrix} \quad (14)$$

Moreover A is partitioned as:

$$\bar{A} = \begin{bmatrix} \bar{A}_1 & \bar{A}_2 & \bar{A}_3 \end{bmatrix} \quad (15)$$

where:

$$\bar{A}_1 = \begin{bmatrix} \bar{A}_{11} \\ \bar{A}_{21} \\ \bar{A}_{31} \end{bmatrix}, \bar{A}_2 = \begin{bmatrix} \bar{A}_{12} \\ \bar{A}_{22} \\ \bar{A}_{32} \end{bmatrix}, \bar{A}_3 = \begin{bmatrix} \bar{A}_{13} \\ \bar{A}_{23} \\ \bar{A}_{33} \end{bmatrix}$$

From theorem 1 described before, the following corollary can be deduced:

Corollary 1 A necessary and sufficient condition for the existence of an observer of order m is:

1. $\text{rank} \begin{bmatrix} \bar{A}_{13} \\ \bar{A}_{23} \end{bmatrix} = \text{rank} \begin{bmatrix} \bar{A}_{13} \\ \bar{A}_{23} \end{bmatrix}$,
2. pair (\bar{A}_{22}, L_2) is detectable.

Proof 1 Starting from (6) and according to the state variable change, (2) has a solution if and only if:

$$\text{rank} \bar{\Sigma} = \text{rank} \begin{bmatrix} \bar{\Sigma} \\ \bar{K}A \end{bmatrix} \quad (16)$$

where $\bar{\Sigma}$ is given by (12). Using (13) and (15) we get:

$$\begin{aligned} KA &= \begin{bmatrix} \bar{A}_{21} & A_{22} & \bar{A}_{23} \end{bmatrix} \\ CA &= \begin{bmatrix} \bar{A}_{11} & A_{12} & \bar{A}_{13} \end{bmatrix} \end{aligned} \quad (17)$$

Thus, (16) becomes:

$$\begin{aligned} \text{rank} \begin{bmatrix} I_l & 0_{l,m} & 0_{l,n-l-m} \\ 0_{m,l} & I_m & 0_{m,n-l-m} \\ \bar{A}_{11} & A_{12} & \bar{A}_{13} \end{bmatrix} &= \\ \text{rank} \begin{bmatrix} I_l & 0_{l,m} & 0_{l,n-l-m} \\ 0_{m,l} & I_m & 0_{m,n-l-m} \\ \bar{A}_{11} & A_{12} & \bar{A}_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix} & \end{aligned}$$

which denotes that then $n - \rho$ last columns of A_{23} must be null or linearly dependent on the $n - \rho$ last columns of A_{13} . Eventually, the existence condition (16) becomes:

$$\text{rank} \begin{bmatrix} \bar{A}_{13} \\ \bar{A}_{23} \end{bmatrix} = \text{rank} \begin{bmatrix} \bar{A}_{13} \\ \bar{A}_{23} \end{bmatrix} \quad (18)$$

which constitutes condition of corollary 1. Concerning the second condition of this corollary, if (18) is fulfilled, (7) is written in the basis defined in (11), so:

$$[J \ D \ V] = \bar{K}A\bar{\Sigma}^{\{1\}} + Z(I_{2l+m} - \bar{\Sigma}\bar{\Sigma}^{\{1\}}) \quad (19)$$

To solve (19), we use the expression of KA defined in (17) and as $\bar{\Sigma}$ is defined by (12), we can choose for $\bar{\Sigma}^{\{1\}}$ (see [18]):

$$\begin{aligned} \bar{\Sigma}^{\{1\}} &= \begin{bmatrix} I_\rho & 0_{\rho,l-\mu} \\ 0_{n-\rho,\rho} & 0_{n-\rho,l-\mu} \end{bmatrix} \Rightarrow \\ I_{2l+m} - \bar{\Sigma}\bar{\Sigma}^{\{1\}} &= \begin{bmatrix} 0_{\rho,\rho} & 0_{\rho,l-\mu} \\ -L & I_{l-\mu} \end{bmatrix} \end{aligned}$$

The particular choice of $\bar{\Sigma}^{\{1\}}$ leads to major simplifications in the design of the LFO.

Remembering that $\rho = l + m + \mu$, $\bar{\Sigma}^{\{1\}}$ can be partitioned as:

$$\bar{\Sigma}^{\{1\}} = \begin{bmatrix} I_l & 0_{l,m} & 0_{l,\mu} & 0_{l,l-\mu} \\ 0_{m,l} & I_m & 0_{m,\mu} & 0_{m,l-\mu} \\ 0_{\mu,l} & 0_{\mu,m} & I_\mu & 0_{\mu,l-\mu} \\ 0_{n-\rho,l} & 0_{n-\rho,m} & 0_{n-\rho,\mu} & 0_{n-\rho,l-\mu} \end{bmatrix} \quad (20)$$

Using the partition of L defined in (14), we have:

$$I_{2l+m} - \bar{\Sigma}\bar{\Sigma}^{\{1\}} = \begin{bmatrix} 0_{\rho,l} & 0_{\rho,m} & 0_{\rho,l-\mu} \\ -L_1 & & I_{l-\mu} \end{bmatrix} \quad (21)$$

This partition induces to write Z as:

$$Z = [Z_1 \ Z_2] \quad (22)$$

where $Z_1 \in \mathfrak{R}^{m \times \rho}$ and $Z_2 \in \mathfrak{R}^{m \times (l-\mu)}$. We will see in the following that only Z_2 has to be considered. Eventually, (19) can be expressed as 3 equations:

$$\begin{aligned} J &= \bar{A}_{21} - Z_2 L_1 \\ D &= \bar{A}_{22} - Z_2 L_2 \\ V &= \bar{A}_{23} \begin{bmatrix} I_\mu & 0_{\mu,l-\mu} \\ 0_{n-\rho,\mu} & 0_{n-\rho,l-\mu} \end{bmatrix} + Z_2 [-L_3] \end{aligned} \quad (23)$$

Thanks to (23), if (\bar{A}_{22}, L_2) is detectable, Z can be determined by using a standard eigenvalue assignment technique (SEAT) which provides D as a Hurwitz matrix.

The last point is to see that the detectability condition is independent of the choice of $\bar{\Sigma}^{\{1\}}$ and this is shown in the appendix.

This standpoint simplifies the design in the minimum case by introducing numerical simplifications in the necessary and sufficient conditions of Theorem 1. All required matrices are obtained without calculation of any generalized inverse.

However when the existence conditions are not fulfilled, we have to study the case where $\rho > m$. This last case constitutes the next section called minimal case.

III. The Minimal Case

Our philosophy for the minimal case is to find, for a given p , the necessary and sufficient existence conditions for an asymptotic observer. The minimal order observer will be obtained by increasing p until these conditions are fulfilled. To study the minimal case we have to consider conditions (3), (4) and (5), using the frame we have already defined for the minimum case.

III.1. Analysis

In order to simplify the design procedure, we suppose that the observer is written in an observable canonical form (see [2]):

$$P = \begin{bmatrix} I_m & 0_{m,p-m} \end{bmatrix}, D = \begin{bmatrix} D_1 & D_2 \end{bmatrix} \quad (24)$$

with :

$$D_2 = \begin{bmatrix} I_{p-m} \\ 0_{m,p-m} \end{bmatrix}$$

$$D_1 = \begin{bmatrix} D_{11} \\ D_{21} \end{bmatrix}, D_2 = \begin{bmatrix} D_{12} \\ D_{22} \end{bmatrix} \quad (25)$$

$$E = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$$

We choose to define the above mentioned partitioning in order to treat (4).

Remark 3. D_{12} and D_{22} are a specific partitioning of the $p - m$ last columns of D_2 . For a given p , these 2 matrices are known.

Using partitions (24) in (4) yields:

$$T_1 A - D_{11} T_1 - D_{12} T_2 = E_1 C \quad (26)$$

$$T_2 A - D_{21} T_1 - D_{22} T_2 = E_2 C \quad (27)$$

According to the definition of matrix P , $T_1 = K - VC$ and by setting $J_1 = E_1 - D_{11}V$ and $J_2 = E_2 - D_{21}V$, (26)-(27) can be written as:

$$KA - D_{12}T_2 = VCA + D_{11}K + J_1C$$

$$T_2A - D_{22}T_2 = D_{21}K + J_2C$$

or equivalently:

$$\begin{bmatrix} J_1 & D_{11} & V \end{bmatrix} \Sigma = KA - D_{12}T_2 \quad (28)$$

$$\begin{bmatrix} J_2 & D_{21} \end{bmatrix} \Phi = T_2A - D_{22}T_2 \quad (29)$$

where $\Phi = \begin{bmatrix} C \\ K \end{bmatrix}$.

(28)-(29), in the new basis, are defined by:

$$\begin{bmatrix} J_1 & D_{11} & V \end{bmatrix} \bar{\Sigma} = \bar{K}A - D_{12}\bar{T}_2 \quad (30)$$

$$\begin{bmatrix} J_2 & D_{21} \end{bmatrix} \bar{\Phi} = \bar{T}_2\bar{A} - D_{22}\bar{T}_2 \quad (31)$$

Thanks to (13), (31) can be expressed as:

$$\begin{bmatrix} J_2 & D_{21} & 0_{p-m,n-l-m} \end{bmatrix} = \bar{T}_2\bar{A} - D_{22}\bar{T}_2 \quad (32)$$

As seen in the proof of corollary 1, we express $\bar{\Sigma}^{(1)}$ by (20) and $I_{2l+m} - \bar{\Sigma}\bar{\Sigma}^{(1)}$ by (21). In addition to the partitioning already defined for L, A and Z respectively in (14), (15) and (22), \bar{T}_2 is partitioned as:

$$\bar{T}_2 = \begin{bmatrix} \bar{T}_{21} & \bar{T}_{22} & \bar{T}_{23} \end{bmatrix} \quad (33)$$

III.2. Design

To solve (30)-(31), existence conditions must be established.

Existence condition of (30) is established thanks to $\{1\}$ -inverses properties. Multiplying (30) by $\bar{\Sigma}^{(1)}\bar{\Sigma}$ leads to:

$$(KA - D_{12}\bar{T}_2)(I_n - \bar{\Sigma}^{(1)}\bar{\Sigma}) = 0 \quad (34)$$

Due to the particular form of $I_n - \bar{\Sigma}^{(1)}\bar{\Sigma}$, (34) leads to:

$$\bar{A}_{23} \begin{bmatrix} 0_{\mu,n-\rho} \\ I_{n-\rho} \end{bmatrix} = D_{12}\bar{T}_{23} \begin{bmatrix} 0_{\mu,n-\rho} \\ I_{n-\rho} \end{bmatrix} \quad (35)$$

(35) constitutes the first relation which constrains \bar{T}_{23} to obtain a solution for (30).

Furthermore, partition of (31) allows to write:

$$0_{p-m,n-l-m} = \bar{T}_2\bar{A}_3 - D_{22}\bar{T}_{23} \quad (36)$$

(36) constitutes the second relation which constrains \bar{T}_{23} to obtain a solution for (31).

So, if T_{23} is such that (35) is fulfilled, a solution for (30) is given by:

$$\begin{bmatrix} J_1 & D_{11} & V \end{bmatrix} = \begin{bmatrix} \bar{K}A - D_{12}\bar{T}_2 \end{bmatrix} \bar{\Sigma}^{(1)} + Z \left(I_{2l+m} - \bar{\Sigma}\bar{\Sigma}^{(1)} \right) \quad (37)$$

where Z is an arbitrary matrix.

By expressing \mathcal{KA} using (17) and substituting $\bar{\Sigma}^{(1)}$ and $I_{2l+m} - \Sigma\Sigma^{(1)}$ in (37), 3 equations are deduced:

$$J_1 = \bar{A}_{21} - D_{12}\bar{T}_{21} - Z_2L_1 \quad (38)$$

$$D_{11} = \bar{A}_{22} - D_{12}\bar{T}_{22} - Z_2L_2 \quad (39)$$

$$V = (\bar{A}_{23} - D_{12}\bar{T}_{23}) \begin{bmatrix} I_\mu & 0_{\mu, l-\mu} \\ 0_{n-\rho, \mu} & 0_{n-\rho, l-\mu} \end{bmatrix} + Z_2 \begin{bmatrix} -L_3 & I_{l-\mu} \end{bmatrix} \quad (40)$$

Using the partitions of \bar{A} and \bar{T}_2 given respectively in (15) and (33), and by considering that (36) is fulfilled, (31) can be written as:

$$J_2 = \bar{T}_2\bar{A}_1 - D_{22}\bar{T}_{21} \quad (41)$$

$$D_{21} = \bar{T}_2\bar{A}_2 - D_{22}\bar{T}_{22} \quad (42)$$

with D_{11} defined (39) and D_{21} (42), D given in (25) can be written as follows:

$$D = \begin{bmatrix} \bar{A}_{22} + \Gamma_1 & D_{12} \\ \Gamma_2 & D_{22} \end{bmatrix}$$

where:

$$\Gamma_1 = -D_{12}\bar{T}_{22} - Z_2L_2 \quad (43)$$

$$\Gamma_2 = \bar{T}_2\bar{A}_2 - D_{22}\bar{T}_{22} \quad (44)$$

D can also be decomposed into the canonical form allowing the application of a SEAT:

$$D = \Psi + \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} \begin{bmatrix} I_m & 0_{m, p-m} \end{bmatrix} \quad (45)$$

where:

$$\Psi = \begin{bmatrix} A_{22} & D_{12} \\ 0_{p-m, m} & D_{22} \end{bmatrix} \quad (46)$$

According to remark 2. Ψ is known, so if the pair $(\Psi, \begin{bmatrix} I_m & 0_{m, p-m} \end{bmatrix})$ is observable, Γ_1 and Γ_2 can be determined by choosing arbitrary eigenvalues and using a SEAT to fulfill (3). If the pair $(\Psi, \begin{bmatrix} I_m & 0_{m, p-m} \end{bmatrix})$ is detectable (3) is fulfilled and Γ_1 and Γ_2 are fixed and not chosen.

III.3. Determination of \bar{T}_2 and Z

Once Γ_1 and Γ_2 are calculated, T_2 and Z must be found by solving (35), (36), (43) and (44) using the Kronecker product (see [19]). Let $\text{vec}(X)$ be the vector value function of a matrix X , defined by:

$$\text{vec}(X) = \begin{bmatrix} x_{11} & \cdots & x_{m1} & x_{12} & \cdots & x_{m2} & x_{1n} & \cdots & x_{mn} \end{bmatrix}^T$$

where X is defined by:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & & \vdots \\ x_{m1} & & x_{mn} \end{bmatrix}$$

So using the following property, $\text{vec}(AXB) = (B^T \otimes A)(X)$ where \otimes is the Kronecker product, (35), (36), (43) and (44). Eventually, we obtain (system (47)):

$$\begin{aligned} \text{vec}(\Gamma_1) &= (I_m \otimes -D_{12})\text{vec}(\bar{T}_{22}) - (L_2^T \otimes I_m)\text{vec}(Z_2) \\ \text{vec}(\Gamma_2) &= (\bar{A}_2^T \otimes I_{p-m})\text{vec}(\bar{T}_2) - (I_m \otimes D_{22})\text{vec}(\bar{T}_{22}) \\ 0 &= (\bar{A}_3^T \otimes I_{p-m})\text{vec}(\bar{T}_2) - (I_{n-l-m} \otimes D_{22})\text{vec}(\bar{T}_{23}) \\ &\left(\begin{bmatrix} 0_{\mu, n-\rho} \\ I_{n-\rho} \end{bmatrix}^T \otimes D_{12} \right) \text{vec}(\bar{T}_{23}) = \text{vec} \left(\bar{A}_{23} \begin{bmatrix} 0_{\mu, n-\rho} \\ I_{n-\rho} \end{bmatrix} \right) \end{aligned}$$

Due to the decomposition of \bar{T}_2 (33), it is assumed that:

$$\begin{aligned} \text{vec}(\bar{T}_{22}) &= G_1 \text{vec}(\bar{T}_2) \\ \text{vec}(\bar{T}_{23}) &= G_2 \text{vec}(\bar{T}_2) \end{aligned} \quad (48)$$

with:

$$G_1 = \begin{bmatrix} 0_{(p-m)m, (p-m)l} & I_{(p-m)m} & 0_{(p-m)m, (p-m)(n-l-m)} \end{bmatrix}$$

and:

$$G_2 = \begin{bmatrix} 0_{(p-m)(n-l-m), (p-m)(l+m)} & I_{(p-m)(n-l-m)} \end{bmatrix}$$

By placing (48) into (47) we get:

$$S_1 \text{vec}(Z_2) = S_2 \quad (49)$$

$$H_1 \text{vec}(\bar{T}_2) = H_2 \quad (50)$$

with:

$$H_1 = \left(\begin{array}{c} \left((\bar{A}_2^T \otimes I_{p-m}) - (I_m \otimes D_{22}) G_1 \right) \\ \left((\bar{A}_3^T \otimes I_{p-m}) - (I_{n-l-m} \otimes D_{22}) G_2 \right) \\ \left(\begin{array}{c} 0 \\ \mu, n-\rho \\ I_{n-\rho} \end{array} \right)^T \otimes D_{12} \end{array} \right) G_2$$

$$\left[\begin{array}{c} \text{vec}(\Gamma_2) \\ 0_{(n-l-m)(p-m),1} \\ \text{vec} \left(\bar{A}_{23} \begin{bmatrix} 0_{\mu, n-\rho} \\ I_{n-\rho} \end{bmatrix} \right) \end{array} \right]$$

$$S_1 = L_2^T \otimes I_m$$

$$S_2 = (I_m \otimes -D_{12}) G_1 \text{vec}(\bar{T}_2) - \text{vec}(\Gamma_1)$$

A solution for T_2 exists if and only if:

$$\text{rank}(H_1) = \text{rank}([H_1 \ H_2]) \quad (51)$$

and is given by:

$$\text{vec}(T_2) = H_1^{\{1\}} H_2 + (I_n - H_1^{\{1\}} H_1) Y \quad (52)$$

where Y is an arbitrary matrix that can be taken equal to zero as shown in the presented example.

Once T_2 is determined, a solution for Z_2 exists if and only if:

$$\text{rank}(S_1) = \text{rank}([S_1 \ S_2]) \quad (53)$$

and a solution is given by:

$$\text{vec}(Z_2) = S_1^{\{1\}} S_2 + (I_n - S_1^{\{1\}} S_1) W \quad (54)$$

where W is an arbitrary matrix. As mentioned above for Y , W can be taken equal to zero.

The previous development leads to propose the following theorem:

Theorem 2. The necessary and sufficient conditions for the existence and stability of the minimal functional observer (2) of order p , for system (1) are:

1. $(\Psi, [J_m \ 0_{m, p-m}])$ is detectable,
2. (51) and (53) are fulfilled.

IV. Design Procedure

The following procedure is proposed:

Minimum LFO

1. Set $p = m$
2. Compute Q with (11) and deduce \bar{S} , \bar{A} and L
3. *Existence condition:*
if (18) is true then go to *step 4*
else go to *step 8*
4. If (\bar{A}_{22}, L_2) is observable, then go to *step 5*
else if (\bar{A}_{22}, L_2) is detectable, then go to *step 6*
else go to *step 8*
5. Choose eigenvalues for D and calculate Z with a SEAT
6. Compute D , V and J which are defined in (19) and compute $E = J + DV$ and $\bar{T} = \bar{K} - V\bar{C}$
7. End: a minimum LFO is designed

Minimal LFO

8. Increase p : $p = p + 1$
if $p < n - l$, then go to *step 9*
else the reduced order observer has to be considered
9. Compute Ψ with (46)
10. If $(\Psi, [I_m \ 0])$ is observable, then go to *step 11*
else if $(\Psi, [I_m \ 0])$ is detectable, then go to *step 12*
else go to *step 8*
11. Choose observer eigenvalue, get Γ_1 and Γ_2 by taking $Y = W = 0$. Calculate D with (45)
12. Existence condition
If (51) and (53) are fulfilled, calculate \bar{T}_2 and Z_2 with (52) and (54)
else go to *step 8*
13. Compute Z , J with (21-38) and V with (40).
Compute E and \bar{T} with $E = \begin{bmatrix} J_1 + D_{11}V \\ J_2 + D_{21}V \end{bmatrix}$ and
 $\bar{T} = \begin{bmatrix} \bar{K} - V\bar{C} \\ \bar{T}_2 \end{bmatrix}$
14. End: a minimal LFO is designed

V. Numerical Examples

V.1. Case 1: A Minimum Order Observer Exists

Considering the following system:

$$A = \begin{bmatrix} -1 & 0 & 0.1 & & -1 & \\ 2 & 1 & 0 & 0.5 & 1 & 2 \\ -1 & 0 & 0 & 0 & 2 & -1 \\ & 0.5 & 1 & 0 & 0.1 & 0.2 \\ 2 & -0.1 & 0.2 & 0.3 & 0.9 & 0 \\ 0 & 0 & 1 & 2 & 0.1 & 0.5 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\text{and } K = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

From this system we get $n=6$, $l=3$ and $m=2$. Using the design procedure we get:

1. Minimum case: $\rho=2$
2. A change in the state variable (Q) in order to compute C in the form (12) is given by:

$$Q = \begin{bmatrix} C \\ K \\ (CA)_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & & 0 & 0 \\ 0 & 0 & 0 & 0 & & 0 \\ -1 & 0 & 0.1 & & -1 & \end{bmatrix}$$

where $(CA)_1$ ($\mu=1$) is the first line of CA linearly independent of $\begin{bmatrix} C \\ K \end{bmatrix}$. So, the matrix A is such that:

$$\bar{A} = QAQ^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \\ 4 & 1 & -0.2 & -1.5 & 3 & 2 \\ -2 & 0 & 0.1 & 1 & 1 & -1 \\ 1.2 & 0.5 & 0.98 & -0.2 & 0.3 & 0.2 \\ 2 & -0.1 & 0.2 & 0.3 & 0.9 & 0 \\ -0.5 & 0.6 & 1.74 & 1.1 & 0.1 & -0.4 \end{bmatrix}$$

Following (9) as $\rho=l+m+\mu=6=n$, (18) is always fulfilled.

3. With Q , $\bar{\Sigma}$ is such that:

$$\bar{\Sigma} = \begin{bmatrix} CQ^{-1} \\ KQ^{-1} \\ CAQ^{-1} \end{bmatrix}$$

$$\bar{\Sigma} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & -0.2 & -1.5 & 3 & 2 & \\ -2 & 0 & 0.1 & & & -1 \end{bmatrix}$$

$$L = \begin{bmatrix} 4 & 1 & -0.2 & -1.5 & 3 & 2 \\ -2 & 0 & 0.1 & 1 & 1 & -1 \end{bmatrix}$$

By partitioning the matrix L , we obtain

$$L_1 = \begin{bmatrix} 4 & 1 & -0.2 \\ -2 & 0 & 0.1 \end{bmatrix}, L_2 = \begin{bmatrix} -1.5 & 3 \\ 1 & 1 \end{bmatrix}$$

$$\text{and } L_3 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

4. Following the partition for A defined in (15), we have:

$$\bar{A}_{22} = \begin{bmatrix} -0.2 & 0.3 \\ 0.3 & 0.9 \end{bmatrix}$$

4. (\bar{A}_{22}, L_2) is observable: A_{22} is a (2×2) matrix and $\text{rank} \left(\begin{bmatrix} L_2 \\ L_2 \bar{A}_{22} \end{bmatrix} \right) = 2$.

5. Eigenvalues for D are chosen in -1 and -2 , it follows that:

$$Z = \begin{bmatrix} -0.1111 & 0.6333 \\ 0.5778 & 1.1667 \end{bmatrix}$$

6. D , V , J , E and T are such that:

$$D = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$J = \begin{bmatrix} 2.7111 & 0.6111 & 0.9144 \\ 2.0222 & -0.6778 & 0.1989 \end{bmatrix}$$

$$V = \begin{bmatrix} 1.0556 & 0 & 0 \\ 0.0111 & 0 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 1.6556 & 0.6111 & 0.9144 \\ 2 & -0.6778 & 0.1989 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -2 & -1 & -2 \\ 0 & 0 & 0 & 0 & 2 & -0.5 & 2 & -1 \end{bmatrix}$$

As $\text{rank} [\bar{A}_{13}] = 2$ and $\text{rank} \begin{bmatrix} \bar{A}_{13} \\ \bar{A}_{23} \end{bmatrix} = 3$, (18) is not fulfilled, then the minimal order observer does not exist, so go to step 8.

3. With $p = 3$, the system is such that $p < n - l$.

4. Ψ is given by (46):

$$\Psi = \begin{bmatrix} 0 & -1.8 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

5. $(\Psi, [I_2 \ 0_{2,1}])$ is observable.

6. By choosing observer eigenvalues in -1 , -2 and -3 , we get Γ_1 and Γ_2 :

$$\Gamma_1 = \begin{bmatrix} -3 & 1.8 \\ 0 & -3 \end{bmatrix}$$

$$\Gamma_2 = [-2 \ 0]$$

so we get:

$$D = \begin{bmatrix} -3 & 0 & 1 \\ 0 & -3 & 0 \\ -2 & 0 & 0 \end{bmatrix}$$

7. T_2 and Z_2 exist as (51) and (53) are fulfilled:

$$\bar{T}_2 = \begin{bmatrix} 2.3802 & 2.1561 & -3.8728 & -2.7875 & 5.0277 \\ -1.7078 & 2.6248 & -0.2414 & -0.1 & 1.8 \end{bmatrix}$$

$$Z_2 = \begin{bmatrix} 0.3423 & -1.185 \\ -1.7143 & 0.8571 \end{bmatrix}$$

8. Matrices J , V , E and T are respectively defined by:

$$J = \begin{bmatrix} -2.3802 & -2.1561 & 3.8728 & 2.7875 \\ 0 & 0 & 0 & 0 \\ 9.4159 & 2.7853 & -3.1298 & -7.2930 \end{bmatrix}$$

$$V = \begin{bmatrix} 2.0876 & -0.459 & 0.3423 & -1.185 \\ -1.4286 & -5.5714 & -1.7143 & 0.8571 \end{bmatrix}$$

$$E = \begin{bmatrix} -8.6432 & -0.7792 & 2.8458 & 6.3426 \\ 4.2857 & 16.7143 & 5.1429 & -2.5714 \\ 5.2406 & 3.7033 & -3.8145 & -4.9229 \end{bmatrix}$$

$$T = \begin{bmatrix} -2.08 & 0.45 & -0.34 & 1.18 \\ 1.42 & 5.57 & 1.71 & -0.85 & 0 \\ 2.38 & 2.15 & -3.87 & -2.78 & 5.02 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1.70 & 2.62 & -0.24 & -0.1 & 1.8 \end{bmatrix}$$

VI. Conclusion

We have shown that the procedure established by [1] can be simplified in the case where $p = m$. Derived from theorem 1, corollary 1 is proposed and summarizes our results. Moreover, when it is not possible to design a LFO in this case, we propose a procedure which enables to design a minimal LFO by increasing order p . Theorem 2 gives necessary and sufficient conditions in this latter case.

Appendix

To verify that the detectability condition is independent of the choice of $\Sigma^{\{1\}}$, we consider two different choices for $\Sigma^{\{1\}}$ denoted $\Sigma^{\{1\}'}$ and $\Sigma^{\{1\}''}$. Supposing that we get two different solutions for (7): $[J' \ D' \ V']$ and $[J'' \ D'' \ V'']$ respectively, we have:

$$[J' \ D' \ V'] = KA\Sigma^{\{1\}'} + Z'(I_{2l+m} - \Sigma\Sigma^{\{1\}'})$$

$$[J'' \ D'' \ V''] = KA\Sigma^{\{1\}''} + Z''(I_{2l+m} - \Sigma\Sigma^{\{1\}''})$$

$\Sigma^{\{1\}'}$ and $\Sigma^{\{1\}''}$ verify:

$$\Sigma^{\{1\}'} = \Sigma^{\{1\}''} + Y - \Sigma^{\{1\}''} \Sigma Y \Sigma \Sigma^{\{1\}''}$$

where Y is an arbitrary matrix with appropriate dimensions:

$$[J' \ D' \ V'] = KA(\Sigma^{\{1\}''} + Y - \Sigma^{\{1\}''} \Sigma Y \Sigma \Sigma^{\{1\}''})$$

$$+ Z'(I_{2l+m} - \Sigma(\Sigma^{\{1\}''} + Y - \Sigma^{\{1\}''} \Sigma Y \Sigma \Sigma^{\{1\}''}))$$

$$= KA\Sigma^{\{1\}''} + KAY - KA\Sigma^{\{1\}''} \Sigma Y \Sigma \Sigma^{\{1\}''} +$$

$$Z' - Z' \Sigma \Sigma^{\{1\}''} + -Z' \Sigma Y + Z' \Sigma \Sigma^{\{1\}''} \Sigma Y \Sigma \Sigma^{\{1\}''}$$

For compatibility reasons $KA\Sigma^{\{1\}''} \Sigma = KA$, so:

$$\begin{aligned} \begin{bmatrix} J' & D' & V' \end{bmatrix} &= K A \Sigma^{(1)''} + K A Y (I_{2l+m} - \Sigma \Sigma^{(1)''}) + \\ Z' (I_{2l+m} - \Sigma \Sigma^{(1)''}) - Z' \Sigma Y (I_{2l+m} - \Sigma \Sigma^{(1)''}) & \\ &= K A \Sigma^{(1)''} + (K A Y + Z' - Z' \Sigma Y) (I_{2l+m} - \Sigma \Sigma^{(1)''}) \end{aligned}$$

If we choose $Z'' = K A Y + Z' - Z' \Sigma Y$, we get $\begin{bmatrix} J' & D & V' \end{bmatrix} = \begin{bmatrix} J'' & D'' & V'' \end{bmatrix}$. If we suppose now that an arbitrary pole placement on D is possible by computing a matrix Z' , it will always be possible to arbitrarily assign the pole of D'' by substituting Z'' for $K A Y + Z' - Z' \Sigma Y$.

References

- [1] M. Darouach, Existence and design of functional observers for linear systems, *IEEE Transactions on Automatic Control*, Vol. AC-45(Issue 5):940-943, 2000.
- [2] D. G Luenberger, An introduction to observers, *IEEE Transactions on Automatic Control*, Vol. AC-16:596-602, 1971.
- [3] M. Aldeen, H. Trinh, Reduced-order linear functional observers for linear systems. *IEE Proc. Control Theory Appl.*, Vol. 146, n. 5:399-405, 1999.
- [4] B.N. Datta, D. Sarkissian, Block algorithms for state estimation and functional observers. IEEE Inter. Symp. on Computer-Aided Control Systems Design, Anchorage, September 2000.
- [5] T. Fortmann, D. Williamson, Design of low-order observers for linear feedback control laws, *IEEE Transactions on Automatic Control*, Vol. AC-17(Issue 3):301-308, 1972.
- [6] H. Kimura, Geometric structure of observers for linear feedback control laws, *IEEE Transactions on Automatic Control*, Vol. AC-22(Issue 5):845-855, 1977.
- [7] E. Kondo, T. Sunaga, A systematic design of single linear functional observers, *Int. Jour. of Control*, Vol. 44-3:597-611, 1986.
- [8] J. B. Moore, G. F. Ledwich, Minimal order observers for estimating linear functions of a state vector, *IEEE Transactions on Automatic Control*, Vol. AC-20(Issue 5):623-632, 1975.
- [9] P. Murdoch, Observer design for a linear functional of the state vector, *IEEE Transactions on Automatic Control*, Vol. AC-18:308-310, 1973.
- [10] T. Raff, P. Menold, C. Ebenbauer, F. Allgöwer, A finite time functional observer for linear systems, 44th IEEE CDC and ECC'05, pp. 198-203, Sevilla, December 2005.
- [11] J. R. Roman, T. E. Bullock, Design of minimal orders stable observers for linear functions of the state via realization theory, *IEEE Transactions on Automatic Control*, Vol. AC-20(Issue 5):613-622, 1975.
- [12] D. W. Russell, T. E. Bullock, A frequency domain approach to minimal-order observer design for several linear functions of the state, *IEEE Transactions on Automatic Control*, Vol. AC-22(Issue 4):600-604, 1977.
- [13] H. R. Sirisena, Minimal order observers for linear functions of a state vector, *Int. Jour. of Control*, Vol. 29, n.2:235-254, 1979.
- [14] H. Trinh, J. Zhang, Design of reduced-order scalar function observers, *Int. Jour. of Innovative Computing, Information and Control*, Vol. 1, n. 4:791-799, 2005
- [15] C. C. Tsui, A new algorithm for the design of multifunctional observers, *IEEE Transactions on Automatic Control*, Vol. AC-30(Issue 1):89-93, 1985.
- [16] C. C. Tsui, On the order reduction of linear function observers, *IEEE Transactions on Automatic Control*, Vol. AC-31(Issue 5):447-449, 1985.
- [17] J. B. Moore, A note on minimal-order observers, *IEEE Transactions on Automatic Control*, Vol. AC-17(Issue 2):255-256, 1972.
- [18] A. Ben Israel, T. N. E. Greville, Generalized Inverses: Theory and Applications (Wiley, 1974).
- [19] J. Brewer, Kronecker products and matrix calculus in system theory, *IEEE Transactions on Automatic Control*, Vol. AC-25(Issue 9):772-781, 1978.

Authors' information



Benoît Larroque is a PhD candidate at the Ecole Nationale Ingénieurs, (ENIT), Tarbes, France. He received the Diploma in engineering from the Ecole Nationale d'Ingénieurs, (ENIT), Tarbes, France, in 2002. His research focuses on observers design for linear time invaring (LTI) and extends his work to the design of observers applied to linear time varying systems. The main field of application of his work is for fault detection and isolation (FDI) problems applied for LTI and LTV systems. He worked as an assistant professor from 2004 to 2007 at the ENIT and from 2007 to 2008 at the "Université de Pau des Pays de l'Adour".



Farid Noureddine was born in Paris, France on January 19, 1958. He obtained his Bachelor and Master degrees in Electrical Engineering at the Beaulieu University, Rennes 1, France. Further he received the Ph.D Degree in Robotics from the I.N.S.A, National Institut of Applied Sciences of Rennes, France in 1987. His major field of study concerns industrial

robots.

He worked from 1987 to 1990 as Assistant Professor at the University of Sciences and Technology in Oran, Algeria, where he contributed to the creation of a Research Laboratory in Robotics. He directed this Laboratory during two years, where he developed a 3D range finder for a mobile robot. In 1990, he joined the E.N.I.T., National Engineering School of Tarbes, France, where he is currently Assistant Professor. His main research interest is in the model based fault detection isolation for dynamical systems, especially in manufacturing systems.

Dr. Noureddine has been Deputy Director of the International Relations of the E.N.I.T the last 2 years



Frédéric Rotella (A'96-M'95-SM'00) was born in France in 1957. In 1981, he received the Diploma in engineering from the Institut Industriel du Nord, Lille, France. He received the Ph.D. and Doctorat d'Etat degrees from the University of Science and Technology, Lille-Flandres-Artois, France, in 1983 and 1987, respectively.

From 1981 to 1994, he was with the Laboratoire d'Automatique et d'Informatique Industrielle, Lille, where his research interests focused on modelization and control of nonlinear systems. During this period, he served at the Ecole Centrale de Lille (ex. Institut Industriel du Nord) as Assistant Professor in Automatic Control. In 1994, he joined the Ecole Nationale d'Ingénieurs, (ENIT), Tarbes, France, as Professor of Automatic Control. Since 1994, he has been in charge of the Department of Electrical Engineering at ENIT. Currently, he is with the Laboratoire de Génie de Production of the ENIT, where he manages the Control Systems Research Group. His research interests include identification and control of nonlinear systems.

Dr. Rotella is coauthor of six french textbooks on automatic control and mathematics.