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# Emergency relief routing models for injured victims considering equity and priority

Li Zhu, Yeming Gong, Yishui Xu, Jun Gu

(Version of working paper)

**Abstract:** In humanitarian aid, emergency relief routing optimization needs to consider equity and priority issues. Different from the general path selection optimization, this paper builds two models differentiated by considerations on the identical and diverse injured degrees, where the relative deprivation cost is proposed as one of the decision-making objectives to emphasize equity, and the in-transit tolerable suffering duration is employed as a type of time window constraint to highlight rescue priority. By proving the NP-hardness of our models, we design a meta-heuristic algorithm based on the ant colony optimization to accelerate the convergence speed, which is more efficient than the commonly-used genetic algorithm. Taking 2017 Houston Flood as a case, we find some results by performing the experimental comparison and sensitivity analysis: First, our models have evident advantages in the fairness of human sufferings mitigation. Second, the role of the in-transit tolerable suffering time window cannot be ignored in humanitarian relief solutions. Various measures are encouraged to extend this type of time window for achieving better emergency relief. Finally, our proposed hybrid transportation strategy aiming at diverse injured degrees stably outperforms the separated strategy, both in operational cost control and psychological sufferings alleviation, especially when relief supplies are limited.

**Keywords:** humanitarian logistics; equity; priority; path selection; meta-heuristic algorithm

## 1 Introduction

Our research is partially motivated by the injured victims relief problem arising from Houston's devastating flooding in Hurricane Harvey of 2017 summer. This fourth largest city in the United States, Houston, was struck by a heavy rain over a 4-day period, with an average rainfall of 33 inches and a maximum of 49.6 inches. More than 136,000 structures suffered from flooding in the entire Harris County, at least nine people were dead reported by Texas officials, and dozens of communities asked for relief help over 2,000 times each day, causing over 30,000 people forced from their homes in Houston (HCFCD, 2017). In such a major flood disaster, there is an urgent need for a quick and effective humanitarian relief logistics. Post-disaster humanitarian logistics relates to various activities, approximately accounting for 80% of disaster relief efforts (Van Wassenhove, 2006; Chiappetta Jabbour et al., 2017), such as locating shelters to provide temporary safe spaces for evacuees, distributing

survival resources from supply origins to disaster areas, and transporting injured victims to emergency medical centers (Sheu 2014). Since closely related with human sufferings and survival probabilities, it is crucial to study the emergency relief routing optimization for injured victims in the destructive disaster.

Different from the commercial path optimization aiming at the shortest distance or the minimal operational cost as decision-making goals, the existing literature of emergency path selection is primarily to minimize relief duration, maximize the number of people whose survival probability exceeds the marginal level, or minimize life losses and human sufferings (Balcik and Beamon, 2008a; Holguín-Veras et al., 2012; Sabouhi et al., 2018). As one of major concerns and vital principles in humanitarian operations, the equity or fairness issues especially from the perspective of human sufferings need to be paid more attention in disaster relief routing planning (Van Wassenhove, 2006; Chiappetta Jabbour et al., 2017). Moreover, in the emergency routing optimization for transporting affected victims, various injury degrees and different survival probabilities are rarely considered and incorporated into path selection decisions. But since most disasters such as this Houston flood catastrophe actually have differentiated impacts on various areas, the priority issue cannot be ignored in emergency relief. Particularly when there are not enough relief resources to satisfy all victims, persons with serious sufferings need to be given a higher priority than slightly injured ones (Özdamar and Ertem, 2015; Tofighi, Torabi, and Mansouri, 2016), which is called as *Triage* in the field of healthcare management (Saghafian et al., 2014).

To the best of our knowledge, few studies on victims' relief path selection deal with the trade-off among physical or economic indicators like distance/cost, and humanitarian considerations such as equity and priority. Thereby, we study the following research question: *How to make an efficient decision on emergency relief routing for those injured victims, with a balance between operational cost minimization and humanitarian considerations on equity and priority simultaneously?* Our research objective is to propose and implement an innovative path selection methodology and a novel transportation strategy, in order to achieve a more efficient and more equitable emergency relief operation when facing with victims in diverse injury degrees.

To answer this question and reach this objective, we first formulate a multi-objective path selection model, reflected as minimization of the transportation cost, and the absolute and relative deprivation cost. The absolute deprivation cost is an economic valuation of human sufferings associated with a lack of access to the emergency relief service (Holguín-Veras et al. 2013). The relative deprivation cost expressed by the absolute value of deviations between any two absolute deprivation costs (Gutjahr and Nolz, 2016), is used to characterize the fairness in victims relief operations, which is similar to the Gini coefficient index (Matl, Hartl and Vidal, 2018) that most widely used for measuring the inequity. Then, for highlighting the priority requirement in

emergency relief, we use heterogeneous upper bounds of the in-transit tolerable suffering duration as constraints to represent diverse injury degrees of victims. Furthermore, we discuss the impacts of equity and priority on optimal relief routing decisions, by comparing the two models that consider or do not consider the absolute and relative deprivation cost as objectives, as well as by comparing the two models that consider or do not consider diverse in-transit tolerable suffering time windows as constraints. Eventually, we provide a hybrid transportation strategy and verify its better relief effect through comparing it with the traditional separated strategy.

The contributions of this paper are as follows. First, we construct a new objective, named the relative deprivation cost, and incorporate it into emergency path selection decisions. Using this relative deprivation cost, we put more emphasis on the relief fairness for injured victims, rather than only seek to minimize human sufferings in all disaster sites. Second, we propose heterogeneous in-transit tolerable suffering time windows to reflect differentiated durations that injured victims can persist until arriving at the medical center, except traditional time window constraints on rescue vehicles' arrival. This new time restriction helps to more effectively distinguish the relief priorities for victims with different injured degrees. Third, we develop a novel hybrid transportation strategy for victims in different injury degrees, and demonstrate its superiority on both operational cost control and human sufferings mitigation.

Several important insights can be given to governments, NGOs and interest groups like enterprises. Whenever it comes to the emergency routing optimization, including the distribution of relief materials, the evacuation of affected victims, and the rescue of injured people, the following managerial suggestions are suited to various stakeholders in the disaster management domain: First, by adopting our proposed relative deprivation cost as one of the objectives, we show that our models are more effective and fairer in mitigating human sufferings. This point is exactly constructive and attractive for governmental agencies that emphasize equity. Further, other than the usual concern on the waiting time window for relief vehicles' arrival, stakeholders are advised to keep an eye on another type of time window within which victims can persist until arriving at the medical center. Otherwise, it may appear invalid rescue solutions. In addition, decision-makers are encouraged to take all possible measures to extend the in-transit tolerable suffering duration of victims, for achieving a better emergency rescue. Last, facing with victims in different injured degrees, various stakeholders are recommended to use our hybrid transportation strategy for improving the operational and humanitarian performance, especially with limited relief resources under serious disasters.

The rest of the paper is organized as follows. Related work is reviewed in Section 2. In Section 3, we describe the relief routing problem for injured victims and formulate two emergency path selection models considering equity and priority. In Section 4, we show that the proposed models are NP-hard and design an ant colony algorithm for the approximate solution. The validation of the ant colony algorithm is also

presented. Section 5 applies our models and solution methods to the case of 2017 Houston Flood. The comparison study and sensitivity analysis are also conducted, followed by presenting how our research results can be used for policy makers. Finally, we conclude this paper along with future work in Section 6.

## **2 Literature Review**

Our study mainly relates to three different streams in the literature, including multiple objectives in emergency relief routing optimization, rescue priorities for victims with different injury degrees, as well as various transportation optimization strategies under differentiated demand.

### **2.1 Multiple Objectives in Emergency Relief Routing Optimization**

Like general humanitarian operations, multi-objective decision-making is a distinguishing feature in emergency relief routing problems, compared with path selection in the commercial context where minimization of cost or distance may be the primary concern. Various optimization goals have been well-recognized in the literature of disaster relief routing, such as efficiency, effectiveness, and equity (Gralla, Goentzel, and Fine, 2014; Huang et al., 2015; Gutjahr and Nolz, 2016; Rezaei-Malek et al., 2016). Efficiency, the traditional performance indicator, mainly refers to the economy measurement shown in cost-based objectives, e.g., distribution or transportation cost in disaster relief (Balcik, Beamon, and Smilowitz, 2008).

Effectiveness is a measure of service quality, which can be reflected in several aspects, including response time, reliability, and mitigation of life loss and sufferings. For example, Campbell et al. (2008) proposed the latest and average arrival time as response indicators to characterize the effectiveness in vehicles routing decisions for delivering critical supplies. Vitoriano et al. (2011) formulated a reliability objective function to capture the probability that none of relief routes fails. Hu and Sheu (2013), Sheu and Pan (2014) addressed the minimization of the psychological cost as the objective function, where psychological feelings refer to some stress, anxiety, grief and depression. Holguín-Veras et al. (2013), Pérez and Holguín-Veras (2015) introduced the deprivation cost concept to capture human sufferings when lacking of relief services, and argued that the minimization of the social cost (additively composed of logistics cost and deprivation cost) may be a more effective strategic goal in emergency scenarios. Wang et al. (2017) proposed a new method to estimate human sufferings for which the deprivation cost is replaced with a deprivation level.

Equity relates to balancing the allocation or utilization of resources, so that victims have the same opportunity to survive in humanitarian aid (Gutjahr and Nolz, 2016). Researchers in the humanitarian field are increasingly aware of the importance of equity or fairness. Different methods are employed to express equity (Karsu and Morton, 2015; Matl et al., 2018), such as the measures of min-max, the range, the mean absolute deviation, the standard deviation and the Gini coefficient. Özdamar et al. (2004) considered the minimization of unmet demand as the equity objective in the dispatching routing decision for relief commodities. Coverage has also been chosen to

model equity, which can be quantified as the percentage of the supplied volume to the overall available amount (Balcik and Beamon, 2008a). Huang et al. (2012) characterized equity in terms of the disparity between service levels among aid recipients in relief routing decisions. Ransikarbum and Mason (2016) addressed equity by a max-min approach, which is maximizing the minimum percentage of satisfied demand in relief distribution.

In general, in these literature of emergency relief routing optimization, the equity considerations are usually evaluated from some quantitative, economic or monetary perspectives, rarely judged by the suffering extent reflected as deprivation, pain and negative emotions of affected persons. But actually negative sufferings are regarded as a type of more appropriate and critical performance measurement, especially in relief operations related to human beings like the rescue of injured people and the evacuation of affected persons. On the other hand, Gutjahr and Fischer (2018) recently demonstrated that relief solutions simply minimizing deprivation costs would entail an arbitrarily high degree of inequity. All these related work gives us a motivation to discuss the equity issues from the perspective of relative sufferings among affected persons, and further explore the trade-off among efficiency, effectiveness and equity in relief path selection decisions.

## **2.2 Priorities for Victims with Different Injury Degrees**

In the literature of healthcare management, according to the severity of different patients, the practical process of rationing medical resources is called Triage (Saghafian et al., 2014; Wex et al., 2014). Similar to Triage, emergency relief priorities have also drawn widespread attention, especially when facing with victims in different injury degrees under limited resources. Three main methods and techniques are proposed in the field of humanitarian operations, which are priority assignment, incorporating survival probabilities into objective functions, and diverse rescue duration constraints.

Priority assignment traditionally starts from the most urgent victims to those who are less urgent (Jacobson et al., 2012). Most of studies differentiate the priorities by assigning different urgency levels or using different serving weights (Yi and Özdamar, 2007), while Sung and Lee (2016) declared that priority assignment policies should have trade-offs among payoff, service time and urgency. Another method depicting priority is to apply diverse survival probabilities to reveal different injury severities. These studies like Jin et al. (2015) usually formulate an objective function to maximize the number of affected people whose survival probability exceeds the marginal level. Specifically, they divided all affected people into several particular types that have differentiated survival probabilities, which depends on each victim's injury level.

Our research follows the papers using different relief duration constraints to express rescue priorities. For characterizing diverse injury degrees, this stream of work

emphasizes that the duration of time waiting for rescue cannot exceed some special hard limits, otherwise relief disutility would occur (Erera et al. 2010; Talarico et al. 2015; Miranda and Conceição, 2016). Apart from the time for waiting and receiving the first-aid treatment on site, the whole emergency response duration also covers the period of transporting victims from the original disaster site to the terminal medical center, and the period of waiting and undergoing treatment at medical centers (Boonmee, Arimura, and Asada, 2017). In particular, it is necessary to pay attention to the in-transit time window within that injured people can tolerate, similar to requirements in the distribution of fresh or perishable products (Hsu, Hung, and Li, 2007; Chen, Hsueh, and Chang, 2009; Govindan et al, 2014). To our best knowledge, most emergency path selection studies that use time constraints to express rescue priorities only focus on diverse arrival time windows, and ignore differentiated in-transit tolerable suffering time windows, which greatly hampers the feasibility and effectiveness of relief solutions.

### **2.3 Transportation optimization strategies under differentiated demand**

Another closely related stream to our paper is about transportation optimization strategies under differentiated demand. Usually in post-disaster environments, the available relief supplies are not sufficient to meet the needs of all those affected areas and all victims. Aid agencies have to decide on how best to allocate the available scarce resources considering differentiated demand characterized by diverse urgency of disaster areas and victims. Through appropriate classification of affected areas and victims, grouped demand is often served separately by the corresponding group-based allocation strategy. For instance, Sheu (2007; 2010) sorted disaster affected areas into several groups, and bounded the sites with similar urgency attributes together, so as to respond efficiently to their rescue needs based on different priorities. Sheu (2014) classified survivors into three groups that are normal people, the elderly, and women with young children, in order to facilitate differentiating the urgency levels of relief services needed by different groups of survivors. Zheng et al. (2014) proposed a method for classifying evacuee population in fire evacuation operations. Most of these papers in humanitarian operations divide the relief demand into different types like urgent and non-urgent one, and then correspondingly implement different kinds of strategies for achieving the respective relief objectives.

Other than separately dealing with differentiated demand, there are actually various transportation optimization strategies in the commercial context, which are proved to improve the service level and operational efficiency. For example, Üster and Kewcharoenwong (2011) provided a strategic design and analysis of the relay truckload transportation network to improve truck utilization, and consequently lead to a higher driver utilization. A consolidation-based transportation strategy was proposed by Zhu, Crainic, and Gendreau (2014), where grouping loads from different shippers, with possibly different origins and destinations, and loading them into the same vehicles for an efficient long-haul transportation. Harks et al. (2016) formulated a comprehensive transportation model by incorporating the possibility for flexible and

cyclic delivery patterns, to improve the transportation efficiency. These implemented optimization strategies aiming at differentiated demand, although did well in improving the transportation performance, are rarely applied into the humanitarian relief context. It gives us a great deal of inspiration and motivation.

### 3 Problem Description and Model Formulation

Taking 2017 Houston Flood disaster as our background, we consider the following emergency relief process: rescue vehicles (e.g., ambulances, lifeboats or some emergency trucks for evacuation) are dispatched from a safe place (e.g., emergency medical centers) to different disaster areas, then pick up some injured persons from these affected sites and finally return to the medical center. Injured persons here do not only refer to those people are really hurt, but also may mean those people that need some simply physical examination after experiencing disasters. To find out the optimal transportation path for victims when considering equity and priority simultaneously, it is required to dispatch each vehicle to certain disaster areas and to sequence the set of areas assigned to each vehicle, to minimize the transportation cost and psychological sufferings, under some specific time window constraints and vehicle capacity constraints. Here, we only consider one medical center for the sake of modeling, a more realistic scenario with several medical centers can be discussed in further study.

#### 3.1 Notations and Settings

We consider such a relief setting that all tours start from the same designated medical center depot. We use both index 1 and  $n + 1$  to express the medical center, but the difference is that all routes start from the beginning point 1 and end at the terminated point  $n + 1$ . We denote the set of disaster areas by  $\mathcal{N} = \{2, \dots, n\}, n = 2, \dots, N$ . There is a network connecting all disaster areas and the medical center, with the arcs of the network corresponding to connections between the nodes. Let  $\mathcal{K}$  be the set of potential rescue vehicles,  $\mathcal{K} = \{1, 2, \dots, k\}, k = 1, 2, \dots, K$ .

A transportation cost  $c_{ijk}$  occurs from node  $i$  to  $j$ ,  $j \neq i$ , with vehicle  $k$ , which is proportional to the distance  $d_{ijk}$  of the arc  $(i, j)$  by  $k$ . It means that  $c_{ijk} = C \cdot d_{ijk} + D$ , where  $C$  and  $D$  are constant cost coefficients. Note that  $d_{ijk}$  is a kind of virtual distance in disaster scenarios, related with the post-disaster dynamic travelling velocity. We define  $V_{ijk}^0$  as the pre-disaster or normal travelling velocity from  $i$  to  $j$ ,  $j \neq i$  with vehicle  $k$ .  $V_{ijk}(t)$  is the post-disaster travelling velocity from  $i$  to  $j$ ,  $j \neq i$  with vehicle  $k$  at time  $t$ . As a reflection of disaster impacts,  $\varepsilon_{ijk}(t)$  is the attenuation coefficient of the travelling velocity between  $i$  and  $j$  with  $k$ , where

$$V_{ijk}(t) = V_{ijk}^0 \cdot \varepsilon_{ijk}(t).$$



All relief vehicles are assumed to be identical, with a fixed capacity  $W_k$  for each vehicle  $k$ . And  $q_{ik}$  is the number of injured persons to be rescued by vehicle  $k$  at each disaster area  $i$  ( $i \in \mathcal{N}$ ). The number of trapped victims in any one disaster area is assumed not to exceed the capacity of one vehicle, which guarantees the relief routes do not need to repeat. Surely, this assumption can be easily relaxed to a split delivery scenario in the future research.  $t_{ijk}$  is the transportation time associated with each arc  $(i, j)$ ,  $j \neq i$  with vehicle  $k$ . The transportation time is nonnegative and satisfies the triangle inequality. In disaster area  $i$  ( $i \in \mathcal{N}$ ), the start of the vehicle rescue activities is supposed to be within a given time interval, called as the time window  $[0, u_i]$ . Rescue vehicles leave the medical center within the time window  $[0, u_1]$  and return within the time window  $[0, u_{n+1}]$ . Without loss of generality, we suppose that  $u_1 = 0$ . In addition to the rescue waiting time windows, other time-related parameters are as follows:  $t_{ik}$  represents the time spent on picking up each injured person at disaster area  $i$  ( $i \in \mathcal{N}$ ) by  $k$ , and  $b_i$  means the longest tolerable suffering time of injured persons at disaster area  $i$  ( $i \in \mathcal{N}$ ) when they are sent to the medical center.

This relief transportation process for victims can be formulated as a vehicle routing problem with several time window constraints, which contains two following types of decision variables. The decision variable  $x_{ijk} (\forall i \neq j, \forall k \in \mathcal{K}, i \neq n+1, j \neq 1)$  equals 1 if vehicle  $k$  travels directly from  $i$  to  $j$ , and 0 otherwise. The decision variable  $\theta_{ik}$  denotes when vehicle  $k$  arrives and starts to provide the relief service at disaster area  $i$  ( $i \in \mathcal{N}$ ), where  $\theta_{1k} = 0 (\forall k)$ , and  $\theta_{n+1,k}$  denotes the arrival time of vehicle  $k$  returning to the medical center. The decision-making goal of our emergency relief problem is to design a set of routes with minimizing the transportation cost and human sufferings, one for each vehicle, so that all injured victims in disaster areas can be rescued in an equitable and efficient manner.

### 3.2 Absolute and Relative Deprivation Costs

The human sufferings in our decision-making objectives are measured by the absolute deprivation cost and the relative deprivation cost. The absolute deprivation cost is depicted as the economic valuation of injured persons' sufferings when lacking of emergency relief services. Incorporating the subjective feelings' changes of victims and some socio-economic characteristics into humanitarian relief operations, our paper proposes an innovative three-stage absolute deprivation cost function. First, when the disaster occurs, injured persons in various affected areas have to wait for vehicles' relief services. During this waiting period, the absolute deprivation cost increases exponentially with the length of waiting time (Holguin-Veras et al., 2013). Then, when a relief vehicle arrives at the disaster area and begins to pick up the injured persons for the medical treatment, the absolute deprivation cost shows a linear reduction (Cantillo et al., 2017). This means that human sufferings can be mitigated linearly once victims receive relief services. Finally, different from Holguin-Veras et al. (2013), after picking up all injured persons in the disaster area, we consider that a

new exponential increase on the absolute deprivation cost appears when victims are transported to the medical center.

Fig. 1 illustrates the three-stage absolute deprivation cost, where the exponential increasing function is based on the function expression experimentally obtained by Holguin-Veras et al. (2013). The growth of the absolute deprivation cost is in a slightly slower increase, after victims receive vehicles' rescue service. We define

$$f(t) = \begin{cases} e^{g_1 t} + e^{h_1} & 0 < t \leq \theta_{ik} \\ -g_2 t + h_2 & \theta_{ik} < t \leq \theta_{ik} + t_{ik} \cdot q_{ik} \\ e^{g_3 t} + e^{h_3} & \theta_{ik} + t_{ik} \cdot q_{ik} < t \leq \theta_{n+1,k} \end{cases} \quad (i \in \mathcal{N}, \forall k \in \mathcal{K})$$

as the probability density function of the absolute deprivation cost. The absolute deprivation cost  $\Gamma_{ik}^a$  is:

$$\Gamma_{ik}^a = \int_0^{\theta_{ik}} (e^{g_1 t} + e^{h_1}) dt + \int_{\theta_{ik}}^{\theta_{ik} + t_{ik} \cdot q_{ik}} (-g_2 t + h_2) dt + \int_{\theta_{ik} + t_{ik} \cdot q_{ik}}^{\theta_{n+1,k}} (e^{g_3 t} + e^{h_3}) dt.$$

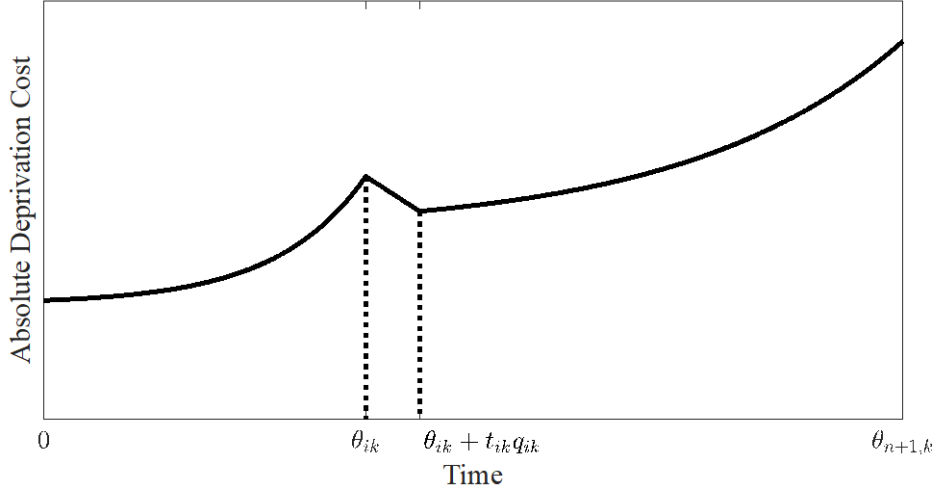


Fig. 1. The three-stage absolute deprivation cost

In addition to minimize the absolute deprivation cost of victims at each disaster site (Holguin-Veras et al. 2013), we also pay attention to the equity or fairness issues among victims in different affected areas in terms of human sufferings mitigation.

Based on  $\Gamma_{ik}^a$ , we propose  $\Gamma_{ijk}^r$  which is the relative deprivation cost of injured persons in area  $i$  and  $j$  visited by vehicle  $k$ , for highlighting the equity concerns in humanitarian operations. Various techniques can be used to formulate a relative deprivation cost function, such as minimizing the maximal absolute deprivation cost, or minimizing the absolute value of deviations between the absolute deprivation costs in any two disaster regions (Itani, 2014). Our paper adopts the latter method, with a relative deprivation cost as follows:  $\Gamma_{ijk}^r = |\Gamma_{ik}^a - \Gamma_{jk}^a|$ .

### 3.3 Emergency Relief Routing Models

First, we formulate a basic emergency relief model, named Model I, assuming that only one type of injury degree in each disaster area. That means the two time

windows factors  $u_i$  and  $b_i$  in Model I can reflect the impacts of the disaster on different areas to a certain extent. Next, by relaxing the assumption on the identical injury degree, we build an extended model, called Model II, to address different injury levels suffered by victims at each disaster site. Aiming to victims with diverse injury degrees, we also propose in Model II a hybrid transportation strategy to improve the effectiveness of emergency relief.

### 3.3.1 Basic Model I

The optimal path selection must be feasible in accordance with vehicles' capacities and victims' tolerable time windows. The mathematical formulation of the basic Model I is given as follows.

$$\min \sum_{k \in \mathcal{K}} \sum_{j \in \{n+1\} \cup \mathcal{N}} \sum_{i \in \{1\} \cup \mathcal{N}} c_{ijk} \cdot x_{ijk} \quad (1)$$

$$\min \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{N}} \Gamma_{ik}^a \quad (2)$$

$$\min \sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{N}} \sum_{i \in \mathcal{N}} \Gamma_{ijk}^r \cdot x_{ijk} \quad (3)$$

$$s. t. c_{ijk} = C \cdot d_{ijk} + D, \quad \forall i \in \{1\} \cup \mathcal{N}, j \in \{n+1\} \cup \mathcal{N}, k \in \mathcal{K} \quad (4)$$

$$d_{ijk} = \int_{\theta_{ik} + t_{ik} \cdot q_{ik}}^{\theta_{jk}} V_{ijk}(t) dt, \quad \forall i \in \{1\} \cup \mathcal{N}, j \in \{n+1\} \cup \mathcal{N}, k \in \mathcal{K} \quad (5)$$

$$V_{ijk}(t) = V_{ijk}^0 \cdot \varepsilon_{ijk}(t), \quad \forall i \in \{1\} \cup \mathcal{N}, j \in \{n+1\} \cup \mathcal{N}, k \in \mathcal{K} \quad (6)$$

$$\Gamma_{ik}^a = \int_0^{\theta_{ik}} (e^{g_1 t} + e^{h_1}) dt + \int_{\theta_{ik}}^{\theta_{ik} + t_{ik} \cdot q_{ik}} (-g_2 t + h_2) dt + \int_{\theta_{ik} + t_{ik} \cdot q_{ik}}^{\theta_{n+1,k}} (e^{g_3 t} + e^{h_3}) dt \quad \forall i \in \mathcal{N}, k \in \mathcal{K} \quad (7)$$

$$\Gamma_{ijk}^r = |\Gamma_{ik}^a - \Gamma_{jk}^a|, \quad \forall i \in \mathcal{N}, j \in \mathcal{N}, k \in \mathcal{K} \quad (8)$$

$$\sum_{k \in \mathcal{K}} \sum_{j \in \{n+1\} \cup \mathcal{N}} x_{ijk} = 1, \quad \forall i \in \{1\} \cup \mathcal{N} \quad (9)$$

$$\sum_{i \in \mathcal{N}} q_{ik} \sum_{j \in \{n+1\} \cup \mathcal{N}} x_{ijk} \leq W_k, \quad \forall k \in \mathcal{K} \quad (10)$$

$$\sum_{j \in \mathcal{N}} x_{1jk} = 1, \quad \forall k \in \mathcal{K} \quad (11)$$

$$\sum_{i \in \mathcal{N}} x_{i\zeta k} - \sum_{j \in \mathcal{N}} x_{\zeta j k} = 0, \quad \forall \zeta \in \mathcal{N}, k \in \mathcal{K} \quad (12)$$

$$\sum_{i \in \mathcal{N}} x_{i,n+1,k} = 1, \quad \forall k \in \mathcal{K} \quad (13)$$

$$\theta_{ik} + t_{ik} \cdot q_{ik} + t_{ijk} \leq \theta_{jk} + M \cdot (1 - x_{ijk}), \quad \forall i \in \mathcal{N}, j \in \mathcal{N}, k \in \mathcal{K} \quad (14)$$

$$\theta_{ik} \leq u_i, \quad \forall i \in \mathcal{N}, k \in \mathcal{K} \quad (15)$$

$$\theta_{n+1,k} - \theta_{ik} \leq b_i \quad \forall i \in \mathcal{N}, k \in \mathcal{K} \quad (16)$$

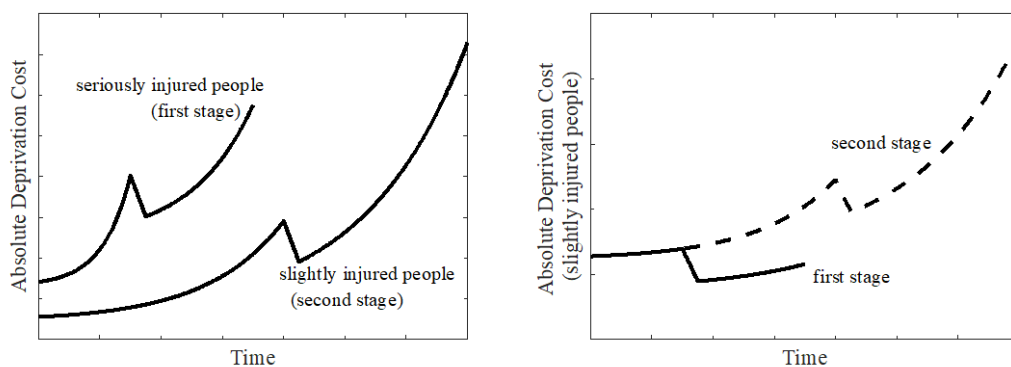
$$x_{ijk} \in \{0,1\} \quad \forall i \in \{1\} \cup \mathcal{N}, j \in \{n+1\} \cup \mathcal{N}, k \in \mathcal{K} \quad (17)$$

The multiple objectives consist of the transportation cost objective (1), the absolute deprivation cost objective (2), and the relative deprivation cost objective (3). Constraint (4) is the relationship between the transportation cost and the virtual travelling distance. The relationship between the virtual distance and the post-disaster travelling velocity is shown in Equation (5). The impacts of the disaster on the travelling velocity are presented in the Constraint (6). Constraints (7) and (8) are

respectively used to define the absolute and relative deprivation cost. Constraint (9) states that each disaster area must be provided the rescue service by exactly one vehicle. We call it as the assignment constraint. Inequality (10) ensures that no relief vehicle can transport more victims than its capacity. Constraint sets (11), (12), and (13) are the standard flow constraints and ensure that all transportation routes leave from and return to the medical center. Constraint (14) states that vehicle  $k$  cannot arrive at disaster site  $j$  before a certain time (i.e.,  $\theta_{ik} + t_{ik} \cdot q_{ik} + t_{ijk}$ ), if it travels from  $i$  to  $j$ . The scalar  $M$  can be an arbitrarily large number. Constraint (15) presents that all the arrival time windows for rescue vehicles should be obeyed. Inequality (16) requires the difference between the feasible arrival time and the final return time not to exceed the maximum tolerable suffering duration of injured persons in disaster area  $i$ . Constraint (17) is the integrality constraint.

### 3.3.2 Model II Considering Diverse Injury Degrees

In Model II, we discuss a more complex emergency relief situation, in which victims are of different injury degrees. Specifically, we consider two types of victims (Talarico, Meisel, and Sørensen, 2015): Seriously injured persons who need the immediate medical treatment and should be given priority to be picked up, as well as slightly injured persons who can wait for a while to receive the medical treatment and be picked up later. Normally, in terms of different rescue priorities, the emergency relief problem for victims with two types of injury degrees is considered to be a two-stage separated transportation process shown as Fig. 2(a), just like Triage: Only seriously injured persons are provided the relief services at the first stage, and the slightly injured persons are rescued at the second stage.



(a) Serious and slight victims under separated strategy (b) Slight victims under hybrid strategy

Fig. 2. Absolute deprivation cost under different transportation strategies

Fig. 2(a) illustrates respectively the absolute deprivation cost of the serious and slight victims, under the two-stage separated transportation strategy. It shows diverse growth rates of the absolute deprivation cost for different injured persons in the entire relief period. From Fig. 2(a), we can see that the absolute deprivation cost of seriously injured persons is growing faster than that of slightly injured ones, whether before or after vehicles' relief services. During the period of picking up victims from disaster sites, the absolute deprivation costs for both types decrease at the same speed.

Different from the traditional separated rescue process in Fig. 2(a), our Model II attempts to explore another type of emergency relief process called hybrid transportation strategy. The hybrid strategy allows some of slightly injured persons in the last accessed areas on some relief paths to be picked up at the first stage, after all seriously injured persons have been rescued. In other words, some slight victims can be rescued from the last visiting areas on some vehicles' paths at the first stage, if there is still room in these vehicles after providing relief services for serious victims. Fig. 2(b) demonstrates the comparative changes with and without adopting the new hybrid strategy, in terms of the absolute deprivation cost of slightly injured persons.

Considering different waiting and in-transit tolerable suffering time windows for diverse victims, differentiated notations  $q_{ik}^s$  and  $q_{ik}^l$  are used to distinguish the pickup number of seriously and slightly injured persons by vehicle  $k$  at disaster area  $i$ .  $u_i^s$  and  $u_i^l$  respectively denote the waiting time windows of relief services for serious and slight victims at disaster area  $i$ . Correspondingly,  $b_i^s$  and  $b_i^l$  are the maximum in-transit tolerable suffering time windows for seriously and slightly injured persons in disaster site  $i$ . In addition, differentiated expressions are also analogously applied into some notations in Model II, such as decision variables  $x_{ijk}^s$

and  $x_{ijk}^l$ ,  $\theta_{ik}^s$  and  $\theta_{ik}^l$ . After substituting these new differentiated notations into all the same objectives and constraints (1)-(17) in Model I, we still need to supplement two following additional constraints for Model II, for capturing the impacts of different injury degrees and the hybrid transportation strategy.

$$\xi_{ik}^{1l} = \min\{W_k - \sum_{i=2}^n q_{ik}^s, q_i^l\} \quad \forall i \in \mathcal{N}, k \in \mathcal{K} \quad (18)$$

$$\theta_{n+1,k}^s - \theta_{i,k}^s + t_{nk} \cdot \xi_{nk}^{1l} \leq b_i^s \quad \forall i \in \mathcal{N}, k \in \mathcal{K} \quad (19)$$

E.q. (18) shows the pickup number of slightly injured victims with vehicle  $k$  in disaster area  $i$  at the first rescue stage, denoted by  $\xi_{ik}^{1l}$ , where  $q_i^l$  represents the total number of slight victims at affected site  $i$ . This constraint is exactly designed to characterize the hybrid transportation strategy, where the serious victims are rescued firstly at all disaster areas, and then a part of slight victims in the last arriving sites on some paths are also allowed to receive relief services at the first stage, as long as the vehicle's capacity is not exceeded. Constraint (19) emphasizes that the appropriate decisions on the vehicle's arrival time for seriously injured persons, not only are constrained by the maximum in-transit tolerable suffering time windows of serious victims in different areas, but also require to consider the pickup time for those slightly injured victims at the vehicle's last arriving area in the first stage.

### 3.3 Application of the Models and Strategy

Our built emergency routing models considering equity and priority simultaneously, not only play an important role in the relief path planning for injured people, but also

can be exactly applied into other similar path selection decisions in pro-active disaster relief operations (Elluru et al., 2017), such as the prepositioning/distribution of relief materials, as well as the evacuation of victims. Apart from the humanitarian context, the equity objective expressed as the relative deprivation cost is also suitable to deal with the fairness considerations in general supply chain allocation scenarios, by presenting the economic valuation of psychological sufferings resulted from the lack of supply services. And the priority constraint represented by different in-transit tolerable suffering time windows, can also be well used to characterize the differentiated distribution requirements in commercial logistics and supply chain environment. In addition to the models, our hybrid strategy succeeded in the commercial context, can be more widely implemented into many humanitarian operations for improving the relief performance, such as the allocation-routing problem for diversified relief materials or the evacuation routing optimization for differentiated victims.

#### **4 An Ant Colony Optimization Algorithm to Solve Models**

Model I and Model II presented above are multi-objective optimization problems, for seeking optimal relief paths to minimize the total transportation cost, as well as the absolute and relative deprivation cost of all victims. Although the Pareto Front solutions for multi-objective problems can be obtained by many approaches (Deb, 2001), this is not the important point of our paper. We adopt one of the simplest methods, called scalarization, to normalize these objectives into a uniform and dimensionless scale, for further comparison analysis. In detail, we normalize each objective function and convert them into the range  $[0,1]$  through  $(f - f_{min}) / (f_{max} - f_{min})$ . After summing up all normalized terms as one dimensionless objective, our models are actually transformed to the time-varied vehicle routing problems with some time window constraints (VRPTW), where the travelling velocity on each arc is a time-dependent continuous change function. Since even the simplest form of the travelling salesman problem (TSP) is known to be NP-hard (Parker and Rardin, 1983), our extended VRPTW models can also be proved to be NP-hard (see Appendix A). A variety of exact and approximate algorithms have been proposed to solve this problem class. The approximate algorithms are usually based on heuristics, which can be further classified as the classical heuristics and meta-heuristics. Due to the inefficiency of general exact methods and their inability to solve optimally within a reasonable time, heuristics, especially the meta-heuristics, has received a lot of attention in solutions (Bräysy and Gendreau, 2005b).

The ant colony optimization (ACO) algorithm is a kind of meta-heuristic optimization technique and has been proven to be an effective approach in solving routing problems (Bell and McMullen 2004). The essence of ACO is the pheromone trail laying and following behavior of real ants which use pheromones as a communication medium (Dorigo and Gambardella, 1997). The ACO was first used in TSP problems, later was also successfully applied to the general vehicle routing problems

(Bullnheimer, Hartl, and Strauss, 1999; Bell and McMullen, 2004; Yuan and Wang, 2009; Ariyasingha and Fernando, 2015; Schyns, 2015), quadratic assignment problems (Maniezzo, 1999), as well as scheduling problems (Merkle, Middendorf, and Schmeck, 2002). We attempt to design an ACO algorithm for solving our models with the following main reasons: The characteristics of the autocatalytic positive feedback and the inherent parallelism in ACO algorithm can accelerate the discovery of global optimal solutions, which is vital in emergency scenarios. The ACO algorithm to solve Models I and II is presented below, and its convergence related proofs are given later.

#### 4.1 The ACO Algorithm

##### Step 1: Initialization.

We set the values of parameters, including the number of ants  $m$ , the pheromone factor  $\alpha$ , the heuristic factor  $\beta$ , the pheromone evaporation rate  $\rho$ , the pheromone strength  $Q$ , and the maximum iteration  $NC_{max}$ . For every edge, we also set an initial pheromone concentration  $\tau_{ij}(0)$  and an initial pheromone updating value  $\Delta\tau_{ij}^k(0)$ .

##### Step 2: Construction of Ant Solutions.

We randomly place all the  $m$  ants on the  $N$  nodes. At node  $i$ , the  $k^{\text{th}}$  ant chooses the next node  $j$  to move to with the probability below:

$$P_{ij}^k = \begin{cases} \frac{(\tau_{ij})^\alpha \cdot (\eta_{ij})^\beta}{\sum_{q \in N_i^k} (\tau_{iq})^\alpha \cdot (\eta_{iq})^\beta}, & j \in N_i^k \\ 0, & j \notin N_i^k \end{cases} \quad (20)$$

where  $N_i^k$  is the neighborhood of ant  $k$  when at node  $i$ .  $\alpha, \beta$  are parameters to weight respectively the relative importance of the pheromone trail and heuristic information.  $\tau_{ij}$ , called the pheromone concentration, is a pheromone trail value associated with each arc  $(i, j)$ .  $\eta_{ij}$  is a heuristic value that measures the desirability of the  $k^{\text{th}}$  ant from node  $i$  to  $j$ . We propose the heuristic function (21) to describe the impacts of various road situations, which is reflected in parameter  $\delta$ : a larger value of  $\delta$  means a more serious road condition, leading to a smaller heuristic value for path seeking.

$$\eta_{ij} = \begin{cases} v_{ij}^0 \cdot \varepsilon_{ij} \cdot e^{-\delta}, & j \in N_i^k \\ 0, & j \notin N_i^k \end{cases} \quad (21)$$

After the ants' arrival to the next node, they have to make a judgement whether both the capacity constraint (10) and the waiting time window restriction (15) in Models I and II are satisfied. If so, go to Step 3 after all ants finishing visiting all nodes; otherwise, repeat Step 2.

##### Step 3: Validation of Feasible Solutions.

In Model I, if any route does not meet the Constraint (16), we set its length to a very large positive value  $M$ . Analogously, if any route in Model II does not meet the Inequality (16) and the additional Constraints (18) (19), a very large positive value  $M$  is also given as its length.

**Step 4:** Updating Feasible Solutions.

The feasible solutions are updated by means of the 2-opt local search method (Croes 1958), which is used to improve the quality of solutions and select the current optimal path for globally updating the pheromone. Specifically, a part of arcs on the path are reversely exchanged. If the length of the route after reversely exchanging is shortened, then the feasible solution is updated.

**Step 5:** Updating Pheromone.

We make a record of the optimal solution in the present cycle. The value of the pheromone trail on each arc is updated with Equations (22) and (23):

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \sum_{k=1}^m \Delta\tau_{ij}^k(t) \quad (22)$$

$$\Delta\tau_{ij}^k(t) = \begin{cases} \frac{Q}{L_k}, & \text{if the } k^{\text{th}} \text{ ant passed arc } (i,j) \text{ in } t^{\text{th}} \text{ cycle} \\ 0, & \text{otherwise} \end{cases} \quad (23)$$

where  $\rho$  ( $0 < \rho \leq 1$ ) is the pheromone evaporation rate.  $\Delta\tau_{ij}^k(t)$ , named the pheromone updating value, is the amount of pheromone that the  $k^{\text{th}}$  ant deposits on the arc  $(i,j)$  at time  $t$ .  $Q$  is the pheromone strength, which means the increased amount of pheromone when one ant finishes one cycle.  $L_k$  is the total tour length of the  $k^{\text{th}}$  ant.

**Step 6:** Termination.

If  $NC < NC_{max}$ ,  $NC = NC + 1$ , go to Step 2; Otherwise, the algorithm is terminated.

**4.2 Validation of ACO Algorithm**

We now validate ACO algorithm by proving the basic type of convergence, convergence in value, which is guaranteeing to find an optimal solution with a probability that can be made arbitrarily close to 1 given enough time. Note that, although convergence in solution is generally a stronger and more desirable result than convergence in value, we are still interested in finding an optimal value. Because once the optimal value has been found, the problem is solved and the algorithm stops. The convergence in value is all that our models need. Before presenting the theorem of convergence in value for AGO algorithm, we have two following propositions.

**Proposition 1:** For any  $\tau_{ij}$ , it holds:

$$\lim_{t \rightarrow \infty} \tau_{ij}(t) \leq \tau_{max} = \frac{q_y(\gamma^*)}{\rho}.$$

**Proof:** The quality function  $q_y(\gamma)$  is non-increasing with respect to  $y$ , that is, if  $y(\gamma_1) > y(\gamma_2)$ , then  $q_y(\gamma_1) \leq q_y(\gamma_2)$ .  $q_y(\gamma^*)$  is the best quality function, which means the maximum possible amount of pheromone added to any arc  $(i,j)$  after any iteration. Due to the pheromone evaporation, the maximum possible pheromone trail at iteration 1 is  $(1-\rho)\tau_0 + q_y(\gamma^*)$ . And the maximum possible pheromone trail at iteration 2 is  $(1-\rho)^2\tau_0 + (1-\rho)q_y(\gamma^*) + q_y(\gamma^*)$ .  $\tau_{ij}^{max}(t)$  can be used to express the maximum possible pheromone trail at iteration  $t$ , that is as follows:



$$\tau_{ij}^{max}(t) = (1 - \rho)^t \tau_0 + \sum_{i=1}^t (1 - \rho)^{t-i} q_y(\gamma^*).$$

As  $0 < \rho \leq 1$ , this maximum possible pheromone trail at iteration  $t$  converges asymptotically to  $\frac{q_y(\gamma^*)}{\rho}$ . Therefore,  $\lim_{t \rightarrow \infty} \tau_{ij}(t) \leq \lim_{t \rightarrow \infty} \tau_{ij}^{max}(t) = \tau_{max} = \frac{q_y(\gamma^*)}{\rho}$ . ■

Proposition 1 shows that the maximum possible pheromone level  $\tau_{max}$  is asymptotically bounded by the pheromone evaporation.

**Proposition 2:** Once an optimal solution  $\gamma^*$  is found, it holds that  $\forall (i, j) \in \gamma^*$ ,  $\lim_{t \rightarrow \infty} \tau_{ij}^*(t) = \tau_{max} = \frac{q_y(\gamma^*)}{\rho}$ , where  $\tau_{ij}^*$  is the pheromone trail value on connections  $(i, j) \in \gamma^*$ .

**Proof:** Once an optimal solution has been found, then  $\forall t \geq 1, \tau_{ij}^*(t) \geq \tau_{min}$ .

Combining with the iteration-best update rule, we find that  $\tau_{ij}^*(t)$  monotonically increases.

After the iteration  $t^*$  in which the first optimal solution  $\gamma^*$  has been found, analogous to Proposition 1, the maximum possible pheromone trail at iteration 1 is  $(1 - \rho)\tau_{ij}^*(t^*) + q_y(\gamma^*)$ . The maximum possible pheromone trail at iteration 2 is  $(1 - \rho)^2\tau_{ij}^*(t^*) + (1 - \rho)q_y(\gamma^*) + q_y(\gamma^*)$ . And  $\tau_{ij}^*(t)$  is the pheromone trail value on connections  $(i, j) \in \gamma^*$ , which is used to express the maximum possible pheromone trail at iteration  $t$ . The formulation of  $\tau_{ij}^*(t)$  is as follows:

$$\tau_{ij}^*(t) = (1 - \rho)^t \tau_{ij}^*(t^*) + \sum_{i=1}^t (1 - \rho)^{t-i} q_y(\gamma^*).$$

As  $0 < \rho \leq 1$ , this maximum possible pheromone trail at iteration  $t$  converges asymptotically to  $\frac{q_y(\gamma^*)}{\rho}$ . Therefore,  $\forall (i, j) \in \gamma^*$ ,  $\lim_{t \rightarrow \infty} \tau_{ij}^*(t) = \tau_{max} = \frac{q_y(\gamma^*)}{\rho}$ . ■

Proposition 2 states that, once an optimal solution has been found, the values of the pheromone trails on all connections in the optimal solution set  $\gamma^*$  converge to  $\frac{q_y(\gamma^*)}{\rho}$ .

**Theorem 1:** Let  $P^*(t)$  be the probability that the algorithm finds an optimal solution at least once within the first  $t$  iterations. Then, for an arbitrarily small  $\varepsilon > 0$  and for a sufficiently large  $t$ , it holds that  $P^*(t) \geq 1 - \varepsilon$ , and  $\lim_{t \rightarrow \infty} P^*(t) = 1$ .

**Proof:** From Equation (20), we can denote the minimum probability of the  $k^{\text{th}}$  ant from node  $i$  to  $j$  as  $p_{min} > 0$ . It holds that,  $\forall (i, j), \tau_{min} \leq \tau_{ij} \leq \tau_{max}$ ,

$$P_{ij}^k \geq p_{min} = \frac{(\tau_{min})^\alpha \cdot (\eta_{ij})^\beta}{(N_c - 1) \cdot (\tau_{max})^\alpha \cdot (\eta_{ij})^\beta + (\tau_{min})^\alpha \cdot (\eta_{ij})^\beta}$$

where  $N_c$  is the cardinality of the solution set.  $p_{min}$  is the worst-case situation: the pheromone trail associated with the desired decision is  $\tau_{min}$ , while all the other feasible choices (at most  $N_c - 1$ ) have an associated pheromone trail  $\tau_{max}$ .

A lower bound for  $P^*(t)$  can be given by  $\hat{P}^*(t) = 1 - (1 - p_{min})^t$ , which is a sufficient condition for one ant to find an optimal solution. By choosing a sufficiently large  $t$ , this probability can be greater than any value of  $1 - \varepsilon$ , and  $\lim_{t \rightarrow \infty} \hat{P}^*(t) = 1$ .

Since the probability  $P^*(t) \geq \hat{P}^*(t)$ , we can derive that  $\lim_{t \rightarrow \infty} P^*(t) = 1$ . ■

In this way, the convergence of our designed ACO algorithm is validated.

## 5 Computational Experiments

In the following computational experiments and case study, we try to: (i) evaluate the effectiveness of our emergency relief routing models and solution procedure; (ii) compare our basic Model I with the path selection model that does not consider equity/fairness in humanitarian relief operations, and compare Model I with the routing model that does not consider priorities reflected in heterogeneous in-transit tolerable suffering time windows; (iii) facing with diverse injury degrees in Model II, verify the superiority of our hybrid transportation strategy over the traditional separated one; and (iv) by sensitivity analysis, study impacts of key parameters (e.g., the waiting and in-transit tolerable suffering time windows) on the optimal path selection.

### 5.1 Case Description and Parameter Estimation

Based on the data from Federal Emergency Management Agency's (FEMA) effective Flood Insurance Rate Map (FIRM) in USA (FEMA, 2017), we conduct computational experiments by taking Houston Flood of Texas in 2017 as the case scenario. As shown in Fig. 3, there are a total of 20 reachable affected sites in the downtown area of Houston, where the pin-shaped icon in the center represents the emergency relief agency, and other stars represent the disaster areas. The Texas Medical Center, as the emergency relief agency, needs to dispatch rescue vehicles to visit the other 19 disaster areas, and pick up the trapped injured persons there back to the center for the medical treatment. Our decision-making aims to select the optimal paths for these relief vehicles when considering equity and priority simultaneously.

In Fig. 3 and Table 1, we mark the total 20 locations with corresponding numbers, including one medical center and 19 disaster sites. Assuming only one type of injury degree in each area under Model I scenario, the number of injured persons in various disaster sites is expressed as "IN" in Table 1, which is calculated from the public disaster data released by government. Combining the Base Flood Elevation (BFE)

from FEMA’s FIRM with the CNN report about Houston Flood (CNN, 2017), we estimate the parameter values of the waiting and in-transit tolerable suffering time windows, listed in Table 1. We use the large values  $u_1 = b_1 = 10,000$  as the time windows and record victims’ number as zero, to distinguish Texas Medical Center with other disaster sites. Under Model II scenario, we supplement the experimental data related to slight victims into Table 1, considering the distinctions between the slight and serious injury. It needs to be mentioned that we specially use “LIN” to represent the number of slightly injured persons in Model II, and adopt the “IN” data of Model I to express the number of seriously injured victims in Model II. Apart from the number of injured victims, we also provide the values of the time windows parameters in Table 1, whose unit is “hour”.

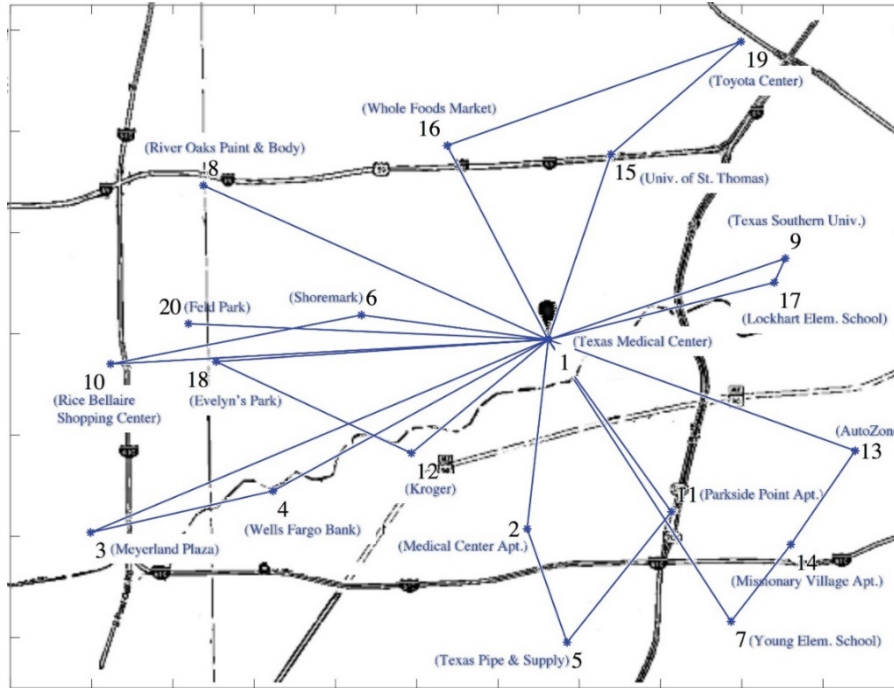


Fig. 3. The optimal relief paths of Houston flood case applying Model I

The maximum capacity of all the identical rescue vehicles is  $W_k = 13$ . It takes 9 minutes to pick up one person onto the rescue vehicle at any disaster area, which means  $t_{ik} = 0.15h$ . All the normal pre-disaster travelling velocities on each arc are assumed to be equal, that is  $V_{ijk}^0 = 60km/h$ . Considering different disaster situations, we technically obtain the velocity attenuation coefficient by generating some random matrices, and set randomly the constant coefficients  $C$  and  $D$  in the transportation cost function. Some of parameters in the absolute deprivation cost function in Model I are  $g_1 = 1.2, h_1 = 3.5, g_2 = -20, g_3 = 0.5$ . To ensure that the three-stage absolute deprivation cost is continuous, the coefficients  $h_2$  and  $h_3$  are not pre-fixed but adjustable according to the function value at the first stage. In Model II, the coefficients of the absolute deprivation cost function for slightly injured persons are given as  $g_1^l = 0.4, h_1^l = 3.2, g_2^l = -20, g_3^l = 0.1$ , and the parameters of seriously injured persons are the same as those in Model I. The weights in the

normalization process are assumed to be equal. In addition, according to the classical ACO algorithm for solving path selection problems (Dorigo and Stützle, 2004), some parameters in our algorithm are set as follows: the number of ants is  $m = 20$ ; the pheromone factor is  $\alpha = 1$ ; the heuristic factor is  $\beta = 1$ ; the pheromone evaporation rate is  $\rho = 0.15$ ; the maximum iteration is  $NC_{max} = 60$ ; the pheromone strength is  $Q = 10$ .

Table 1 The affected areas and relevant parameters

Area	1	2	3	4	5	6	7	8	9	10	
Place Name	Texas Medical Center	Medical Center Apt.	Meyerland Plaza	Wells Fargo Bank	Texas Pipe & Supply	Shoremark	Young Elem. School	River Oaks Paint & Body	Texas Southern Univ.	Rice Bellaire Shopping Center	
Model I	IN	0	3	1	3	2	2	3	4	3	3
	$u_i$	10000	1.5	3	1.5	3	1.5	3	0.5	1.5	1.5
	$b_i$	10000	0.5	2.5	0.5	2.5	0.5	2.5	0.3	0.5	0.5
Model II	LIN	0	6	2	6	4	4	6	8	6	6
	$u_i^l$	20	9.5	11	9.5	11	9.5	11	8.5	9.5	9.5
	$b_i^l$	10000	2	6	2	6	2	6	1.3	2	2
Area	11	12	13	14	15	16	17	18	19	20	
Place Name	Parkside Point Apt.	Kroger	AutoZone	Missionary Village Apt.	Univ. of St. Thomas	Whole Foods Market	Lockhart Elem. School	Evelyn's Park	Toyota Center	Feld Park	
Model I	IN	1	2	2	3	3	1	3	2	2	4
	$u_i$	6	3	3	1.5	1.5	3	1.5	1.5	1.5	0.5
	$b_i$	6	2.5	2.5	0.5	0.5	2.5	0.5	0.5	0.5	0.3
Model II	LIN	2	4	4	6	6	2	6	4	4	8
	$u_i^l$	12.5	11	11	9.5	9.5	11	9.5	9.5	9.5	8.5
	$b_i^l$	12	6	6	2	2	6	2	2	2	1.3

## 5.2 Model Solution and Algorithm Verification

Taking 2017 Houston Flood as the case background, we first use MATLAB R2010b to solve Model I by implementing the proposed ACO algorithm in Section 4.1, and display the optimal emergency relief paths in Fig. 3 and Table 2. To demonstrate the stability of the ACO algorithm, we test the convergence results under different scenarios with 20, 25, 30, 35, 40 disaster areas (or nodes), shown in Fig. 4.

To evaluate the efficiency of the ACO algorithm, we also execute the genetic algorithm (GA) on MATLAB R2010b with the same computer to get the solutions of Model I. GA is another evolutionary meta-heuristics that has also been often used in solving many path selection problems (Baker and Ayechev, 2003), which serves as the reference algorithm in our computational experiments. The comparison results using ACO and GA are given in Table 2.

From Table 2, the convergence of ACO costs about 33 seconds, while the run time of GA is nearly 186s. This means that our ACO algorithm uses fewer iterations to converge, which is more suitable for urgent relief situations. Due to the random cost coefficients, we choose the shortest total travelling distance instead of the transportation cost as one of the performance objectives for comparison. From the perspective of performance objectives, although GA shows better than ACO in the shortest total travelling distance and the occupied number of rescue vehicles, ACO is evidently superior to GA in terms of sufferings mitigation and equity issues emphasized in humanitarian relief operations, which are reflected as a lower absolute deprivation cost and a lower relative deprivation cost with ACO in Table 2.

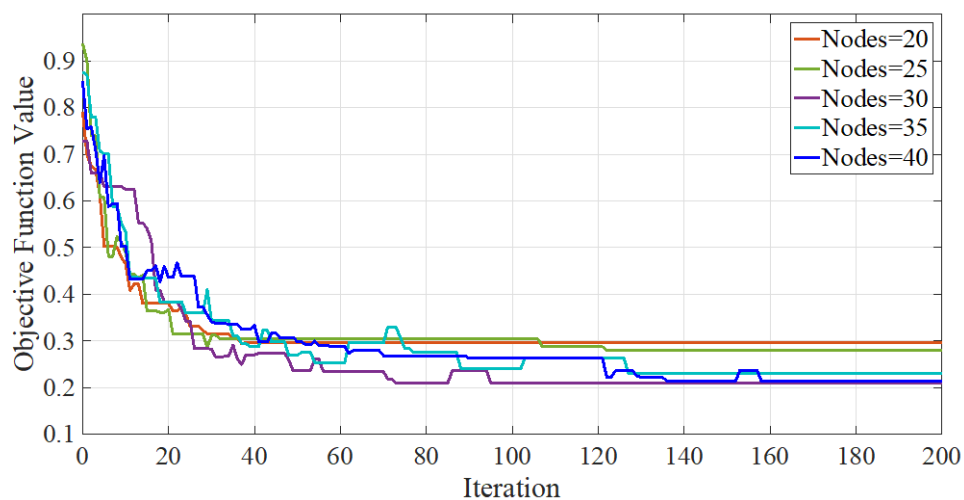


Fig. 4. Convergence of the ACO algorithm

Table 2 Comparison between ACO and GA

Path No. / Performance	ACO	GA
1	1→4→3→1	1→2→5→7→14→13→1
2	1→8→1	1→8→1
3	1→7→14→13→1	1→6→18→1
4	1→15→19→16→1	1→16→19→15→17→1
5	1→20→1	1→20→1
6	1→2→5→11→1	1→10→1
7	1→18→12→1	1→12→4→3→11→1
8	1→17→9→1	1→9→1
9	1→10→6→1	
The Shortest Total Travelling Distance (km)	55	48.7
The Minimum Absolute Deprivation Cost (USD)	1525.49	2106.45
The Minimum Relative Deprivation Cost (USD)	245.17	471.15
Run Time (s)	33.102	186.01

Furthermore, for illustrating the impacts of different disaster scenarios, we perform the comparison tests regarding various performance objectives, using ACO and GA under 20, 25, 30, 35, 40 disaster nodes respectively. As shown in Fig. 5, with the

increase in the scale of disasters, the advantages of GA on saving operational costs become less apparent. When the number of disaster areas rises up to 40, the shortest total travelling distance and the occupied number of rescue vehicles using GA are almost the same to the results with ACO. On the other hand, ACO shows its steady superiority on both the minimum absolute and relative deprivation cost, which especially demonstrates more obvious advantages in the case of more nodes. Thereby, it is verified that ACO can be effectively used to solve our emergency routing optimization models. We perform the ACO algorithm for the following comparison and sensitivity analysis.

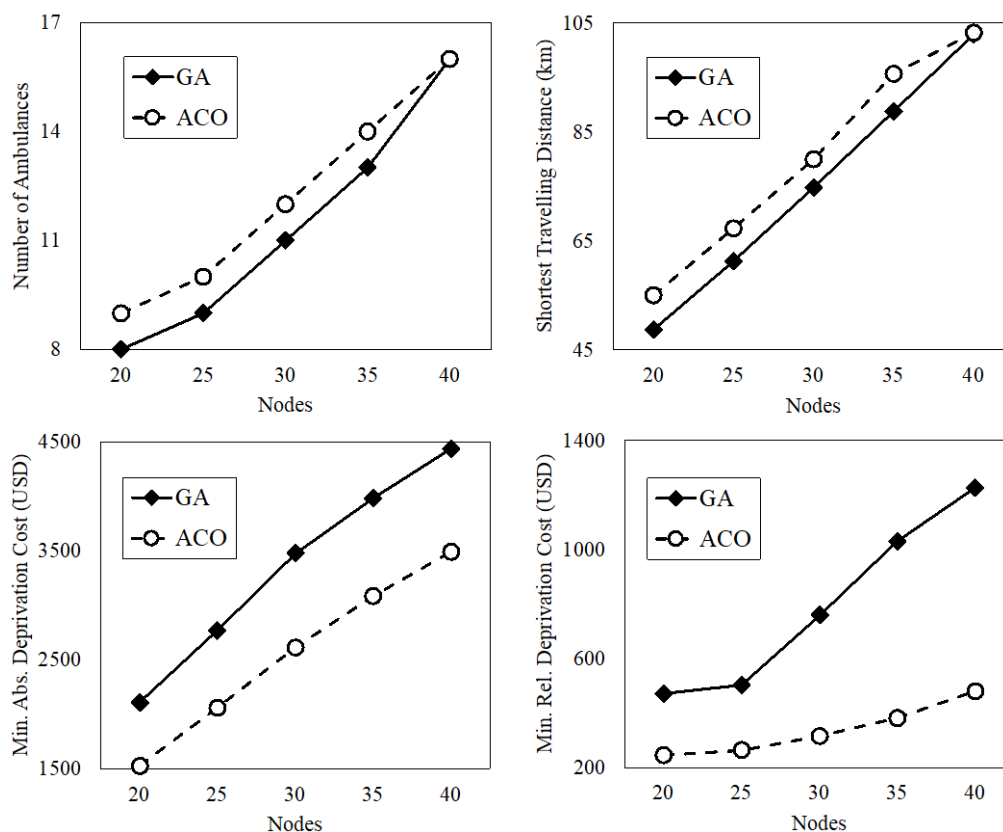


Fig. 5. The different performance comparisons using ACO and GA

### 5.3 Comparison Analysis

To discuss the necessity of equity and priority considerations in humanitarian operations, we make some comparisons between Model I and some traditional path selection models. Then, taking the equity and priority into account, we perform a comparison analysis on the two-stage separated and the hybrid transportation strategy in Model II.

#### 5.3.1 Comparison Based on Model I

Model I<sup>a</sup> is the path selection model that does not consider the objectives of the absolute and relative deprivation cost in Model I, and Model I<sup>b</sup> is the one that does not consider the constraints of heterogeneous in-transit tolerable suffering time windows in Model I. We perform a comparison among Model I, Model I<sup>a</sup> and Model I<sup>b</sup>, and

demonstrate all the optimal paths and the corresponding objective values in Table 3. The optimal relief routes under Model I<sup>a</sup> and Model I<sup>b</sup> are also respectively shown in Fig. B.1(a) and (b) of Appendix B. Taking a rough look at Table 3, more rescue vehicles are required in our Model I. The main reason is that the equity considerations in human sufferings lead to a higher demand for relief supplies.

### 1. Model I v.s. Model I<sup>a</sup>

We first compare the performance objectives under Model I and Model I<sup>a</sup>. As shown in Table 3, the human sufferings reflected as deprivation costs are significantly mitigated in model I, with a relatively acceptable increase in the travelling distance (i.e. from 47.6km to 55km). More specifically, the minimum absolute deprivation cost drops from \$2,077.57 to \$1,525.49, and the minimum relative deprivation cost falls from \$682.99 to \$245.17 at the same time. It indicates that our Model I guarantees an effective alleviation on human sufferings and provides a good improvement on equity in humanitarian relief.

Then, we observe the comparison of optimal routes under Model I and Model I<sup>a</sup>. From Table 3, we find that it does not appear one path with so many visited areas under Model I, like 1→13→14→11→7→5→1 or 1→12→18→10→6→1 in Model I<sup>a</sup>. In other words, there are more evenly dispersed disaster areas on each vehicle's path under our Model I, which leads to a shorter suffering period for injured victims who are waiting for rescue vehicles or already in vehicles. It is verified again that Model I can achieve a good result in the mitigation of human sufferings, from the perspective of selected optimal relief paths.

Table 3 Comparison among Model I, Model I<sup>a</sup> and Model I<sup>b</sup>

Path No. / Performance	Optimal Routes		
	Model I	Model I <sup>a</sup>	Model I <sup>b</sup>
1	1→4→3→1	1→13→14→11→7→5→1	1→12→4→3→11→1
2	1→8→1	1→8→1	1→8→10→1
3	1→7→14→13→1	1→9→17→1	1→9→17→1
4	1→15→19→16→1	1→15→19→16→1	1→15→19→16→1
5	1→20→1	1→20→1	1→20→18→6→1
6	1→2→5→11→1	1→2→3→1	1→2→5→7→14→13→1
7	1→18→12→1	1→12→18→10→6→1	
8	1→17→9→1		
9	1→10→6→1		
The Shortest Total Travelling Distance (km)	55	47.6	43.5
The Minimum Absolute Deprivation Cost (USD)	1,525.49	2,077.57	2,337.46
The Minimum Relative Deprivation Cost (USD)	245.17	682.99	454.83

### 2. Model I v.s. Model I<sup>b</sup>

Similarly, we firstly compare the three performance objectives under Model I and Model I<sup>b</sup>. It is shown in Table 3 that although both of the minimum absolute and

relative deprivation cost under Model I are lower, the travelling distance under Model I<sup>b</sup> is shorter. One of possible reasons is that the in-transit tolerable suffering time constraints in Model I trigger an increased demand for rescue vehicles. The more occupied vehicles subsequently lead to a longer total travelling distance and a higher transportation cost. Thereby, at the expense of increased transportation costs, Model I can exactly better mitigate human sufferings and improve the equity issues in disaster relief.

Next, we pay attention to the different optimal relief routes under Model I and Model I<sup>b</sup> in Table 3. Taking the second route for instance, the optimal path is 1→8→1 under Model I, while it becomes 1→8→10→1 under Model I<sup>b</sup>. According to Table 1, the acceptable waiting and in-transit tolerable suffering time windows of victims in area 8 are very small, that means River Oaks Paint & Body is a severe flood impacted zone where the injured persons need to be promptly transported to the medical center for treatment. So it is not feasible in real situations that one rescue vehicle departs from the Texas Medical Center, subsequently visits the disaster area 8, then goes through area10, and finally returns to the center. Another similar example is the fifth route under Model I and Model I<sup>b</sup>. It is also because of the narrow in-transit tolerable suffering time windows for injured persons in area 20, one rescue vehicle is specially arranged to Feld Park in our Model I. Therefore, those path selection models that do not consider the constraints of the in-transit tolerable suffering time windows for victims may lead to invalid decisions in humanitarian relief operations.

Through comparison analysis based on Model I, we present the first managerial insight here: emergency decision-makers are suggested to simultaneously incorporate the equity and priority considerations into humanitarian relief decisions, in order to better alleviate human sufferings in a fairer manner. Furthermore, policy makers need to pay attention to various in-transit tolerable suffering time windows for achieving feasible and effective path selection decisions. This result is also consistent with Chen, Hsueh, and Chang (2009) and Govindan et al (2014), in which they emphasized that the time-sensitive products like perishable foods must be delivered within allowable time windows.

### **5.3.2 Comparison Based on Model II**

In this subsection, considering victims with different injury degrees, we compare the two-stage separated and the hybrid transportation strategy based on Model II. We still implement the case scenario in Table 1 by applying the ACO algorithm, and show the comparison results in Table 4, where STD, ADC and RDC represent respectively the shortest total travelling distance, the minimum absolute deprivation cost and the minimum relative deprivation cost. The optimal relief routes under the separated and hybrid strategy are also respectively shown in Fig. B.2(a1),(a2) and (b1),(b2) of Appendix B.



First, we observe the comparison on the number of vehicles under the two strategies. From Table 4, fewer rescue vehicles are required in our hybrid transportation strategy, with the total number of vehicles being reduced from 18 to 15. This result illustrates that the hybrid transportation strategy, which has been often used to improve the vehicle loading efficiency in commercial environments, can also play an important role in the humanitarian relief context. In particular, since our models considering equity and priority require more rescue vehicles as shown in Table 3, the proposed hybrid transportation strategy is more beneficial to humanitarian operations when supplies are limited.

Table 4 Comparison between the separated and the hybrid transportation strategy

		1	2	3	4	5	6	7	8	9	-	STD	ADC	RDC
													(km)	(USD)
Separated Transportation Strategy	Path	1	2	3	4	5	6	7	8	9	-	STD	ADC	RDC
	Stage	→1	1	→13→1	6→1	1→8→1	11→1	12→1	9→1	→16→1	-	55	1525.49	245.17
Separated Transportation Strategy	Path	10	11	12	13	14	15	16	17	18	-	-	-	-
	Stage	6→1	14→1	→1	1→8→1	18→1	→16→1	→1	17→1	→3→1	-	57.1	6649.20	836.46
Hybrid Transportation Strategy	Path	1	2	3	4	5	6	7	8	9	10	STD	ADC	RDC
	Stage	4→3→1	1	→16→1	18→1	13→1	11→1	1→6→1	1→8→1	→1	→1	58.7	2159.08	245.08
Hybrid Transportation Strategy	Path	11	12	13	14	15	-	-	-	-	-	-	-	-
	Stage	12→1	→1	→1	15→1	→1	-	-	-	-	-	34.4	3707.74	416.41

Second, we compare the performance objectives under the two different strategies. From Table 4, when applying the hybrid strategy, the shortest total travelling distance at the first stage is slightly longer (i.e. 58.7km v.s. 55km), and the minimum absolute deprivation cost at the first stage is much higher (i.e. \$2,159.08 v.s. \$1,525.49). The longer distance is due to one more vehicle occupied in the first phase under the hybrid strategy. And two main reasons can be used to explain the higher minimum absolute deprivation cost at the first stage under the hybrid strategy: (1) By our hybrid strategy, some slightly injured persons are probably to be picked up in the last accessed areas of vehicles at the first stage, which makes those serious victims already in vehicles have to suffer a longer deprivation time and a higher absolute deprivation cost. (2) Also because some slight victims may be rescued at the first stage under the hybrid strategy, the absolute deprivation cost in the first phase naturally includes the cost for these minor injured persons. But it is worth noting that all the performance objectives at the second stage under the hybrid strategy are improved greatly. Specifically, the shortest total travelling distance is reduced from 57.1km to 34.4km, and the minimum absolute deprivation cost decreases from \$6,649.2 to \$3,707.74 as well as the minimum relative deprivation cost drops from \$836.46 to \$416.41. Moreover, from the overall perspective of two stages, there are a shorter traveling distance and a lower

absolute/relative deprivation cost under the hybrid strategy. It demonstrates that our proposed hybrid transportation strategy can not only better alleviate the psychological sufferings of differentiated injured victims and better achieve relief fairness in humanitarian operations, but also better control the operational costs.

Last, we focus on the optimal relief routes under the two different strategies. Obviously, under the separated strategy, all the disaster areas are revisited at the second stage. This is because a strict rule exists in the separated strategy, which is only seriously injured persons can be rescued at the first stage while all the slightly injured ones have to wait to be provided with relief service later. According to Table 1, since there are slight victims in each disaster area, all the disaster sites need to be visited again at the second stage when applying the separated strategy. Comparatively, when the hybrid transportation strategy works, disaster areas 3, 6, 8, 11, 13, 14, 16, 17, 18 and 20 do not appear on the relief routes of the second stage. This means that all slightly injured persons in the ten areas have already been taken away at the first stage under the hybrid strategy, and the rescue vehicles only need to visit the remaining nine disaster areas at the second stage. It can also serve as a reasonable explanation about why the reduction in the shortest total travelling distance and the decrease in the number of occupied rescue vehicles appear under the hybrid transportation strategy.

According to these comparison results based on Model II, we demonstrate the second managerial insight: When facing with differentiated demand, such as victims with different injury degrees, evacuees with different priorities, and relief materials with different urgency levels, emergency decision-makers are recommended to apply our proposed hybrid transportation strategy into the optimal path selection. The hybrid transportation strategy originating from commercial environments, can well improve the sufferings mitigation and the operational performance in humanitarian relief, especially when emergency supplies are limited.

#### **5.4 Sensitivity Analysis**

After verifying the effectiveness of our models and the hybrid strategy, we conduct a sensitivity analysis on some key time window parameters, including the waiting time windows and the victims' in-transit tolerable suffering time windows. These parameters are influenced by many complex realistic factors, such as severities of disasters, locations of disaster areas, and decision-makers' risk attitudes. We first observe the impacts of changing time windows on the optimal relief routing in Model I, and then discuss the choice of different transportation strategies under diverse time window parameters in Model II.

##### **5.4.1 Impacts of Changing Time Windows in Model I**

We implement an increase or a decrease on the waiting time windows  $u_i$  and the in-transit tolerable suffering time windows  $b_i$ , and show their influences on different performance objectives of Model I in Table 5. LTD in Table 5 represents the longest

travelling distance among all vehicles' routes. Fig. C.1 in Appendix C illustrates the optimal relief paths under various time windows scenarios.

First, we focus on the impacts of changes in both  $u_i$  and  $b_i$  on the operational performance, mainly reflected in the number of vehicles and the travelling distance. From Table 5, when  $u_i$  and  $b_i$  are decreased by 20%, both the number of occupied vehicles and the shortest total travelling distance show an increase trend. The main reason is that the smaller waiting and in-transit time windows for victims result in there are fewer disaster sites on each rescue vehicle's path, which can also be used to explain why the longest travelling distance is reduced from 10.1km to 9.5km. Furthermore, from comparison between Fig. 3 and Fig. C.1(a1) in Appendix C, under a narrower time window scenario, more rescue vehicles specially provide relief service only to one disaster site, just like areas 2, 8, 9, 10, 17 and 20 in Fig. C.1(a1). On the contrary, when a 20% increase in both  $u_i$  and  $b_i$ , the longest travelling distance rises up from 10.1km to 11.2km, while the shortest total travelling distance declines due to fewer occupied rescue vehicles.

Table 5 The impacts of changing  $u_i$  and  $b_i$  in Model I

Influence on Different Performance Objectives	Houston Flood	Both Changes of $u_i$ and $b_i$		Only Change of $u_i$		Only Change of $b_i$	
		Decreased	Increased	Decreased	Increased	Decreased	Increased
		by 20%	by 20%	by 20%	by 30%	by 20%	by 30%
Number of Vehicles	9	12	8	10	9	11	8
LTD/STD (km)	10.1/55	9.5/65.5	11.2/49	9.5/56.9	11.6/53	9.5/61	11.6/50.8
ADC (USD)	1525.49	1193.37	1772.78	1351.39	1574.98	1272.39	1688.97
RDC (USD)	245.49	114.54	374.85	241.02	277.18	195.99	315.2656

Second, we observe the impacts of changes in both  $u_i$  and  $b_i$  on the humanitarian performance, involving the minimum absolute and relative deprivation cost. Surprisingly, with the stricter time windows, the sufferings of victims are better mitigated in a fairer way, which reflected as the decrease of ADC and RDC in Table 5. Correspondingly, Table 5 also demonstrates that the more relaxed time windows even cause the increase of minimum absolute and relative deprivation cost. This result seems a paradox at a rough glance, but really does not mean that a shorter time window in emergency relief is more favor. From another perspective, even in very urgent situations, our Model I simultaneously considering equity and priority would not be too bad in terms of humanitarian performance. Besides, from the last four columns of Table 5, we find that changing either  $u_i$  or  $b_i$  alone shows the same influences as above on the operational and humanitarian performance.

Finally, we discuss the respective effects of changing  $u_i$  and  $b_i$  alone, by comparing the corresponding change rates in those operational and humanitarian performance. An interesting result from Table 5 is that an adjustment in  $b_i$  brings about a greater change to the performance objectives, compared with the impacts of changing  $u_i$ . This finding confirms again that the in-transit tolerable suffering time

window for victims cannot be ignored in emergency path selection decisions, apart from the traditionally emphasized vehicles' arrival efficiency. Given the important role of the in-transit tolerable suffering time window, the third managerial insight that we are aiming to suggest is that emergency decision-makers should make efforts to improve the medical treatment levels in rescue vehicles. Exemplified by strengthening the configuration of medical instruments and emergency medicine in vehicles, we can slow down the growth rate of victims' in-transit sufferings, and realize a more effective emergency relief operation.

#### 5.4.2 Impacts on Changing Time Windows in Model II

Now we compare different transportation strategies under diverse time window parameters based on Model II. Table 6 presents the operational and humanitarian performance at two stages under the separated and hybrid transportation strategy, when facing with changes in the waiting time windows  $u_i$  and the in-transit tolerable suffering time windows  $b_i$ . The corresponding optimal relief paths are displayed in Fig. C.2 of Appendix C.

Table 6 The impacts of changing  $u_i$  and  $b_i$  in Model II

Influence on Different Performance Objectives		STD (km)		ADC (USD)		RDC (USD)		Number of Vehicles
		First Stage	Second Stage	First Stage	Second Stage	First Stage	Second Stage	
		Total		Total		Total		
Separated Transportation Strategy	Houston	55	57.1	1525.49	6649.20	245.17	836.46	18
	Flood	112.1		8174.69		1081.63		
	Decreased	65.5	58.6	1193.37	5486.14	114.54	597.72	22
	by 20%	124.1		6679.51		712.26		
	Increased	49	56.9	1772.78	6690.90	374.85	890.51	17
	by 20%	105.9		8463.68		1265.36		
Hybrid Transportation Strategy	Houston	58.7	34.4	2159.08	3707.74	245.08	416.41	15
	Flood	93.1		5866.82		661.49		
	Decreased	61.7	28.6	2024.67	3285.21	184.61	323.61	15
	by 20%	90.3		5309.88		508.22		
	Increased	52.4	38.8	2219.33	4249.20	335.17	429.54	14
	by 20%	91.2		6468.53		764.71		

First, we observe the impacts of changes in  $u_i$  and  $b_i$  on the performance objectives from two separate stages. Shown in Table 6, although their impacts at the first stage display the same trend as the above analysis in Model I, one special change needs to be noted at the second stage. Under the hybrid transportation strategy, we find that the impacts of adjusting  $u_i$  and  $b_i$  on the shortest total travelling distance at the second stage are opposite to those influences in Model I. Taking the 20% decrease of  $u_i$  and  $b_i$  in Table 6 as an example, the shortest total traveling distance at the second stage under the hybrid strategy becomes shorter with the tighter time windows. As for this abnormal situation, we may give the following interpretation: Due to the shorter time windows, under the hybrid strategy, each rescue vehicle can only pick up fewer

seriously injured persons, leaving more space for the slightly injured victims in the last visiting area on each vehicle's path at the first stage. This inference can be exactly confirmed from data in Table 1 and Table 4, where it shows that the total number of rescued slightly injured persons increase from 46 to 50 at the first stage under the hybrid strategy. So the emergency relief pressure at the second stage is alleviated greatly when the time windows become shorter, leading to a shorter travelling distance at the second stage. Thereby, even in the emergency relief scenarios with strict time windows, our hybrid strategy can also perform well both in operational and humanitarian aspects, mainly thanks to its reasonable equity and priority considerations together with the commercial optimization technology.

Then, from the overall rescue effect of two stages in Table 6, regardless of the increase or decrease in the time windows, our hybrid strategy is evidently superior to the separated one, by demonstrating a more effective emergency relief with less operational consumption. Therefore, we provide the last managerial insight of our paper: since the proposed hybrid transportation strategy has a stable advantage over the separated one, we suggest once again that emergency decision-makers should adopt appropriately the hybrid strategy when facing with victims with different injured degrees, for a better humanitarian relief operation. The similar idea has been proven by many commercial logistics papers, such as Zhu, Crainic, and Gendreau (2014), where they verified that a consolidation-based transportation strategy can improve vehicles utilization and ultimately achieve better efficiency. Our results indicate that this similar technology also holds for the humanitarian context.

## **6 Concluding Remarks**

In this paper, we proposed an approach to study the emergency relief routing optimization for injured victims with considerations of equity and priority. For highlighting the equity or fairness requirements, we introduced a relative deprivation cost as one of decision-making objectives, based on the concept of the absolute deprivation cost. In order to more accurately characterize rescue priority, we used heterogeneous upper bounds of in-transit tolerable suffering duration as one of the time windows constraints. Then, in terms of the identical and different injury degrees for victims, we constructed Model I and Model II respectively, and proposed a hybrid transportation strategy to improve the humanitarian aid efficiency. Taking Houston Flood as a case, an ant colony meta-heuristic algorithm was developed for models solution. The validation and verification demonstrated the effectiveness of our designed algorithm. By comparison analysis, our models dealing with equity and priority can help better alleviate human sufferings in emergency scenarios, and our hybrid strategy can simultaneously improve the operational and humanitarian performance. We also conducted a sensitivity analysis to observe the influence of diverse time windows on the optimal relief paths and different performance objectives.

In future work, it is interesting to study other absolute and relative deprivation cost functions to test the robustness of our results. Or some assumptions in our models can be relaxed. For instance, the number of victims in each disaster area may exceed the capacity of each rescue vehicle, or there are several medical centers in the relief network. Uncertain scenarios in the humanitarian context are also one of further directions, such as unknown locations of disaster areas and unknown number of victims. Another important idea is that we can explore the application of our reactive models and strategies in pro-active disaster relief operations. In detail, we can attempt to incorporate the equity objectives and the in-transit tolerable time constraints into optimal decisions on the pre-disaster relief supplies prepositioning and distribution, the pre-disaster victims' evacuation, or the emergency facilities location, to achieve more effective disaster mitigation and preparation.

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## Appendix A

### NP-hardness Proof of Models I and II

By the following proposition, we prove the relationship between our Models (VRPTW) and the travelling salesman problem (TSP).

**Proposition A.1:** The VRPTW is at least as hard as TSP.

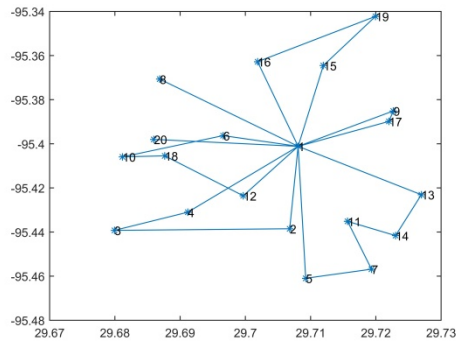
**Proof:** VRPTW is considered as a problem with additional time window constraints based on the general vehicle routing problem (VRP). VRPTW is equivalent to VRP in the case of unlimited time windows. In this sense, VRP is a special case of VRPTW.

Then, we review the definitions of VRP and TSP. VRP tries to answer “what are the optimal routes for a group of vehicles when serving a given set of customers, where vehicles initially-located at a depot are dispatched to customers and return to the origin depot?”, while TSP refers to “given a list of customers and a starting depot, what is the optimal route for one vehicle when services each customer exactly once and returns to the origin depot?”. So, VRP can be treated as a generalization of TSP. When the number of vehicles is 1, VRP becomes TSP. In other words, TSP is a special case of VRP.

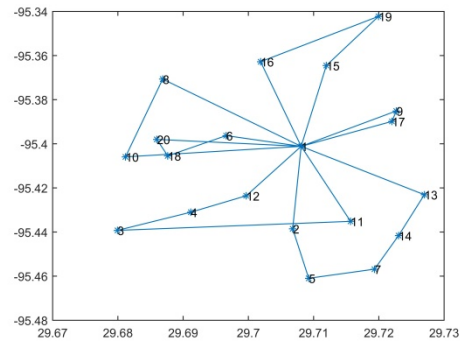
Therefore, if TSP is NP-hard, both VRPTW and VRP are naturally NP-hard. That is to say, VRPTW and VRP are at least as hard as TSP. ■

It is well-known that TSP is NP-hard, since the Hamiltonian Cycle (HC) that is NP-complete can be reducible to TSP in polynomial time (Rahman and Kaykobad, 2005). Thereby, we prove that Models I and II are NP-hard according to Proposition A.1.

## Appendix B Comparison Analysis Based on Model I and Model II

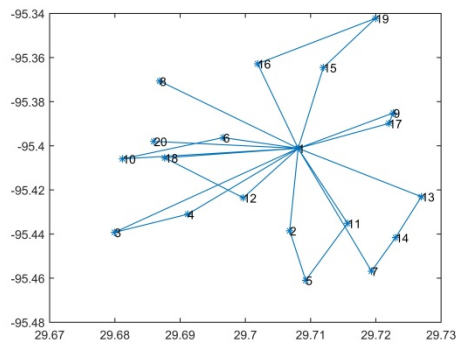


(a) No deprivation cost objective

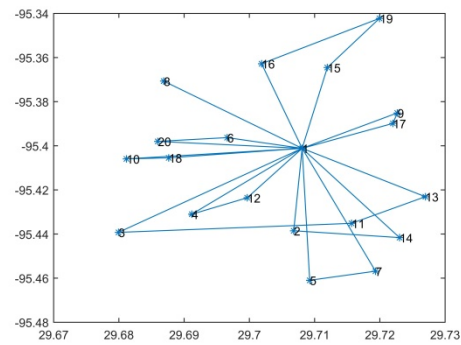


(b) No tolerable suffering time constraint

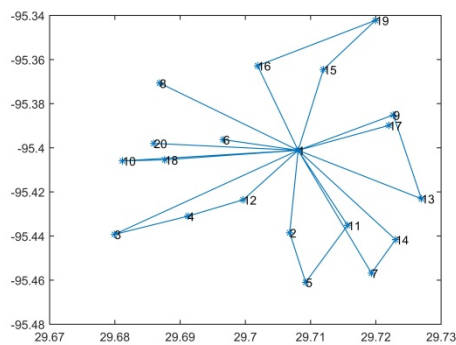
Fig. B.1. The optimal paths of traditional path selection models



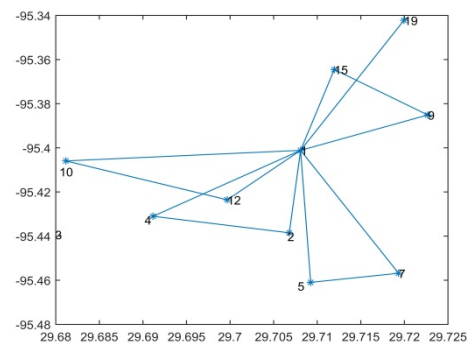
(a1) The 1<sup>st</sup> stage paths in separated strategy



(a2) The 2<sup>nd</sup> stage paths in separated strategy



(b1) The 1<sup>st</sup> stage paths in hybrid strategy

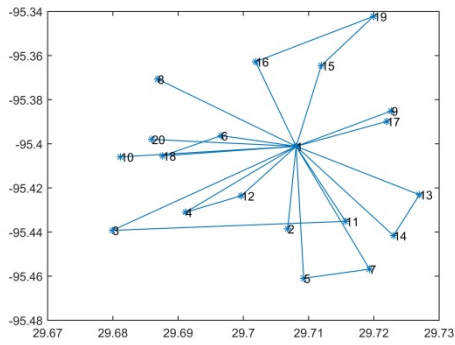


(b2) The 2<sup>nd</sup> stage paths in hybrid strategy

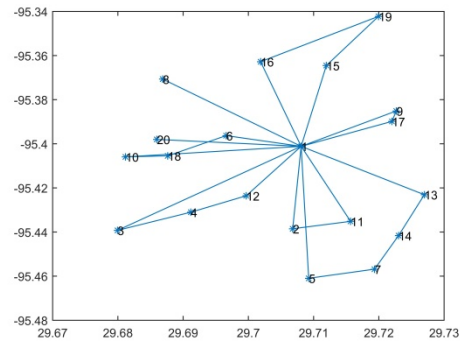
Fig. B.2. The optimal paths under different transportation strategies

## Appendix C

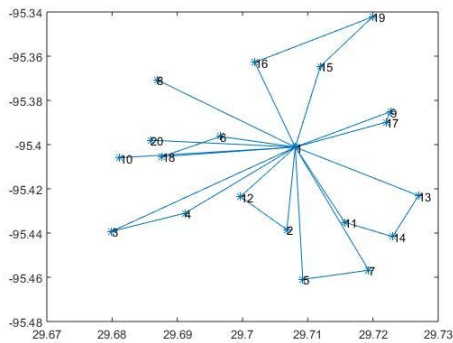
### Sensitivity Analysis Based on Model I and Model II



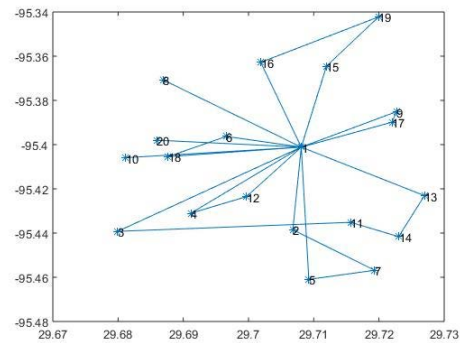
(a1) Both  $u_i$  and  $b_i$  decreased by 20%



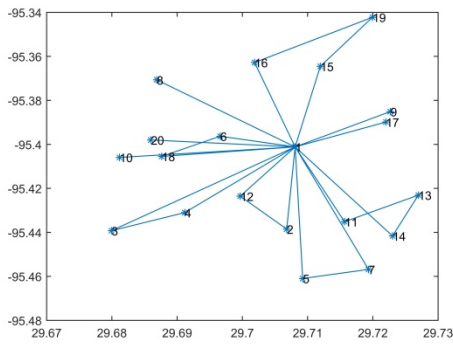
(a2) Both  $u_i$  and  $b_i$  increased by 20%



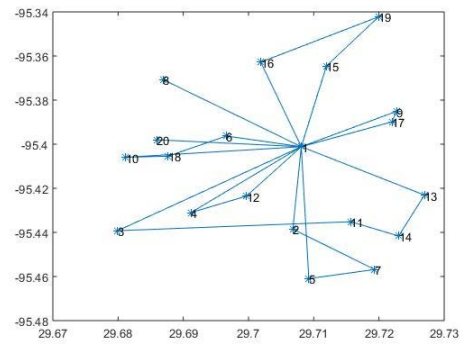
(b1) Only  $u_i$  decreased by 20%



(b2) Only  $u_i$  increased by 30%

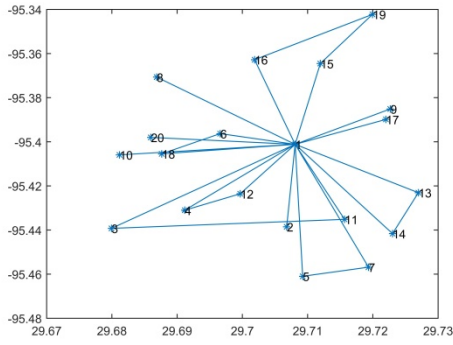


(c1) Only  $b_i$  decreased by 20%

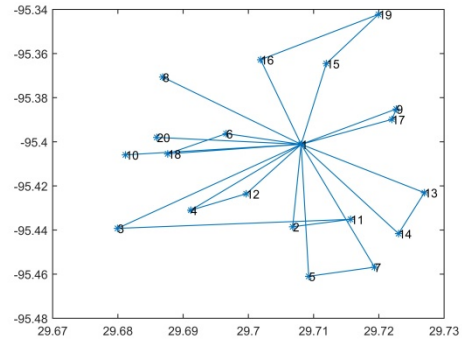


(c2) Only  $b_i$  increased by 30%

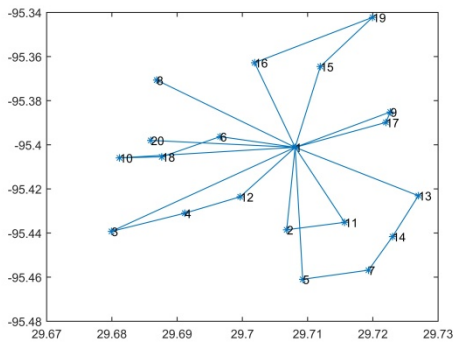
Fig. C.1. The optimal paths under different changes of  $u_i$  and  $b_i$  in Model I



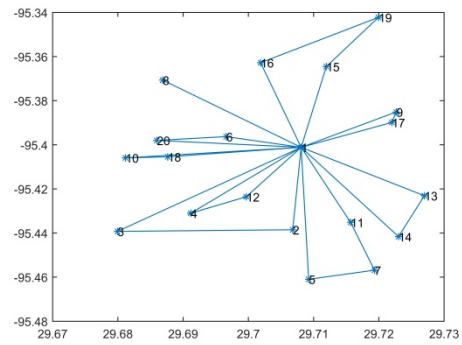
(a1) The 1<sup>st</sup> stage paths in separated strategy with 20% decrease of  $u_i$  and  $b_i$



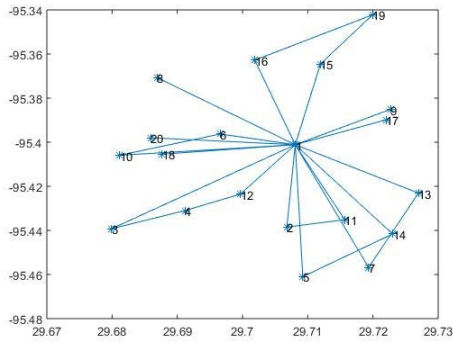
(a2) The 2<sup>nd</sup> stage paths in separated strategy with 20% decrease of  $u_i$  and  $b_i$



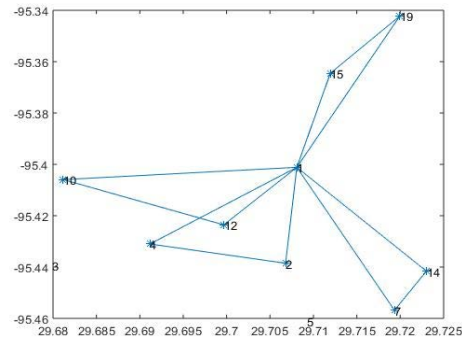
(b1) The 1<sup>st</sup> stage paths in separated strategy with 20% increase of  $u_i$  and  $b_i$



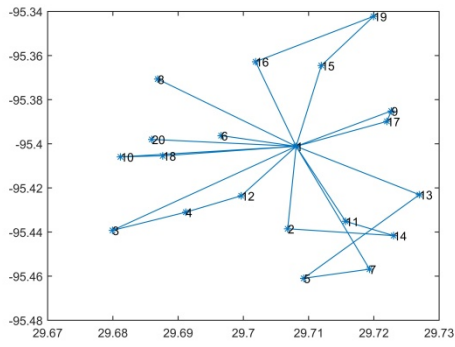
(b2) The 2<sup>nd</sup> stage paths in separated strategy with 20% increase of  $u_i$  and  $b_i$



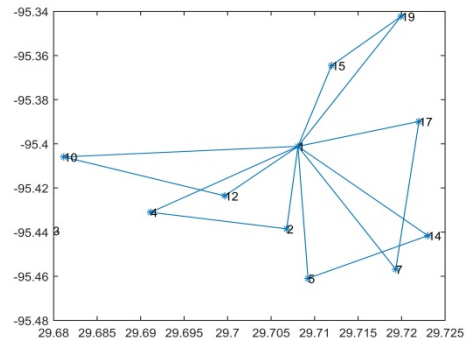
(c1) The 1<sup>st</sup> stage paths in hybrid strategy with 20% decrease of  $u_i$  and  $b_i$



(c2) The 2<sup>nd</sup> stage paths in hybrid strategy with 20% decrease of  $u_i$  and  $b_i$



(d1) The 1<sup>st</sup> stage paths in hybrid strategy with 20% increase of  $u_i$  and  $b_i$



(d2) The 2<sup>nd</sup> stage paths in hybrid strategy with 20% increase of  $u_i$  and  $b_i$

Fig. C.2. The optimal paths under different changes of  $u_i$  and  $b_i$  using two transportation strategies in Model II