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Wall-pressure fluctuations of Laminar Separation 1 Bubble based on Direct Numerical Simulation and 2 Experiments over a Hydrofoil at Re=450,000 3 4

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Abstract 9

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The objective of this paper is to investigate wall-pressure fluctuations 10 downstream of a Laminar Separation Bubble (LSB) on a laminar NACA66 11 hydrofoil section, based on Direct Numerical Simulation (DNS) and exper-12 imental analysis. DNS is performed using the massively parallelized open 13 source code NEK5000, which uses the high order spectral element method 14 to solve the incompressible Navier-Stokes equations. It is compared to mea-15 surements performed in a hydrodynamic tunnel at the French Naval Academy 16 Research Institute (IRENav). A Reynolds numbers of Re = 450,000 with 17 an angle of attack of $\alpha = 4^{\circ}$ is considered, which induces an LSB to the rear 18 of the maximum thickness of the boundary layer. The local wall pressures 19 at three different locations are compared: just downstream of the LSB at 20 x/c=0.7, in the breakdown region of the LSB at x/c=0.8 and in the fully 21 turbulent region at x/c=0.9. The DNS results compare very well with the 22 measured wall pressures. The DNS captures efficiently the periodic fluctu-23 ations downstream of the LSB, up to the random behavior induced by the 24

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transition to turbulence. Analysis of the boundary layer flow obtained by DNS suggests that an H-type or K-type transition occurs, characterized by 2D TS waves and typical Λ structures, that break down and generate turbulence. Moreover, it shows that intense and localized pressure fluctuations occur in the breakdown region.

30 Key words: laminar separation bubble, wall pressure fluctuation,

³¹ transition, laminar propeller section, spectral element method

32 1. Introduction

For many marine applications operating at low angles of attack, laminar 33 profiles can be chosen for the propeller blade in order to reduce friction at 34 the body surface and hence improve performance. However, this leads to 35 development of further transitional regimes, even at high Reynolds numbers 36 where the boundary layer is usually considered to be fully turbulent. These 37 transitional regimes can induce large pressure fluctuations and hence wide 38 variation in the hydrodynamic loadings. Hence, numerical and experimental 39 investigations are still necessary to help predict performance in the context of 40 marine propeller sizing. Moreover, the present study can help to improve the 41 physical understanding of (i) cavitation inception at the blade surface, which 42 is known to occur inside Laminar Separation Bubbles (LSB) in the case of 43 transitional flows, and (ii) laminar separation-induced structural vibrations. 44 45

Typically, this type of transition occurs at low to moderate Reynolds numbers and is triggered by a laminar separation and a reversed flow, caused by an adverse pressure gradient. The development of the turbulent flow induces

a momentum transfer in the direction normal to the wall, which reattaches 49 the flow. An LSB or transitional separation bubble is then generated ([16]), 50 which is characterized by a dead region near the laminar separation, and an 51 unstable region near the reattachment point. This point is usually subjected 52 to a low frequency oscillation in the shear layer (known as *flapping*), which 53 is responsible for the first destabilization of the LSB. Meanwhile, it has been 54 shown that primary instabilities (Kelvin-Helmholtz vortex and/or Tollmien-55 Schlichting waves) lead to LSB vortex shedding. Upstream of the LSB, the 56 flow is highly unstable and is governed by complex mechanisms identified as 57 primary and secondary instabilities that lead to transition to turbulence. 58

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The physical understanding of laminar to turbulent transition over lift-60 ing profiles is derived primarily from aerodynamic applications on small-scale 61 devices, where the transition comprises a large portion of the chord and dom-62 inates the boundary layer flow. The associated Reynolds numbers are typi-63 cally in the range 20,000 < Re < 100,000. In order to study the transition to 64 turbulence at these high Reynolds numbers, DNS studies of transitional flows 65 were introduced approximately a decade ago, supported by experimental ob-66 servations. It has been shown both experimentally [17] and numerically [22] 67 that the transitional region appears near the trailing edge for low to mod-68 erate angles of attack, and moves towards the leading edge as the angle of 69 attack increases. The transition on the SD7003 geometry has been exten-70 sively investigated for Reynolds numbers in the range 20000 < Re < 66000, 71 and for different angles of attack using experimental [3, 40, 30, 29] and nu-72 merical methods [39, 38]. At Re = 20000, transition is observed near the 73

trailing edge and a vortex associated with a Kelvin-Helmholtz instability is 74 convected into the foil's wake, which strongly interacts with the well-known 75 von Kármán instability. In [13], the characteristic frequency of LSB vortex 76 shedding matches the Strouhal number found in the work of [33], defined for 77 a flat plate with a pressure gradient. For Re = 66000, transition is stronger 78 and a burst phenomenon is associated with a fast transition to turbulence 79 [3, 39], although Kelvin-Helmholtz instabilities may still be observed. Jones 80 et al. [21] performed a numerical analysis of an LSB in the case of an airfoil at 81 $\alpha = 5^{\circ}$ with a low Reynolds number, Re = 50000. The authors investigated 82 the instability mechanisms leading to the breakdown to turbulence, linking 83 the breakdown to the combination of two absolute instabilities: an elliptic 84 instability and a mode-B or hyperbolic instability. An important conclusion 85 was that, in the absence of a convectively driven transition effect within the 86 shear layer, an absolute instability can take place, weakening the influence of 87 free stream turbulence. More recently, an interesting DNS study investigated 88 the effect of aspect ratio on laminar to turbulent transition on a NACA0012 89 profile ([41]). Due to computational costs, the simulation used a reduced 90 span and periodic boundary conditions. The results suggest that different 91 aspect ratios result in close predictions of the time-averaged aerodynamic 92 quantities, although small aspect ratios tend to under-predict the turbulent 93 fluctuations near the separation point but over-predict them inside the sep-94 aration bubble. This work highlighted the importance of setting span length 95 correctly to predict the development of turbulence. However, the effect of aspect ratio on the initial development of coherent structures downstream 97 from the LSB has not been investigated.

The NACA4412 airfoil profile has also been extensively investigated, mainly 99 for wind turbine applications. In the experiments of [26], the location and 100 formation of LSB were affected both by Reynolds number (ranging from 101 Re = 25,000 to Re = 50,000) and angle of attack. Moreover, the near wake 102 seems to be affected by LSB and trailing-edge separation, as also observed 103 by [13]. This profile has been also investigated numerically for aircraft appli-104 cations involving a much higher Reynolds number, Re = 400,000 [19]. The 105 authors use the spectral element code NEK5000, and set over 3.2 billion grid 106 points, with an element order of 12 and a full domain. This work showed 107 the ability of the spectral element method to predict the transition and the 108 development of turbulence. The authors also showed that adverse pressure 109 gradients have a great influence on the development of the boundary layer 110 and turbulent kinetic energy. 111

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As far as hydrodynamic applications are concerned, very few papers have 113 addressed the prediction of transitional flows on marine propellers. Those 114 that do usually use RANS based codes with empirical transition models, and 115 hence no LSB dynamic is captured. In [2], the authors show that the use of 116 a transition model leads to an improvement in the flow pattern and slight 117 increases in thrust. However, prediction of the transition location is strongly 118 dependent on the values for turbulence inlet used by the model. [34] investi-119 gated how application of a transition model interest the flow prediction for a 120 cavitating marine propeller. The results clearly correlated cavitation incep-121 tion with the location of transitional regions, however, conclusions could not 122 be drawn concerning the influence of transition modeling on cavitation de-123

velopment and global propeller performance. Similar conclusions are drawn 124 in [14]. More academically, experimental studies on LSB have been carried 125 out on marine propeller sections (i.e. hydrofoils) at the Naval Academy Re-126 search Institute (IRENav), France. Several of these studies concerned the 127 NACA66 section, for which highly transitional flows have been observed for 128 relatively high Reynolds numbers (Re=300,000 to 1,000,000). It has been 129 shown that a very strong and localized transition to turbulence occurs inside 130 the boundary layer, which induces intense pressure fluctuations ([11]) that 131 strongly influence hydrodynamic performance [8]. Moreover, it has been 132 demonstrated that this type of transition induces important structural vi-133 brations. [12] showed that there could be a strong interaction between these 134 vibrations and the physics of LSB vortex shedding. In particular, additional 135 frequencies close to that of LSB shedding and structural modes were observed 136 in the vibration spectra in transitional flows, as well as a global increase in 137 amplitude, which could be the result of a complex fluid structure interac-138 tion phenomenon. This work also showed that when the frequency of LSB 139 vortex shedding approaches a natural vibration mode, resonance with the 140 natural frequencies of the blade can be obtained and additional harmonics 141 of the main LSB shedding frequency are observed. It should be noted that 142 this behaviour is quite different from that of transition-induced vibrations on 143 membrane wings, which is a major field of investigation of transition-induced 144 vibration. The latter case is characterized by much lower Reynolds numbers 145 and large deformations of the structure, which is seen to significantly change 146 the LSB characteristics [9]. Moreover, the laminar profiles of a marine pro-147 peller are obtained by moving the maximum thickness to the center of the 148

chord, which reduces the adverse pressure gradient and hence increases the critical Reynolds number of transition, Re_{θ} , see Figure 1. This results in a delay of transition compared with conventional, non-laminar profiles, and a modification of the Re_{θ} slope after transition occurs, which can induce different physical processes compared to existing aerodynamic studies.

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Finally, although a link between LSBs and cavitation inception has been 155 clearly identified by John Carlton in his book about marine propellers [5], 156 it remains poorly understood. It is known that the effect of LSB strongly 157 depends on the scale of the propeller, its surface roughness and the nuclei den-158 sity. For a model scale with low surface roughness, the position of cavitation 159 inception is related to the transition point, i.e. the location of the LSB. For 160 high angles of attack (before stall), cavitation germs may be trapped inside 161 the LSB to form cavitation patches at the leading edge. Further develop-162 ment of these patches typically results in sheet cavitation. In [1], the authors 163 studied cavitation on a two-dimensional Eppler hydrofoil. They found some 164 indication of the presence of a separation bubble through pressure measure-165 ments. At the suction side, a region with a near-constant pressure aft of the 166 pressure peak was observed, and cavitation inception is observed as a band 167 along the leading edge, similar to the type of inception associated with lam-168 inar separation. Moreover, examination of the spanwise vorticity indicates a 169 thin region of vorticity production which could be an additional indicator of 170 the presence of a thin leading edge flow separation, where cavitation appears. 171 172

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For lower angles of attack, the LSB is located from the mid-chord up to the

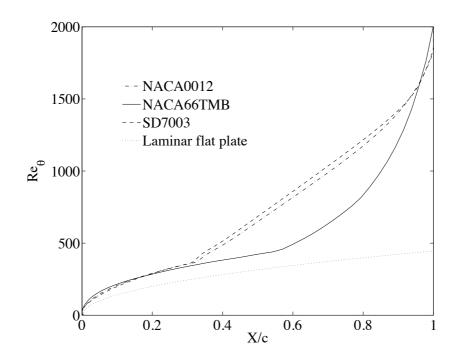


Figure 1: Critical Reynolds number Re_{θ} along the chord obtained with Xfoil. Comparison of laminar (NACA66) and non-laminar (NACA0012 and sd7003) profiles at Re = 450,000.

trailing edge, depending on the inflow conditions and hydrofoil geometry, and
cavitation typically occurs in this region, which is mainly correlated with the
adverse pressure gradient and the location of minimum pressure. Because
there is no negative pressure peak, bubbles can grow and form cavitation
bubbles, which appear quite randomly around the minimum pressure at the
surface. As a consequence, further studies investigating the link between
LSB, wall pressure and cavitation are essential.

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Therefore, the objective of the present work is to characterize the wall pressure downstream from an LSB on a laminar NACA66 section operat-

ing at a low angle of attack and at Re = 450,000, through a combination 184 of numerical and experimental analysis. Because of the moderate Reynolds 185 number considered, a simplified DNS domain is used and RANS velocity pro-186 files are implemented for the inflow conditions. A mesh sensitivity analysis 187 was first carried out at a lower Reynolds number (Re = 225,000), and a 188 similar resolution is used for the target Reynolds number (Re = 450,000) by 189 increasing the elements order. To check the validity of the model, mean flow 190 is compared with the RANS calculation. Then, the wall pressure fluctua-191 tions are investigated both experimentally and numerically at three different 192 locations between the LSB and the fully turbulent boundary layer. Finally, 193 the flow is analyzed numerically, and conclusions are drawn concerning the 194 particularity of bubble dynamics and its effects on wall pressure fluctuations 195 over laminar hydrofoil sections. 196

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¹⁹⁸ 2. Experimental method

Measurements were carried out in the cavitation tunnel at IRENav, France. 199 The test section was 1 m long with a h = 0.192 m square section. The ve-200 locity ranged between 0 and 15m/s and the pressure from 30 mbar to 3bars. 201 The hydrofoil was a NACA 66 with a NACA a=0.8 camber type, a camber 202 ratio of 2% and a relative thickness of 12% [27]. It was mounted horizon-203 tally in the tunnel test section. The chord was c = 0.150m and the span is 204 b = 0.191m, yielding a low aspect ratio b/c = 1.3 and a confinement param-205 eter h/c = 1.28. Flow visualizations show that about 80 to 90% of the foil 206 surface could be considered to be 2D flow ([27]), depending on the angle of 207

208 incidence.

Pressure measurements were carried out using seventeen piezo-resistive 209 transducers (Keller AG 2 MI PAA100-075-010) of 10 bars maximum pres-210 sure. The pressure transducers were mounted into small cavities with a 0.5211 mm diameter pinhole at the hydrofoil surface. The wall pressure spectrum 212 measured by the transducer was attenuated at the theoretical cut-off fre-213 quency $f_c = 9152$ Hz. Experiments used a sample frequency of f = 20kHz. A 214 main set of ten transducers was aligned along the chord on the suction side 215 at mid-span, where the flow was considered to be quasi-2D: from the leading 216 edge, at the reduced coordinate x/c = 0.1, to the trailing edge, at coordinate 217 x/c = 0.90, with a step of 0.10c. 218

Measurements were performed using an angle of attack $\alpha = 4^{\circ}$ and upstream velocities ranging from $U_{\infty} = 1.5$ m/s to 10m/s, corresponding to Reynolds numbers ranging from Re = 225,000 to 1,500,000. Under these conditions, laminar flow was maintained over about 60% to 80% of the chord and hence pressure fluctuations were observed starting at x/c = 0.7. It was also observed that the signal to noise ratio leads to accurate measurements of the physical pressure fluctuations for $U_{\infty} \geq 3$ m/s, i.e. $Re \geq 450,000$.

227 3. Computational methods

228 3.1. Direct Numerical Simulation

The dynamics of a three-dimensional incompressible flow of a Newtonian fluid are described by the Navier-Stokes equations:

$$\dot{\mathbf{U}} = -(\mathbf{U} \cdot \nabla)\mathbf{U} - \nabla P + Re^{-1}\Delta\mathbf{U}$$
(1)
$$\nabla \cdot \mathbf{U} = 0$$

where $\mathbf{U} = (U_x, U_y, U_z)^T$ is the velocity vector and P the pressure term. The Reynolds number is defined by $Re = U_{\infty}c/\nu$, where ν is the kinematic viscosity of the considered fluid, c is the chord length and U_{∞} is the upstream velocity.

The Navier-Stokes equations were solved using the flow solver NEK5000 235 developed at Argonne National Laboratory by Fischer et al. [15]. The method 236 is based on the spectral elements method (SEM), introduced by Patera [32], 237 which provides spectral accuracy in space while allowing for the geometrical 238 flexibility of finite element methods. Spatial discretization is obtained by 239 decomposing the physical domain into spectral elements within which the 240 velocity is defined on Gauss-Lobatto-Legendre (GLL) nodes and the pres-241 sure field on Gauss-Legendre (GL) nodes. The solution to the Navier-Stokes 242 equations is then approximated within each element as a sum of Lagrange 243 interpolants defined by an orthogonal basis of Legendre polynomials up to 244 degree N. The results presented in this paper were obtained with a poly-245 nomial order from N = 8 to N = 12. The convective terms were advanced 246 in time using an extrapolation of order 3, whereas the viscous terms used a 247

backward differentiation, also of order 3, resulting in the time-advancement 248 scheme labeled BDF3/EXT3. NEK5000 employs the MPI standard for par-249 allelism [36, 37]. For further details about the spectral element method, the 250 reader is referred to the books by [10] and [24]. The calculations were per-251 formed on the French National Server named Institute for Development and 252 Resources in Intensive Scientific Computing (IDRIS). IBM Blue Gene/Q was 253 the super computer used, which has high energy efficiency of 2.17 Gflops/W. 254 The main characteristics of this machine include six racks, each having 1024 255 nodes and a total of 98,304 cores with 1GB per core. 256

The scalability of NEK5000 was tested with the present case up to 32768 257 processors and 326 million grid points, and compared with a combustion 258 case with the same code, using a blue Gene/P([25]), see Figure 2. The re-259 sults show a large decrease in computing efficiency when the ratio between 260 the number of grid points N and the number of processors P decreases be-261 low 20000. To achieve an efficiency greater than 90%, the following number 262 of processors were chosen for the simulations: P = 16384 at order O(12), 263 P = 8192 at order O(10) and P = 2048 at order O(8). 264

265 3.2. Numerical setup

Because of the moderate Reynolds number considered for DNS (Re = 450,000), the DNS domain was restricted to the near wall region, and velocity boundary conditions were imposed at the domain inlet boundaries (left, top and bottom regions) from a transitional RANS calculation to reproduce the velocity gradient external to the hydrofoil boundary layer, see Figure 3. $\nabla \mathbf{U} \cdot \mathbf{x} = 0$ was set at the outlet, whereas a no-slip condition was imposed on the vertical

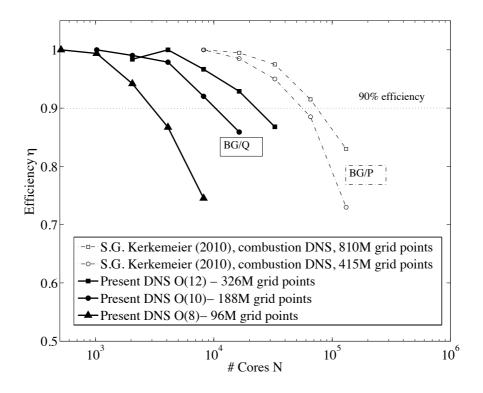


Figure 2: Scalability tests of NEK5000 as function of number of grid points

side planes of the domain. The total height of the DNS domain was 0.25c,
with 0.5c in the wake. The span was reduced to 0.05c. The domain of the
RANS calculation corresponds to the dimensions of the experimental test
section of the hydrodynamic tunnel at IRENav.

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The spectral mesh was determined using the following method: having restricted the domain to the near wall region, the spatial resolution was determined using the wall friction from the RANS calculations, according to the general requirements for a DNS at Re = 450,000, namely

 $\Delta y_{min} < 0.2, \Delta x_{max} = \Delta z_{max} < 10$. Then, the span length was set accord-282 ing to the computational time allocated for the project, i.e. 0.05c. It is 283 important to point out that the reduced span can force development of 3D 284 boundary layer flow in the transition region, related to the development of 285 "hairpin structures". Although the influence of Reynolds number on the 286 transverse wavelength has not been fully characterized in the literature, a 287 direct dependence has been shown in the experiments of [4] on a sd7003 air-288 foil. They show first that LSB thickness linearly depends on boundary layer 289 thickness. They also show that the transverse wavelength of what they call 290 C-shape structures is approximately 0.2c at Re = 20,000, compared to about 291 0.1c at Re = 60,000. It is then reasonable to estimate that this parameter 292 may be proportional to the boundary layer thickness in the proximity of the 293 LSB, i.e a laminar boundary layer thickness, which is proportional to $1/\sqrt{Re}$. 294 Using the current Reynolds number of Re = 450,000, this gives an estima-295 tion of about 0.04c for the transverse wavelength of hairpin structures, so 296 0.05c was considered a minimum for the present study. 297

The spectral mesh is shown in figure 4. It is refined close to the wall to obtain a low y^+ value, whereas it is almost constant in the x (chordwise) and z (spanwise) directions. This leads to 188480 spectral elements, leading to approximately O(8) at Re = 225,000 (mesh sensivity analysis) and O(12) at Re = 450,000 (study case).

To obtain the DNS, a progressive increase of the element order was performed to advance in time, up to the target mesh. As the DNS code is semi-implicit, it requires the local CFL number to be strictly $CFL = u\Delta t/\Delta x < 0.75$. The computations were carried out with $CFL \approx 0.5$.

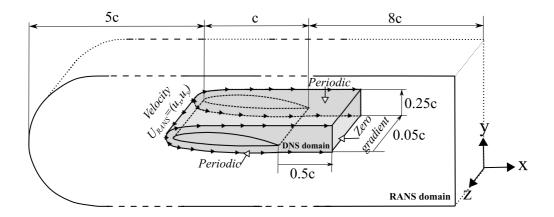


Figure 3: Computational domain of the NACA66 hydrofoil.

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308 3.3. Isotropic von Kármán turbulence model

In order to add the perturbation at the inlet of the computational do-309 main, an isotropic von Kármán turbulence model ([23]) was implemented 310 into the NEK5000 code. This model is based on the energy spectra of the 311 perturbations. The energy spectrum is set numerically by using random 312 number-generating algorithms [31] to generate random angles, resulting in a 313 randomized field. The von Kármán isotropic turbulence model considers the 314 linear velocity and the angular velocity components of the disturbances as 315 spatially varying stochastic processes and specifies power spectral density to 316 each of them. 317

³¹⁸ For this study, a modified von Kármán spectrum was utilized:

$$E(\kappa) = c_E \frac{u_{rms}^2}{\kappa_e} \frac{(\kappa_1/\kappa_e)^4}{[1 + (\kappa_1/\kappa_e)^2]^{17/6}} e^{[-2(\kappa_1/\kappa_\eta)^2]}$$
(2)

Here, $\kappa_1 = (\kappa_i \kappa_i)^{1/2}$ and $\kappa_\eta = \epsilon^{1/4} \nu^{-3/4}$ are obtained from the kinetic energy

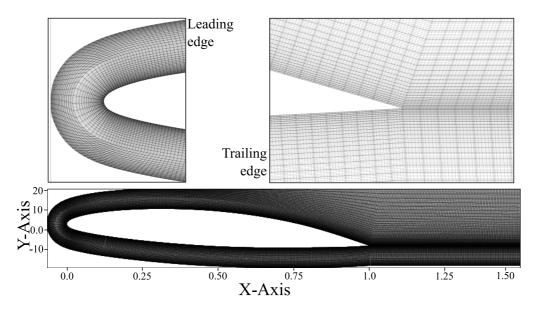


Figure 4: Spectral element mesh of the NACA66 hydrofoil, N = 188, 480.

by integrating the energy spectrum over all the wave numbers, and then utilizing the gamma functions. κ_e is linked to the smallest wave number, and c_E is a constant. The mathematical details and derivations can be found in [18], and details about the von Kármán spectra used here can be found in [7].

The von Kármán model is applied only to a portion of the inlet domain but covers the entire hydrofoil boundary layer. The disturbances are convected downstream by the free-stream flow, destabilizing the boundary layer and triggering the turbulence. The integral length scale is set to 0.006c, corresponding to 50% of the boundary layer thickness in the transition region, whereas the smallest scale corresponds to the smallest cell at the inlet, i.e. 0.0018c.

332 3.4. Sensitivity analysis

A sensitivity analysis was performed for a lower Reynolds number (Re =225,000) for element orders from O(6) to O(10). The effect of mesh resolution was investigated in the unsteady region of the LSB where vortex shedding occurs, and in the turbulent region. After selecting the required resolution, the element order was increased to simulate the higher Reynolds number (Re = 450,000), assuming that the LSB dynamics would be similar.

First, the flow in the transition region was analyzed. Figure 5 shows the 340 instantaneous velocity field in the unsteady region of the LSB. Although 341 the figures cannot be directly compared, since they were taken at different 342 instants with different mesh resolutions, their prediction of the LSB shed-343 ding process can be qualitatively compared. The lower order mesh (O(6)) is 344 clearly too coarse to capture the LSB vortex shedding correctly. Numerical 345 instabilities disturb the unsteady region of the LSB, resulting in the for-346 mation of smaller structures, and hence further numerical instabilities form 347 downstream. From order O(8), this numerical effect seems to be suppressed 348 and the shedding process is correctly captured. 349

This observation is confirmed in Figure 6(a), which shows the pressure spectra downstream from the LSB. Due to numerical instabilities, the O(6) produces a pressure peak at 125 Hz compared to 85 Hz for O(8) and O(10). The effect of mesh resolution on the development of turbulence is also observed near the trailing edge. The spectra of streamwise velocity V_X clearly shows that increasing element order tends to capture a larger range of turbulent scales, as indicated by the frequency range over which the slope is

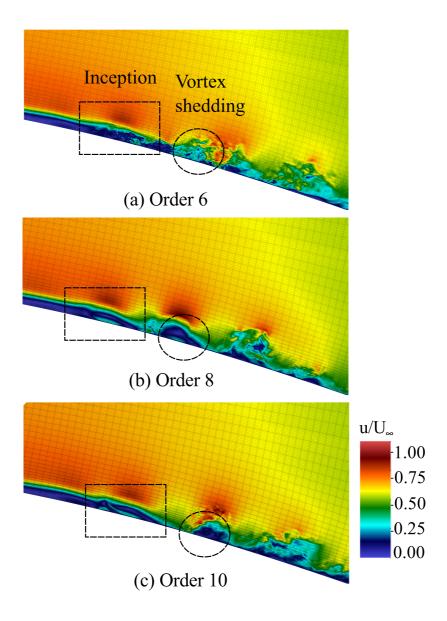


Figure 5: Convergence of the LSB shedding. Velocity contours downstream of the LSB for three different element orders, Re = 225000.

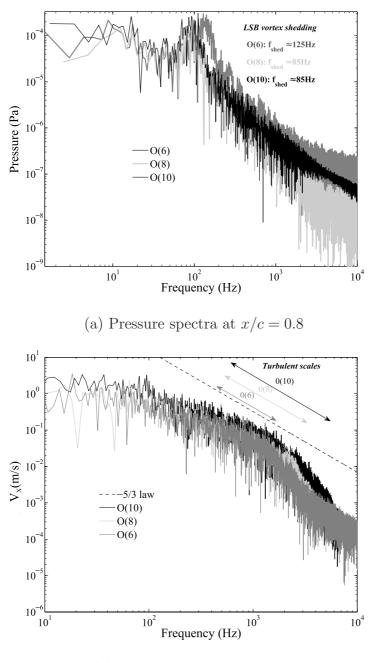
Element order	C_L	C_D
O(6)	0.8198 ± 0.055	0.0213 ± 0.0049
O(8)	0.8272 ± 0.061	0.0219 ± 0.0034
O(10)	0.8274 ± 0.062	0.0215 ± 0.00057

Table 1: Convergence of lift and drag coefficients as a function of the element order, Re = 225000.

-5/3, corresponding to the characteristic slope of turbulent cascade. Since
the primary objective of the present paper is to investigate LSB dynamics,
it is not necessary to achieve convergence on the turbulent spectra.

Finally, the effect of element order on components of hydrofoil performance was tested, see Table 1. It was observed that drag converged for all mesh resolutions, whereas lift is slightly under-predicted for the lower order O(6) mesh, as compared to the O(10) mesh.

According to this convergence study, at Re = 225,000 the mesh with order 364 O(8) is sufficient to fully capture the LSB dynamics, and the transition to 365 turbulence. The total number of points was N = 96,501,760. The element 366 order was then increased to 12 to simulate the case at Re = 450,000. Fig-367 ure 7 shows the final mesh resolution along the chord for the two Reynolds 368 numbers at the wall in the tangential, normal and spanwise directions, re-369 spectively. It is taken from the friction velocity, which is time and spanwise 370 averaged. Because of the non-uniform distribution of the spectral nodes, 371 Δx^+ and Δz^+ present maximum and minimum values, which are taken from 372 the largest and smallest cells within one spectral element. This corresponds 373 to the maximum resolution as it is at the wall, and because the spectral ele-374



(b) Spectra of Vx at x/c = 1.0

Figure 6: Effect of element order on (a) LSB vortex shedding and (b) turbulent scales, Re = 225000.

ments are uniformly distributed through the domain along the x and z axes. 375 The final resolution for O(8), Re = 225000 leads to $\Delta x_{max}^+ = \Delta z_{max}^+ = 3$ 376 and $\Delta y_{min}^+ = 0.1$, and for O(12) it is $\Delta x_{max}^+ = \Delta z_{max}^+ \approx 7$ and $\Delta y_{min}^+ = 0.2$, 377 corresponding to 326 million grid points at Re = 450000. Even though the 378 mesh resolution is lower for the higher Reynolds number, it is considered to 379 be sufficient for analyzing the transitional flow. As a comparison, a similar 380 method was recently presented in [19] for a DNS around a wing section at a 381 similar Reynolds number (Re = 400,000), using Nek5000 on a NACA4412 382 over a full domain. This study obtained 3.2 billion grid points with strong 383 wall refinements, to obtain approximately the same mesh resolution. 384

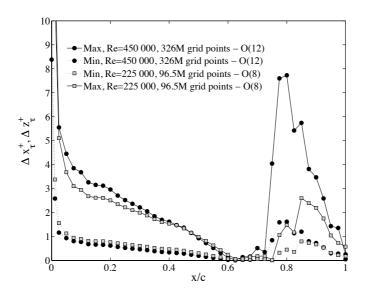
385 4. Results and discussion

$_{386}$ 4.1. Mean flow analysis: DNS at Re=450,000

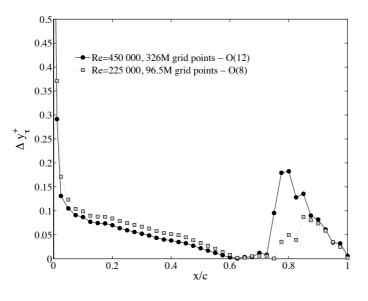
To check the validity of the RANS velocity boundary conditions of the DNS domain, the mean flow obtained with DNS was compared to the transitional RANS calculations. The velocity fields are shown in Figure 8. Good agreement is observed overall. The stagnation at x/c = 0.0055 is correctly predicted by DNS, and the velocity gradient is reproduced in the DNS up to the LSB, after which point differences are observed because the RANS solver does not capture the LSB unsteadiness.

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Local velocity profiles along the chord are shown in Figure 9. Good agreement is observed within the boundary layer; however the DNS calculates higher external velocities (by approximately 5%) compared to the RANS calculation, due to added confinement. This numerical error is corrected in



(a) Streamwise (x-axis) and spanwise (z-axis) directions



(b) wall normal direction (y-axis)

Figure 7: Mesh resolution at the hydrofoil surface for the two cases considered

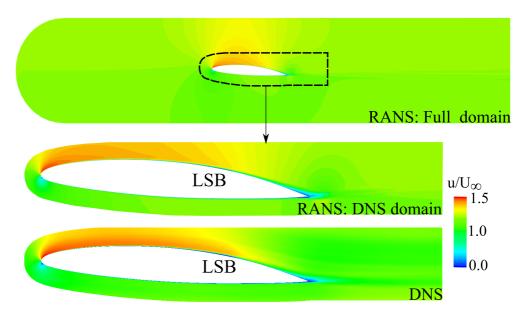


Figure 8: Comparison of the RANS and DNS velocity fields, Re=450,000.

the results section, to ensure that the pressure coefficient calculations and the shedding frequencies compare with experimental values. As expected, the velocities are the same at the upper boundary as RANS velocity profiles are imposed in the DNS calculations.

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The method was tested for the lower Reynolds number case at Re=225,000, which was used for the sensivity analysis. As shown in Figure 10, the DNS calculation of external boundary layer velocities was greater than for the higher Reynolds number, at 8%. This difference is attributed to the increase in boundary layer thickness, which magnifies the confinement effects.

Table 2 compares the LSB characteristics between RANS and DNS. Since
 DNS captures the velocity gradient well, its calculation of laminar separation

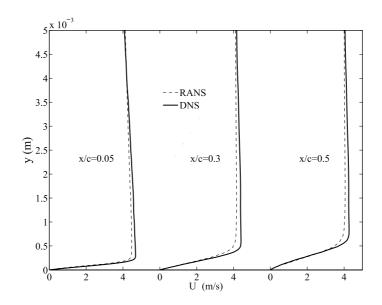


Figure 9: Comparison of the RANS and DNS velocity profiles for three different locations, Re=450,000.

is very close to the RANS value. It can be seen that the LSB vortex shedding 411 frequency obtained from DNS (f_{shed}) compares well with the experimental 412 value $(f_{exp-shed})$ extracted from wall pressure measurements. This is consis-413 tent with the work of [33], which used the Strouhal number, $St_{\theta} = \frac{f_{shed}\theta_{sep}}{u_{sep}}$, 414 to characterize the shedding frequency, considering it to be dependent only 415 on the boundary layer quantities at the laminar separation, captured nu-416 merically by DNS. The computed Strouhal number is $St_{\theta} = 0.0090$ for 417 Re = 450,000, as compared to $St_{\theta} = 0.0068$ found by [33]. This differ-418 ence is attributed to the different case studied by [33], namely a flat plate 419 submitted to a pressure gradient. 420

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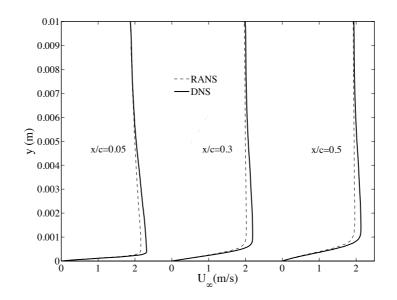


Figure 10: Comparison of the RANS and DNS velocity profiles for three different locations, Re=225,000.

Case	x_{sep}	x_{re}	$f_{shed}(\mathrm{Hz})$	$f_{exp-shed}(\mathrm{Hz})$
DNS - Re=450,000	0.62	0.72	335	300
RANS - Re=450,000	0.65	0.72	-	-

Table 2: Comparison of RANS and DNS LSB characteristics. x_{sep} and x_{re} are the laminar separation and turbulent reattachment location, f_{shed} (Hz) and $f_{exp-shed}$ are the numerical and experimental shedding frequencies.

422 4.2. Investigation of pressure fluctuations: experiments vs DNS at Re =423 450,000

Experiments in the cavitation tunnel were first performed at $\alpha = 4^{\circ}$ for 424 $U_{\infty} = 1.5 \text{ m/s}$ to 10 m/s in steps of 0.5m/s. The results are shown in Figure 425 11, where frequency spectra of wall pressure at x/c = 0.7 and x/c = 0.8 are 426 plotted for all the velocities considered. To illustrate the evolution of wall 427 pressure fluctuations with the LSB dynamics, Figure 12 shows the average 428 position of the LSB (in blue), the region of LSB vortex shedding where peri-429 odic fluctuation occurs (in green), and the fully turbulent region (in red), as 430 the upstream velocity increases. 431

First, it is observed that the frequency of LSB shedding increases linearly as 432 Reynolds number (i.e. U_{∞}) increases, from about 100 Hz for $U_{\infty} = 1.5$ m/s, 433 up to about 1000 Hz for $U_{\infty} = 7$ m/s. The evolution of this frequency from 434 x/c = 0.8 to x/c = 0.7 visible in Figure 11 illustrates the displacement of the 435 LSB along the leading edge. The frequency peak vanishes after $U_{\infty} > 3.5 \text{m/s}$ 436 at x/c = 0.8 (Figure 11 (b)) and appears starting $U_{\infty} > 4$ m/s at x/c = 0.7437 (Figure 11 (a)), which clearly means that periodic fluctuation due to LSB 438 shedding move toward the leading edge. Figure 12 illustrates that this vari-439 ation (in green) is primarily due to the decrease in LSB size ($U_{\infty} < 4$ m/s 440 to $4m/s < U_{\infty} < 8m/s$), which is due to an increase in Reynolds number. 441 Secondly, a decrease in viscous effects slightly increases the adverse pressure 442 gradient, which tends to shift the LSB toward the leading edge. The trailing 443 edge vortex shedding frequency identified in the dotted area is illustrated in 444 Figure 12 at $U_{\infty} > 8$ m/s. Due to a large displacement of the LSB toward 445 the leading edge and the early establishment of turbulent boundary layer in 446

a relatively strong adverse pressure gradient region, the turbulent boundary 447 layer tends to detach at the trailing edge for the highest velocities, although 448 the wall friction increases. This phenomenon had been investigated numeri-449 cally previously, in [11]; in this study, an abrupt transition to turbulent flow 450 caused by a leading edge LSB was associated with the sudden generation of a 451 vortex at the trailing edge, caused by detachment of the turbulent boundary 452 layer (see also Figure 12 at $U_{\infty} > 8$ m/s, in orange). Finally, the additional 453 frequencies around 400Hz for $4.5 \text{m/s} < U_{\infty} < 8 \text{m/s}$ are natural vibrations of 454 the hydrofoil excited by LSB vortex shedding, which are not investigated in 455 this paper. 456

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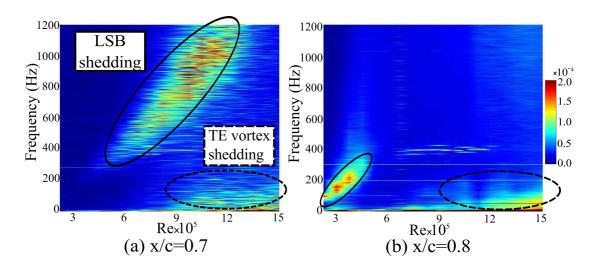


Figure 11: Experimental frequency spectra of wall pressure at (a) x/c=0.7 and (b) x/c=0.8 as function of Reynolds number, $\alpha = 4^{\circ}$.

Figure 13 compares the local wall pressure coefficients measured experimentally to those obtained with DNS at $Re = 450,000, U_{\infty} = 3$ m/s, for

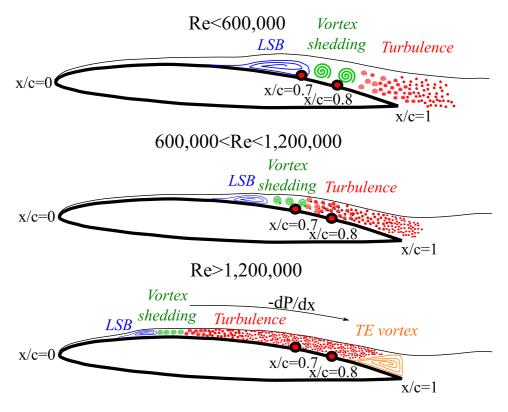


Figure 12: Evolution of LSB dynamics with the Reynolds number. TE=trailing edge.

x/c = 0.7, x/c = 0.8 and x/c = 0.9. The full DNS is plotted in time, and 460 adjusted temporally to be synchronized with the experimental signal. Over-461 all there is good agreeement between the DNS and experiments, in terms 462 of temporal behavior and amplitudes. In the transition region at x/c = 0.7463 (Figure 13 (a)), the flow is highlighted by the development of coherent struc-464 tures and the pressure fluctuations show a strong periodic and intermittent 465 behavior for both DNS and experiments. The amplitudes and modulations 466 are fully captured by the present computations. With the development of 467 turbulence, the pressure signal shows smaller fluctuations with random be-468

havior; however the signal is still periodic for x/c = 0.8. Near the trailing edge, i.e. at x/c = 0.9, the fluctuations are more random with significantly lower amplitudes, clearly demonstrating turbulent fluctuation.

472

In order to refine the comparison between the measured and predicted 473 wall pressures, a fast fourier transform was performed on the signals shown 474 in Figure 13. The results are shown in Figure 14. Again, there was good 475 agreement between DNS and measurements for all spectra. At x/c = 0.7, 476 DNS captures the main frequency peak of LSB vortex shedding, with good 477 agreement in both amplitude and frequency range. A secondary peak around 478 500Hz is also observed, resulting from intermittency. At higher frequencies, 479 there is a rapid decay in amplitude. At x/c = 0.8 and x/c = 0.9, the 480 main peak is still observed, but with a progressive reduction in amplitude, 481 as also observed in experiments. Moving closer to the trailing edge, an in-482 crease in amplitude at higher frequencies is observed, due to development of 483 turbulence. Again, this matches the experimental data, demonstrating the 484 accuracy of the simulation. 485

486

An isosurface of λ_2 (introduced in [20]) is shown in Figure 15 for 0.6 < x/c < 1, colored with pressure coefficient contours. This figure illustrates a correlation between the evolution of wall pressure, and the boundary layer transition to turbulence. It shows where coherent structures start to form downstream from the LSB, and where the transition to turbulence occurs. Coherent structures associated with the development of TS waves appear at x/c = 0.65, which is very close to the point of laminar separation at

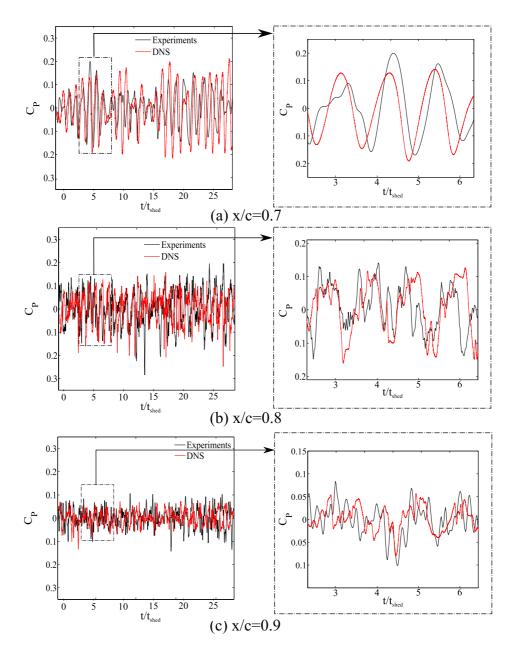


Figure 13: Comparison of computed and measured wall pressure coefficient, Re=450,000, $U_{\infty}=3 {\rm m/s}$

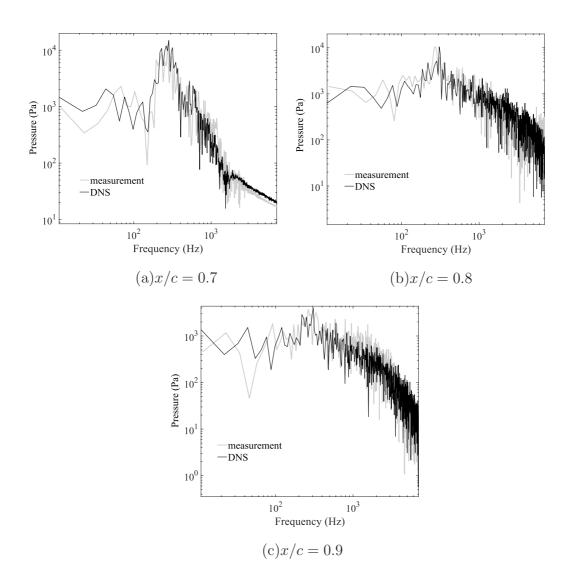


Figure 14: Spectral comparison of measured and DNS pressure signals for different chordwise locations, $\mathrm{Re}{=}450~000$

494 x/c = 0.62, see Table 2. The dynamics remain approximately 2D until 495 x/c = 0.7. This is consistent with figure 13 (a), which shows that wall pres-496 sure is fully periodic in this location. In the region 0.7 < x/c < 0.8, the 2D 497 vortex deforms and breaks down. Coherent structures are still present, in 498 agreement with the periodicity in the pressure signal shown in Figure 13 (b). 499 Subsequently, only turbulent structures are observed up to trailing edge.

While experiments allow a wide range of velocity conditions, they are limited

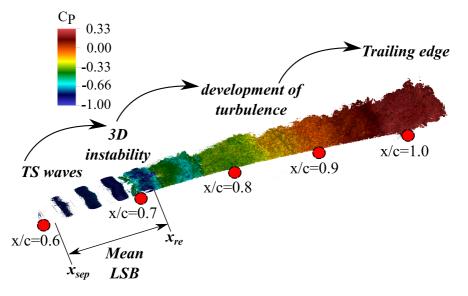


Figure 15: Coherent structures downstream from the LSB, isosurface of λ_2 colored with pressure coefficient, Re = 450,000

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⁵⁰¹ spatially to only three points placed in the transition region, and generally ⁵⁰² only one pressure transducer to record the bubble dynamic. In contrast, the ⁵⁰³ DNS cannot be used to investigate a wide range of Reynolds numbers be-⁵⁰⁴ cause of the cost of a single computation, but can offer a complete spatial ⁵⁰⁵ description of the flow. As such, a complete map of the wall pressure coef-⁵⁰⁶ ficients along the chord (0.6 < x/c < 1) over 25 periods is shown in Figure

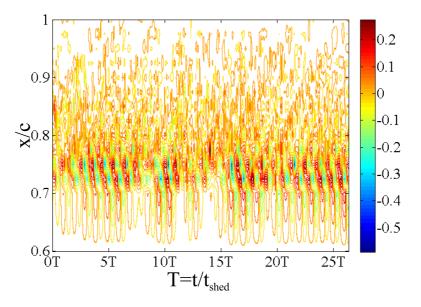


Figure 16: Time evolution of computed wall pressure coefficient fluctuations along the chord of the hydrofoil, Re=450,000

⁵⁰⁷ 16. Periodic pressure fluctuations appear starting x/c = 0.62, i.e. near the ⁵⁰⁸ laminar separation of LSB. For 0.7 < x/c < 0.75, the pressure fluctuations ⁵⁰⁹ increase in amplitude, after which they quickly lose their periodicity, reach-⁵¹⁰ ing fully turbulent flow after x/c = 0.85, in agreement with the numerical ⁵¹¹ observations in Figure 15.

512 4.3. Analysis of transition behavior: DNS at Re=450,000

Figure 17 shows the instantaneous velocity field and RMS of the spanwise velocity extracted from the mean flow. It is clear that the RMS increases in the unsteady region of the LSB, and is maximum near the turbulent reattachment. It remains high in the transition region, then decreases when turbulence is established. The instantaneous velocity field shows that the

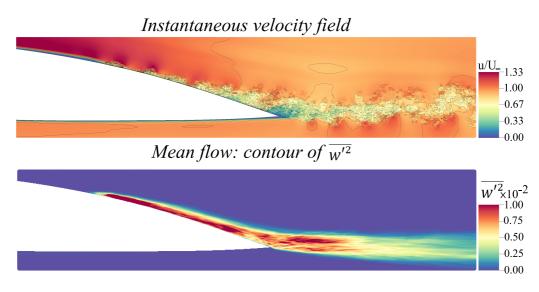


Figure 17: Evolution of unsteadiness downstream from the LSB, Re=450,000

⁵¹⁸ suction side boundary layer increases in thickness with the development of
⁵¹⁹ turbulent boundary layer, which interacts with the pressure side boundary
⁵²⁰ layer at the trailing edge to form a turbulent wake instability.

Figure 18 shows the spectra of V_x at three locations along the chord: 521 x/c = 0.7, just downstream from the LSB; x/c = 0.85, between the transi-522 tional and turbulent regions; and x/c = 1, in the turbulent region. Velocities 523 were recorded outside of the boundary layer. At x/c = 0.7, only the LSB 524 vortex shedding frequency is observed on the spectra. At x/c = 0.85, this 525 peak is still observed, and additionally a high amplitude is observed, cor-526 responding to the increased RMS shown in Figure 17. The development of 527 turbulence also leads to the inception of higher frequency components. Fi-528 nally, at x/c = 1, the spectra shows a classical -5/3 slope resulting from the 529 turbulent cascade. 530

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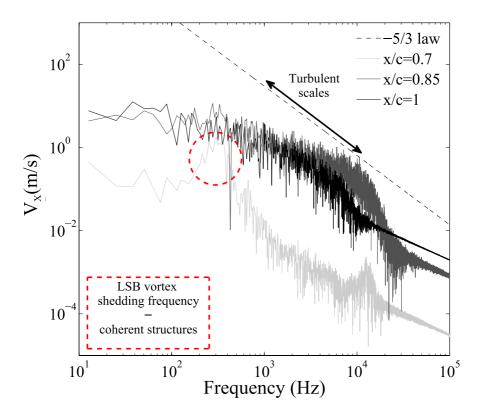


Figure 18: Spectra of V_X for three chordwise locations, Re=450,000

Finally, the transition mechanism for Re = 450000 is detailed in Fig-532 ure 19, which shows the isosurfaces of λ_2 colored with velocity and pressure 533 coefficient contours. The top and bottom subfigures were produced using 534 different values of λ_2 in order to highlight the different mechanisms: devel-535 opment of TS waves for higher λ_2 , and development of hairpin structures and 536 turbulence downstream from the LSB for lower λ_2 . First, it can be observed 537 on Figure 19(a) that the flow exhibits a typical H-type or K-type transition, 538 as shown for the case of a flat plate with forcing by [35]. The distinction 539

between these two types of transition cannot be identified in this paper, as 540 only one Λ shape structure was captured spanwise, so it is not possible to 541 determine whether it is aligned (K-type) or staggered (H-type). The top 542 view of Figure 19 shows that TS waves appear downstream from the LSB, 543 and progressively deform with the inception of a 3D instability known as an 544 elliptic primary instability ([28]). The development of this instability leads 545 to the formation of a Λ-shaped structure, also known as a "hairpin" struc-546 ture, whose breakdown leads to development of turbulent flow. It should be 547 noted that the Reynolds number Re_X at which this transition takes place 548 is comparable to that identified in [35], albeit advanced somewhat due to 549 the angle of attack and the curvature of the hydrofoil: as an example, the 550 TS waves start at Re = 310000, compared to Re = 360000 for [35]. The 551 velocity field in Figure 19(b) shows that the breakdown of Λ structures is 552 due to development of secondary instabilities, which progressively lead to 553 turbulence. The investigation of the pressure field in Figure 19(c) shows that 554 the pressure is globally maintained (dark blue) during development of the 555 hairpin structure. This means that the vortex convects along the chord with 556 a quasi-constant angular velocity, i.e. it extends the pressure plateau from 557 the LSB region. In the breakdown region, mixing leads to an increase in 558 pressure fluctuations, characterized by more random behavior. In the tur-559 bulent region, the pressure increases almost linearly up to the trailing edge. 560 561

(a) *Transition mechanism*

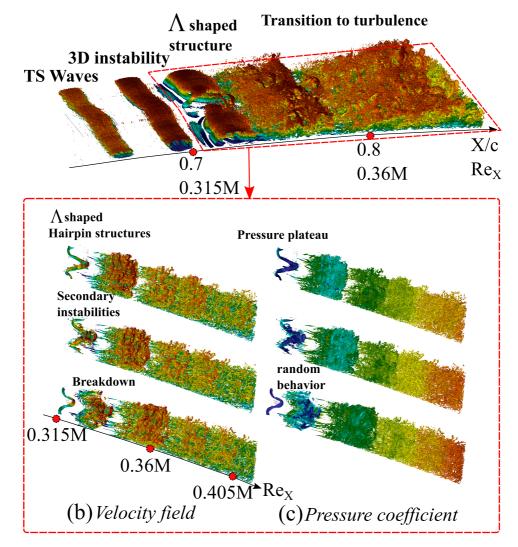


Figure 19: Time evolution of computed wall pressure coefficient fluctuations along the chord of the hydrofoil, isosurfaces of λ_2 colored with velocity contours (top and bottom left) and pressure coefficients contours (bottom right).

562 5. Conclusion

In this paper, the laminar to turbulent transition and the resulting wall pressure fluctuations were analyzed on a laminar NACA66 propeller section. Despite the moderate Reynolds number investigated (Re = 450,000), DNS resolution was successfully obtained by using a simplified fluid domain, which considered the near wall region by imposing RANS velocity profiles at the boundaries. The results were compared with wall pressure measurements performed in a hydrodynamic tunnel.

The DNS correctly reproduced the velocity field as found from transitional 570 RANS calculations, allowing the main characteristics of the LSB to be cap-571 tured. Experimental investigations of wall pressure fluctuations for a wide 572 range of Reynolds numbers (225,000 < Re < 1,500,000) at $\alpha = 4^{\circ}$ showed 573 that the frequency of vortex shedding downstream from the LSB increases 574 linearly with increasing Reynolds number. Further, the fluctuations move 575 progressively from x/c = 0.8 to x/c = 0.7, which is associated with a de-576 crease of the LSB length and its shift toward the leading edge. The DNS 577 results for wall pressure fluctuations at Re = 450,000 showed good agree-578 ment with experiment throughout the different stages, from development of 579 the LSB, through the transition process, up to fully turbulent flow. 580

Following this validation of the DNS, the interaction between boundary layer flow and the evolution of wall pressure fluctuations was investigated. It was observed that the flow exhibit a classical H-type or K-type transition to turbulence, highlighted by the inception of 2D TS waves, which deform to hairpin structures due to a 3D instability. A secondary instability subsequently causes degradation of the hairpin structures, and the flow transitions quickly to turbulence. The wall pressures were observed to respond to these different
 stages of transition:

- from the inception of TS waves at x/c = 0.65 to the development of hairpin structures at x/c = 0.72, the wall pressures display periodic intermittent behavior with one main period that corresponds to LSB vortex shedding.
- The breakdown around x/c = 0.75 has a mixing effect associated with maximum amplitude of wall pressure fluctuations, which subsequently decrease quickly for 0.75 < x/c < 0.8. Periodic pressure fluctuations are still observed, but with a greater degree of randomness due to multi-scaled structures passing through the local surface.
- 598 599
- In the region of fully developed turbulence (x/c > 0.85) the pressure signal reveals random fluctuations of lower amplitude.

It should be noted that because of the complex transitional flow captured, 600 boundary layer flow cannot be fully characterized due to certain numerical 601 effects. From laminar separation up to the inception of TS waves, typical 602 boundary layer flow is reproduced, suggesting the DNS is free of numerical 603 effects, mainly because it is a 2D process. As shown in [33], LSB shedding 604 LSB depends mainly on the boundary layer characteristics at laminar sepa-605 ration, which appear to be correctly predicted by the DNS, as indicated by 606 the comparison of wall pressures with experiments. On the other hand, in 607 the transition region, where secondary instabilities play an important role 608 in the destabilization of the LSB, the numerical setup can strongly influence 609 the development and breakdown of 3D structures. In particular, because of 610

the reduced span used in the present simulation, it captures only one coherent hairpin structure in the spanwise direction, so the flow may be forced and hence the actual wavelength is probably not fully accurate. This aspect will be investigated in future work by extending the DNS domain size in the normal and spanwise directions. The present setup is also currently used to study fluid structure interaction in transitional flows.

617

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