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► **To cite this version:**

Mohammad Abobala. On Some Special Substructures of Refined Neutrosophic Rings. International Journal of Neutrosophic Science, 2020, 5, pp.59 - 66. 10.5281/zenodo.3789325 . hal-02876518

HAL Id: hal-02876518

<https://hal.science/hal-02876518v1>

Submitted on 10 Aug 2020

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On Some Special Substructures of Refined Neutrosophic Rings

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Abstract

The objective of this article is to define and study the concepts of refined neutrosophic AH-ideal and AHS-ideal in refined neutrosophic rings. We investigate the elementary properties of these concepts.

Keywords: Refined neutrosophic ring, Refined neutrosophic AH-ideal, Refined neutrosophic AHS-Ideal, Refined AHS-homomorphism.

1. Introduction

Neutrosophy as a new branch of philosophy can be applied into the algebraic systems, which leads to a better comprehension and evolution of these systems. The notion of neutrosophic groups and rings was defined by Kandasamy and Smarandache in [10], and studied widely in [4, 5, 7, 8]. Studies were carried out on neutrosophic rings and neutrosophic hyperring. See [1, 3, 4, 6].

Refined neutrosophic rings were defined and studied carefully in [2, 3], where special substructures such as refined neutrosophic subrings and refined neutrosophic ideals are defined. Many interesting results were proved. In [1] concepts as AH-ideal and AHS-ideal were defined and studied as interesting substructures of neutrosophic ring. Some related concepts such as weak principal, maximal, and prime AH-ideals were introduced. These concepts have many properties which are similar to classical case of rings. In this paper, we try to define concepts such as AH-ideal and AHS-ideal in refined neutrosophic ring with some related concepts such as weak prime, principal and maximal refined neutrosophic AH-ideals. Also, we introduce the notion of refined AHS-homomorphism in similar way to AHS-homomorphism defined in [1].

Motivation

This paper is the continuation of the work began in the paper entitled "On Some Special Substructures of Neutrosophic Rings and Their properties".

2. Preliminaries

Definition 2.1:[5]

Let $(R, +, \times)$ be a ring then $R(I) = \{a + bI; a, b \in R\}$ is called the neutrosophic ring where I is a neutrosophic element with the condition $I^2 = I$.

Remark 2.2: [2]

The element I can be split into two indeterminacies I_1, I_2 with conditions:

$$I_1^2 = I_1, I_2^2 = I_2, I_1 I_2 = I_2 I_1 = I_1.$$

Definition 2.3: [2]

If X is a set then $X(I_1, I_2) = \{(a, bI_1, cI_2); a, b, c \in X\}$ is called the refined neutrosophic set generated by X, I_1, I_2 .

Definition 2.4: [2]

Let $(R, +, \times)$ be a ring, $(R(I_1, I_2), +, \times)$ is called a refined neutrosophic ring generated by R, I_1, I_2 .

Definition 2.5: [2]

Let $(R(I_1, I_2), +, \times)$ be a refined neutrosophic ring; it is called commutative if

$$x \times y = y \times x, \forall x, y \in R(I_1, I_2).$$

Theorem 2.6: [2]

Let $(R(I_1, I_2), +, \times)$ be a refined neutrosophic ring then it is a ring.

Definition 2.7: [3]

Let $(R(I_1, I_2), +, \times)$ be a refined neutrosophic ring and J be a nonempty subset of $R(I_1, I_2)$ then J is called a neutrosophic refined ideal if:

- (a) J is a refined neutrosophic subring of $R(I_1, I_2)$.
- (b) For every $x \in J$ and $r \in R(I_1, I_2)$ then $x \times r \in R(I_1, I_2)$.

Definition 2.8:[1]

Let $R(I)$ be a neutrosophic ring and $P = P_0 + P_1 I = \{a_0 + a_1 I; a_0 \in P_0, a_1 \in P_1\}$.

- (a) We say that P is an AH-ideal if P_0, P_1 are ideals in the ring R .
- (b) We say that P is an AHS-ideal if $P_0 = P_1$.

Definition 2.9:[1]

Let $R(I), T(J)$ be two neutrosophic rings and the map $f: R(I) \rightarrow T(J)$; we say that f is a neutrosophic AHS-homomorphism :

Restriction of the map f on R is a ring homomorphism from R to T , i.e $f_R: R \rightarrow T$ is homomorphism and

$$f(a + bI) = f_R(a) + f_R(b)J.$$

We say that $R(I), T(J)$ are AHS-isomorphic neutrosophic rings if there is a neutrosophic AHS-homomorphism

$f: R(I) \rightarrow T(J)$ which is a bijective map; i.e ($R \cong T$), we say that f is a neutrosophic AHS-isomorphism.

Definition 2.10:[1]

Let $R(I)$ be a neutrosophic ring and $P = P_0 + P_1I$ be an AH-ideal, we define the AH-factor as:

$$R(I)/P = R/P_0 + R/P_1I.$$

Theorem 2.11:[1]

Let $R(I)$ be a neutrosophic ring and $P = P_0 + P_1I$ be an AH-ideal then $R(I)/P$ is a ring.

Theorem 2.12:[1]

Let $R(I), T(J)$ be two neutrosophic rings and $f: R(I) \rightarrow T(J)$ is a neutrosophic ring AHS-homomorphism, let $P = P_0 + P_1I$ be an AH-ideal of $R(I)$ and $Q = Q_0 + Q_1J$ be an AH-ideal of $T(J)$, we have:

- (a) $f(P)$ is an AH-ideal of $f(R(I))$.
- (b) $f^{-1}(Q)$ is an AH-ideal of $R(I)$.
- (c) If P is an AHS-ideal of $R(I)$, $f(P)$ is an AHS-ideal of $f(R(I))$.
- (d) $AH - kerf = kerf_R + kerf_R I$ is an AHS-ideal; f_R is the restriction of f on the ring R .
- (e) The AH-factor $R(I)/kerf$ is AHS - isomorphic to $f(R(I))$.

Definition 2.13: [1]

Let $R(I)$ be a neutrosophic commutative ring and $P = P_0 + P_1I$ be an AH-ideal, we say that:

- (a) P is a weak prime AH-ideal if P_0, P_1 are prime ideals in R .
- (b) P is a weak maximal AH-ideal if P_0, P_1 are maximal ideals in R .
- (c) P is a weak principal AH-ideal if P_0, P_1 are principal ideals in R .

3. Main concepts and discussion

Definition 3.1:

Let $(R(I_1, I_2), +, \times)$ be a refined neutrosophic ring, and P_0, P_1, P_2 be three ideals in the ring R then the set

$$P = (P_0, P_1I_1, P_2I_2) = \{(a, bI_1, cI_2); a \in P_0, b \in P_1, c \in P_2\}$$
 is called a refined neutrosophic AH-ideal.

If $P_0 = P_1 = P_2$ then P is called a refined neutrosophic AHS-ideal.

Definition 3.2:

Let $(R, +, \times), (T, +, \times)$ be two rings and $f_R: R \rightarrow T$ is a homomorphism :

DOI: 10.5281/zenodo.3789325

The map $f: R(I_1, I_2) \rightarrow T(I_1, I_2); f(x, yI_1, zI_2) = (f_R(x), f_R(y)I_1, f_R(z)I_2)$ is called a refined AHS-homomorphism.

It is easy to see that for all $x, y \in R(I_1, I_2)$, we have $f(x + y) = f(x) + f(y), f(x \times y) = f(x) \times f(y)$.

Example 3.3:

Suppose that $R = (Z_6, +, \times), T = (Z_{10}, +, \times)$ are two rings and $f_R: R \rightarrow T; f(a) = 5a$ is homomorphism, the related refined AHS-homomorphism can be defined:

$$f: R(I_1, I_2) \rightarrow T(I_1, I_2); f(x, yI_1, zI_2) = (5x, 5yI_1, 5zI_2).$$

The previous example shows that refined AH-homomorphism is not a refined neutrosophic homomorphism in general because:

$$f(I_1) \neq I_1$$

Definition 3.4:

(a) Let $f: R(I_1, I_2) \rightarrow T(I_1, I_2)$ be a refined AHS-homomorphism, we define refined AH-Kernel of f by:

$$AH - Ker f = \{(a, bI_1, cI_2); a, b, c \in Ker f_R\} = (Ker f_R, Ker f_R I_1, Ker f_R I_2).$$

(b) Let $S = (S_0, S_1 I_1, S_2 I_2)$ be a subset of $R(I_1, I_2)$, then: $f(S) = (f_R(S_0), f_R(S_1)I_1, f_R(S_2)I_2) = \{(f_R(a_0), f_R(a_1)I_1, f_R(a_2)I_2); a_i \in S_i\}$.

(c) Let $S = (S_0, S_1 I_1, S_2 I_2)$ be a subset of $T(I_1, I_2)$, then:

$$f^{-1}(S) = (f_T^{-1}(S_0), f_T^{-1}(S_1)I_1, f_T^{-1}(S_2)I_2).$$

Definition 3.5:

Let $f: R(I_1, I_2) \rightarrow T(I_1, I_2)$ be a refined AHS-homomorphism we say that f is a refined AHS-isomorphism if it is a bijective map, $R(I_1, I_2), T(I_1, I_2)$ are called AHS-isomorphic refined neutrosophic rings.

It is easy to see that restriction f_R will be an isomorphism between R, T .

Theorem 3.6:

Let $f: R(I_1, I_2) \rightarrow T(I_1, I_2)$ be a refined AHS-homomorphism we have:

(a) $AH - Ker f$ is a refined neutrosophic AHS-ideal of $R(I_1, I_2)$.

(b) If P is a refined neutrosophic AH-ideal of $R(I_1, I_2)$, $f(P)$ is a refined neutrosophic AH-ideal of $T(I_1, I_2)$.

(c) If P is a refined neutrosophic AHS-ideal of $R(I_1, I_2)$, $f(P)$ is a refined neutrosophic AHS-ideal of $T(I_1, I_2)$.

Proof:

(a) Since $Ker f_R$ is an ideal of $R, AH - ker f = (Ker f_R, Ker f_R I_1, Ker f_R I_2)$ is a refined neutrosophic AHS-ideal of $R(I_1, I_2)$.

(b) Suppose that $P = (P_0, P_1 I_1, P_2 I_2)$ is a refined neutrosophic AH-ideal of $R(I_1, I_2)$. Since $f_R(P_i)$ is an ideal of $T, f(P) = (f_R(P_0), f_R(P_1)I_1, f_R(P_2)I_2)$ is a refined neutrosophic AH-ideal.

(c) The proof is similar to (b).

Definition 3.7:

Let $(R(I_1, I_2), +, \times)$ be a refined neutrosophic ring and $P = (P_0, P_1 I_1, P_2 I_2)$ be a refined neutrosophic AH-ideal then:

- (a) We say that P is a weak prime refined neutrosophic AH-ideal if $P_i; i \in \{0, 1, 2\}$ are prime ideals in R .
- (b) We say that P is a weak maximal refined neutrosophic AH-ideal if $P_i; i \in \{0, 1, 2\}$ are maximal ideals in R .
- (c) We say that P is a weak principal refined neutrosophic AH-ideal if $P_i; i \in \{0, 1, 2\}$ are principal ideals in R .
- (d) We define the refined neutrosophic AH-factor as:

$$R(I_1, I_2)/P = (R/P_0, R/P_1 I_1, R/P_2 I_2) = \{([x_0 + P_0], [x_1 + P_1] I_1, [x_2 + P_2] I_2); x_0, x_1, x_2 \in R\}.$$

Theorem 3.8:

Let $f: R(I_1, I_2) \rightarrow T(I_1, I_2)$ be a refined AHS-homomorphism and $P = (P_0, P_1 I_1, P_2 I_2)$ be a refined neutrosophic AH-ideal of $R(I_1, I_2)$, let $Q = (Q_0, Q_1 I_1, Q_2 I_2) \neq T(I_1, I_2)$ be a refined neutrosophic AH-ideal of $T(I_1, I_2)$, assume that $\text{Ker} f_R \leq P_i \neq R$ then:

- (a) P is a weak prime refined neutrosophic AH-ideal of $R(I_1, I_2)$ if and only if $f(P)$ is a weak primerefineneutrosophic AH-ideal in $f(R(I_1, I_2))$.
- (b) P is a weak maximal AH-ideal of $R(I_1, I_2)$ if and only if $f(P)$ is a weak maximal in $f(R(I_1, I_2))$.
- (c) Q is a weak prime AH-ideal of $T(I_1, I_2)$ if and only if $f^{-1}(Q)$ is a weak prime in $R(I_1, I_2)$.
- (d) Q is a weak maximal AH-ideal of $T(I_1, I_2)$ if and only if $f^{-1}(Q)$ is a weak maximal in $R(I_1, I_2)$.

Proof:

The proof is similar to the Theorem 3.15 in [1].

It is easy to see that conditions (a), (b) are still true if P is an AHS-ideal and conditions (c), (d) are still true if Q is an AHS-ideal.

Theorem 3.9:

The refined neutrosophic AH-factor $R(I_1, I_2)/P$ is a ring with respect to the following operations:

Let $x = (x_0 + P_0, (x_1 + P_1) I_1, (x_2 + P_2) I_2)$, $y = (y_0 + P_0, (y_1 + P_1) I_1, (y_2 + P_2) I_2)$, be two arbitrary elements in $R(I_1, I_2)$ then:

$$x + y = ([x_0 + y_0] + P_0, ([x_1 + y_1] + P_1) I_1, ([x_2 + y_2] + P_2) I_2),$$

$$x \times y = ([x_0 \times y_0] + P_0, ([x_1 \times y_1] + P_1) I_1, ([x_2 \times y_2] + P_2) I_2).$$

Proof:

The proofs similar to the Theorem 3.9 in [1].

Example 3.10:

Let $R = (Z_6, +, \times)$, $T = (Z_{10}, +, \times)$ be two rings, and f be the refined neutrosophic AHS-homomorphism defined in Example 3.3, we have the following:

(a) $P_0 = \{0, 2, 4\}$, $P_1 = \{0, 3\}$ are two ideals in Z_6 thus $P = (P_0, P_0I_1, P_1I_2)$ is a refined neutrosophic AH-ideal of $R(I_1, I_2)$.

(b) $f(P) = (f(P_0), f(P_0)I_1, f(P_1)I_2) = \{(0, 0, 0), (0, 0, 5I_2)\}$ is a refined neutrosophic AH-ideal in $T(I_1, I_2)$.

(c) $Q_0 = \{0, 2, 4, 6, 8\}$ is a maximal ideal in Z_{10} and $f_T^{-1}(Q_0) = \{0, 2, 4\}$, so $Q = (Q_0, Q_0I_1, Q_0I_2)$ is a weak maximal refined neutrosophic AHS-ideal in $T(I_1, I_2)$, we have $f^{-1}(Q) = (\{0, 2, 4\}, \{0, 2, 4\}I_1, \{0, 2, 4\}I_2)$ is a weak maximal refined neutrosophic AHS-ideal in $R(I_1, I_2)$.

Example 3.11:

(a) In the ring $(Z, +, \times)$, $P = \langle 3 \rangle$, $Q = \langle 2 \rangle$ are two prime and maximal ideals, thus $M = (P, QI_1, QI_2) = \{(3a, 2bI_1, 2cI_2); a, b, c \in Z\}$ is a weak maximal/prime refined neutrosophic AH-ideal of $(Z(I_1, I_2), +, \times)$.

(b) The map $f_Z : Z \rightarrow Z_6; f(a) = a \text{ mod } 6$ is a homomorphism so the related refined neutrosophic AHS-homomorphism is

$f : Z(I_1, I_2) \rightarrow Z_6(I_1, I_2); f(a, bI_1, cI_2) = (a \text{ mod } 6, (b \text{ mod } 6)I_1, (c \text{ mod } 6)I_2)$, $AH - \ker f = (6Z, 6ZI_1, 6ZI_2) \leq M$ since $6Z \leq P, Q$.

(c) $f(M) = (\{0, 3\}, \{0, 2, 4\}I_1, \{0, 2, 4\}I_2)$ is a weak maximal/prime refined neutrosophic AH-ideal of $Z_6(I_1, I_2)$.

Definition 3.12:

A refined neutrosophic ring $R(I_1, I_2)$ is called weak principal refined neutrosophic AH-ring if every refined neutrosophic AH-ideal is weak principal.

Theorem 3.13:

Let R be a principal ideal ring then $R(I_1, I_2)$ is weak principal refined neutrosophic AH-ring.

Proof:

Let $P = (P_0, P_1I_1, P_2I_2)$ be a refined neutrosophic AH-ideal of $R(I_1, I_2)$. Since P_i are ideals in R and then principal this implies that P is a weak refined neutrosophic AH-ideal; thus $R(I_1, I_2)$ must be weak principal refined neutrosophic AH-ring.

Example 3.14:

The ring $(Z, +, \times)$ is principal ideals ring; thus $Z(I_1, I_2)$ is weak principal refined neutrosophic AH-ring.

Definition 3.15:

Let $(R(I_1, I_2), +, \times)$ be a refined neutrosophic ring and $P = (P_0, P_1I_1, P_2I_2)$, $Q = (Q_0, Q_1I_1, Q_2I_2)$ be two refined neutrosophic AH-ideals of $R(I_1, I_2)$, then we define:

(a) $P \cap Q = (P_0 \cap Q_0, [P_1 \cap Q_1]I_1, [P_2 \cap Q_2]I_2)$.

(b) $P + Q = (P_0 + Q_0, [P_1 + Q_1]I_1, [P_2 + Q_2]I_2)$.

$$(c) P \times Q = (P_0 \times Q_0, [P_1 \times Q_1]I_1, [P_2 \times Q_2]I_2).$$

Theorem 3.16:

Let $(R(I_1, I_2), +, \times)$ be a refined neutrosophic ring and $P = (P_0, P_1I_1, P_2I_2)$, $Q = (Q_0, Q_1I_1, Q_2I_2)$ be two refined neutrosophic AH-ideals of $R(I_1, I_2)$, then:

$P \cap Q, P + Q, P \times Q$ are refined neutrosophic AH-ideals of $R(I_1, I_2)$.

Proof:

As a result of Theorem 2.5 in [1], we have $P_i + Q_i, P_i \cap Q_i, P_i \times Q_i$ are ideals of R , thus the proof is complete.

Remark 3.17:

Theorem 3.16 is still true if P and Q are refined neutrosophic AHS-ideals.

Example 3.18:

Let $R(I_1, I_2) = Z_8(I_1, I_2)$ and $Q = \{0, 4\}$, $S = \{0, 2, 4, 6\}$ be two principal ideals in R , then:

(a) $P = (S, QI_1, SI_2)$ is a refined neutrosophic AH-ideal of $R(I_1, I_2)$, the related refined neutrosophic AH-factor is:

$$R(I_1, I_2)/P = (R/S, R/Q I_1, R/S I_2) = (\{S, 1+S\}, \{Q, 1+Q, 2+Q, 3+Q\}I_1, \{S, 1+S\}I_2).$$

To clarify addition and multiplication on $R(I_1, I_2)/P$ we take:

$x = (1 + S, (1 + Q)I_1, SI_2)$, $y = (S, (2 + Q)I_1, (1 + S)I_2)$, we have:

$$x + y = ([1 + 0] + S, ([1 + 2] + Q) I_1, ([0 + 1] + S)I_2) = (1+S, (3+Q)I_1, (1 + S)I_2).$$

$$x \times y = ([1 \times 0] + S, ([1 \times 2] + Q) I_1, ([0 \times 1] + S)I_2) = (S, (2+Q) I_1, SI_2).$$

Conclusion

In this article we defined concepts of refined neutrosophic AH-ideal/ AHS-ideal in a refined neutrosophic ring. We studied some of elementary properties of these concepts. Also, notions as weak maximal, prime and principal refined neutrosophic AH-ideal and refined AHS-homomorphisms were introduced and checked.

Funding: "This research received no external funding."

Conflicts of Interest: "The authors declare no conflict of interest."

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