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# Modeling macroscopic unsteady inertial flow in porous media should take into account the process history

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## Introduction

- Unsteady flow in porous media is relevant in applications ranging from seismic waves to super fluid flow.
- This process is usually modeled using the heuristic momentum equation inferred from Darcy's law:
$$\rho \frac{\partial \langle \mathbf{v} \rangle^\beta}{\partial t} = -\nabla \langle p \rangle^\beta - \mu \varepsilon \mathbf{H}^{-1} \cdot \langle \mathbf{v} \rangle^\beta$$
- Many analyses have been performed in the frequency but not in the time domain.

## Pore-scale equations

$$\begin{aligned} \nabla \cdot \mathbf{v} &= 0 \\ \rho \frac{D\mathbf{v}}{Dt} &= -\nabla \tilde{p} + \mu \nabla^2 \mathbf{v} - \underbrace{\nabla \langle p \rangle^\beta}_{source} \\ \mathbf{v} &= \mathbf{0}, \text{ at the solid-fluid interface} \\ \mathbf{v} &= \underbrace{\mathbf{v}_0}_{source}, \text{ when } t = 0 \\ \psi(\mathbf{r} + \mathbf{l}_i) &= \psi(\mathbf{r}), \psi = \mathbf{v}, \tilde{p}, i = 1, 2, 3 \\ \langle \tilde{p} \rangle^\beta &= 0 \end{aligned}$$

Formal solution

$$\begin{aligned} \mathbf{v} &= \frac{1}{\mu} \left( - \int_{t_0=0}^{t_0=t} \frac{\partial \mathbf{D}}{\partial t} \Big|_{t-t_0} \cdot \nabla \langle p \rangle^\beta \Big|_{t_0} dt_0 + \mathbf{m}_0 \right) \\ \tilde{p} &= - \int_{t_0=0}^{t_0=t} \frac{\partial \mathbf{d}}{\partial t} \Big|_{t-t_0} \cdot \nabla \langle p \rangle^\beta \Big|_{t_0} dt_0 + n_0 \end{aligned}$$

## Upscaled model

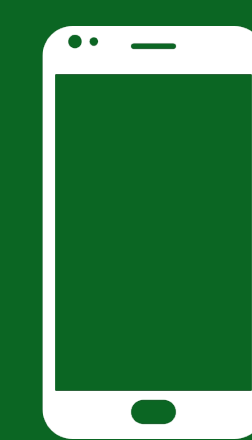
$$\begin{aligned} \nabla \cdot \langle \mathbf{v} \rangle &= 0 \\ \langle \mathbf{v} \rangle &= -\frac{1}{\mu} \int_{t_0=0}^{t_0=t} \frac{\partial \mathbf{H}_t}{\partial t} \cdot \nabla \langle p \rangle^\beta dt_0 + \boldsymbol{\alpha} \end{aligned}$$

where  $\mathbf{H}_t = \langle \mathbf{D} \rangle$  and  $\boldsymbol{\alpha} = \langle \mathbf{m}_0 \rangle / \mu$ .

# H21L-1923: Upscaling unsteady inertial flow in homogeneous porous media



# Modeling macroscopic unsteady interial flow in porous media should take into account the process history.



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## Closure problems

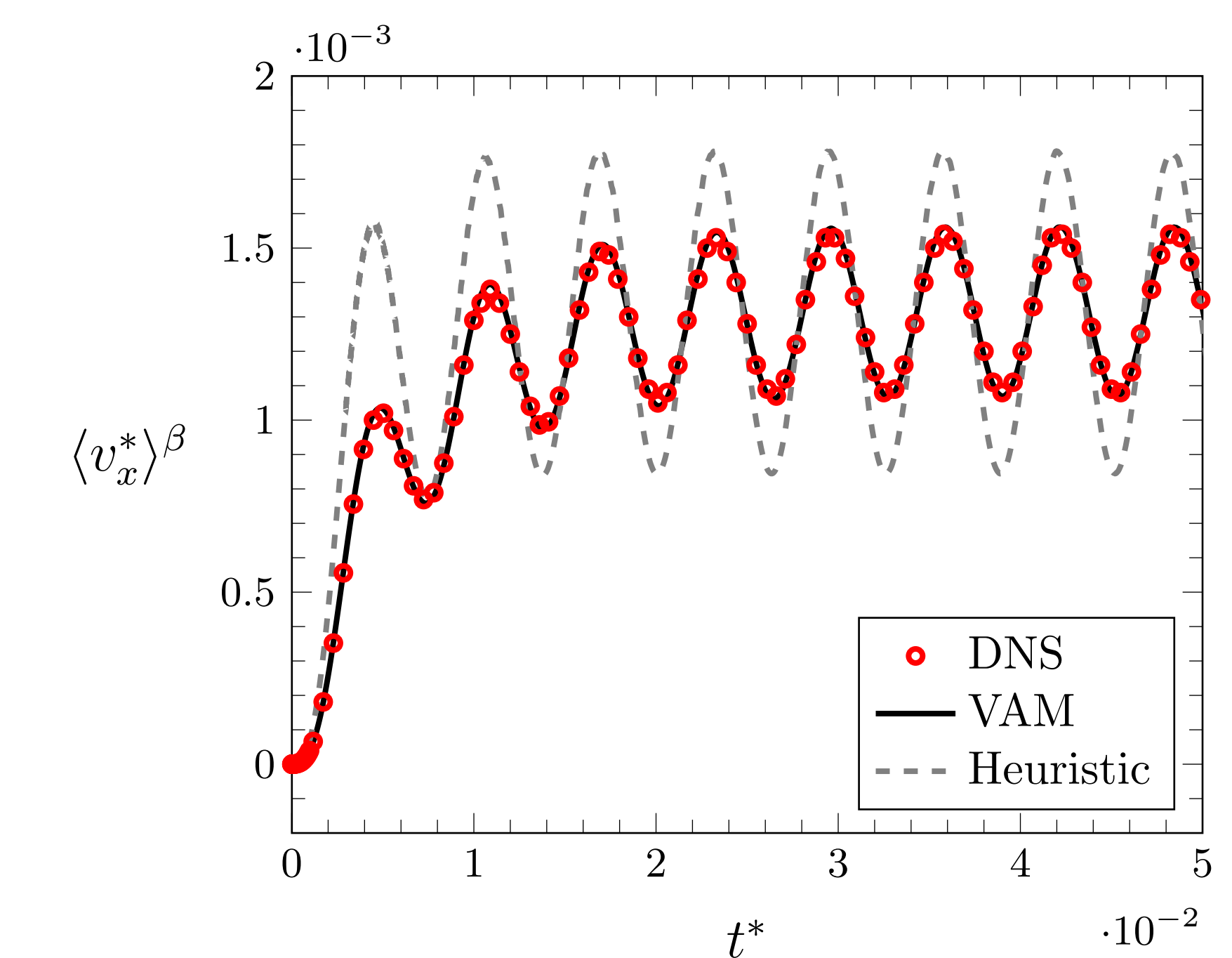
Problem I

$$\begin{aligned} \nabla \cdot \mathbf{D} &= \mathbf{0} \\ \frac{\rho D\mathbf{D}}{\mu Dt} &= -\nabla \mathbf{d} + \nabla^2 \mathbf{D} + \mathbf{I} \\ \mathbf{D} &= \mathbf{0}, \text{ at the solid-fluid interface} \\ \mathbf{D} &= \mathbf{0}, t = 0 \\ \langle \mathbf{d} \rangle^\beta &= \mathbf{0} \\ \psi(\mathbf{r} + \mathbf{l}_i) &= \psi(\mathbf{r}), \psi = \mathbf{D}, \mathbf{d}, i = 1, 2, 3 \end{aligned}$$

Problem II

$$\begin{aligned} \nabla \cdot \mathbf{m}_0 &= \mathbf{0} \\ \frac{\rho D\mathbf{m}_0}{\mu Dt} &= -\nabla n_0 + \nabla^2 \mathbf{m}_0 \\ \mathbf{m}_0 &= \mathbf{0}, \text{ at the solid-fluid interface} \\ \mathbf{m}_0 &= \mu \mathbf{v}_0, t = 0 \\ \langle n_0 \rangle^\beta &= 0 \\ \psi(\mathbf{r} + \mathbf{l}_i) &= \psi(\mathbf{r}), \psi = \mathbf{m}_0, n_0, i = 1, 2, 3 \end{aligned}$$

## Comparison with DNS



Dynamics of the average velocity for an oscillatory macroscopic pressure gradient for a porosity of 0.4 and  $Re = 10^6$ .

## Conclusions

- Unsteady flow is described by a macroscopic model that is non-local in time and that considers the influence of the initial condition.
- The performance and accuracy of the model is validated with DNS.
- The heuristic model is inappropriate in the general case.