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Andrei P. Kirilyuk

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UNIVERSAL COMPLEXITY IN ACTION: ACTIVE CONDENSED MATTER, INTEGRAL MEDICINE, CAUSAL ECONOMICS AND SUSTAINABLE GOVERNANCE

ANDREI P. KIRILYUK

Institute of Metal Physics, 36 Vernadsky Avenue, 03142 Kyiv, Ukraine
E-mail: Andrei.Kirilyuk@Gmail.com

Abstract.

We review the recently proposed universal concept of dynamic complexity and its new mathematics based on the unreduced interaction problem solution. We then consider its progress-bringing applications at various levels of complex world dynamics, including complex-dynamical nanometal physics and living condensed matter, unreduced nanobiosystem dynamics and the integral medicine concept, causally complete management of complex economical and social dynamics, and the ensuing concept of truly sustainable world governance.

1. Introduction: Universal dynamic complexity and its causally complete new mathematics. As shown previously [K1, K2, K3, K4, K5, K6, K7, K8, K9, K10, K11, K12, K13, K14, K15, K16, K17, K18, K19, K20, K21, K22, K23, K24, K25, K26, K27, K28, K29], the unreduced solution of arbitrary many-body interaction problem provides much richer (and *causally complete*) dynamical structure and behaviour, in the form of multiple and incompatible system realisations permanently replacing one another in truly random order. We can thus introduce the *absolutely universal* concept and definition of (*dynamic*) *complexity* as a growing function of realisation number or rate of their change, which proves its universality in various applications. In this report we summarise both the universal complexity formalism (in this section 1) and its applications (in the following sections 2–4), including some new and perspective ones, thus additionally confirming the concept universality, which is always missing in conventional complexity theory (because it is restricted to only one system realisation of standard “exact” or perturbative problem solutions).

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We generalise the starting mathematical expression of arbitrary real interaction process in the form of the system “existence equation”, reproducing the Hamiltonian formalism of well-known fundamental equations (such as the Schrödinger equation for quantum systems or the Hamilton-Jacobi equation for classical ones) and providing the eventually confirmed unified expression of any real system dynamics (see below in this section) [K1, K3, K5, K7, K9, K10, K23, K25]:

$$\left\{ \sum_{k=0}^N \left[h_k(q_k) + \sum_{l>k}^N V_{kl}(q_k, q_l) \right] \right\} \Psi(Q) = E\Psi(Q) , \quad (1)$$

where $h_k(q_k)$ is the “generalised Hamiltonian” (specified below as a measure of dynamic complexity) for the k -th system component, q_k is the degree(s) of freedom of the k -th component, $V_{kl}(q_k, q_l)$ is the potential of (arbitrary) interaction between the k -th and l -th components, $Q \equiv \{q_0, q_1, \dots, q_N\}$, $\Psi(Q)$ is the system state-function expressing its configuration, E is the generalised energy (generalised Hamiltonian eigenvalue), and summations cover all (N) system components. With evident transformations, this dynamic equation actually includes the less fundamental case of time-dependent formalism.

A more convenient form of the initial existence equation (1) is obtained if we explicitly separate some “common” degree(s) of freedom, $q_0 \equiv \xi$, usually characterising system’s spatial configuration:

$$\left\{ h_0(\xi) + \sum_{k=1}^N \left[h_k(q_k) + V_{0k}(\xi, q_k) + \sum_{l>k}^N V_{kl}(q_k, q_l) \right] \right\} \Psi(\xi, Q) = E\Psi(\xi, Q) , \quad (2)$$

where now $Q \equiv \{q_1, \dots, q_N\}$ and $k, l \geq 1$. For a natural problem expression, we can use the state-function expansion in terms of known eigenfunctions of system elements:

$$h_k(q_k) \varphi_{knk}(q_k) = \varepsilon_{nk} \varphi_{knk}(q_k) , \quad (3)$$

$$\Psi(\xi, Q) = \sum_n \psi_n(\xi) \varphi_{1n_1}(q_1) \varphi_{2n_2}(q_2) \dots \varphi_{Nn_N}(q_N) \equiv \sum_n \psi_n(\xi) \Phi_n(Q) , \quad (4)$$

where $\{\varphi_{knk}(q_k), \varepsilon_{nk}\}$ is the complete set of orthonormal eigenfunctions and eigenvalues of the k -th component Hamiltonian $h_k(q_k)$, $n \equiv \{n_1, \dots, n_N\}$ runs through all eigenstate combinations, and $\Phi_n(Q) \equiv \varphi_{1n_1}(q_1) \varphi_{2n_2}(q_2) \dots \varphi_{Nn_N}(q_N)$ by definition.

Inserting expansion (4) into existence equation (2), we obtain the system of equations for $\{\psi_n(\xi)\}$ in a standard way, using the eigenfunction orthonormality [K1, K3, K7, K14]:

$$\begin{aligned} [h_0(\xi) + V_{00}(\xi)] \psi_0(\xi) &+ \sum_n V_{0n}(\xi) \psi_n(\xi) = \eta \psi_0(\xi) \\ [h_0(\xi) + V_{nn}(\xi)] \psi_n(\xi) &+ \sum_{n' \neq n} V_{nn'}(\xi) \psi_{n'}(\xi) = \eta_n \psi_n(\xi) - V_{n0}(\xi) \psi_0(\xi) , \end{aligned} \quad (5)$$

where $n, n' \neq 0$ (also below), $\eta \equiv \eta_0 = E - \varepsilon_0$, $\eta_n = E - \varepsilon_n$, $\varepsilon_n = \sum_k \varepsilon_{nk}$,

$$V_{nn'}(\xi) = \sum_k \left[V_{k0}^{nn'}(\xi) + \sum_{l>k} V_{kl}^{nn'} \right] , \quad (6)$$

$$V_{k0}^{nn'}(\xi) = \int_{\Omega_Q} dQ \Phi_n^*(Q) V_{k0}(q_k, \xi) \Phi_{n'}(Q) , \quad (7)$$

$$V_{kl}^{nn'}(\xi) = \int_{\Omega_Q} dQ \Phi_n^*(Q) V_{kl}(q_k, q_l) \Phi_{n'}(Q) , \quad (8)$$

and we have separated the equation for $\psi_0(\xi)$ describing the generalised “ground state”, specified eventually as the state with minimum dynamic complexity. The resulting system of equations (5) is equivalent to the starting existence equation (1)–(2), but the problem can now be efficiently analysed due to its “natural” dynamic variables.

In order to solve the “nonintegrable” system of equations (5) we use the generalised effective, or optical, potential method [D, K22], where one expresses $\psi_n(\xi)$ through $\psi_0(\xi)$ from the equations for $\psi_n(\xi)$ using the standard Green function technique and then inserts the result into the equation for $\psi_0(\xi)$, obtaining thus the *effective existence equation*, which contains *explicitly* only “integrable” degrees of freedom (ξ) [K1, K2, K3, K7, K11]:

$$h_0(\xi) \psi_0(\xi) + V_{\text{eff}}(\xi; \eta) \psi_0(\xi) = \eta \psi_0(\xi) , \quad (9)$$

where the operator of *effective potential (EP)*, $V_{\text{eff}}(\xi; \eta)$, is obtained as

$$V_{\text{eff}}(\xi; \eta) = V_{00}(\xi) + \hat{V}(\xi; \eta) , \quad \hat{V}(\xi; \eta) \psi_0(\xi) = \int_{\Omega_{\xi}} d\xi' V(\xi, \xi'; \eta) \psi_0(\xi') , \quad (10)$$

$$V(\xi, \xi'; \eta) = \sum_{n,i} \frac{V_{0n}(\xi) \psi_{ni}^0(\xi) V_{n0}(\xi') \psi_{ni}^{0*}(\xi')}{\eta - \eta_{ni}^0 - \varepsilon_{n0}} , \quad \varepsilon_{n0} \equiv \varepsilon_n - \varepsilon_0 , \quad (11)$$

and $\{\psi_{ni}^0(\xi), \eta_{ni}^0\}$ is the complete eigen-solution set for a *truncated* system of equations:

$$[h_0(\xi) + V_{nn}(\xi)] \psi_n(\xi) + \sum_{n' \neq n} V_{nn'}(\xi) \psi_{n'}(\xi) = \eta_n \psi_n(\xi) . \quad (12)$$

The eigenfunctions, $\{\psi_{0i}(\xi)\}$, and eigenvalues, $\{\eta_i\}$, of the formally “integrable” equation (9) are used to obtain other state-function components:

$$\psi_{ni}(\xi) = \hat{g}_{ni}(\xi) \psi_{0i}(\xi) \equiv \int_{\Omega_{\xi}} d\xi' g_{ni}(\xi, \xi') \psi_{0i}(\xi') , \quad (13)$$

$$g_{ni}(\xi, \xi') = V_{n0}(\xi') \sum_{i'} \frac{\psi_{ni'}^0(\xi) \psi_{ni'}^{0*}(\xi')}{\eta_i - \eta_{ni'}^0 - \varepsilon_{n0}} , \quad (14)$$

and the total system state-function, $\Psi(q_0, q_1, \dots, q_N) = \Psi(\xi, Q)$ (see (4)):

$$\Psi(\xi, Q) = \sum_i c_i \left[\Phi_0(Q) + \sum_n \Phi_n(Q) \hat{g}_{ni}(\xi) \right] \psi_{0i}(\xi) , \quad (15)$$

where coefficients c_i should be found from the state-function matching conditions at the dynamic boundary with vanishing EP. The measured system density $\rho(\xi, Q)$ is obtained as state-function squared modulus, $\rho(\xi, Q) = |\Psi(\xi, Q)|^2$ (for “wave-like” complexity levels), or state-function itself, $\rho(\xi, Q) = \Psi(\xi, Q)$ (for “particle-like” structures) [K1].

Even though the EP problem formulation, (9)–(11), remains “nonintegrable” and equivalent to the initial interaction problem, (1), (2), (5), the *dynamical links* of the unreduced interaction process appearing explicitly in the effective version reveal the *qual-*

itatively new properties of the unreduced problem solution, permitting its reconstitution in the complete form [K1, K2, K3, K4, K5, K6, K7, K8, K9, K10, K14, K22, K23].

The key property of the unreduced (any real) interaction result (9)–(15) is its *dynamic multivaluedness, or redundance*, meaning that one has a *redundant* number of individually complete and therefore *mutually incompatible* solutions describing *equally real* system configurations, or *realisations*. This major property of *dynamic* realisation (solution) plurality, underlying the *new mathematics of complexity* [K1, K3, K6, K7, K10, K21, K23, K24, K25] (see also below), is due to the nonlinear and self-consistent dependence of the unreduced EP, (9)–(11), on the solutions to be found, which leads to the easily calculated growth of the highest power of the characteristic equation determining the total eigenvalue number and reflects the physically obvious plurality of interacting eigen-mode combinations [K1, K2, K3, K4, K5, K6, K7, K8, K9, K10, K22, K23].

If N_ξ and N_q are the numbers of terms in sums over i and n in equation (11), then the total number of eigenvalues of equation (9) is $N_{\max} = N_\xi(N_\xi N_q + 1) = (N_\xi)^2 N_q + N_\xi$, which gives the N_ξ -fold redundance of the usual “complete” set of $N_\xi N_q$ eigen-solutions of equations (5) plus an additional, “incomplete” set of N_ξ solutions. It means that the total number of “regular”, locally complete system realisations is $N_{\mathfrak{R}} = N_\xi$, whereas the additional set of N_ξ solutions forms a special, “intermediate” realisation that plays the role of transitional state during inevitable system jumps between regular realisations and provides thus the universal, *causally complete* (physically real) extension of the quantum-mechanical *wavefunction* and classical (*probability*) *distribution function* [K1, K3, K6, K7, K12, K14, K20, K23] (see further details below).

This key feature of fundamental dynamic multivaluedness of any real, unreduced interaction result is confirmed by both graphical analysis of EP equation solutions [K1, K2, K22, K23] and various simplified transparent schemes [K3, K10, K23].

Note the essential difference between thus revealed dynamic multivaluedness of the unreduced problem solution and its imitations within the *dynamically single-valued, or unitary*, schemes of usual, perturbative or “exact-model” theory, including its complexity versions, such as “attractors”, “unstable orbits”, or “multistability” (of the single-valued trajectory), as well as deeply flawed perturbative notions of linear stability and Lyapunov exponents. All such unitary schemes of usual theory, including standard EP applications (see e. g. [D]) and scholar “complexity science”, use one or another version of perturbation theory (or “exact model”), which tries to produce a single, closed-form solution just “killing” all other, *redundant* solutions by eliminating the *dynamically* emerging nonlinear interaction links and retaining *only one*, “averaged” solution, which expresses only trivial, small or artificially inserted, deviations from the initial system configuration (hence the absence of the key property of explicit *emergence* in usual theory, as opposed to our unreduced solution, see below).

Since in our unreduced interaction description we obtain many *incompatible*, but *equally real* system realisations resulting from the same, *totally regular* system interaction, we arrive at the *universal* property of *causal, or dynamic, randomness* in the form of *permanently changing* system realisations that replace one another in *truly random* (and *therefore* fundamentally *unpredictable, undecidable* and *noncomputable*) order

thus naturally defined. This *omnipresent* and fundamental origin of randomness in any, even *externally* regular system behaviour is equivalent to the universal and totally consistent version of (*dynamical*) *chaos*, which is essentially different from its usual, unitary versions, inevitably reduced to “sophisticated regularity”, including *incorrectly* assumed “exponential noise amplification” (as a result of unjustified extension of perturbation theory results) [K1].

We therefore obtain the complete *general solution* of arbitrary interaction problem as the *dynamically probabilistic* sum of measured quantity (system density) values for different realisations:

$$\rho(\xi, Q) = \sum_{r=1}^{N_{\mathfrak{R}}} \oplus \rho_r(\xi, Q), \quad (16)$$

where the summation covers all system realisations, $N_{\mathfrak{R}}$ is their number (generally equal to the number of system eigenmodes, $N_{\mathfrak{R}} = N$), and the sign \oplus designates the special, dynamically probabilistic meaning of the sum. It implies that any measured quantity (16) is *intrinsically unstable* (even for a totally isolated system) and its current value will unpredictably change to another one, corresponding to another, *randomly* chosen realisation. This *permanently unstable* dynamics is often explicitly observed in nature and explains, in particular, specific features of living organism behaviour [K1, K3, K5, K6, K7, K20, K23, K25, K26], but remains misunderstood in unitary theory and usual technological systems, where it is associated with ambiguous linear “noncomputability” (e. g. [P]) and technical failure. Needless to say, the externally regular behaviour of unitary dynamics in reality hides within it the plurality of chaotically changing, but “self-organised” realisations with very similar configurations (see below for more details).

Thus obtained intrinsic, causal randomness of the generalised EP formalism and unreduced problem solution it provides (9)–(16) naturally includes the *dynamic probability definition*. Since elementary realisations have equal “rights to emerge”, the dynamically derived, *a priori probability* of r -th realisation emergence, α_r , is given by

$$\alpha_r = \frac{1}{N_{\mathfrak{R}}}, \quad \sum_r \alpha_r = 1. \quad (17)$$

For a general case of actually observed dense “self-organised” groups of similar elementary realisations, the dynamic probability of r -th compound realisation is determined by the number, N_r , of elementary realisations it contains:

$$\alpha_r(N_r) = \frac{N_r}{N_{\mathfrak{R}}} \left(N_r = 1, \dots, N_{\mathfrak{R}}; \sum_r N_r = N_{\mathfrak{R}} \right), \quad \sum_r \alpha_r = 1. \quad (18)$$

The (quasi)stationary *expectation value*, $\rho_{\text{exp}}(\xi, Q)$, is easily obtained from (16)–(18) for statistically large event numbers:

$$\rho_{\text{exp}}(\xi, Q) = \sum_r \alpha_r \rho_r(\xi, Q). \quad (19)$$

However, our *dynamically derived* randomness and probability concept does not rely on that “statistical” situation and its empirically based definitions, with our basic expressions (16)–(18) remaining valid for any *single* event of realisation emergence and even *before* it

occurs. Note that the realisation/event probability distribution can be obtained also in another way involving the *generalised wavefunction (or distribution function)* and *Born's probability rule* (see below) [K1, K3, K6, K7, K12, K14, K20, K23].

Another essential feature of the unreduced interaction result and problem solution is the *dynamic entanglement* of system components (degrees of freedom). It is described by the dynamically weighted products of eigenfunctions depending on different degrees of freedom (ξ, Q) in the state-function expression (15) and provides the *mathematically exact* notion of the tangible *quality* of emerging interaction result (system structure), which is absent in unitary models producing only abstract, “immaterial” entities.

The obtained *dynamically multivalued entanglement* of the unreduced interaction result describes a *living* kind of structure, *permanently probabilistically changing* and *adapting* its tangible configuration, which provides a well-specified basis for bio-inspired applications reviewed below. The properties of dynamically multivalued entanglement and adaptability are amplified due to the complex-dynamical, *probabilistic fractality* of the unreduced general solution [K1, K3, K5, K6, K7, K20, K23] (see below for more details).

Using thus specified unreduced problem solution, we can now rigorously and *universally* define the unreduced *dynamic complexity*, C , of *any* real system or interaction process as a growing function of the number of its *explicitly obtained* realisations or rate of their change, equal to zero for the unrealistic case of only one realisation [K1, K2, K3, K6, K7, K9, K10, K12, K14, K16, K17, K18, K19, K20, K21, K23, K24, K26, K27]:

$$C = C(N_{\mathfrak{R}}) , \quad dC/dN_{\mathfrak{R}} > 0 , \quad C(1) = 0 . \quad (20)$$

Examples include $C(N_{\mathfrak{R}}) = C_0 \ln N_{\mathfrak{R}}$, $C(N_{\mathfrak{R}}) = C_0 (N_{\mathfrak{R}} - 1)$, generalised energy/mass (temporal rate of realisation change) and momentum (spatial rate of realisation emergence) (see below). We see again that the entire *dynamically single-valued* paradigm and results of canonical theory (including its versions of “complexity”, “chaos” and *imitations* of “multistability” in *abstract* “spaces”) correspond to exactly *zero* value of unreduced complexity equivalent to effectively zero-dimensional, point-like projection of reality.

The proposed universal concept of complexity and its applications appear thus as the well-specified, *causally complete extension* of usual theory to the unreduced, dynamically multivalued picture of reality. It naturally includes also such usually vague and separated, but now clearly specified, universal and intrinsically unified notions as chaoticity, with its genuine and always dynamic randomness (fundamental unpredictability), uncertainty, nonintegrability, nonseparability, noncomputability, undecidability and non-unitarity. *All* real, interaction-based objects, systems and processes, starting from elementary particles, are therefore characterised by both *positive (and high) dynamic complexity* ($C(N_{\mathfrak{R}}) > 0$, $N_{\mathfrak{R}} \gg 1$) and related *intrinsic chaoticity*, the latter appearing in a variety of plain (uniform) or confined (self-organised) regimes (see below). This can be compared to various artificial, ill-defined and separated classes of complex and chaotic systems in usual theory remaining within the Newtonian, mechanistic science paradigm, even for complexity studies or “new physics” mysteries (now causally explained as unreduced dynamic complexity manifestations [K1, K2, K3, K10, K11, K12, K13, K14, K15, K23]).

Universal dynamic complexity features involve the *essential, or dynamic, nonlinearity* of all real systems revealed within the unreduced problem solution. It originates in the in-

teraction feedback links expressed by the EP dependence on the problem eigen-solutions (see (9)–(11)). It is the *dynamically emerging* nonlinearity, since it appears for a formally “linear” initial problem expression (1)–(2), (5), whereas any usual, mechanistic “non-linearity” is but a perturbative reduction and artificially introduced imitation of this emerging nonlinearity of real interaction dynamics (see also below). Essential nonlinearity leads to irreducible *dynamic instability* within *any*, dynamically multivalued system state: both are determined by the same mechanism of dynamic feedback development.

We proceed now to more elaborated complexity features and its emergent evolution. One of essential, new features of unreduced interaction result is the *dynamically multivalued, or probabilistic, fractal* of its multilevel structure. It appears as a result of remaining incompleteness of the above first-level solution (9)–(19) relying on the unknown solutions of the truncated system of equations (12). In order to reduce the uncertainty, we can apply the same unreduced EP approach to solution of this truncated problem, which gives the second-level effective equation, similar to the first-level equation (9):

$$[h_0(\xi) + V_{\text{eff}}^n(\xi; \eta_n)] \psi_n(\xi) = \eta_n \psi_n(\xi) , \quad (21)$$

where the second-level EP $V_{\text{eff}}^n(\xi; \eta_n)$ resembles its first-level version (10)–(11):

$$V_{\text{eff}}^n(\xi; \eta_n) \psi_n(\xi) = V_{nn}(\xi) \psi_n(\xi) + \sum_{n' \neq n, i} \frac{V_{nn'}(\xi) \psi_{n'i}^{0n}(\xi) \int_{\Omega_\xi} d\xi' \psi_{n'i}^{0n*}(\xi') V_{n'n}(\xi') \psi_n(\xi')}{\eta_n - \eta_{n'i}^{0n} + \varepsilon_{n0} - \varepsilon_{n'0}} , \quad (22)$$

and $\{\psi_{n'i}^{0n}(\xi), \eta_{n'i}^{0n}\}$ is the eigen-solution set of the second-level truncated system:

$$h_0(\xi) \psi_{n'}(\xi) + \sum_{n'' \neq n'} V_{n'n''}(\xi) \psi_{n''}(\xi) = \eta_{n'} \psi_{n'}(\xi) , \quad n' \neq n, 0 . \quad (23)$$

Similarity of equations (21)–(23) to the first-level EP expressions (10)–(12) implies that its second-level version is also split into many incompatible realisations (numbered by index r') due to the self-consistent dependence on the eigen-solutions to be found, with respective splitting of solutions of the first-level truncated system (12):

$$\{\psi_{ni}^0(\xi), \eta_{ni}^0\} \rightarrow \{\psi_{ni}^{0r'}(\xi), \eta_{ni}^{0r'}\} . \quad (24)$$

This dynamical splitting of emerging system structure continues with ever more truncated auxiliary systems of equations until the last, exactly solvable equation. Substituting the dynamically multivalued solutions of each truncated system into expressions of the previous-level EP solutions, we get the now *totally complete problem solution* in the form of multilevel (fractal) hierarchy of probabilistically changing realisations:

$$\rho(\xi, Q) = \sum_{r, r', r'', \dots}^{N_{\mathfrak{R}}} \oplus \rho_{rr'r'' \dots}(\xi, Q) , \quad (25)$$

where indexes r, r', r'', \dots enumerate realisations at consecutive fractal levels. Similar to the a priori realisation probabilities of the first level, (17)–(18), we obtain the hierarchy of *causal realisation probabilities* $\{\alpha_{rr'r'' \dots}\}$ for all levels of dynamically multivalued fractal:

$$\alpha_{rr'r'' \dots} = \frac{N_{rr'r'' \dots}}{N_{\mathfrak{R}}} , \quad \sum_{rr'r'' \dots} \alpha_{rr'r'' \dots} = 1 . \quad (26)$$

The *expectation value* of the observed density of the dynamically probabilistic fractal of emerging system structure is then obtained as:

$$\rho_{\text{exp}}(\xi, Q) = \sum_{r, r', r'', \dots}^{N_{\mathfrak{R}}} \alpha_{rr'r''\dots} \rho_{rr'r''\dots}(\xi, Q). \quad (27)$$

It is important to emphasize the key difference of the dynamically probabilistic fractal from usual, abstract fractals: the latter are not solutions to any real interaction problems and possess only simplified, unitary and regular “scale symmetry” (which cannot be modified by artificial probability insertion). By contrast, our dynamically multivalued fractal would not generally show that regular scale invariance (with rare and approximate exceptions) and realises instead a much deeper symmetry of complexity (see below). It is also different from any conventional, approximate (and usually diverging) “series expansion”: the possibly long, but finite sums of the dynamically probabilistic fractal solution (25), (27) provide the *exact* version of *real* multilevel system structure. It is made by permanently probabilistically moving and growing/disappearing fractal branches, which reproduce natural structure dynamics, as opposed to any unitary fractal models.

Moreover, the entire world structure emerges in that way as a gigantic, but unique and physically unified dynamically multivalued fractal of the underlying simplest interaction between two initially structureless primordial entities (“protofields”), with all the observed properties and laws at all its levels rigorously derived as emergent features, without any artificially inserted abstract “principles” of conventional theories [K1, K2, K3, K4, K5, K6, K7, K8, K9, K10, K11, K12, K13, K14, K15, K16, K17, K18, K19, K20, K21, K23, K27] (see also below). Among those emergent properties one may mention *dynamic adaptability* directly related to the interactive dynamic origin of probabilistic realisation change in fractal structure (and therefore absent in any unitary description). The high efficiency of the related “intelligent search” of optimal structure creation underlies the “magic” properties of life and intelligence, reflecting the huge exponential growth of fractal realisation number $N_{\mathfrak{R}}$ and thus complexity (20) (see below).

The unified multivalued fractal dynamics is related also to the origin and classification of *all possible dynamic regimes*, between the limiting cases of strong chaos and (external) regularity. Indeed, it is physically evident that the highly chaotic “free search” in fine fractal branches is transformed into much more “confined” or “heavy” (quasi-regular) branch dynamics as a result of restrictive interactions leading to higher-level structure emergence (which in its turn may give rise to chaotic higher sublevels).

As shown in [K1, K2, K3, K7, K8, K14, K22, K23], one limiting case of complex dynamics, called *uniform, or global, chaos*, emerges from the main EP formalism (9)–(15) as sufficiently different realisations with a quasi-homogeneous probability distribution (i. e. $N_r \approx 1$ and $\alpha_r \approx 1/N_{\mathfrak{R}}$ for all r in (18)). It is realised when major interaction parameters (represented by energy level separations or frequencies of intra-component and inter-component motions) are close to each other, which leads to a strong “conflict of interests” and the ensuing “big disorder”, without any dominant ordering motion.

The opposite limiting regime of *dynamically multivalued self-organisation or self-organised criticality (SOC)* emerges for sufficiently different interaction frequencies, so

that, as easily seen from (9)–(15), a rigid, low-frequency component “enslaves” a great number of high-frequency and rapidly changing realisations with similar configurations (i. e. the realisation probability distribution is highly inhomogeneous, $N_r \sim N_{\mathfrak{R}}$), while the EP (9)–(10) and state-function (15) approach quasi-local functions [K1, K3, K7, K8, K14]. However, the difference of that extended, dynamically multivalued SOC from usual self-organisation and SOC is essential: despite the rigid *external* shape of system configuration in this regime, it confines an intense “internal life” and *chaos* of randomly changing “enslaved” realisations (which are *not* superposable or coexisting unitary “modes” or states). This is the key to consistent solution of well-known entropy-growth problems for any ordered structure formation (see also below). Another important advance is that this real, multivalued self-organisation unifies the extended versions of separated unitary models, including usual “self-organisation” (or “synergetics”), SOC, “synchronisation”, “control of chaos”, “attractors”, and “mode locking”.

All real dynamic regimes fall between these limiting cases of uniform chaos and multivalued SOC (including their multi-level combinations). The point of transition to the global chaos regime is expressed by the *universal criterion of global chaos onset* derived from the basic EP formalism (9)–(15) as the mentioned frequency equality condition:

$$\kappa \equiv \frac{\Delta\eta_i}{\Delta\eta_n} = \frac{\omega_\xi}{\omega_q} \cong 1, \quad (28)$$

where κ is the introduced *chaoticity* parameter, $\Delta\eta_i$, ω_ξ and $\Delta\eta_n \sim \Delta\varepsilon$, ω_q are energy-level separations and frequencies for the inter-component and intra-component motions respectively. At $\kappa \ll 1$ one has the externally quasi-regular multivalued SOC regime, which degenerates into global chaos as κ grows from 0 to 1, and the maximum irregularity at $\kappa \approx 1$ is again transformed into a SOC kind of structure at $\kappa \gg 1$ (but with the “inverse” system configuration). One can compare the universal and physically transparent criterion of chaos onset (28) with non-universal and contradictory criteria of chaoticity from unitary theory, such as “overlapping resonances”, “(positive) Lyapunov exponents”, “multistability”, “coexisting attractors”, or “unstable periodic orbits”, all of them referring to the dynamically single-valued and thus basically regular problem description (see [K1, K3] for more details). In particular, our criterion (28) remains valid for the *quantum chaos* case, where it describes the emergence of *genuine quantum dynamic randomness*, in full agreement with the quantum-classical correspondence principle [K1, K2, K3, K23], whereas usual theory inevitably fails to find any true quantum chaos.

The obtained unified criterion of chaos (28) provides also the extended meaning of the “well-known” phenomenon of resonance as the condition of system dynamics chaoticity. The same analysis of the unreduced EP equations reveals a similar role of higher resonances as sources of increased chaoticity, so that when chaoticity κ grows from 0 (quasi-regularity) to 1 (global chaos), the degree of randomness makes a jump each time κ passes through a higher resonance value, $\kappa = m/n$, with integer $n > m$ [K1, K2, K3, K14, K23]. As those higher resonances constitute a dense network of rational values of κ , we obtain a well-specified concept of the “fractal structure of chaos”.

The dynamically multivalued fractal is thus the *unified structure* of the world or any its part, exactly expressed by the unreduced interaction problem solution (9)–(27)

and containing various dynamic regimes between global chaos and multivalued SOC. There is also the *universal law* of dynamic existence and development of this unified structure, the *complexity conservation law*. It stems already from the fact that the total system realisation number determining its dynamic complexity according to (20) is fixed by the initial system structure (e.g. by the number of component eigenmodes or their combinations) and therefore remains unchanged during any further system evolution. However, while the total dynamic complexity remains constant, something should change in the process of system structure development. As branches and levels of dynamically multivalued fractal progressively emerge in this process, the *potential form* of interaction complexity, or dynamic information I , is transformed to its realised, *unfolded form* of dynamic entropy S , so that their sum, the *total dynamic complexity* $C = I + S$ remains unchanged, $\Delta C = 0$, $\Delta I = -\Delta S < 0$ [K1, K3, K6, K7, K9, K10, K13, K14, K23].

Note that unlike unitary conservation laws, here the dynamic *symmetry* between changing realisations and their number *conservation* are naturally unified from the beginning, so that there is no difference any more between a “symmetry” and the respective “conservation law” (cf. “Noether’s theorem”), and we obtain the *universal symmetry of complexity* implying *complexity conservation by permanent transformation from dynamic information to dynamic entropy*. Another difference from unitary symmetries is that the latter reflect “ideal” (regular) structure transformations and therefore often become “broken” or inexact, while the universal symmetry of complexity does the opposite by relating quite *irregular* realisation structures within the *absolutely exact* symmetry of complexity, which thus is *never violated* (as it should be the case for a genuine, rigorous law). It also unifies the extended, complex-dynamical versions of *all* (correct) symmetries and laws (see below), remaining separated in unitary theory and mathematics.

In order to obtain a useful dynamic expression of the universal symmetry of complexity, we first introduce the unified elementary forms of complexity known as *time* and *space*, now *explicitly emerging* from the unreduced interaction analysis and absent in the initial system structure and problem formulation (as opposed to their empirically based postulation in unitary theory) [K1, K3, K6, K7, K9, K10, K12, K13, K14, K23].

Physically real, naturally unstoppable and irreversible *time flow* is provided by permanent, interaction-driven change of mutually incompatible system realisations in dynamically random order, so that such real time is practically equivalent to the fundamental dynamic multivaluedness of the unreduced interaction process (see (9)–(18)) and therefore *cannot* be understood within usual, dynamically single-valued description (where it is introduced artificially). Note the fundamental role of *dynamic randomness* in real, complex-dynamical time concept and flow, explaining its *irreversibility* and contrasting with usually assumed, but finally only external, regularity of time. The equally real and emergent, naturally discrete *space structure*, dynamically related to this physically real time, is given simply by realisations themselves (forming the generalised physical space “points”) and system jumps between realisations (determining the elementary “length”).

While space is a tangible, textured entity (reflecting the tangible realisation structure made by the *dynamic entanglement* of interacting entities, mentioned above), time is a really flowing, but “immaterial” entity, reflecting realisation change *process* and related

to material space structure only dynamically (as opposed to the postulated mechanistic “mixture” of abstract space and time variables within a space-time “manifold” in unitary theory). Because of the naturally multilevel, dynamically fractal structure of developing interaction complexity, the physically real time and space will also possess multilevel and fractal structure reproducing that of the universal complex-dynamical fractal. The lowest level of space and time emerges in interaction between two primordial protofields and constitutes the physically real version of fundamental, (externally) smooth, “embedding” and “empty” space and homogeneously flowing time from traditional, Newtonian science [K1, K3, K10, K11, K12, K13, K14, K15, K23], while their higher levels demonstrate space discreteness and irreversible time flow on all scales, giving the observed diversity of world structures and dynamics.

In terms of rigorous mathematics, the *space element*, or *elementary size*, Δx , is given, up to a measurement unit, by the eigenvalue separation of the unreduced EP formalism (9)–(12), $\Delta x = \Delta\eta_i^r$, where the separation of eigenvalues (numbered by i) within the same realisation provides the *space point size*, $r_0 \simeq \Delta x_i = \Delta_i\eta_i^r$, while the characteristic separation of eigenvalues from neighbouring realisations (numbered by r) gives the elementary length (the smallest distance between points), $\lambda \simeq \Delta x_r = \Delta_r\eta_i^r$. The *elementary time interval*, Δt , is obtained as *intensity*, specified as *frequency*, ν , of universally defined *events* of realisation change, $\Delta t = \tau = 1/\nu$. Whereas the events and thus the time flow result causally from the dynamic multivaluedness of unreduced interaction process, a practically useful expression for $\Delta t = \tau$ can be obtained from the above elementary length $\lambda = \Delta x_r$ and the (known) velocity v_0 of signal propagation in the material of interaction components (from a lower complexity level), $\tau = \lambda/v_0$.

Since the dynamically emerging time and space intervals characterise the realisation change process, while the unreduced dynamic complexity (20) is universally defined as a growing function of realisation number, it becomes evident that a *fundamental complexity measure* is provided by the simplest combination of (independent) space and time variables, known as *action*, \mathcal{A} , which acquires now the extended, universal and complex-dynamical meaning [K1, K3, K6, K7, K9, K10, K13, K14, K23, K25, K27]:

$$\Delta\mathcal{A} = p\Delta x - E\Delta t, \quad (29)$$

where the coefficients p and E are recognised as generalised *momentum* and *energy*:

$$p = \left. \frac{\Delta\mathcal{A}}{\Delta x} \right|_{t=\text{const}} \simeq \frac{\mathcal{A}_0}{\lambda}, \quad (30)$$

$$E = - \left. \frac{\Delta\mathcal{A}}{\Delta t} \right|_{x=\text{const}} \simeq \frac{\mathcal{A}_0}{\tau}, \quad (31)$$

\mathcal{A}_0 being the characteristic action magnitude at the considered complexity level, and the evident vector versions of these and further relations are implied if necessary. We see that in its extended meaning action is a universal *integral complexity measure*, while momentum and energy are equally universal *differential complexity measures*.

Due to the dynamically irreversible time flow ($\Delta t > 0$) obtained above and positive total energy ($E > 0$), action can only decrease with time, $\Delta\mathcal{A} < 0$ (see (31)). It measures a *consumable*, irreversibly decreasing form of *potential complexity* introduced

above as dynamic information I , $\mathcal{A} = I$ (we shall call it *complexity-action*). The above conservation, or symmetry, of total complexity $C = I + S$ can now be expressed as

$$\Delta C = \Delta \mathcal{A} + \Delta S = 0, \quad \Delta S = -\Delta \mathcal{A} > 0, \quad (32)$$

where the dynamic entropy, or *complexity-entropy*, S , describing the already created tangible structures, can only grow, at the expense of complexity-action \mathcal{A} , providing another expression of irreversible time direction, as well as the extended, *absolutely universal* versions (and genuine meaning) of the *second law of thermodynamics* (energy degradation principle) and the *least-action principle*, applicable to *any* real system dynamics [K1, K3, K6, K7, K9, K10, K13, K14, K23].

We can now obtain the desired dynamic expression of the universal symmetry of complexity by dividing its initial integral expression (32) by $\Delta t|_{x=\text{const}}$:

$$\frac{\Delta \mathcal{A}}{\Delta t} \Big|_{x=\text{const}} + H \left(x, \frac{\Delta \mathcal{A}}{\Delta x} \Big|_{t=\text{const}}, t \right) = 0, \quad H = E > 0, \quad (33)$$

where the *generalised Hamiltonian*, $H = H(x, p, t)$, considered as a function of emerging space coordinates x , momentum $p = (\Delta \mathcal{A}/\Delta x)|_{t=\text{const}}$ (see equation (30)) and time t , expresses the unfolded, entropy-like form of differential complexity, $H = (\Delta S/\Delta t)|_{x=\text{const}}$, while the last inequality reflects the above generalised second law (or the time arrow direction), in agreement with the generalised energy definition (31). We obtain thus the differential dynamic expression of the universal symmetry of complexity in the form of *generalised*, rigorously derived and universal *Hamilton-Jacobi equation*, with its true, *complex-dynamical* origin and meaning. The finite-increment form of equation (33) reflects the natural discreteness of complex interaction dynamics and will tend to continuous limit for suitable cases and problem scales. The generalised Hamilton-Jacobi equation takes a simpler form for conservative (closed) systems, where the Hamiltonian does not depend explicitly on time:

$$H \left(x, \frac{\Delta \mathcal{A}}{\Delta x} \Big|_{t=\text{const}} \right) = E, \quad (34)$$

with the conserved total energy E defined by equation (31).

The dynamic entropy growth law, constituting an integral part of the universal symmetry of complexity (32)–(33), can be further amplified with the help of *generalised Lagrangian*, L , defined as the total (discrete) time derivative of complexity-action \mathcal{A} :

$$L = \frac{\Delta \mathcal{A}}{\Delta t} = \frac{\Delta \mathcal{A}}{\Delta t} \Big|_{x=\text{const}} + \frac{\Delta \mathcal{A}}{\Delta x} \Big|_{t=\text{const}} \frac{\Delta x}{\Delta t} = pv - H, \quad (35)$$

where $v = \Delta x/\Delta t$ is the velocity of global system motion. Irreducible dynamic randomness of realisation choice at every step of system dynamics implies the unconditional *decrease* of dynamic information, or complexity-action, (32), meaning that

$$L < 0, \quad E, H(x, p, t) > pv \geq 0. \quad (36)$$

As noted above, it is important that in the dynamic multivaluedness paradigm this “generalised second law” refers, due to its universality, to both externally chaotic and *externally regular* structure emergence (the conclusion clearly beyond the limits of conventional unitarity unable to solve its respective entropy-growth problems).

The generalised Hamilton-Jacobi equation (33)–(34) describing the evolution and behaviour of the ensemble of “regular” system realisations has an important complement dealing with the dynamics of special, “intermediate” realisation revealed above in the unreduced EP formalism and forming the transitional state of briefly disentangled, quasi-free system components before they take the next regular, properly entangled realisation. This intermediate realisation and state, the *generalised wavefunction* $\Psi(x)$, is a realistic and universal extension of the quantum-mechanical wavefunction and various distribution functions from unitary theory. It has a chaotically fluctuating internal structure reflecting the dynamically random emergence of each regular realisation, with the dynamic probability obeying both the main rule of the unreduced EP formalism (17)–(18) and the *generalised Born rule*, causally following from this transitional role of the generalised wavefunction and rigorously obtained from the above matching conditions for the state-function coefficients c_i in (15) [K1, K3, K10, K12, K13, K14]:

$$\alpha_r = \alpha(x_r) = |\Psi(x_r)|^2, \quad (37)$$

where x_r is the r -th realisation configuration and for particle-like complexity levels one should imply the value of the generalised distribution function itself at the right-hand side instead of its modulus squared for wave-like complexity levels.

In order to find the dynamic equation for $\Psi(x)$ similar to the Hamilton-Jacobi equation (33)–(34) for regular realisations, we can use the *causal quantisation condition*, following from the symmetry (conservation) of complexity, but applied now to one entire cycle of transition from the wavefunction to a regular realisation and back [K1, K3, K10, K12, K13, K14, K20, K23]:

$$\Delta(\mathcal{A}\Psi) = 0, \quad \Delta\mathcal{A} = -\mathcal{A}_0 \frac{\Delta\Psi}{\Psi}, \quad (38)$$

where \mathcal{A}_0 is a characteristic complexity-action magnitude that here may contain also a numerical constant reflecting specific features of the considered complexity sublevels (thus, at quantum sublevels $\mathcal{A}_0 = i\hbar$, where $\hbar = h/2\pi$ is Planck’s constant). Using relation (38) in the Hamilton-Jacobi equation (33), we obtain the causally derived *universal Schrödinger equation* for the *realistically interpreted* generalised wavefunction at *any* complexity level (starting from the lowest, quantum levels, now liberated from all postulated “mysteries” [K1, K3, K10, K12, K13, K14, K15, K23]):

$$\mathcal{A}_0 \frac{\Delta\Psi}{\Delta t} \Big|_{x=\text{const}} = \hat{H}(x, \hat{p}, t) \Psi(x, t), \quad \hat{p} = -\mathcal{A}_0 \frac{\Delta}{\Delta x} \Big|_{t=\text{const}}, \quad (39)$$

where the momentum operator \hat{p} and the Hamiltonian operator, $\hat{H}(x, \hat{p}, t)$, are obtained from momentum p and the Hamiltonian function $H = H(x, p, t)$ of equations (30), (33) by the same causal quantisation (38). For the closed (conservative) system case we similarly obtain from (34) the respective reduced form of the universal Schrödinger equation:

$$\hat{H}(x, \hat{p}) \Psi(x) = E\Psi(x). \quad (40)$$

This causally derived and now complete dynamic expression of the universal symmetry of complexity, the *universal Hamilton-Schrödinger formalism* (33)–(40), does apply, together with the initial “global” expression (32), to any real system dynamics (thus justifying also the Hamiltonian form of the initial existence equation (1)) and therefore

underlies *any* (correct) law, “principle” and dynamic equation from unitary theory (where it is typically postulated in a semi-empirical way). In order to demonstrate this in more detail, we can, for example, expand the Hamiltonian $\hat{H}(x, \hat{p}, t)$ in (39) in a power series of \hat{p} (and Ψ), which gives (for the ordinary, continuous-derivative version):

$$\frac{\partial \Psi}{\partial t} + \sum_{m=0}^{\infty} h_{mn}(x, t) [\Psi(x, t)]^m \frac{\partial^n \Psi}{\partial x^n} + \sum_{m=0}^{\infty} h_{m0}(x, t) [\Psi(x, t)]^{m+1} = 0, \quad (41)$$

where $h_{mn}(x, t)$ are arbitrary functions, while the dependence on Ψ may arise from the effective potential. We can see that various usual model equations are but particular cases of (41), providing thus their true, causally specified origin, including the complex-*dynamic* origin of *any*, usually postulated *nonlinearity* (with similar results for a series expansion in (33), (34) and (40)). Details for quantum, relativistic and other laws can be found elsewhere [K1, K3, K7, K9, K10, K12, K13, K14, K16, K17, K18, K19, K20, K23].

Arriving thus at the end of major framework of the universal science of complexity, we should emphasize its key mathematical distinctions from usual, unitary theory, or the *new mathematics of complexity and emergence* [K1, K3, K6, K7, K10, K21, K23, K24]:

- (i) *Non-uniqueness* of any real problem solution, in the form of *fundamental dynamic multivaluedness (redundance)* of mutually incompatible system realisations revealed within the unreduced EP method (9)–(27), with the ensuing *unceasing, dynamically random (chaotic) internal change* as the unique way of real object existence.
- (ii) *Absence and impossibility of self-identity postulate* and property, which leads to the omnipresent feature of (structure) *emergence* and the origin of *physically real time flow*: $\mathfrak{A} \neq \mathfrak{A}$, for any structure \mathfrak{A} , due to the same universal dynamic multivaluedness of \mathfrak{A} , leading to its permanent internal change.
- (iii) *Rigorous, universal and irreducible expression of material quality (texture)* of any described (emerging) system structure. It is obtained in the form of *dynamically multivalued fractal entanglement* of interacting system components in the unreduced problem solution (9)–(18), (21)–(26).
- (iv) The universal and omnipresent origin of *intrinsic, dynamic and genuine randomness* within *any* real structure, process and evolution, at any reality (complexity) level. It is due to the same major feature (i)–(ii) of fundamental dynamic multivaluedness, which now reveals internal randomness within any real structure, together with the actually synonymous notion of universal dynamic complexity (20) and the true meaning of other vague notions of usual theory, such as *nonintegrability, nonseparability, noncomputability, uncertainty (indeterminacy), undecidability, stochasticity, broken symmetry, free will, etc.* [K1, K2, K3, K7, K10, K23] (cf. [P]).
- (v) *Dynamic discreteness, or causal quantisation*, of unreduced interaction results (and thus all structures), due to the *holistic* character of *unreduced* interaction, with its feedback loops and finite realisations. It gives rise to a deeply structured, *qualitatively inhomogeneous, or nonunitary*, character of any system configuration and evolution, summarised in the dynamic origin of *fundamentally discrete* (and fractally structured) *space*.

It is important to emphasize that these distinctive features of the new mathematics of complexity and emergence (i)–(v) result from the universal complete solution of arbitrary interaction problem and therefore are *unified* within the *unique structure* of dynamically multivalued (probabilistic) fractal and *unique law* of the universal symmetry of complexity representing *all* the real world structures and laws. This means, in particular, that unlike unitary theory, the new mathematics of complexity can be applied with *equal rigour* to description of objects and processes from conventional “exact”, “natural” or “social” sciences and arts [K1, K3, K5, K6, K10, K16, K17, K18, K19, K20, K21, K23, K26, K27], as demonstrated e. g. by applications outlined below.

One can add to this unified mathematics of complexity several important corollaries of the universal symmetry of complexity, or *complexity principles*, related to applied aspects of the universal science of complexity [K1, K3, K7, K10, K16, K17, K20, K23]:

- (I) The *complexity correspondence principle* implies efficient or sensible interaction mainly between systems of comparable dynamic complexity. It means, in particular, that a system of certain complexity can be efficiently designed and controlled only by systems and techniques of higher, but not lower, dynamic complexity, with numerous critically important applications to modern real-world problems, from unitary “quantum computers” (impossible as such) to various computer applications, artificial intelligence, global sustainability and other development problems [K1, K3, K7, K10, K16, K17, K18, K19, K20, K21, K23].
- (II) The *complex-dynamical control principle* is based on the complexity development aspect of the universal symmetry of complexity and states that any efficient, sustainable control necessarily implies suitable *complexity development* (of both controlled and controlling systems), with inevitable *partially random change*, in contrast to “limiting” or “fixing” approach of usual, unitary control theory (including its explicitly complex-dynamical aspects, such as “chaos control”).
- (III) The *unreduced (free) interaction principle* refers to the exponentially huge power and efficiency of natural multicomponent interaction processes, as opposed to their power-law efficiency from unitary-model projection [K3, K6, K7, K10, K16, K17, K18, K20, K23, K25]. Referring to the dynamically multivalued fractal of unreduced interaction process, one can see that its operation power P_{real} , determined by the total realisation number $N_{\mathfrak{R}}$ (proportional to the unreduced complexity C), can be estimated as the number of system mode or link combinations:

$$P_{\text{real}} \propto N_{\mathfrak{R}} = L! \rightarrow \sqrt{2\pi L} \left(\frac{L}{e}\right)^L \sim L^L \gg \gg L, \quad (42)$$

where the number of system links L can already be a very large number, essentially exceeding the number of interacting system components N (thus for both human brain and genome $N > 10^{10}$, $L > 10^{14} \gg N$, see below). The obtained exponentially huge power of unreduced complex dynamics P_{real} , dramatically exceeding its unitary-model estimates, $P_{\text{reg}} \propto L^\beta$, $\beta \sim 1$, $P_{\text{real}}/P_{\text{reg}} \sim L^{L-\beta} \rightarrow \infty$, provides the origin of the “miraculous” properties of life, intelligence and consciousness.

2. Complex-dynamical nanometal physics and active condensed matter. As mentioned in the previous section, the unified probabilistically fractal structure of the world emerges dynamically from the initial simplest interaction configuration of homogeneous attraction between two originally structureless media, the gravitational and electromagnetic protofields, which progressively and causally gives rise to all known elementary particle species, with their dynamically obtained intrinsic properties, naturally discrete space and irreversibly flowing time, as well as rigorously derived and intrinsically unified laws of quantum and relativistic dynamics [K1, K3, K10, K11, K12, K13, K14, K15, K23].

In this paper we want to outline in more detail the structural and dynamic properties of many-body interaction results at the next higher complexity level, i. e. interaction between simplest bound states of elementary particles (atoms) and their further agglomerates in *structures of nanoscale size*. As shown elsewhere [K1, K3, K10, K12, K13, K14], atoms demonstrate the complex-dynamical transition to *permanently localised, classical type of behaviour* due to the strongly chaotic dynamics of their components, where essentially quantum behaviour can transiently reappear in further interactions of atoms.

As a result, those arbitrary many-body nanosystem interactions represent a complicated mixture of permanently changing, complex-dynamical quantum and classical processes, with a dominating degree of *strongly chaotic behaviour*. The latter feature follows rigorously from our unified criterion of global chaos (28), where the nanoscale length of driving interactions limits participating frequency values to comparable magnitudes determined e. g. by the atomic Bohr frequency, so that the chaoticity parameter κ of (28) cannot deviate significantly from the global chaos value of 1, $\kappa \sim 1$.

This conclusion is qualitatively different from conventional theory, which does not contain the intrinsic origin of chaos and cannot find any true chaoticity in essentially quantum systems. As a result, its strongly limited, dynamically single-valued models predict fantastically high performance possibilities for *quantum computer* systems within the unitary interaction scheme devoid of any true chaoticity. However, if we take into account the unreduced, dynamically multivalued character of real quantum interaction processes, then the above nanoscale limitation of essential chaoticity proves the *impossibility* of unitary quantum computer operation, even in the absence of any noisy influences (usually evoked as major obstacles to quantum computer creation) [K3, K4].

This negative conclusion is compensated by a positive possibility of highly efficient, but *complex-dynamical, rather than unitary, quantum and classical machines* on the nanoscale interaction range, based on the unreduced interaction complexity principle (III), which reveals the exponentially huge efficiency (42) of complex interaction dynamics, just due to its chaoticity. Such machines are designed as a combination of essentially quantum, hybrid, and classical complex-dynamical processes, with permanent transitions between them [K3, K4]. They can reproduce the huge efficiency of their natural, biological prototypes, underlying all real “magic” properties of life and intelligence.

While trying to approach practical realisation of those essentially complex-dynamical nanosystems, we may first recall that usual, macroscopic systems often perform their functions due to metallic atom properties, with their ability to liberate few external, “valent” electrons. Those quasi-free electrons become “collective” and determine the “bulk”

material properties used in applications. By contrast, natural, biological (and therefore essentially complex-dynamical) systems use the same properties of individual metal atoms in a qualitatively different way, where atoms preserve their individuality, while their electronic reactivity is used to initiate complex chain processes of high efficiency.

The idea of the *new, complex-dynamical nanometal physics* is to reproduce artificially the “method” of biological structures by using individual metal atoms or their molecular clusters as efficient interaction centres driving complex-dynamical interaction networks, rather than influencing bulk properties in usual metal physics. Note that this new nanometal physics will still be a physics of metals relying essentially on metal atom features, but used “individually” and complex-dynamically, rather than “collectively” and perturbatively (linearly) in usual solid-state physics. Particular complex nanosystem dynamics and properties can then be described by the above universal formalism (section 1), where manifestations of *unreduced dynamic multivaluedness* and related *true chaoticity* will be essential for the expected superior efficiency (42).

The ultimately wide generalisation of this idea leads to the *new, essentially complex-dynamical condensed matter design* that can be called *active, or living, condensed matter*, in which unreduced dynamically multivalued interactions of properly arranged interaction units create the environment of permanent adaptive change and intelligent response to external influences due to suitably chaotic internal dynamics of superior efficiency. These processes would often start at the nanoscale interactions of small atomic and molecular groups with causal quantum-classical transitions [K1, K3, K10, K12, K13, K14] and other complex-dynamical features described above and governed by the complexity principles (I)–(III). Depending on the application, they can include all larger scale interactions, determining e. g. mechanical properties, involving structure defects, etc.

Most promising applications of active condensed matter structures include nanobiosystems, molecular electronics, artificial intelligence and any condensed matter systems with “intelligent” behaviour. Direct use of the above *causally complete complexity science*, with its new mathematics (i)–(v) and complexity principles (I)–(III), is *indispensable* for efficient progress in these applications, as demonstrated by the mentioned conclusion about impossibility of unitary quantum computation, which should be replaced by essentially complex-dynamical nanomachines [K3, K4]. The tendency of modern solid-state physics to strong-interaction cases also involves essentially complex-dynamical behaviour, which needs now to be explicitly recognised as such, including the extended and causally complete approach to strong-interaction problem solution (section 1), otherwise remaining unachievable within the standard unitary theory. One can cite high-temperature superconductivity, magnetoelectric multiferroics, high-entropy alloys and other multi-component systems with strong interaction as examples of that kind of solid-state systems.

Note finally that there can be no other way of new progress in fundamental condensed-matter science, since all conventional weak-interaction and “exact-solution” cases have practically exhausted their possibilities, while those of essentially complex-dynamical and active condensed matter structures are yet to be explored within the emerging wide spectrum of applications, with extremely promising prospects rigorously demonstrated by the above extended framework of the universal science of complexity.

3. Complex-dynamical nanobioscience, causal genetics and integral medicine.

With the reference to previous work [K3, K4, K5, K6, K23, K25], here we mention only briefly this group of life-science applications of our universal complexity concept, also in order to emphasize its close relation to the above ideas of nanometal physics and active condensed matter. Indeed, as already noticed above, the latter kind of system to be created is but an artificial reproduction of the unreduced living matter structure and dynamics. We certainly must also use the proposed causally complete understanding of unreduced interaction complexity as the unified foundation for reliable genetics and rigorously correct, integral medicine, as opposed to conventional purely empirical trial-and-error search combined with the prevailing unitary interpretation approach, which leads *inevitably* to fatal errors in life-science applications that deal with superior levels of *unreduced* dynamic complexity. As a result of this unreduced description of life complexity, one can also imagine numerous “mixed” applications of thus obtained unified nanobioscience, where suitable combinations of unreduced natural and artificial structure complexity can produce creative, life extending directions of integral medicine.

We first recall that the unified structure of the unreduced dynamically probabilistic fractal (section 1) provides already major properties of living systems, including autonomous dynamic adaptability by the chaotic interactive search process, intrinsic development ability (or “*élan vital*”) by the universal symmetry of complexity, and superior operation efficiency (42) of the free interaction complexity principle (III).

If now we apply these general results to human genome analysis, we arrive at the important conclusion that it constitutes a system of strongly interacting components, where in average each elementary nucleotide, or base (pair), interacts essentially with any other one, so that human genome is realised as a space of permanent intense interaction (giving rise, in particular, to highly nonlinear effects), rather than a mainly sequential, linear programme similar to unitary computer operation in the usual genetics paradigm [K6, K23, K25]. Indeed, as the effective number of strong interaction links in genome, L_{genome} , cannot be smaller than that in the brain, $L_{\text{genome}} > L_{\text{brain}}$ (due to the symmetry of complexity), we find, for $L_{\text{brain}} = N_{\text{neuron}} n_{\text{syn}} \approx 10^{10} \times 10^4 = 10^{14}$, that the number of effective interaction links per gene $n_{\text{eff}} > L_{\text{brain}}/N_{\text{gene}} \approx 3 \times 10^9 \approx N_{\text{base}}$, where $N_{\text{base}} \approx 3 \times 10^9$ is the number of human genome nucleotide bases, $N_{\text{gene}} \approx 3 \times 10^4$ is its number of genes, $N_{\text{neuron}} \approx 10^{10}$ is the number of brain neurons, and $n_{\text{syn}} \approx 10^4$ is the number of synaptic interactions per neuron. We see thus that every gene should in average interact not only with any other gene, but also with any individual nucleotide pair, which proves the above conclusion about the genome being the omnipresent strong interaction space. It explains the key role and great proportion of the famous “noncoding” DNA parts occupying almost the entire DNA length (98 % in the human genome), which remains otherwise mysteriously big in the framework of conventional paradigm.

And we finally find the exponentially huge efficiency (42) of *both* human genome [K6, K25] and human brain [K20, K23] activity, with the effective number of interaction links $L \geq 10^{14}$, explaining the “magic” properties of life and consciousness. The ensuing practical conclusion is that this huge efficiency can be properly managed (and combined with respective artificial nanobiosystems) only if one knows the detailed structure and

complex-dynamical fractal links of at least major interaction chains involved. In the absence of such knowledge (which is close to its practical state today), any simplified, effectively blind manipulation with the genome structure within the dominating unitary science paradigm can easily lead to unpredictable negative results for the corresponding living organism dynamics, in addition with the unpredictably delayed time of their explicit appearance [K6, K23, K25]. It is but another, inevitable manifestation of the complexity correspondence principle (I).

The idea of *integral medicine* is actually obtained as extension of this unreduced complexity dynamics from the level of genome interactions to higher levels of macroscopic organism dynamics, where the latter is also represented by the dynamically probabilistic fractal of its unreduced interactions, properly specified for each *individual* organism. It is then surveyed and managed as such, in the full richness of its essential dynamical dimensions, instead of a very limited projection to the visible system of direct 3D links and structures in the conventional medicine paradigm fundamentally limited by the same complexity correspondence principle. The universal features of dynamic complexity structure, regimes, laws and principles described above (section 1) should serve as indispensable guiding lines for the genuine understanding and truly efficient management of this unreduced, dynamically fractal life complexity.

4. Causal economics, efficient risk management and sustainable governance after the globalisation complexity threshold. Passing now to the superior world complexity level of global economic and social development, we note the modern extraordinary situation of the growing omnipresent global crisis, where all efforts to understand and efficiently manage these “human” complexity levels end up in a disturbing state of sporadic and useless trial-and-error efforts, leading to the catastrophic complexity degradation, with the emerging fatal consequences for the world development (see e.g. [C, R, F, S, G, K23]).

This kind of situation is causally described in our universal complexity development concept (as a part of the symmetry of complexity, section 1) as the necessary transition to the superior complexity level opening new possibilities for development and problem solutions, in the absence of which the system (global civilisation in this case) enters the “death branch” of destructive complexity development [K18, K19, K20, K21, K23]. This necessary *complexity transition* occurring after a well-defined *complexity threshold* of “globalisation” (recently attained by the world civilisation) illustrates again the complexity correspondence (I) and complex-dynamical control (II) principles from section 1, according to which the growing civilisation complexity can be efficiently managed only within the unreduced understanding of its complexity suggesting, in particular, the creative, complexity-transformation kind of control and risk minimisation.

While conventional unitary theories inevitably fail to provide even the consistent and fundamental *definition of risk* in economic, financial, and social systems, our unreduced interaction analysis shows that the desired “stability” of any real system, opposed to “crisis” and other “risks”, is nothing but permanent *progressive* changes of its complexity-entropy (or sustainability thus rigorously defined) based on the intrinsic instability of real,

multivalued interaction dynamics and related causally random changes [K1, K7, K18, K19, K20, K23, K29]. Therefore our universal principle of complex-dynamical control (II) emphasizes the *optimal growth of complexity-entropy* through *dynamically random* changes, instead of the false unitary control strategy of mechanistic restrictions in order to preserve the desired (external and illusive) maximum regularity and status quo.

Based on the universal complexity development paradigm (32), with the discrete transformation of potential complexity-action $\mathcal{A}(x, t)$ to the explicit form of complexity-entropy $S(x, t)$, $\Delta S(x, t) = -\Delta \mathcal{A}(x, t) > 0$, we can propose the *universal definition* of “undesired”, negative instability kind referred to as *risk* in the form of increased (maximum) probability of *destructive* complexity-entropy growth along the “the death branch” on the universal complexity evolution curve [K18, K19], with the *quantitative risk magnitude*, R , equal (up to a coefficient) to the reciprocal complexity-entropy growth rate, or reciprocal generalised energy E (cf. (31)) [K23, K29]:

$$R = \left(\frac{\Delta S}{\Delta t} \Big|_{x=\text{const}} \right)^{-1} = \left(-\frac{\Delta \mathcal{A}}{\Delta t} \Big|_{x=\text{const}} \right)^{-1} = \frac{1}{E} \simeq \frac{\tau}{\mathcal{A}_0}, \quad (43)$$

with the opposite *sustainability magnitude*, $\mathfrak{S} = 1/R$, coinciding (again up to a coefficient) with the total development energy E :

$$\mathfrak{S} = \frac{1}{R} = \frac{\Delta S}{\Delta t} \Big|_{x=\text{const}} = -\frac{\Delta \mathcal{A}}{\Delta t} \Big|_{x=\text{const}} = E \simeq \frac{\mathcal{A}_0}{\tau}, \quad (44)$$

where \mathcal{A}_0 and τ are the characteristic values of complexity-action magnitude (or variation) and change period respectively.

As generalised energy E characterises the temporal rate of complexity-action transformation to complexity-entropy, i.e. actually the *rate of progress*, it is natural that the positive, progress-bringing result of unstable multivalued dynamics, or sustainability \mathfrak{S} , is proportional to the rate of progress, while the negative result of the same omnipresent instability, in the form of degradation probability, or risk R , is inversely proportional to the same progress rate E , so that $R\mathfrak{S} = 1$. The risk magnitude R is high (on any scale) during stages of weak complexity-entropy growth (between its step-wise transition jumps) and especially during and after the fatal establishment of the death branch of slow, destructive complexity-entropy growth (starting as “stagnation”), while it is minimal during rapid system transitions to higher complexity levels. Correspondingly, all risks will grow with slowing down (higher τ) of transformation of smaller amounts of complexity-action \mathcal{A}_0 to complexity-entropy, in accord with (44).

It is important that unified dynamic complexity (S, \mathcal{A}) and its growth rate in the above risk definition originate in the *unreduced, multivalued interaction dynamics* (1)–(27) with permanent probabilistic change of incompatible system realisations and thus include the *totality* of chaotically occurring events. The obtained *unified* risk magnitude definition (43) shows that the truly reliable (and universal) way of risk reduction can only be based on *sustainable, intrinsically progressive complexity development* liberated from crises and impasses of unitary organisation. While the definite establishment of that genuine sustainability regime of intrinsically low risks occurs only after the transition to the superior complexity level of the Harmonical System [K1, K18, K19, K20, K23], the proposed risk definition and criterion remains valid at any complexity level.

Whereas the main way of risk reduction (in particular in economy and finance) is the search for further *progressive growth of unreduced complexity-entropy*, the *unreduced* interaction dynamics analysis at its given complexity level (for example during phases of slow complexity-entropy growth) also provides universal understanding of the origin of current risky events and ways of their probability reduction. We can see, in particular, that any relatively stable, low-risk system operation mode corresponds to the unified *SOC regime* of internally chaotic, but externally quasi-regular complex dynamics (section 1), while the opposite limiting regime of *global chaos* implies maximum risk values. Recalling the *unified criterion of global (or partial) chaos regime* in terms of major frequency resonances between system operation modes (28), we arrive at the rigorously substantiated conclusion that in order to reduce risk *one should avoid frequency resonances* between major repeated operations involving essential quantities of money or other exchange matter, i. e. one should avoid the condition $m\omega_q \cong n\omega_\xi$, where m, n are small integers and ω_q, ω_ξ are major system operation frequencies involving essential exchanges (e. g. frequencies characterising respectively internal operation cycles and external interactions of an economic system unit). While in some cases such rules may be implemented by intuitively felt empirical considerations, we provide the *universal and rigorously substantiated criterion* that cannot be avoided in any real system dynamics.

In ordinary situations with relatively smooth complexity transformation we can apply our unified Hamilton-Schrödinger formalism (33)–(41) in order to describe the inhomogeneous, highly nonlinear and chaotic dynamics, distribution and evolution of the introduced risk magnitude R (43). Thus, a usual classical Hamiltonian leads to the following generalised Hamilton-Jacobi equation (33) describing the unreduced dynamics of complexity-action $\mathcal{A}(X, t)$:

$$\frac{\partial \mathcal{A}}{\partial t} + \sum_{i=1}^N \left\{ \frac{1}{2m_i} \left(\frac{\partial \mathcal{A}}{\partial x_i} \right)^2 + \left[U_i(x_i, t) + \sum_{j=1, j \neq i}^N V_{ij}(x_i, x_j) \right] \mathcal{A}(X, t) \right\} = 0, \quad (45)$$

where $X = (x_1, x_2, \dots, x_n)$ is the vector of all relevant agent coordinates (configuration measures) x_i , $U_i(x_i, t)$ is the generally time-dependent external influence potential acting on the i -th agent with the generalised mass m_i , $V_{ij}(x_i, x_j)$ is the potential of interaction between the i -th and j -th agents, and N is the total number of agents. Using our unreduced interaction analysis with the help of the generalised EP formalism (1)–(27), we can find the complete, dynamically multivalued, or suitable approximate solution for complexity-action $\mathcal{A}(X, t)$ and then using (43) find the corresponding, in general also multivalued (i. e. probabilistically distributed (25)), risk distribution and evolution in time and space of relevant (financial, economic, technological, social, or political) variables, $R(X, t) = -(\partial \mathcal{A} / \partial t)^{-1}$. This approach opens up absolutely new, mathematically exact and causally substantiated possibilities for the *totally objective* analysis of risk dynamics and evolution in arbitrary real system at any level of its (growing) complexity. Due to the direct link between our objectively defined risk and sustainability values (44), we thus obtain actually the *causally complete, mathematically rigorous and therefore totally reliable tool of entire economic development control and management* (to be compared with critically growing problems of usual economics [W, B]).

As noted above (see also [K1, K18, K19, K20, K23, K29]), progressive complexity-entropy growth of the global civilisation system towards lower-risk dynamics is possible today only by a step-like transition to the superior complexity level corresponding to vanishing economical and social risk values, with the single, already dangerously growing alternative of destructive, death-branch complexity evolution, which realises the opposite tendency of dramatically growing risks. Today we have therefore the nontrivial, historically significant bifurcation point leading to that qualitatively big and always growing difference between the emerging near-future low-risk and high-risk levels and (incompatible) evolution tendencies. Using the universal complexity laws (I)–(III) driven by the unified symmetry of complexity (section 1), we can specify major features of the desired superior-complexity level of development also referred to as the Harmonical System, or genuine sustainability [K1, K7, K18, K19, K20, K21, K23]. Those features include the new, *emergent* kind of social structure and corresponding *intrinsically progressive, reason-based governance, creative (complexity-increasing) production processes, new kind of settlement, infrastructure and lifestyle*, and certainly *new organisation, content and role of intrinsically complete knowledge* of (growing) unreduced dynamic complexity.

The *qualitatively new, reason-based (global) governance system* [K28] deserves a special mention here as the concrete and indispensable element to be urgently introduced in addition to (and then increasingly instead of) traditional, unitary governance. In accord with the universal complexity correspondence principle (I), it appears in this higher-complexity tendency as a superior governance structure oriented to explicit, rigorously substantiated and clearly presented problem solutions (as illustrated by the above risk and sustainability dynamics formalism (43)–(45)), which need not immediately replace the existing, unitary governance structures preserving their full decision power. Now, however, these traditional governments and populations they govern are provided with the *causally substantiated and openly presented* guidelines of *objectively complete*, provably consistent problem solutions in the spirit of intrinsically progressive complexity growth. And although they are not formally obligatory for realisation, they clearly demonstrate the extended possibilities of intrinsically progressive complexity development.

We obtain thus the superior-complexity level of unified *social conscious intelligence* of the planetary “organism” (including all its interactive and omnipresent global networks), which is absent in the known traditional social structures relying only on the empirical, “animal” and very short-sighted kind of intelligence. However, the latter “spontaneous”, “invisible-hand” kind of development of social structure is now totally exhausted at the attained high level of “globalised” civilisation complexity (marking the fundamental *complexity threshold* [K23]), after which further progress can only be realised with the help of explicit social consciousness layer, providing also the advanced, *superior-complexity version of traditional unitary democracy* (usually erroneously considered to be the best possible one, within its own paradigm). This superior-complexity, explicitly conscious democratic system is not limited in its intrinsically progressive, low-risk development and provides its ever growing, creative liberties due to the qualitatively extended, causally complete kind of knowledge and the new system of science of unreduced dynamic complexity [K1, K7, K18, K19, K20, K21, K23, K28, K29].

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