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Multi-Criteria Single Batch Machine Scheduling Under Time-of-Use Tariffs

Junheng Cheng, Feng Chu, and Peng Wu

Abstract Most of the industrialized countries are moving towards the fourth stage of industrialization. This development has provided immense opportunities for industrial sustainability. As the largest energy consumer in the world, most of the industrial sector's consumption is in the form of electricity. In recent years, to strengthen the peak load regulation capability, time-of-use (ToU) pricing has been implemented in many countries to encourage consumers to shift their use from peak to mid- and off-peak periods such that their energy bills can be reduced. In this chapter, we first introduce a basic single machine batch scheduling problem under ToU electricity tariffs. Then it is extended by further considering machine on/off switching. Finally, a single machine batch scheduling problem under ToU tariffs in a continuous processing environment is investigated. For the three considered problems, appropriate mathematical models are established, and their problem properties and complexities are demonstrated.

11.1 Introduction

The growing population brings great opportunities for economic and social development, but also presents enormous challenges to limited resources. Globally, facing the contradiction between the continuously increasing worldwide demand for consumer goods and sustainable evolvement of human existing environment,

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industrial value creation must turn towards sustainability (Stock and Seliger 2016). Recent technological leaps have paved way for highly mechanization and automation in industry, most of the early industrialized countries are moving towards the era of Industry 4.0. This development provides tremendous opportunities for the realization of sustainable manufacturing, where energy saving takes a very important position. Industrial sector is the largest energy consumer in the world. Improving their energy usage efficiency is highly important to enhance the competitiveness of manufacturing enterprises in the new era and promote sustainable development of the economy.

Electricity is one of the most widely used energies, which accounts for about 30% of total energy consumption in the area of Asia Pacific Economic Cooperation (Fang et al. 2011). As a renewable resource, it is cleaner, safer, and more convenient than many nonrenewable resources, such as coal and oil. With the acceleration of electrification process, electricity will play a more important role in modern manufacturing industry. However, electricity cannot be efficiently stored. Thus its production, transmission, and consumption has to be conducted simultaneously. In addition, electricity demand distribution is uneven over time. To improve peak load regulation ability, demand response (DR) strategy has been widely implemented in the world. It encourages customers to change their normal consumption patterns to respond to the varying electricity prices and save total power cost. ToU electricity tariffs are one of the most important DR strategies. It provides varying electricity prices, for example, low price in off-peak periods and high price in on-peak periods.

Under ToU tariffs, manufacturing enterprises can simultaneously achieve the optimization on production efficiency and energy cost via more reasonable production scheduling, and this topic has attracted increasing interest from researchers recently. For single-machine environment, Shrouf et al. (2014) proposed a genetic algorithm for jobs with a given processing order to optimize total electricity cost with turn-on/off strategy under ToU electricity prices. Fang et al. (2016) considered energy cost saving for uniform-speed and speed-scalable machines and proposed several heuristics to obtain near-optimal solution. Che et al. (2016) developed a mixed-integer linear programming (MILP) model and a greedy insertion heuristic to minimize total electricity cost. For parallel machines, Moon et al. (2013) investigated an unrelated parallel machine scheduling problem under varying electricity prices. Ding et al. (2016) presented a MILP model and a column generation based heuristic to minimize total electricity cost respecting a given makespan. Later, Cheng et al. (2018) improved Ding's model. For flow shops, Luo et al. (2013) developed a novel ant colony optimization based meta-heuristic to minimize both electricity cost and makespan. Zhang et al. (2014) formulated a time-indexed MILP model to minimize total electricity cost and carbon emissions while ensuring the production throughput at the same time. All of the above studies focused on classical machine environment.

Batch processing manufacturing system representing a typical production environment has been widely encountered in modern manufacturing industries, such as steel manufacturing (Tang et al. 2001; Wang et al. 2019), semiconductor manufacturing (Jia et al. 2015; Uzsoy et al. 1994), and aircraft industry (Xu and Bean 2016), and most of them are energy-intensive ones. A specific feature of batch processing is that a processing machine can process multiple jobs at a time.

As a result, batch scheduling is usually more complex than traditional production scheduling, because it needs to optimally group the jobs into batches and schedule the formed batches. A majority of batch scheduling problems have been proved to be NP-hard, even under single-machine environment. Considering ToU tariffs, a traditional batch processing machine scheduling problem will be more complicated; hence, it is significant to investigate such problems in theoretical perspective to guide the practice in manufacturing industries. This chapter discusses some most recent advances and issues in this field including single batch machine scheduling under ToU tariffs (SBMS-ToU) (Cheng et al. 2016), SBMS-ToU considering machine on/off switching (SBMS-ToU-on/off) (Cheng et al. 2017), and SBMS-ToU in continuous processing (SBMS-ToU-CP) (Cheng et al. 2016). Their problem descriptions and formulations will be introduced in the following sections.

11.2 Single Batch Machine Scheduling Under ToU Tariffs

Single batch machine is a basis of more complicated batch processing systems and has been widely encountered in real production environments, such as semiconductor manufacturing industry (Lee et al. 1992; Wang and Uzsoy 2002) and shoe manufacturing industry (Fanti et al. 1996). A variety of single batch machine scheduling (SBMS) problems have been proved to be NP-hard, where most of them focus on optimizing production efficiency, e.g., makespan, total completion time, maximum tardiness. Taking ToU tariffs into consideration, a traditional SBMS problem will be further complicated, since it has to not only group jobs into batches and determine batch processing sequence, but also position batch processing time period. In order to simultaneously optimize environmental benefits as well as production efficiency, total electricity cost and makespan are considered as the two optimization objectives.

11.2.1 Problem Description

A bi-objective single batch machine scheduling problem under ToU tariffs (SBMS-ToU) can be described as follows:

A given set of $J = \{1, 2, \dots, |J|\}$ jobs is to be processed on a single batch processing machine within a horizon $I = \{1, 2, \dots, |I|\}$. The duration of period $i \in I$ is denoted as S_i . Job $j \in J$ is nonpreemptive and has a processing time p_j . Any p_j is less than the duration of any period i ; i.e., $S_i \gg p_j, \forall j \in J, \forall i \in I$. Without loss of generality, we assume that the jobs are numbered in nonincreasing order of the processing times; i.e.,

$$p_1 \geq p_2 \geq \dots \geq p_{|J|}.$$

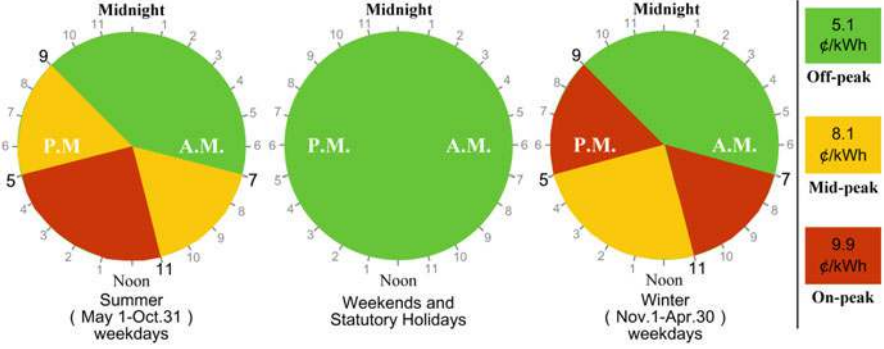


Fig. 11.1 An example of time-of-use tariffs (Source: Ontario Energy Board)

The jobs can be regrouped to $|B|$ (to be optimized) batches and each batch can contain at most C jobs. Therefore, we must have $\lceil |J|/C \rceil \leq |B| \leq |J|$ (Ikura and Gimple 1986). The processing time of a batch is determined by the longest processing time job in the batch.

The processing of a batch should be completed before the end of a period or it must wait for the beginning of another period. Each period can be regarded as a workshift, whose unit electricity cost can be calculated according to the tariff information. Take the TOU tariffs of Ontario (see Fig. 11.1) for example, for a three-shift in a work day: 8–16 h, 16–0 h, 0–8 h, the corresponding unit electricity costs are as follows: $e_{8h-16h} = (11.4 * 3 + 14.0 * 5)/8 \text{ ¢/kWh} * 1 \text{ kWh/h} = 13.0250 \text{ ¢/h}$, $e_{16h-0h} = 9.4125 \text{ ¢/h}$, and $e_{0h-8h} = 8.1625 \text{ ¢/h}$. Generally, a work day is often composed of two or three work shifts according to the types of tariffs.

11.2.2 Mathematical Model

The parameters and decision variables for model formulation can be summarized as follows:

Indices

j : index of jobs, $j \in J = \{1, 2, \dots, |J|\}$;

b : index of batches, $b \in B = \{1, 2, \dots, |B|\}$;

i : index of periods, $i \in I = \{1, 2, \dots, |I|\}$;

Parameters

J : set of all jobs, $J = \{1, 2, \dots, |J|\}$;

B : set of batches, $B = \{1, 2, \dots, |B|\}$;

I : set of time periods on the planning horizon, $I = \{1, 2, \dots, |I|\}$;

C : capacity of a batch;

p_j : processing time of job j , $\forall j \in J$;
 s_i : starting time of period i , $\forall i \in I$;
 S_i : duration of period i , $\forall i \in I$, in which $S_i = s_{i+1} - s_i$;
 e_i : unit electricity cost of period i , $\forall i \in I$;

Decision Variables

$|B|$: number of batches;
 $x_{j,b,i} = 1$, if job j is assigned to batch b and processed in period i ; otherwise 0;
 $\forall j \in J, \forall b \in B, \forall i \in I$;
 $y_{b,i} = 1$, if batch b is assigned to period i ; otherwise 0; $\forall b \in B, \forall i \in I$;
 $z_i = 1$, if at least one batch is assigned to period i ; otherwise 0; $\forall i \in I$;
 $P_{b,i} = P_b = \max\{p_j \mid j \in b\}$, if batch b is processed in period i , where P_b is the processing time of batch b ; otherwise 0; $\forall b \in B, \forall i \in I$;
 E : total electricity cost for completing all jobs;
 C_{max} : completion time of the last job.

In the work, the number of batches $|B|$ is initially set as $|J|$. A batch is opened if there is at least one job allocated to the batch. On the contrary, a batch is closed without any job and its corresponding processing time equals to 0, i.e., $P_b = 0$. The considered problem can be formulated as the following bi-objective MILP model \mathcal{P}'_1 :

$$\mathcal{P}'_1 : \min E \quad (11.1)$$

$$\min C_{max} \quad (11.2)$$

$$s.t. \sum_{i \in I} \sum_{b \in B} x_{j,b,i} = 1, \forall j \in J \quad (11.3)$$

$$\sum_{i \in I} y_{b,i} = 1, \forall b \in B \quad (11.4)$$

$$\sum_{j \in J} x_{j,b,i} \leq C y_{b,i}, \forall b \in B, \forall i \in I \quad (11.5)$$

$$x_{j,b,i} p_j \leq P_{b,i}, \forall j \in J, \forall b \in B, \forall i \in I \quad (11.6)$$

$$\sum_{b \in B} P_{b,i} \leq S_i z_i, \forall i \in I \quad (11.7)$$

$$\sum_{i \in I} e_i \sum_{b \in B} P_{b,i} \leq E \quad (11.8)$$

$$s_i z_i + \sum_{b \in B} P_{b,i} \leq C_{max}, \forall i \in I \quad (11.9)$$

$$x_{j,b,i}, y_{b,i}, z_i \in \{0, 1\}, \forall j \in J, \forall b \in B, \forall i \in I \quad (11.10)$$

$$P_{b,i} \geq 0, E \geq 0, C_{max} \geq 0, \forall b \in B, \forall i \in I. \quad (11.11)$$

Objective (11.1) is to minimize the total electricity cost E on the horizon I . Objective (11.2) is to minimize the makespan C_{max} , which is the completion time of the last batch. Constraints (11.3) ensure that job j , $\forall j \in J$, is assigned to only one batch. Constraints (11.4) guarantee that each batch b , $\forall b \in B$, is processed in only one period. Constraints (11.5) assume that the number of jobs assigned to any batch should not exceed the batch capacity C , and any job j , $\forall j \in J$, cannot be assigned to period i if its corresponding batch is not processed in this period. Constraints (11.6) limit processing time of each batch. Constraints (11.7) ensure that the total processing time of batches in period i , $\forall i \in I$, should not exceed its duration, and $z_i = 1$ if there is at least one batch assigned to period i . Constraint (11.8) calculates the total electricity cost. Constraint (11.9) defines the makespan C_{max} . Constraints (11.10)–(11.11) enforce the restrictions on decision variables.

11.2.3 Property Analysis and Improved Model

As mentioned above, the number of batch $|B|$ equals to the number of jobs $|J|$ in model \mathcal{P}'_1 . According to the preliminary results, it is very time-consuming to directly solve model \mathcal{P}'_1 . Now we try to analyze properties of the problem to reduce solution space. We show that the formation of batches can be solved independent of the scheduling of batches, with two objectives considered in the problem.

A solution of the problem is uniquely defined by $(|B|, \{J_b, 1 \leq b \leq |B|\}, \{\tau_b, 1 \leq b \leq |B|\})$, where $|B|$ is the number of batches, J_b and τ_b are the set of jobs involved in the batch b and the period the batch is processed in, respectively.

We consider in particular those solutions where the batches are formed with a so-called LPT-based rule. In this rule, any job j with $(b-1)C < j \leq bC$ and $1 \leq b \leq \lceil |J|/C \rceil - 1$ is put into batch b and the remaining jobs to batch $\lceil |J|/C \rceil$, where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x . Thus, the processing time of batch b equals to that of job $(b-1)C + 1$. Figure 11.2 gives a simple example to illustrate the rule.

The following theorem shows that we only need to consider such solutions in order to find the Pareto front.

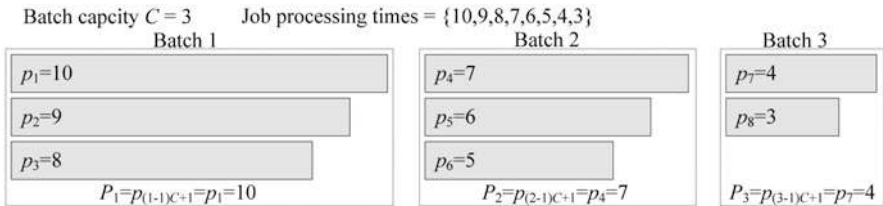


Fig. 11.2 An example of batches formed with the LPT-based rule

Theorem 11.1 *Any solution in which the batches are different from those formed with the LPT-based rule is (at least weakly) dominated.*

Proof. To facilitate the proof, the following notations are used:

J_b : the set of jobs contained in batch b , $J_b \subseteq J$;

$n(J_b)$: the serial number of the least indexed job (thus with the largest processing time) in set J_b , i.e., $n(J_b) = \min\{j | j \in J_b\}$;

$P(J_b)$: the processing time of batch b (the processing time of the least indexed job in J_b), i.e., $P(J_b) = \max_{j \in J_b} p_j = p_{n(J_b)}$.

Let $|B^*|$ and J_b^* represent the number of batches formed with LPT-based rule and the set of jobs involved in batch b ($1 \leq b \leq |B^*|$), respectively. We have

$$|B^*| = \lceil |J|/C \rceil,$$

$$J_b^* = \{(b-1)C + 1, (b-1)C + 2, \dots, bC\}, \quad b = 1, 2, \dots, |B^*| - 1,$$

$$J_{|B^*|}^* = \{(|B^*| - 1)C + 1, (|B^*| - 1)C + 2, \dots, |J|\}.$$

With the above construction, we have the following equations for batch b , $1 \leq b \leq |B^*|$:

$$n(J_b^*) = (b-1)C + 1, \quad (11.12)$$

$$P(J_b^*) = p_{(b-1)C+1}. \quad (11.13)$$

Consider a feasible solution \hat{S} with $(|\hat{B}|, \{\hat{J}_b, 1 \leq b \leq |\hat{B}|\}, \{\hat{t}_b, 1 \leq b \leq |\hat{B}|\})$ in which the batches are different from those formed with the LPT-based method. Obviously, we must have

$$|\hat{B}| \geq \lceil |J|/C \rceil = |B^*|. \quad (11.14)$$

In other words, there are at least as many batches as those formed with the LPT-based method. Without loss of generality, we renumber the batches in an increasing order of $n(\hat{J}_b)$'s; i.e.,

$$n(\hat{J}_1) < n(\hat{J}_2) < \dots < n(\hat{J}_{|\hat{B}|}). \quad (11.15)$$

Then for any batch b such that $1 \leq b \leq |\hat{B}|$, there exists

$$n(\hat{J}_b) = n(\tilde{J}), \text{ where } \tilde{J} = \hat{J}_b \cup \hat{J}_{b+1} \cup \dots \cup \hat{J}_{|\hat{B}|}. \quad (11.16)$$

Owing to the fact that for any subset \tilde{J} belonging to set $J = \{1, 2, \dots, |J|\}$, we have

$$n(\tilde{J}) \leq |J| - |\tilde{J}| + 1, \quad \forall \tilde{J} \subseteq J, \quad (11.17)$$

where $|J|$ denotes the number of jobs contained in set J . As for schedule \hat{S} , in which the job set has $|\hat{B}|$ disjoint subsets that are sorted as (11.15). According to (11.16) and (11.17), we must have

$$n(\hat{J}_b) = n(\tilde{J}) \leq |J| - \sum_{\beta=b}^{|\hat{B}|} |\hat{J}_\beta| + 1, \quad \tilde{J} \subseteq J. \quad (11.18)$$

Due to $J/\tilde{J} = J/\{\hat{J}_b \cup \hat{J}_{b+1} \cup \dots \cup \hat{J}_{|\hat{B}|}\} = \hat{J}_1 \cup \hat{J}_2 \cup \dots \cup \hat{J}_{b-1}$, Equation (11.18) can be further written as:

$$n(\hat{J}_b) \leq \sum_{\beta=1}^{b-1} |\hat{J}_\beta| + 1 \leq \sum_{\beta=1}^{b-1} C + 1 = (b-1)C + 1. \quad (11.19)$$

Since the jobs are indexed in nonincreasing order of their processing times, thus Eq. (11.19) implies that

$$P(\hat{J}_b) = p_{n(\hat{J}_b)} \geq p_{(b-1)C+1} = P(J_b^*), \quad 1 \leq b \leq |B^*|. \quad (11.20)$$

In other words, the processing time of the batches is at least as long as those formed with the LPT-based method.

Construct a new solution by removing batches $|B^*| + 1, \dots, |\hat{B}|$, if any, and replacing each batch \hat{J}_b ($1 \leq b \leq |B^*|$) by the corresponding one formed with the LPT-based method (i.e., batch J_b^*), without changing the starting time. In other words, consider solution $(|B^*|, \{J_b^*, 1 \leq b \leq |B^*|\}, \{\hat{\tau}_b, 1 \leq b \leq |B^*|\})$. Relation (11.20) implies that this new solution is also feasible. Furthermore, due to the fact that some batches are removed and the processing times of the remaining batches are reduced, neither the electrical consumption cost nor the makespan is increased, which means that the initial solution is (at least weakly) dominated by the new one. \square

As a consequence, by considering batches formed with the LPT-rule as new jobs, the problem is transformed into a classical production scheduling problem without a batching machine. Due to the fact that each batch (new job) should be entirely executed in one period, these new jobs are nonpreemptive. There is an (infinitely short) unavailability period between two successive periods. This latter problem has been proved to be NP-hard in the strong sense, even when the single objective is to minimize the makespan. Hence, we have the following property.

Property 11.1 The SBMS under ToU is strongly NP-hard.

According to Theorem 11.1, optimal solutions will not lose by separately solving batch formation and batch scheduling. That is, jobs can be first batched with LPT-based rule, i.e., $P_b = p_{(b-1)C+1}$, $b \in B^*$, and $B^* = \{1, 2, \dots, \lceil |J|/C \rceil\}$. Then

decision variables can be restricted to $y_{b,i}$'s and z_i 's. The initial model can be equivalent to the following one \mathcal{P}_1 :

$$\begin{aligned} \mathcal{P}_1 : \quad & \min E \\ & \min C_{max} \\ s.t. \quad & \sum_{i \in I} y_{b,i} = 1, \forall b \in B^* \end{aligned} \quad (11.21)$$

$$\sum_{b \in B^*} P_b y_{b,i} \leq S_i z_i, \forall i \in I \quad (11.22)$$

$$\sum_{i \in I} \sum_{b \in B^*} e_i P_b y_{b,i} \leq E \quad (11.23)$$

$$s_i z_i + \sum_{b \in B^*} P_b y_{b,i} \leq C_{max}, \forall i \in I \quad (11.24)$$

$$y_{b,i}, z_i \in \{0, 1\}, \forall b \in B, \forall i \in I, E \geq 0, C_{max} \geq 0. \quad (11.25)$$

Constraints (11.21) ensure that a formed batch $b, \forall b \in B^*$ is allocated into exactly one period. Constraint (11.22) guarantees that the total processing time of the batches in period i does not exceed its duration and $z_i = 1$ if there is at least one batch allocated to period $i, \forall i \in I$. Constraints (11.23) and (11.24) restrict the total electricity cost and makespan, respectively. Constraint (11.25) specifies the restrictions on the variables.

Remark 11.1 Compared with the initial model \mathcal{P}'_1 , the improved model \mathcal{P}_1 reduces $|I| \cdot |J|^2 + (|J| - \lceil |J|/C \rceil) \cdot |I|$ binary variables and $|I| \cdot |J|$ real variables as well as $(2 + |I| \cdot |J| + |I|) \cdot |J| - (|J| \cdot |I| + |I| + 1) \cdot \lceil |J|/C \rceil$ constraints.

Taking an instance with $|J| = 100, C = 10$, and $|I| = 10$ as an example, \mathcal{P}_1 can reduce 100900 binary variables, 1000 real variables, and 91090 constraints compared with \mathcal{P}'_1 . Owing to such reduction of variables and constraints, the search space for Pareto optimal solutions via the improved model \mathcal{P}_1 is significantly reduced.

11.3 SBMS-ToU Considering Machine On/Off Switching

By observing the scheduling results of problem SBMS-ToU, we can find that an idle duration may exist in some periods. In some circumstances, machine turn-on and -off may not consume energy and it is always turned off when finishing processing in each period and is restarted when needed. This kind of problem can be directly solved by the model \mathcal{P}'_1 . However, turning on machines can consume a great amount of energy in some manufacturing environments, e.g., steel manufacturing. As indicated by Mouzon et al. (2007), the resulted non-processing energy (NPE)

consumption related to machine turn-on, turn-off, and idling constitutes a significant part (over 30%) of the total energy consumption for certain scheduling environment. Remarkably, it has been shown in Mouzon and Yildirim (2008), Mouzon et al. (2007), and Yildirim and Mouzon (2012), the NPE consumption can be significantly reduced by rationalized machine turn-on/off. Thus, to optimize the whole electricity consumption cost including processing cost as well as machine on/off switching cost is highly important in practice. In other words, it is worthwhile to extend the problem SBMS by considering machine on/off switching strategy.

This section introduces a bi-criteria single machine batch scheduling problem with machine on/off switching under TOU pricing, SBMS-ToU-on/off in short. It can be described as follows. A batch processing machine with C jobs' capacity is associated with processing energy consumption rate P^{proc} and idling energy consumption rate P^{idle} . It is assumed that $P^{proc} > P^{idle}$. Turning on the machine needs a relatively short time and consumes P^{on} units electricity, while turning off the machine is assumed to require no energy. A given set $J = \{1, 2, \dots, |J|\}$ independent nonpreemptive jobs has to be processed on the single batch processing machine within a scheduling horizon with $|I|$ periods. Each job j is available at time 0 and has a processing time p_j . Without loss of generality, we assume that all the jobs are numbered in nonincreasing order of the processing times, i.e., $p_1 \geq p_2 \geq \dots \geq p_{|J|}$. All jobs can be regrouped into $|B|$ (to be determined) batches, where $\lceil |J|/C \rceil \leq |B| \leq |J|$. The processing time of a batch is determined by the largest job processing time in the batch.

Similarly to the problem SBMS in the previous section, a period $i \in I$ is considered as a work shift with duration S_i . Let s_i denote the starting time of period i , respectively. The length of the scheduling horizon is $s_{|I|+1}$. Job processing time p_j is less than the period duration $S_i, \forall i \in I$. The unit electricity price of period i , denoted by Pr_i , is calculated as the average price of period i based on the tariffs information as problem SBMS. Thus, the unit electricity cost for processing jobs in period i , denoted by e_i^p , is calculated as $e_i^p = P^{proc} \times Pr_i$. Similarly, the unit electricity cost of machine staying idle is $e_i^s = P^{idle} \times Pr_i$ and cost for machine turning on is $e_i^o = P^{on} \times Pr_i$, respectively. To determine the NPE cost, we need to analyze the turn-on/off strategies between any two adjacent periods. All cases are analyzed as follows:

- Case 1:** there exists processing in period $i \in I \setminus \{|I|\}$ while period $i + 1$ has not, then the machine will be turned off when finishing processing in period i and thus the machine idling cost in period i and the NPE cost in period $i + 1$ are 0;
- Case 2:** no processing exists in period $i \in I \setminus \{|I|\}$ while period $i + 1$ has, then the machine will be turned on in period $i + 1$ as it is in a shut-down state in period i and thus the NPE cost in period i is 0 and there exists turn-on energy cost e_{i+1}^o in period $i + 1$;
- Case 3:** there exists processing in both periods $i \in I \setminus \{|I|\}$ and $i + 1$, then if $e_i^s d_i^{idle} > e_{i+1}^o$, where d_i^{idle} denotes the idling duration in period i , then the machine will be turned off in period i when finishing processing and thus the machine idling cost in period i is 0 and there exists turn-on energy cost e_{i+1}^o in

period $i + 1$; otherwise, the machine will be kept idling in period i and thus the machine idling cost in period i is $e_i^s d_i^{idle}$ and there is no turn-on energy cost in period $i + 1$.

The objective of the problem is to find an optimal schedule that consists of batching the jobs, allocating the batches to periods, and deciding whether the machine should be turned off or kept running idle for an idle duration in order to optimize the total electricity cost (E) and the makespan (C_{max}) simultaneously.

11.3.1 Mathematical Formulation

The problem SBMS-ToU-on/off considered in this section is a natural extension of problem SBMS and continues to use part of notations in the previous section, which include indices j, b, i , parameters C, p_j, s_i, S_i , and decision variables $x_{j,b,i}, y_{b,i}, z_i, |B|, P_{b,i}$. New parameters and decision variables for SBMS-ToU-on/off are listed below.

Parameters

- e_i^p : electricity cost for processing jobs per unit time in period $i, \forall i \in I$;
- e_i^s : electricity cost for machine idling per unit time in period $i, \forall i \in I$;
- e_i^o : electricity cost produced by turning on the machine in period $i, \forall i \in I$;

Decision Variables

- v_i : 0 if the machine is turned off or no job is processed in period i , or no job is to be processed in period $i + 1$; 1 otherwise, $\forall i \in I, v_0 = 0$;
- u_i : > 0 if the machine is not turned off in period i ; 0 otherwise, $\forall i \in I \setminus \{I\}$.

With the notations and variables defined above, the investigated problem can be formulated as the following model \mathcal{P}'_2 :

$$\mathcal{P}'_2 : \min E \quad (11.26)$$

$$\min C_{max} \quad (11.27)$$

s.t. Constraints (11.3)–(11.7),

$$z_i \leq \sum_{b \in B} y_{b,i}, \forall i \in I \quad (11.28)$$

$$v_i \leq z_{i+1}, \forall i \in I \setminus \{I\} \quad (11.29)$$

$$v_i \leq z_i, \forall i \in I \quad (11.30)$$

$$u_i \geq e_i^s (S_i v_i - \sum_{b \in B} P_{b,i}), \forall i \in I \quad (11.31)$$

$$\sum_{i \in I} \sum_{b \in B} e_i^p P_{b,i} + \sum_{i \in I} e_i^o (z_i - v_{i-1}) + \sum_{i \in I \setminus \{I\}} u_i \leq E \quad (11.32)$$

$$s_i z_i + \sum_{b \in B} P_{b,i} \leq C_{max}, \forall i \in I \quad (11.33)$$

$$x_{j,b,i}, y_{b,i}, z_i, v_i \in \{0, 1\}, \forall j \in J, \forall b \in B, \forall i \in I \quad (11.34)$$

$$P_{b,i} \geq 0, u_i \geq 0, E \geq 0, C_{max} \geq 0, \forall b \in B, \forall i \in I. \quad (11.35)$$

Objectives (11.26) and (11.27) are to minimize total electricity cost E and makespan C_{max} , respectively. Constraints (11.3)–(11.7) state the limitations on allocating jobs to batches and periods. The difficulty of model formulation is controlling the machines' on/off status, which can be achieved by constraints (11.28)–(11.35). Equation (11.28) exactly determines if there is any batch processed in each period. Specifically, $z_i = 1$ if any batch is processed in period i , otherwise 0. Binary variable $v_i, i \in I$ equals to 1 if the machines stay running idle in period i according to its definition. Thus we use constraints (11.29) and (11.30) to, respectively, guarantee Cases 1 and 2 of turn-on/off strategies. That is, $v_i = 0$ if no batch is processed in period i or $i + 1$. Constraints (11.31) calculate the idling electricity cost in period $i \in I$. It only works when the machines stay running idle, i.e., $v_i = 1$. Constraint (11.32) calculates total electricity cost, which includes processing, machine turn-on, and idling cost. Constraint (11.33) defines the makespan. Constraints (11.34) and (11.35) are the restrictions on decision variables. Note that the number of batches $|B|$ is initially considered as its upper bound $|J|$ to derive a linear model.

11.3.2 Optimal Batch Rule Analysis

In this section, we devote our attention to reducing the search space for optimal solutions by analyzing the properties of the problem. In what follows, we demonstrate that batch formation can still be solved independent of batch allocation with LPT-based batch rule.

A solution of the problem $TOU, 1|on/off, B|E, C_{max}$ can be uniquely defined by $(|B|, \{J_b, 1 \leq b \leq |B|\}, \{\tau_b, 1 \leq b \leq |B|\}, \{v_i, i \in I\})$, where $|B|$, J_b , and τ_b are the number of batches, the set of jobs allocated into batch b ($J_1 \cup J_2 \dots \cup J_{|B|} = J$), and the period in which batch b is processed, respectively. v_i denotes the machine status in the idle duration of period i , i.e., the machine is kept idling or turned off. Theorem 11.2 shows that we only need to consider the solutions with LPT-based batch formation to derive the Pareto front of the considered problem.

Theorem 11.2 *Any solution of SBMS-ToU-on/off in which the batches differ from those formed with the LPT-based rule is (at least weakly) dominated.*

Proof. To facilitate the proof, we first recall the following notations:

J_b : the set of jobs contained in batch b , $J_b \subseteq J$;

$n(J_b)$: the serial number of the least indexed job (thus with the largest processing time) in set J_b , i.e., $n(J_b) = \min\{j | j \in J_b\}$;

$P(J_b)$: the processing time of batch b (the processing time of the least indexed job in J_b), i.e., $P(J_b) = \max_{j \in J_b} p_j = p_{n(J_b)}$.

Let $|B^*|$ and J_b^* represent the number of batches formed with LPT-based method and the set of jobs involved in the batch b ($1 \leq b \leq |B^*|$), respectively. We have

$$|B^*| = \lceil |J|/C \rceil,$$

$$J_b^* = \{(b-1)C + 1, (b-1)C + 2, \dots, bC\}, \quad b = 1, 2, \dots, |B^*| - 1,$$

$$J_{|B^*|}^* = \{(|B^*| - 1)C + 1, (|B^*| - 1)C + 2, \dots, n\}.$$

With the above construction, we have

$$n(J_b^*) = (b-1)C + 1, \quad (11.36)$$

$$P(J_b^*) = p_{(b-1)C+1}. \quad (11.37)$$

Suppose there is a feasible schedule \hat{S} with solution $(|\hat{B}|, \{\hat{J}_b, 1 \leq b \leq |\hat{B}|\}, \{\hat{\tau}_b, 1 \leq b \leq |\hat{B}|\}, \{\hat{v}_i, i \in I\})$, in which the batches differ from those formed with LPT-based rule. With similar proof of Theorem 11.1 for problem SBMS from formulas (11.14) to (11.19), we can conclude that the processing time of the batches are at least as long as those formed with the LPT-based rule, i.e.,

$$P(\hat{J}_b) = p_{n(\hat{J}_b)} \geq p_{(b-1)C+1} = P(J_b^*), \quad 1 \leq b \leq |B^*|. \quad (11.38)$$

Renew schedule \hat{S} to S^* with the batches formed with LPT-based rule, the new solution $(|B^*|, \{J_b^*, 1 \leq b \leq |B^*|\}, \{\hat{\tau}_b, 1 \leq b \leq |\hat{B}|\}, \{\hat{v}_i, i \in I\})$ can be achieved by removing batches $|\hat{B}| + 1, \dots, |\hat{B}|$, if any, and replacing each batch \hat{J}_b ($1 \leq b \leq |\hat{B}|$) by the corresponding one formed with the LPT-based method (i.e., batch J_b^*) without changing the starting time. Relation (11.38) indicates that the new schedule S^* is also feasible. Because some batches are removed and the processing time of the rest batches is reduced, the makespan is not increased. Next, we prove that the total electricity cost is also not increased.

For any period $i \in I$ that involves job-processing, i.e., $z_i = 1$, according to (11.38), the total processing time in period i of schedule \hat{S} , calculated by $\sum_{b=1}^{|\hat{B}|} \hat{P}_{b,i}$, and that of schedule S^* , calculated by $\sum_{b=1}^{|B^*|} P_{b,i}^*$, must have the following relation:

$$\sum_{b=1}^{|\hat{B}|} \hat{P}_{b,i} \geq \sum_{b=1}^{|B^*|} P_{b,i}^*. \quad (11.39)$$

Let \hat{E}_i (resp. E_i^*) denote the total processing and idling cost of period i and the turn-on cost of period $i + 1$ of schedule \hat{S} (resp. S^*). Since $z_i = 1$, the magnitude relationship of \hat{E}_i and E_i^* can be analyzed through the following three cases:

Case 1: $z_{i+1} = 0$, then, $\hat{E}_i - E_i^* = e_i^p \sum_{b=1}^{|\hat{B}|} \hat{P}_{b,i} - e_i^p \sum_{b=1}^{|B^*|} P_{b,i}^* \geq 0$.

Case 2: $z_{i+1} = 1$ and $e_i^s(S_i - \sum_{b=1}^{|\hat{B}|} \hat{P}_{b,i}) > e_{i+1}^o$, then, we have

$$\hat{E}_i = e_i^p \sum_{b=1}^{|\hat{B}|} \hat{P}_{b,i} + e_{i+1}^o. \quad (11.40)$$

According to (11.39), we have

$$e_i^s(S_i - \sum_{b=1}^{|B^*|} P_{b,i}^*) \geq e_i^s(S_i - \sum_{b=1}^{|\hat{B}|} \hat{P}_{b,i}) > e_{i+1}^o,$$

thus, for the solution of E_i^* , we have

$$E_i^* = e_i^p \sum_{b=1}^{|B^*|} P_{b,i}^* + e_{i+1}^o. \quad (11.41)$$

Comparing (11.40) with (11.41), it is obvious that $\hat{E}_i - E_i^* \geq 0$.

Case 3: $z_{i+1} = 1$ and $e_i^s(S_i - \sum_{b=1}^{|\hat{B}|} \hat{P}_{b,i}) \leq e_{i+1}^o$, then we have

$$\begin{aligned} & \hat{E}_i - E_i^* \\ &= e_i^p \sum_{b=1}^{|\hat{B}|} \hat{P}_{b,i} + e_i^s(S_i - \sum_{b=1}^{|\hat{B}|} \hat{P}_{b,i}) - (e_i^p \sum_{b=1}^{|B^*|} P_{b,i}^* + \min\{e_i^s(S_i - \sum_{b=1}^{|B^*|} P_{b,i}^*), e_{i+1}^o\}) \\ &\geq e_i^p \sum_{b=1}^{|\hat{B}|} \hat{P}_{b,i} + e_i^s(S_i - \sum_{b=1}^{|\hat{B}|} \hat{P}_{b,i}) - (e_i^p \sum_{b=1}^{|B^*|} P_{b,i}^* + e_i^s(S_i - \sum_{b=1}^{|B^*|} P_{b,i}^*)) \\ &= (e_i^p - e_i^s) \left(\sum_{b=1}^{|\hat{B}|} \hat{P}_{b,i} - \sum_{b=1}^{|B^*|} P_{b,i}^* \right). \end{aligned}$$

Since $e_i^p > e_i^s$ and $\sum_{b=1}^{|\hat{B}|} \hat{P}_{b,i} \geq \sum_{b=1}^{|B^*|} P_{b,i}^*$, thus we have $\hat{E}_i - E_i^* \geq 0$.

The above results of the three cases indicate that the total electricity cost of the new schedule S^* is not greater than that of schedule \hat{S} . Consequently, neither electricity cost nor makespan is increased in the schedule with batches formed with LPT-based rule, which means that the initial schedule \hat{S} is (at least weakly) dominated by the new one. \square

Besides, the following property also holds.

Property 11.2 The batch scheduling problem SBMS-ToU-on/off is strongly NP-hard.

Proof. Consider a special case that $e_i^s = 0, \forall i \in I$ and $e_i^o = 0, \forall i \in I$, problem SBMS-ToU-on/off reduces to problem SBMS, which has been proved to be NP-hard in the strong sense in the previous section. Therefore, the problem SBMS-ToU-on/off is also strongly NP-hard. \square

11.3.3 An Improved MILP Model

By pre-processing the batches of SBMS-ToU-on/off with the LPT-based rule according to Theorem 11.2, we have $P_b = p_{(b-1)C+1}$ and $|B^*| = \lceil |J|/C \rceil$, a new MILP model, denoted by \mathcal{P}_2 , can be derived as follows:

$$\begin{aligned} \mathcal{P}_2 : \quad & \min E \\ & \min C_{max} \\ \text{s.t.} \quad & \sum_{i \in I} y_{b,i} = 1, \forall b \in B^* \end{aligned} \quad (11.42)$$

$$\sum_{b \in B^*} P_b y_{b,i} \leq S_i z_i, \forall i \in I \quad (11.43)$$

$$z_i \leq \sum_{b \in B^*} y_{b,i}, \forall i \in I \quad (11.44)$$

$$u_i \geq e_i^s (S_i v_i - \sum_{b \in B^*} P_b y_{b,i}), \forall i \in I \quad (11.45)$$

$$\sum_{i \in I} e_i^p \sum_{b \in B^*} P_b y_{b,i} + \sum_{i \in I} e_i^o (z_i - v_{i-1}) + \sum_{i \in I/|I|} u_i \leq E \quad (11.46)$$

$$s_i z_i + \sum_{b \in B^*} P_b y_{b,i} \leq C_{max}, \forall i \in I \quad (11.47)$$

$$y_{b,i}, z_i, v_i \in \{0, 1\}, \forall b \in B^*, \forall i \in I \quad (11.48)$$

$$u_i \geq 0, C_{max} \geq 0, E \geq 0, \forall i \in I \quad (11.49)$$

and constraints (11.29) and (11.30),

where $B^* = \{1, 2, \dots, |B^*|\}$ is the set of batches formed with the LPT-based method, $P_b = p_{(b-1)C+1}$. Constraints (11.42) state that a formed batch $b, \forall b \in B^*$ should be entirely processed in one period. Constraints (11.43) ensure that total processing time in period $i \in I$ cannot exceed its duration. Constraints (11.44) ensure that variable z_i takes the value of 1 only if there are batches to be processed in period

$i, \forall i \in I$. Constraints (11.45) denote the total electricity cost when the machine is left running idle. Constraints (11.46) and (11.47) calculate total electricity cost E and makespan C_{max} , respectively. Constraints (11.48) and (11.49) enforce the restrictions on decision variables. Since part of variables and constraints are removed, the search space for Pareto optimal solutions of the initial problem is significantly reduced. To be more specific, model \mathcal{P}_2 reduces $|I| \cdot |J|^2 + (|J| - \lceil |J|/C \rceil) \cdot |I|$ binary variables, $|I| \cdot |J|$ real variables, and $(2 + |I| \cdot |J| + |I|) \cdot |J| - (|J| \cdot |I| + |I| + 1) \cdot \lceil |J|/C \rceil$ constraints comparing to model \mathcal{P}'_2 .

11.4 SBMS Under ToU in Continuous Processing

The two problems introduced in the previous sections both assumed each job has to be completed in one work shift. However, continuously processing manufacturing systems are more widespread and practical in real production environments. In this section, we consider a batch scheduling problem under ToU tariffs where a nonpreemptive job is allowed to be processed in multiple periods. For the convenience of expression, we denote the problem as SBMS-ToU-CP.

11.4.1 Problem Description

Problem SBMS-ToU-CP can be described as follows. There are J jobs to be processed on a single BPM within a given planning horizon H . The machine and all the jobs are available from time 0 to H . The processing time of job j , $1 \leq j \leq J$, is denoted by p_j . Without loss of generality, we label the jobs in nonincreasing order of their processing times, i.e., $p_1 \geq p_2 \geq \dots \geq p_J$.

The machine is able to process up to C jobs simultaneously. Thus, all jobs can be grouped into B ($1 \leq B \leq \lceil J/C \rceil$) batches (to be determined). The processing time of batch b , $1 \leq b \leq B$, is given by the longest processing time job in this batch. Once a batch is being processed, it cannot be interrupted until its processing is completed.

In the planning horizon, unit electricity price varies over the time according to ToU tariffs or real-time electricity pricing. In other words, electricity cost incurred by the processing in unit time is calculated based on the present electricity price and power rate of the given machine. It is high in peak periods and low in off-peak periods.

The scheduling is to determine the batch formation and the processing position of each batch in the horizon, such that total electricity cost E and makespan C_{max} of the jobs are minimized.

It is obvious that batch formation is one of the key decisions for solving SMBSC-ToU. According to the preliminary analysis, we find that LPT-based rule can be

applicable to batch formation for the considered problem, and we have the following lemma.

Lemma 11.1 *Any solution of SBMS-ToU-CP in which the batches differ from those formed with LPT-based rule is (at least weakly) dominated.*

Proof. The theorem can be proved in a similar way as Theorem 11.1. To be specific, it has been proved by Eqs. (11.14)–(11.19) that the processing time of randomly formed batches are at least as long as those formed with LPT-based rule. Consequently, for any schedule with the batches that do not satisfy LPT-based rule, adjusting the jobs with LPT-based method will not deteriorate the schedule. In other words, Lemma 11.1 holds. \square

11.4.2 Mathematical Models

With the above theorem, optimal solutions will not lose by performing the batches with LPT-based rule. By considering the formed batches as new jobs, SMBSC-ToU is reduced to a single machine scheduling problem under ToU tariffs, and the number of new jobs equals to the number of batches B^* .

To calculate total electricity cost, the planning horizon has to be divided into several segments to position the start processing times of the batches. Based on two different division ways, two models, respectively, named as time-index-based model and time-interval-based model, are developed. The common parameters for the two models are as follows:

H : the duration of a given planning horizon;

b, c : index of batches;

i, k : index of time periods or intervals;

B^* : total number of batches;

P_b : processing time of batch b , $1 \leq b \leq B^*$.

11.4.2.1 Time-Index-Based MILP Model

This is an intuitive modeling way. Specifically, we first discrete the scheduling horizon H into $|H|$ unit time periods, then exactly determine the processing position of each batch on the horizon while making sure that the processing is not interrupted and adjacent jobs are not overlapped. For each time period i , $1 \leq i \leq H$, the duration and the electricity cost are 1 and e^i , respectively. Note that e_i may equal to e_{i+1} . Then the schedule can be achieved by determining the following decision variables:

$x_{b,i}$: equal to 1 if batch b is in processing at time period i , 0 otherwise; $1 \leq b \leq B^*$; $1 \leq i \leq H$;

y_{bc} : equal to 1 if batch b is processed (maybe not immediately) before batch c , 0 otherwise; $1 \leq b, c \leq B^*$;
 t_b : the start time of batch b ; $1 \leq b \leq B^*$;
 t'_b : the completion time of batch b ; $1 \leq b \leq B^*$.

With the above notations, time-index-based model is presented as follows:

$$\mathcal{P}_3 : \quad \min E \quad (11.50)$$

$$\min C_{max} \quad (11.51)$$

$$\text{s.t. } t_b + P_b = t'_b, \forall b \in B^* \quad (11.52)$$

$$i \geq t_b - (1 - x_{b,i})|H|, \forall i \in H; \forall b \in B^* \quad (11.53)$$

$$i \leq t'_b - 1 + (1 - x_{b,i})|H|, \forall i \in H; \forall b \in B^* \quad (11.54)$$

$$\sum_{i \in I} x_{b,i} = P_b, \forall b \in B^* \quad (11.55)$$

$$y_{bc} + y_{cb} = 1, \forall b, c \in B^*, \forall b \neq c \quad (11.56)$$

$$t'_b \leq t_c + (1 - y_{bc})|H|, \forall b, c \in B^*, \forall b \neq c \quad (11.57)$$

$$E = \sum_{i \in H} \sum_{b \in B^*} e^i x_{b,i} \quad (11.58)$$

$$C_{max} \geq t'_b, \forall b \in B^* \quad (11.59)$$

$$x_{b,i}, y_{bc} \in \{0, 1\}, \forall i \in H; \forall b, c \in B^*, b \neq c \quad (11.60)$$

$$t_b, t'_b \in \mathbb{Z}^+, \forall b \in B^*. \quad (11.61)$$

The objective functions (11.50) and (11.51) are to minimize total electricity cost E and makespan C_{max} . Constraints (11.52) imply that the processing of a batch is not interrupted, i.e., its completion time is the sum of its starting time and processing time. Constraints (11.53) and (11.54) determine the processing time periods of each batch by limiting the value of binary variable $x_{b,i}$. That is, the two constraints imply that if processing of batch b is not started or has been completed, $x_{b,i}$ must be 0. Constraints (11.55) guarantee that $x_{b,i}$ takes value 1 if time period i is between the start time and completion time of batch b . To sum up, constraints (11.52)–(11.55) ensure that the processing of each batch is not interrupted. Constraints (11.56) and (11.57) make sure that the processing of adjacent batches do not overlap. Specifically, constraints (11.56) express that batch b either precedes or follows batch c . Constraints (11.57) imply that if batch b precedes batch c , the start processing time of batch c must be equal or larger than the completion time of b . Constraints (11.60) and (11.61) denote the restrictions on variables. In model \mathcal{P}_3 , there are $[|B^*|^2 + (|H| + 2)|B^*|]$ variables and $[2|B^*|^2 + (2|H| + 3)|B^*| + 1]$ constraints to be determined.

11.4.2.2 Time-Interval-Based MILP Model

Another dividing way for the planning horizon is based on time-of-use pricing information. That is, divide the horizon into $|I|$ pricing intervals, where interval i , $1 \leq i \leq |I|$, is associated with a starting time s_i , a duration $S_i = s_{i+1} - s_i$, and a unit electricity cost e_i , note that $e_i \neq e_{i+1}$. Obviously, $s^{|I|+1} = |H|$. Then SMBS-ToU can be solved by determining how long each batch is processed in each interval; meanwhile, the uninterrupted processing and machine availability are guaranteed. For the modeling, the following decision variables are defined:

$x_{b,i}$: equals to 1 if batch b is processed in interval i , 0 otherwise; $\forall i \in I; \forall b \in B^*$;
 $w_{b,i}$: equals to 1 if batch b is simultaneously processed in interval i and $i + 1$, 0 otherwise; $\forall i \in I; \forall b \in B^*$;
 $t_{b,i}$: processing duration of batch b in interval i ; $\forall i \in I; \forall b \in B^*$.

Now the time-interval-based model, called Model 4 hereafter, can be formulated as follows:

$$\begin{aligned} \mathcal{P}_4 : \quad & \min E \\ & \min C_{max} \\ \text{s.t.} \quad & t_{b,i} \leq x_{b,i} P_b, \forall i \in I; \forall b \in B^* \end{aligned} \quad (11.62)$$

$$\sum_{i \in I} t_{b,i} = P_b, \forall b \in B^* \quad (11.63)$$

$$\sum_{b \in B^*} t_{b,i} \leq s_{i+1} - s_i, \forall i \in I \quad (11.64)$$

$$w_{b,i} \geq x_{b,i} + x_{b,i+1} - 1, \forall i \in I/\{|I|\}; \forall b \in B^* \quad (11.65)$$

$$\sum_{b \in B^*} w_{b,i} \leq 1, \forall i \in I/\{|I|\} \quad (11.66)$$

$$\begin{aligned} t_{b,i} + s_i x_{b,k} - s_{i+1} x_{b,i} + t_{b,k} &\leq P_b + (2 - x_{b,i} - x_{b,k})H, \\ &\forall i \in I/\{|I|\}; \forall k \in I; \forall b \in B^* \end{aligned} \quad (11.67)$$

$$E = \sum_{i \in I} \sum_{b \in B^*} e_i t_{b,i} \quad (11.68)$$

$$x_{b,i}, w_{b,i} \in \{0, 1\}, \forall i \in I; \forall b \in B^* \quad (11.69)$$

$$t_{b,i} \geq 0, \forall i \in I; \forall b \in B^*. \quad (11.70)$$

The objectives are to optimize total electricity cost and makespan. Constraints (11.62) restrict that processing duration of batch b in interval i does not exceed batch processing time, and processing duration $t_{b,i}$ takes value 0 if batch b is not processed in interval i . Constraints (11.63) guarantee that all the batches are completed within the planning horizon. Constraints (11.64) limit that the total processing time in a

given interval does not exceed the interval duration. Constraints (11.65) and (11.66) state that any two consecutive intervals can be crossed by only one batch. Constraint (11.67) ensures that the processing of any job is not interrupted. It states that once a batch b simultaneously processed in interval i and k ($i < k$), the total processing time of the job in the two intervals plus the distance between the ending time of interval i and starting time of interval k should be less than the processing time of batch b . Constraints (11.69) and (11.70) enforce the restrictions on decision variables.

11.5 Conclusion

The goal of this chapter is to provide an insight into the domain of batch scheduling under ToU tariffs. Three bi-objective batch scheduling problems under ToU electricity tariffs are introduced, which aims to design production plans for batch processing machines under fluctuating electricity prices, with the objectives of simultaneously optimizing total electricity cost and production efficiency. For each of the considered problem, appropriate mathematical model is formulated and the problem property is analyzed.

In the future, more complicated problems under ToU tariffs are worth of further investigation, such as the problems involving other machine environments (e.g., unrelated parallel machines, flow shop, job shop), job characteristics (e.g., dynamic release times, non-identical due dates), more regular objective functions (e.g., total completion time, maximum lateness), and production features (e.g., serial batching, maintenance activity).

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