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Exact invariant solutions for coherent turbulent motions
in Couette and Poiseuille flows

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Abstract

The dynamical systems approach recently applied to understand subcritical transitions in wall-bounded shear flows is combined with the use of large-eddy simulations to investigate the nature of large-scale coherent motions in turbulent Couette and Poiseuille flows. Exact invariant solutions of the filtered Navier-Stokes (LES) equations are computed by using the Smagorinsky model to parametrize small-scale motions. These solutions can be continued into exact solutions of the Navier-Stokes equations, therefore providing a bridge between coherent large-scale motions in wall-bounded fully developed turbulent flows and invariant solutions appearing in transitional flows.

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Keywords:

Turbulence; ; coherent structures; dynamical systems.

1. Introduction

The understanding of subcritical transition in wall-bounded shear flows has been greatly improved by the computation and the analysis of invariant solutions of the Navier-Stokes equations disconnected from the laminar basic state. A number of steady and travelling wave solutions of the Navier-Stokes equations have been computed in canonical flows such as the plane Couette flow^{1,2,3}, the plane channel^{4,5} and the pipe flow^{6,7}. These solutions typically appear in saddle-node bifurcations at low Reynolds numbers and represent saddles in phase space. Typically, lower branch solutions are related to the laminar-turbulent transition boundary, while upper branch solutions display features consistent with the turbulent flow arising in the transition process.

It was originally hoped that, at least at low Reynolds numbers, turbulent solutions would spend a significant time in the neighbourhood of a few of these saddle solutions. However, careful analysis did later show that this is not the case^{8,9}. The next step has consisted in the computation of (unstable) periodic and relative periodic orbits. Global bifurcations of relative periodic orbits have been shown to be related to the transition to chaotic dynamics in plane Couette flow¹⁰ and in magnetohydrodynamic Keplerian shear flows¹¹. However, to this date, attempts to approximate turbulent statistics with expansions based on these periodic solutions^{12,13} have not been completely successful¹⁴. Furthermore, the number of invariant solutions increases rapidly and their spatial structures become more and more

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complex when the Reynolds number is increased above transitional values. The relevance of using a few invariant solutions to approximate the statistics of fully developed turbulent wall-bounded flows might therefore not be straightforward to prove at high Reynolds number.

One of the prominent features of fully developed wall-bounded turbulent flows is the ubiquity of streaky motions. It has long been known¹⁵ that the buffer-layer is populated by very active streaky structures which have an average spanwise streak-spacing of $\lambda_z^+ \approx 100$ (in wall units). Streaky motions also exist in the logarithmic and the outer regions. ‘Large-scale motions’ (LSM) have typical streamwise and spanwise sizes of $\lambda_x \approx 2\delta - 3\delta$ and $\lambda_z \approx \delta - 1.5\delta$ respectively. Very large scale motions (VLSM) have been shown^{17,18,19} to exist with streamwise scales extending up to $\lambda_x \approx O(10\delta)$. An increasingly significant amount of the turbulent kinetic energy and Reynolds stress in the outer region is indeed associated to these large scale and very large scale motions¹⁶.

Using overdamped large-eddy simulations it has been recently shown that large-scale and very-large-scale motions can sustain themselves in the turbulent plane channel and Couette flows even when small-scale structures (including buffer-layer and log-layer structures) are artificially quenched^{20,21}. Motions with intermediate scales, which populate the logarithmic region, have been also shown to self-sustain and to be approximately self-similar and scale with their spanwise wavelength λ_z or, equivalently, with their distance y from the wall^{22,23} which is compatible with the classical concept of attached eddies²⁴.

It makes therefore sense to use the same large-eddy simulation approach to calculate invariant solutions pertaining to large-scale coherent motions which contain most of the energy at high Reynolds numbers. Following this approach, small-scale motions are modelled in order to take only their ‘locally averaged’ (by the filtering action) effect into account. This would for instance permit to compute steady large-scale solutions even if small-scale motions are unsteady but their local average is steady. This paper is about the computation of these ‘large-eddy’ invariant solutions.

2. Background

We follow the approach taken in previous investigations^{20,22,21,25} to compute the self-sustained dynamics of coherent large scale motions. The filtered Navier-Stokes equations routinely used in large eddy simulations are the usual ones^{26,27}:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0; \quad \frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{\partial \bar{q}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j^2} - \frac{\partial \bar{\tau}_{ij}^r}{\partial x_j}, \quad (1)$$

where an over-bar indicates filtered quantities. The anisotropic residual stress tensor $\bar{\tau}^r = \bar{\tau}^R - \text{tr}(\bar{\tau}^R)\mathbf{I}/3$, with $\bar{\tau}_{ij}^R = \bar{u}_i \bar{u}_j - \bar{u}_i \bar{u}_j$ and $\bar{q} = \bar{p} + \text{tr}(\bar{\tau}^R)/3$, is modelled using the Smagorinsky model²⁸ based on the eddy viscosity ν_t : $\bar{\tau}_{ij}^r = -2\nu_t \bar{S}_{ij}$. \bar{S}_{ij} is the rate of strain tensor associated with the filtered velocity field. The eddy viscosity is modelled as $\nu_t = D(C_s \bar{\Delta})^2 \bar{S}$, where $\bar{S} \equiv (2\bar{S}_{ij}\bar{S}_{ij})^{1/2}$, $\bar{\Delta} = (\bar{\Delta}_x \bar{\Delta}_y \bar{\Delta}_z)^{1/3}$. The wall (damping) function $D = 1 - e^{-(y^+/A^+)^2}$ with $A^+ = 25$ is used to avoid non-zero residual velocity and shear stress.

Steady and travelling-wave solutions are computed by means of a Newton-Krylov method implemented in the code `peanuts`^{29,11} which relies on repeated calls to numerical time-integrations of eq. (1). These integrations are performed with the code `diablo` which implements the fractional-step method based on a semi-implicit time integration scheme and a mixed finite-difference and Fourier discretization in space³⁰. The computational domain, which extends from 0 to L_x and from 0 to L_z in the streamwise and spanwise directions respectively and from $-h$ to h in the wall-normal direction, is discretized with $N_x \times N_y \times N_z$ points in respectively the streamwise (x), wall-normal (y) and spanwise (z) directions. Grid stretching is applied in the wall-normal direction in order to refine the grid near the wall. No-slip boundary conditions are applied at the walls and periodic boundary conditions at other boundaries.

The Smagorinsky constant is used as a quenching parameter in temporal simulations and a continuation parameter for the invariant solutions. The value $C_s = 0.05$ which is known to provide the best performance for *a posteriori* tests³¹ is used as a reference value. Solutions can be obtained by increasing the Smagorinsky constant above its reference value ($C_s > 0.05$, ‘overdamped’ LES) therefore quenching the production term of an increasing range of small-scale motions^{20,22,21}. Solutions can also be continued up to $C_s = 0$, where the eddy viscosity is zero and the filtered equations reduce to the (unfiltered) Navier-Stokes equations.

3. Large-eddy exact coherent solutions (LECS)

The first exact invariant solutions of the filtered Navier-Stokes equations (1) have been computed²¹ in plane Couette flow in a horizontally periodic domain having the same size of turbulent large-scale motions $L_x \times L_z = 10.9h \times 5.5h$ (where h is the half-width of the channel). The initial guess for the Newton iterations was found by computing the edge state of coherent large-scale motions at $Re = 750$ (roughly twice the transitional Reynolds number) and $C_s = 0.14$ (overdamped simulation) by edge-tracking. The edge state was found to be a non-trivial steady solution of the filtered equations with all the characteristics of a lower branch solution. Continuation to higher values of C_s revealed that the computed edge state could be connected to an upper branch of solutions via a saddle-node bifurcation (see figure 1). Upper branch solutions of the filtered equations could then be computed up to $Re = 2150$ using specific paths in the $Re - C_s$ parameter plane (see figure 1). Both upper and lower branch solutions were then connected²¹ to Navier-Stokes solutions by continuation to $C_s = 0$. These Navier-Stokes solutions belong to the Nagata-Clever-Busse-Waleffe branch of solutions^{1,2,3} as is verified by continuation in the Reynolds number at $C_s = 0$.

The fact that invariant solutions of the filtered equations representing coherent large-scale turbulent motions could be connected to invariant solutions of the Navier-Stokes equations has opened an exciting path for additional investigations. If a solution can be continued from higher C_s values to zero, the reverse must also be true. It is therefore possible, in principle to use Navier-Stokes invariant solutions (ECS) as starting point for a continuation to higher C_s .

We have initially applied this method to plane Poiseuille flow where a recently computed²⁵ multi-streaks travelling-wave solution of the Navier-Stokes equations ($C_s = 0$) in the periodic domain $L_x \times L_z = 2\pi h \times 5.5h$ has been continued from $C_s = 0$ to $C_s = 0.05$ at $Re = 2000$. The continuation was straightforward and the solution at $C_s = 0.05$ is very similar to that at $C_s = 0$.

Next, we have considered the P4 Navier-Stokes travelling-wave solutions recently computed by Park & Graham³² in plane Poiseuille flow in the periodic domain of size $L_x = \pi h$, $L_z = \pi h/2$. This solution had the interesting property that the upper-branch mean flow profile approaches a logarithmic profile for increasing Reynolds numbers, while the lower-branch mean flow profile approaches the one corresponding to Virk's minimum drag regime. A first continuation of this solution is initiated at $Re = 1650$ and $C_s = 0$. Convergence is obtained in a few iterations to solutions which are substantially unchanged at $C_s = 0.05$. A saddle-node bifurcation, connecting upper and lower C_s -branches of solutions is found at $C_s = 0.13$, similarly to what found in plane Couette flow. It is similarly possible to continue to even higher values of C_s the upper-branch solution at $Re = 3500$ which is connected to a lower a lower branch via a turning point at $C_s = 0.29$. The latter results, obtained in collaboration with JS Park and MD Graham, are currently under further investigation and will be published elsewhere³³.

All the computed solutions consist of a combination of quasi-streamwise vortices and streaks (see figures 2 and 3) which, on average, self-sustain via a coherent self-sustained process at large scale^{20,22,34}. The computed 'large-eddy' solutions (LECS) of the filtered equations take into account the effect of small scales but only through their averaged effect. This approach makes possible to compute steady coherent solutions despite the fact that motions at smaller scale are unsteady and therefore focus on the relevant dynamics of the large-scale coherent solutions without the complications associated to smaller-scales motions. The spatial and Re dependence of the eddy viscosity associated with the averaged (residual) small-scale motions is naturally embedded in the computed solutions (see figures 2 and 3).

4. Conclusions

It has recently been shown that that coherent motions of medium large-scale and very-large scale self-sustain in wall-bounded turbulent flows even if active motions of smaller scales are artificially quenched^{20,22,21}. A deeper understanding of the dynamics of these coherent self-sustained motions is gained by computing invariant solutions of the *filtered* Navier-Stokes equations where the residual motions are suitably modelled, i.e. of the equations used in large-eddy simulations. We have labelled these solutions 'large-eddy' exact coherent solutions (LECS) to distinguish them from the much studied 'exact coherent structures' (ECS) which are invariant solutions of the (unfiltered) Navier-Stokes equations and which of course do not take into account any Reynolds stress associated to smaller scale motions.

An early study²¹ where steady state LECS solutions were computed in plane Couette flow, showed that upper and lower-branch LECS solutions are connected by continuation in the Smagorinsky constant C_s via a turning point

and that they can be connected to Navier-Stokes ECS solutions when C_s is driven to zero. While in this early study LECS were initialized with a 'large-eddy' edge state computed using edge-tracking on large-eddy simulations (finite C_s), and then continued to ECS solutions (corresponding to $C_s = 0$), it was expected that the reverse would also be possible. We have shown that this is the case and that it is possible to compute two families of (LECS) solutions of the filtered Navier-Stokes equations by continuation of previously known^{32,25} travelling wave (ECS) solutions of the (unfiltered) Navier-Stokes equations from $C_s = 0$ to finite values of C_s .

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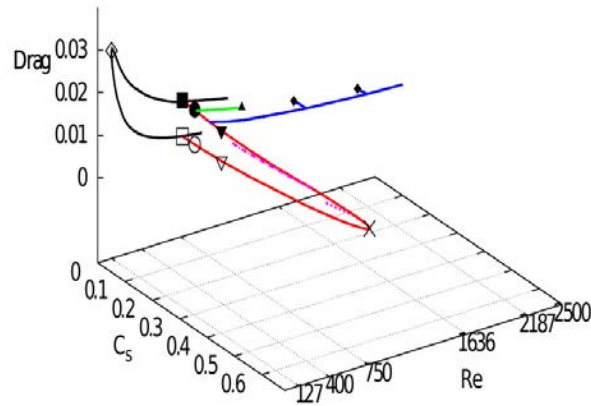


Fig. 1. Continuation paths of the steady solutions of the filtered equations (LES) for plane Couette flow. The C_s continuation in C_s at $Re = 750$ (red line) shows how upper-branch solutions can be accessed in this way. Continuation to $C_s = 0$ and then in Re (black line, dashed) show that solutions continued from the turbulent case (square symbols) belong to the Nagata-Busse-Clever-Waleffe branch of Navier-Stokes solutions. Coherent steady solutions at higher Reynolds numbers are found by first continuing the solutions obtained for $C_s = 0.1$ to higher Re (blue line, dashed-dotted) and then reducing C_s .

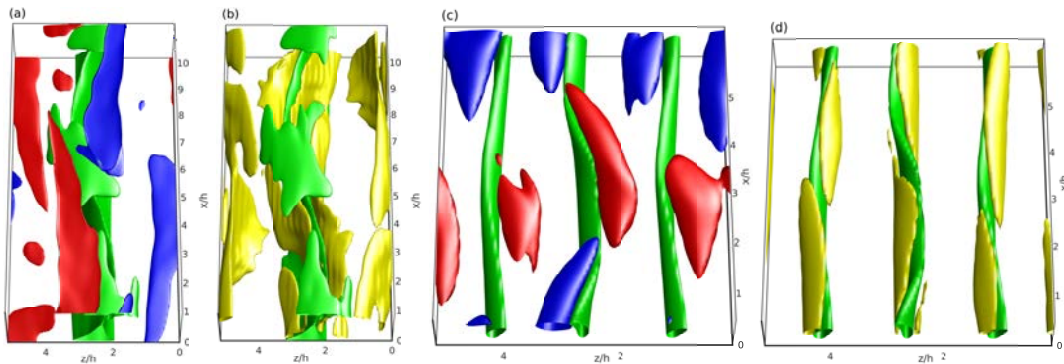


Fig. 2. Visualisation of the upper branch solutions of the filtered equations (LES) obtained in plane Couette ($Re = 2150$, $C_s = 0.05$, steady solution, panels *a* and *b*) and Poiseuille ($Re = 2000$, $C_s = 0.05$, travelling-wave solution, panels *c* and *d*) flow. Panels *a* and *c* represent the large-scale coherent (i.e. filtered) streaks (green) and quasi-streamwise vorticity (blue if negative, red if positive) while panels *b* and *d* report the relative eddy-viscosity ν_t/ν associated to the residual small-scale motions.

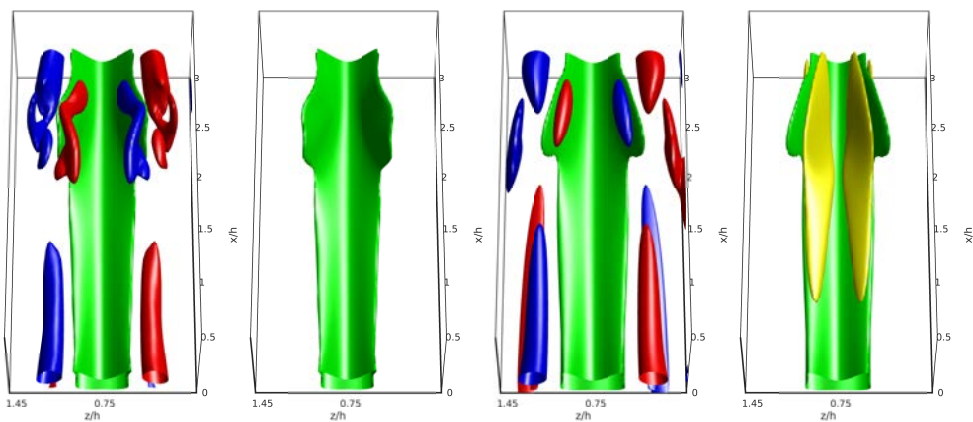


Fig. 3. Visualisation of the upper branch P4 ECS and LECS solutions obtained with, respectively, $C_s = 0$ (Navier-Stokes solutions, panels *a* and *b*) and $C_s = 0.2$ (panels *c* and *d*) at $Re = 3500$. Panels *a* and *c* represent the large-scale coherent (i.e. filtered) streaks (green) and quasi-streamwise vorticity (blue if negative, red if positive) while panels *b* and *d* report the relative eddy-viscosity ν_t/ν associated to the residual small-scale motions.