# A Tractable Model for Coverage in Non-full Interference Cellular Networks with Cell Center/Edge Users

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***Abstract*—This paper presents a non-full interference analysis to derive the coverage probability when the shared spectrum allocation technique is applied to the homogeneous Poisson point process (PPP) network. Each cell is partitioned into two regions based on the strength of the signal to interference ratio (SINR): the cell center region if the SINR is more significant than a threshold and cell edge region otherwise. In our model, a given resource block (RB) is allocated to one user in a cell and cannot be shared by another user. The RB is hence divided into two sub-bands, which will be used exclusively according to whether the user is a cell center or cell edge user, letting a part of the RB unused in order to reduce the interference to the other cell. This scenario implies that the interfering set of base stations (BS) depends on the coverage probability of the typical user. We prove that the interfering BS set is a thinned version of the original PPP and is related to the probability of coverage of the cell center region. Results show that the scheme increases the network's global coverage probability.**

***Index Terms*—Cell center user, cell edge user, coverage probability, Poisson point process.**

## I. Introduction

### A. Motivation

In a dense cellular deployment scenario, cell edge users (CEU) experience a weaker signal to interference plus noise ratio (SINR) than cell center users (CCU) due to weaker received signal strength and higher interference power, which may lead to low performance for CEU. To avoid or attenuate this effect, a classical approach is to divide the total physical resource available, e.g., the system bandwidth, and to allocate different sub-bands to contiguous cells and the same one to cells that are farther from each other. A derivative of that idea is to partition the bandwidth to allocate a fraction to CCU, that may be used by the nearest interfering cell but only by CCU in that cell, and the other part to CEU. This strategy relies first on the CCU/CEU classification, where two approaches can be remarked: the location-based approach [1], and the SINR-based approach [2].

The former approach is purely geometric and hence does not consider the SINR that may be larger at a point located farther from the base station (BS) than the nearest one, at a given time, therefore in this paper, the SINR-based classification is followed. However, the studies that dealt with CEU/CCU resource partitioning always considered a full-load interference scenario, i.e. whatever the resource partitioning and the typical user classification, CEU/CCU, it is interfered by all the BS operating in the network. In other words, CCU and CEU are interfered by thinned versions of the original PPP, but these thinnings are independent of the user classification.

In the non-full interference model, on the other hand, the BS activity is driven by the user position within each cell. In this case, CCU/CEU classification leads to BS classification, which means that the density of the interfering BS set is correlated with the user position in the interfering cells and depends on the location of the user where the SINR is measured. We prove that the correlated interfering scenario can be analyzed as a thinning process: the original PPP is split into two thinned complementary processes. In this paper, we quantify the thinning factor and evaluate the coverage probability and rate of the non-full interference network for the central and edge users.

### B. Related Work

User classification approaches mainly focus on two aspects: the location-based [1], [3]–[6], and SINR based classification [2], [7]. The location-based classification relies on the geometric properties of the PPP in order to classify a user as CCU or CEU. For instance, in [3], [4], authors computed the ratio of the distance between the typical user and the serving BS to the distance between the typical user to the nearest interfering BS. If the ratio is larger than a threshold, then the user is a CEU; otherwise, it is a CCU. Based on this kind of classification, some authors have investigated base stations cooperation techniques to enhance the CEU coverage [1], [5], [6]. However, the received SINR contains the relevant statistic for the communication link performance analysis, and it may be interesting to use it as a classification criterium. In [2], the whole frequency band has been divided into cell center and cell edge frequency sub-bands to improve the CEU coverage using spectrum access techniques such as fractional and soft frequency reuse. In [7], the authors used the instantaneous SINR based classification and got an approximation of the coverage probability of the typical CEU for a PPP-modeled 3-tier heterogeneous network. All these previous works have been done with full interference assumption, i.e. considering that all BS, or a fraction of them but non-related to the coverage probability, act as interferers to the typical user whatever the bandwidth it uses.

In this paper, the coverage probability with non-full interference assumption is tackled. In our work, the non-full interference setting refers to the fact that the interference in a given sub-band depends on the coverage probability of a CCU or CEU according to the sub-band considered. Section II presents our system model. Our main results are stated in Theorems 1 and 2 and special cases are considered in Section III. Section IV presents the numerical results and conclusions are drawn in Section V.

**Notations:** Random variables are denoted in capital font while their realizations remain in small font. Moreover, $\mathbb{P}[\cdot]$ and $\mathbb{E}[\cdot]$ are the probability and the expectation operators. $1(\cdot)$ is an indicator function, and the Laplace transform (LT) of a random variable $X$ in $s$ is denoted as $\mathcal{L}_X(s)$.

## II. System Model

Let us consider an OFDMA-based single-tier downlink cellular network where BS deployment is modeled as a homogeneous PPP $\Phi \subset \mathbb{R}^2$ with density $\lambda$. A user is associated with its nearest BS, and all BS and users are equipped with a single antenna. Similarly to work in [8], one randomly chosen user is considered to communicate with its BS on a given time/frequency resource. We assume that users experience Rayleigh fading. In each cell, a user can be classified as a CCU or CEU, depending on the value of its received SINR relatively to a threshold that, in turn, depends on the location of other BS and channel conditions. We consider that the time/frequency resource is a resource block (RB) allocated to one user in each cell. Contrarily to 4G or 5G systems, RB is divided into two sub-bands, i.e. $B_c$ and $B_e$, the former will be used if the user is a CCU and the later if the user is a CEU. However, since the entire RB is dedicated to the given user, the BS cannot allocate the remaining part of RB, e.g., $B_c$ if the user is CEU or $B_e$ if the user is CCU, to another user in the cell. This setting leads to a non-full load context for interference in each sub-band because only a part of the RB, i.e. $B_e$ or $B_c$, is used in a given cell.

In that context, the typical user, located at the origin $(0,0)$, is classified as a CCU if its SINR is larger than a threshold $\theta_c$, i.e. $\Gamma_c \geq \theta_c$:

$$\Gamma_c = \frac{H^c_{0,0} R_0^{-\alpha}}{\sigma^2 + \sum\limits_{i>0} H^c_{0,i} R_i^{-\alpha} 1(\Gamma_{c,i} \geq \theta_c)}, \tag{1}$$

and

$$\Gamma_{c,i} = \frac{H^c_{i,i} \|X_i - Y_i\|^{-\alpha}}{\sigma^2 + \sum\limits_{j\neq i} H^c_{i,j} \|X_j - Y_i\|^{-\alpha} 1(\Gamma_{c,j} \geq \theta_c)}, \tag{2}$$

where $R_i$ is the distance between BS $i$ and the typical user, $X_i$ is the position of BS $i$, $\Gamma_{c,i}$ is the SINR of a randomly selected user located at $Y_i$ within the cell $i$ whose BS is located at $X_i$, $H^c_{i,j}$ is the channel gain between the CCU $i$ and BS $j$, and $\alpha > 2$ is the path loss exponent. Note that, a spectrum mask constraint is considered for the OFDMA system. The SINR in (1) and (2) do not depend on the bandwidth $B_e$ and $B_c$, and $\sigma^2$ is the normalized additive white Gaussian noise power for a unit bandwidth.

The indicator function in (1) and (2) ensures that the typical user experiences interference only from BS that serves another CCU. This model enlightens the fact that this is an interdependent thinning process, and we prove in this paper that it is equivalent to a thinning process on BS PPP by a factor depending on the coverage probability. Also, if the typical user is not covered as a CCU, it becomes a CEU and will be covered if its SINR be larger than a threshold $\theta_e$, i.e. $\Gamma_e \geq \theta_e$, where

$$\Gamma_e = \frac{H^e_{0,0} R_0^{-\alpha}}{\sigma^2 + \sum\limits_{i>0} H^e_{0,i} R_i^{-\alpha} 1(\Gamma_{c,i} < \theta_c)}. \tag{3}$$

Since separated frequency sub-bands are allocated to CCU and CEU, we assumed that they do not experience the same channel gains. Hence the typical CEU experiences a new direct and interference channel gains, i.e., $H^e_{0,0}$ and $H^e_{0,i}$, respectively, exponentially distributed and independent from $H^c_{0,0}$ and $H^c_{0,i}$.

## III. Performance Metrics

### A. Coverage Probability

Consider two real numbers, $\theta_c > 0$ and $\theta_e > 0$, as the cell center and cell edge thresholds, respectively. The coverage probability is the probability that the SINR of the typical user exceeds a threshold [8]. Since the classification in CCU or CEU we consider is related to SINR, the probability to be covered for a CCU is equal 1 and the probability to be a CCU is

$$p_c(\theta_c, \lambda, \alpha) \triangleq \mathbb{P}[\Gamma_c \geq \theta_c]. \tag{4}$$

This probability is also the probability of being CCU and be covered; let's call it the central coverage probability. Further, the edge coverage probability is defined as

$$p_e(\theta_e, \theta_c, \lambda, \alpha) \triangleq \mathbb{P}[\Gamma_e \geq \theta_e | \Gamma_c < \theta_c]. \tag{5}$$

The global coverage probability of a typical general user randomly located in the considered model is then

$$p(\theta_e, \theta_c, \lambda, \alpha) \triangleq \mathbb{P}[\Gamma_c \geq \theta_c] + p_e(\theta_e, \theta_c, \lambda, \alpha)\mathbb{P}[\Gamma_c < \theta_c]. \tag{6}$$

**Theorem 1.** The central coverage probability in the non-full interference context is well approximated by

$$p_c(\theta_c, \lambda, \alpha) = \pi\lambda \int_0^\infty e^{-\pi\lambda v(1+\rho(\theta_c,\lambda,\alpha))} e^{-\theta_c \sigma^2 v^{\alpha/2}} \mathrm{d}v, \tag{7}$$

where $\rho(\theta_c, \lambda, \alpha) = p_c(\theta_c, \lambda, \alpha)\mathcal{K}(\theta_c, \alpha)$, $\delta = 2/\alpha$ and $\mathcal{K}(\theta, \alpha) = \theta^\delta \int_{\theta^{-\delta}}^\infty \frac{1}{1+x^{1/\delta}} \mathrm{d}x$.

*Proof.* See Appendix A. ☐

**Theorem 2.** The edge coverage probability in the non-full interference context is well approximated by (8), on the top of the next page, where

$$\begin{cases}\rho\left(\theta_e,\lambda,\alpha\right)=\left(1-p_c(\theta_c,\lambda,\alpha)\right)\mathcal{K}(\theta_e,\alpha),\\ \rho\left(\theta_e,\theta_c,\lambda,\alpha\right)=\rho\left(\theta_c,\lambda,\alpha\right)+\rho\left(\theta_e,\lambda,\alpha\right).\end{cases} \quad (9)$$

*Proof.* See Appendix B. □

Some closed-form expressions can be further obtained when considering particular cases.

*1) Noise, $\alpha=4$:* Theorem 1 simply becomes

$$p_c(\theta_c,\lambda,4)=\frac{\pi^{3/2}\lambda}{\sqrt{\theta_c\sigma^2}}e^{\frac{(\pi\lambda\beta(\theta_c,\lambda))^2}{4\theta_c\sigma^2}}Q\left(\frac{\pi\lambda\beta(\theta_c,\lambda)}{\sqrt{2\theta_c\sigma^2}}\right), \quad (10)$$

where $Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{\infty}\exp\left(-y^2/2\right)\mathrm{d}y$ is the standard Gaussian tail probability and $\beta(\theta_c,\lambda)=1+\rho(\theta_c,\lambda,4)$. Also, Theorem 2 can be given by (11) on the top of the next page, where $\beta(\theta_e,\lambda)=1+\rho(\theta_e,\lambda,4)$, and $\beta(\theta_e,\theta_c,\lambda)=1+\rho(\theta_e,\theta_c,\lambda,4)$. As an asymptotic analysis of the global coverage probability, it can be found that for $\theta_c\to\infty$, then $p_c(\infty,\lambda,\alpha)\to 0$ and

$$p(\theta_e,\infty,\lambda,4)=\frac{\pi^{3/2}\lambda}{\sqrt{\theta_e\sigma^2}}e^{\frac{(\pi\lambda\beta(\theta_e,\lambda))^2}{4\theta_e\sigma^2}}Q\left(\frac{\pi\lambda\beta(\theta_e,\lambda)}{\sqrt{2\theta_e\sigma^2}}\right). \quad (12)$$

The reason is that increasing $\theta_c$ shrinks the center region, thereby makes the global coverage probability a function of the edge threshold $\theta_e$. Also, setting $\theta_c\to 0$ makes all users as CCU, then the global coverage will be equal to the central coverage probability and $p_c(\theta_c\to 0,\lambda,\alpha)\to 1$.

*2) No Noise, $\alpha>2$:* It can be shown that the central and edge coverage probabilities become

$$p_c(\theta_c,\lambda,\alpha)=\frac{\sqrt{1+4\mathcal{K}(\theta_c,\alpha)}-1}{2\mathcal{K}(\theta_c,\alpha)}, \quad (13)$$

and

$$p_e(\theta_e,\theta_c,\lambda,\alpha)=\frac{1}{1-p_c(\theta_c,\lambda,\alpha)}\times \left(\frac{1}{1+\rho\left(\theta_e,\lambda,\alpha\right)}-\frac{1}{1+\rho\left(\theta_e,\theta_c,\lambda,\alpha\right)}\right). \quad (14)$$

*3) No Noise, $\alpha=4$, unique $\theta$:* The global coverage probability simplifies to

$$p(\theta,\theta,\lambda,4)=\frac{1}{1+\frac{\lambda_c}{\lambda}\mathcal{K}(\theta,4)}+\frac{1}{1+(1-\frac{\lambda_c}{\lambda})\mathcal{K}(\theta,4)}-\frac{1}{1+\mathcal{K}(\theta,4)}, \quad (15)$$

where $\lambda_c=p_c(\theta,\lambda,\alpha)\lambda$ is the equivalent density of BS dedicated to CCU and $\mathcal{K}(\theta,4)=\sqrt{\theta}\left(\frac{\pi}{2}-\arctan(\frac{1}{\sqrt{\theta}})\right)$. Since $\frac{\lambda_c}{\lambda}\le 1$, the derived coverage probability in (15) is larger than the coverage probability of the full interference network model with a single type user given in [8]. As the coverage does not take to account the resource used, we need another measure to evaluate and compare the proposed method fairly. So, we investigate spectral efficiency as follows.

## *B. Spectral Efficiency*

If only one kind of user is considered, the average rate for the typical user is $\tau=(B_e+B_c)\mathbb{E}[\ln(1+\mathrm{SINR})]$ [8]. In our case, the normalized spectral efficiency (SE) in nats/s/Hz is defined as ($i\in\{c,e\}$)

$$\eta_i(\theta,\lambda,\alpha)\triangleq\begin{cases}\delta\mathbb{E}[\ln\left(1+\Gamma_c\right)|\Gamma_c\ge\theta], & \text{for CCU}\\ (1-\delta)\mathbb{E}[\ln\left(1+\Gamma_e\right)|\Gamma_c<\theta], & \text{for CEU}\end{cases} \quad (16)$$

where $\delta=\frac{B_c}{B_c+B_e}$. Since for a positive random variable $Z$, $\mathbb{E}[Z]=\int_{t>0}P(Z>t)\mathrm{d}t$, the SE of a typical CCU straightforwardly follows from Theorems 1 and 2 as

$$\eta_c(\theta,\lambda,\alpha)=\frac{\delta}{p_c(\theta,\lambda,\alpha)}\mathbb{E}_{R_0}\left[\int_{t>0}e^{-\theta_{max}R_0^{\alpha}\sigma^2}\mathcal{L}_{I_c}(\theta_{max}R_0^{\alpha})\mathrm{d}t\right], \quad (17)$$

where $\theta_{max}=\max(e^t-1,\theta)$, and

$$\mathcal{L}_{I_c}(\theta_{max}R_0^{\alpha})=e^{-\pi\lambda R_0^2 p_c(\theta,\lambda,\alpha)\mathcal{K}(\theta_{max},\alpha)}. \quad (18)$$

The SE for a typical CEU is

$$\eta_e(\theta,\lambda,\alpha)=\frac{1-\delta}{1-p_c(\theta,\lambda,\alpha)}\mathbb{E}_{R_0}\left[\int_{t>0}\left(e^{-s_tR_0^{\alpha}\sigma^2}\mathcal{L}_{I_e}(s_tR_0^{\alpha})-e^{-(s_t+\theta)R_0^{\alpha}\sigma^2}\mathcal{L}_{I_e,I_c}(s_tR_0^{\alpha},\theta R_0^{\alpha})\right)\mathrm{d}t\right], \quad (19)$$

where $s_t=(e^t-1)$, and

$$\begin{cases}\mathcal{L}_{I_e}(s_tR_0^{\alpha})=e^{-\pi\lambda R_0^2\left(1-p_c(\theta,\lambda,\alpha)\right)\mathcal{K}(s_t,\alpha)},\\ \mathcal{L}_{I_e,I_c}(s_tR_0^{\alpha},\theta R_0^{\alpha})=\mathcal{L}_{I_e}(s_tR_0^{\alpha})\mathcal{L}_{I_c}(\theta R_0^{\alpha})\end{cases} \quad (20)$$

The global SE in the non-full interference network is then

$$\eta(\theta,\lambda,\alpha)=\mathbb{P}[\Gamma_c\ge\theta]\eta_c(\theta,\lambda,\alpha)+\mathbb{P}[\Gamma_c<\theta]\eta_e(\theta,\lambda,\alpha). \quad (21)$$

# IV. Numerical Results

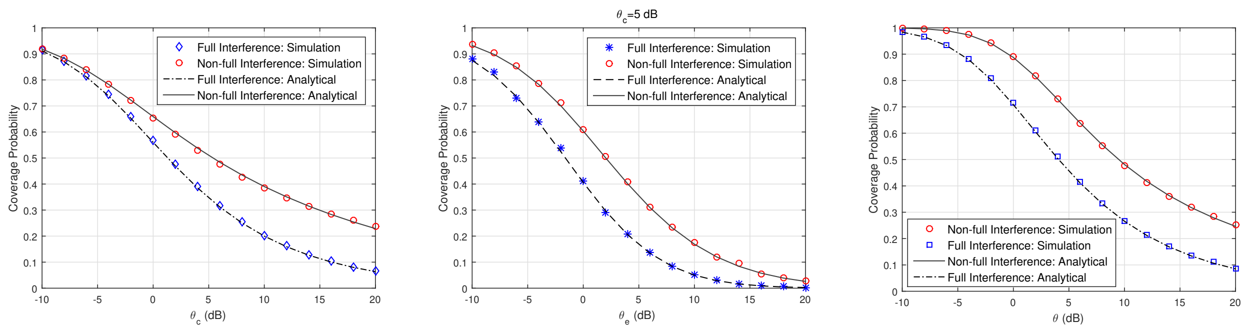
An interference limited network is assumed, i.e. $\sigma^2=0$ and $\lambda=0.25$ and $\alpha=4$ [8]. Fig. 1(a) compares the central coverage probability considering CCU/CEU classification under full and non-full interference models. Non-full interference model thins the PPP that reduces the interference to the typical central user by a factor depending on the coverage probability itself and $\theta_c$ as it can be inferred from (13). For $\theta_c<-8$ dB, the proportion of CCU is high compared to the one of CEU, generating lot of interference on sub-band $B_c$, that converges to the full interference assumption.

In Fig. 1(b), the cell edge coverage probability $p_e$ is plotted versus $\theta_e$ for specific CCU target threshold $\theta_c=5$ dB. As observed in Fig. 1(b), the coverage probability of CEU is enhanced when the sub-band $B_e$ is only used by CEU of the other cells compared to the full-load interference. Fig. 1(c) presents the global coverage probability of a typical user when a unique $\theta$ is considered, i.e. the CCU target threshold is equal to the CEU one. The gain of considering non-full interference strategy, i.e. when a sub-band is dedicated to a single kind of user is clear from this figure.

Fig. 2 represents the SE for CCU, CEU in the non-full interference network versus a user classification threshold

$$p_e(\theta_e,\theta_c,\lambda,\alpha) = \frac{\pi\lambda}{1-p_c(\theta_c,\lambda,\alpha)}\int_0^{\infty}\left(e^{-\pi\lambda v(1+\rho(\theta_e,\lambda,\alpha))}e^{-\theta_e\sigma^2 v^{\alpha/2}} - e^{-\pi\lambda v(1+\rho(\theta_e,\theta_c,\lambda,\alpha))}e^{-(\theta_c+\theta_e)\sigma^2 v^{\alpha/2}}\right)\mathrm{d}v. \quad (8)$$

$$p_e(\theta_e,\theta_c,\lambda) = \frac{\pi^{3/2}\lambda}{1-p_c(\theta_c,\lambda,4)}\left(\frac{\exp\left(\frac{(\pi\lambda\beta(\theta_e,\lambda))^2}{4\theta_e\sigma^2}\right)}{\sqrt{\theta_e\sigma^2}}Q\left(\frac{\pi\lambda\beta(\theta_e,\lambda))}{\sqrt{2\theta_e\sigma^2}}\right) - \frac{\exp\left(\frac{(\pi\lambda\beta(\theta_e,\theta_c,\lambda))^2}{4(\theta_e+\theta_c)\sigma^2}\right)}{\sqrt{(\theta_e+\theta_c)\sigma^2}}Q\left(\frac{\pi\lambda\beta(\theta_e,\theta_c,\lambda))}{\sqrt{2(\theta_e+\theta_c)\sigma^2}}\right)\right). \quad (11)$$




(a) (b) (c)

Figure 1. Coverage probability versus SINR thresholds.

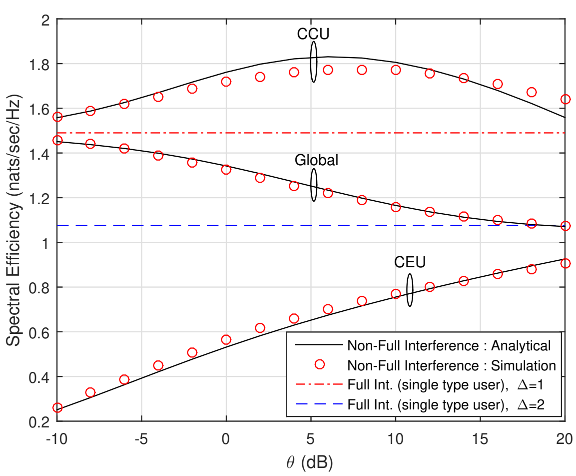



Figure 2. SE versus $\theta$ for SINR-proportional [2] resource sharing.

$\theta$. The SINR-proportional spectral resource sharing in [2] is applied. SE of CEU increases with $\theta$ since the number of CEU increases. On the other hand, SE of CCU first increases and then decreases after 8 dB, because the number of CCU decreases and it is not compensated by the gain of being closer to the BS. The result is that global SE, in (21), is nonetheless decreasing with $\theta$ because of the waste of the resource to create band with less interference. However, global SE is compared with the fractional frequency reuse (FFR) technique with reuse factor $\Delta$ presented in [8], under full interference network, i.e. only one type of user. The global SE is between the values obtained for $\Delta = 1$ and $\Delta = 2$ in FFR approach. On the other hand, since the typical CCU benefits from coverage-dependent fraction of resource and suffers from the same fraction of the interference, it has higher SE than FFR with $\Delta = 1$.

## V. Conclusion

This paper has presented a complete characterization of the downlink global coverage probability of typical user in the non-full interference homogeneous PPP network. Closed-form expressions of the coverage probability of CCU/CEU based on the received SINR level for typical user have been presented. In further works, we intend to investigate system performance when resource allocation is performed according to the number of users in the cells which implies correlations among two PPPs.


## Acknowledgement

This work has been partially funded by the Region Bretagne, France, under the project ARED CEREAL.


## Appendix

### A. *Proof of Theorem 1*

For coverage probability of the typical cell center user in (4), by conditioning on $R_0$, we have

$$\int_0^{\infty}\mathbb{P}\left[\frac{H_{0,0}^c R_0^{-\alpha}}{\sigma^2+I_c}\geq\theta_c\middle| R_0=r_0\right]f_{R_0}(r_0)\mathrm{d}r_0, \quad (22)$$

where $I_c=\sum_{i>0}H_{0,i}^c R_i^{-\alpha}1(\Gamma_{c,i}\geq\theta_c)$. Conditioned on $r_0$, others BS follow a Palm distribution described by the point process $\Phi\cap b^c(0,r_0)$, where $b(0,r_0)$ is the ball centered at the origin and of radius $r_0$. The PDF of the distance to the nearest BS is $f_R(r) = 2\pi\lambda r \exp(-\pi\lambda r^2)$.

$$\mathbb{P}\left[H^c_{0,0} \geq \theta_c r_0^\alpha (\sigma^2 + I_c)|r_0\right] \stackrel{a}{=} e^{-s\sigma^2}\mathcal{L}_{I_c}(s). \tag{23}$$

where $s = \theta_c r_0^\alpha$ and

$$\begin{aligned}
\mathcal{L}_{I_c}(s) &= \mathbb{E}_{\Phi,\{H^c_{0,i}\}}\left[\exp\Big(\sum_{i>0} -sH^c_{0,i}R_i^{-\alpha}1(\Gamma_{c,i} \geq \theta_c)\Big)\right] \\
&\stackrel{b}{=} \mathbb{E}_{\Phi,\{H^c_{0,i}\}}\left[\prod_{i>0}\Big(1 - \mathbb{E}[1(\Gamma_{c,i} \geq \theta_c)](1 - e^{-sH^c_{0,i}R_i^{-\alpha}})\Big)\right] \\
&\stackrel{c}{=} \exp\left(-2\pi\lambda\int_{r_0}^\infty \mathbb{E}\big[1(\Gamma_{c,i} \geq \theta_c)\big]\left(1 - \frac{1}{1+sr^{-\alpha}}\right)r\mathrm{d}r\right) \\
&\stackrel{d}{=} \exp\left(-2\pi\lambda p_c(\theta_c, \lambda, \alpha)\int_{r_0}^\infty\left(1 - \frac{1}{1+sr^{-\alpha}}\right)r\mathrm{d}r\right),
\end{aligned} \tag{24}$$

where (a) follows the i.i.d. exponential distribution of $H_{0,0}$ with mean 1 and is similar to [8], (b) comes from the law of total expectation, $\mathbb{E}_X[f(X)] = \mathbb{E}_Y[\mathbb{E}_X[f(X)|Y]]$, the independence of $\{\Gamma_{c,i}\}_i$, which is a reasonable assumption whose accuracy has been verified by simulations, and finally by factoring out $\mathbb{E}[1(\Gamma_{c,i} > \theta_c)]$, (c) follows from the probability generating function (PGFL) [9] of the PPP and identically distributed $\{\Gamma_{c,i}\}_i$, and (d) refers to the definition of the central coverage probability in (4).

### B. Proof of Theorem 2

Starting with (5) and using the Bayes rule, we have

$$p_e(\theta_e, \theta_c, \lambda, \alpha) = \frac{\mathbb{P}\left[\frac{H^e_{0,0}R_0^{-\alpha}}{\sigma^2+I_e} \geq \theta_e, \frac{H^c_{0,0}R_0^{-\alpha}}{\sigma^2+I_c} < \theta_c\right]}{\mathbb{P}\left[\frac{H^c_{0,0}R_0^{-\alpha}}{\sigma^2+I_c} < \theta_c\right]} \tag{25}$$

where $I_e = \sum_{i>0} H^e_{0,i}R_i^{-\alpha}1(\Gamma_{c,i} < \theta_c)$. Conditioned on $R_0$, the joint coverage probability in numerator of (25) becomes

$$\int_0^\infty \mathbb{P}\left[\frac{H^e_{0,0}R_0^{-\alpha}}{\sigma^2+I_e} \geq \theta_e, \frac{H^c_{0,0}R_0^{-\alpha}}{\sigma^2+I_c} < \theta_c \middle| R_0 = r_0\right] f_{R_0}(r_0)\mathrm{d}r_0. \tag{26}$$

Since $H^c_{0,0}, H^e_{0,0} \sim \exp(1)$ it comes

$$\begin{aligned}
&\mathbb{P}\Big(H^e_{0,0} \geq s_1(\sigma^2 + I_e), H^c_{0,0} < s_2(\sigma^2 + I_c)|r_0\Big) \qquad (27) \\
&\stackrel{a}{=} \mathbb{E}_{I_c,I_e}\Big[\mathbb{P}\left(H^e_{0,0} \geq s_1(\sigma^2 + I_e)|r_0, I_e\right) \\
&\qquad\qquad \times \mathbb{P}\left(H^c_{0,0} < s_2(\sigma^2 + I_c)|r_0, I_c\right)\Big] \\
&\stackrel{b}{=} \mathbb{E}_{I_e}\Big[\mathbb{P}\left(H^e_{0,0} \geq s_1(\sigma^2 + I_e)|r_0, I_e\right)\Big] \\
&\qquad - \mathbb{E}_{I_c,I_e}\Big[\mathbb{P}\left(H^e_{0,0} \geq s_1(\sigma^2 + I_e)|r_0, I_e\right) \\
&\qquad\qquad \times \mathbb{P}\Big(H^c_{0,0} \geq s_2(\sigma^2 + I_c)|r_0, I_c\Big)\Big] \\
&= e^{-s_1\sigma^2}\mathcal{L}_{I_e}(s_1) - e^{-(s_1+s_2)\sigma^2}\mathcal{L}_{I_e,I_c}(s_1, s_2)
\end{aligned}$$

where $s_1 = \theta_e r_0^\alpha$, $s_2 = \theta_c r_0^\alpha$, (a) comes from that conditioned on the interference, the two r.v. are independent, and (b) follows from the probability of a complementary event. Following the same derivation as in (24), $\mathcal{L}_{I_e}(s_1)$ can be expressed as $\mathcal{L}_{I_e}(s_1) = \exp(-\pi\lambda\rho(\theta_e, \lambda, \alpha)r_0^2)$. Also, $\mathcal{L}_{I_e,I_c}(s_1, s_2)$ captures the joint LT between $\Gamma_c$ and $\Gamma_e$ and can be derived as

$$\begin{aligned}
&\mathcal{L}_{I_e,I_c}(s_1, s_2) = \mathbb{E}_{I_c,I_e}[e^{-s_1I_e}e^{-s_2I_c}] \qquad (28) \\
&= \mathbb{E}_{\Phi,\{H^c_{0,i}\},\{H^e_{0,i}\}}\left[\exp\Big(-s_1\sum_{i>0}H^e_{0,i}R_i^{-\alpha}1(\Gamma_{c,i} < \theta_c)\Big)\right. \\
&\qquad \left.\times \exp\Big(-s_2\sum_{i>0}H^c_{0,i}R_i^{-\alpha}1(\Gamma_{c,i} \geq \theta_c)\Big)\right] \\
&\stackrel{a}{=} \mathbb{E}_{\Phi,\{H^c_{0,i}\},\{H^e_{0,i}\}}\left[\prod_{i>0}\Big(\mathbb{E}\big[1(\Gamma_{c,i} \geq \theta_c)\big]e^{-s_2H^c_{0,i}R_i^{-\alpha}}\right. \\
&\qquad \left.+ \big(1 - \mathbb{E}\big[1(\Gamma_{c,i} \geq \theta_c)\big]\big)e^{-s_1H^e_{0,i}R_i^{-\alpha}}\Big)\right] \\
&= \mathbb{E}_\Phi\left[\prod_{i>0}\Big(\frac{\mathbb{E}[1(\Gamma_{c,i} \geq \theta_c)]}{1+s_2R_i^{-\alpha}} + \frac{1 - \mathbb{E}[1(\Gamma_{c,i} \geq \theta_c)]}{1+s_1R_i^{-\alpha}}\Big)\right] \\
&= \exp\Big(-2\pi\lambda\int_{r_0}^\infty(1 - \frac{\mathbb{E}[1(\Gamma_{c,i} \geq \theta_c)]}{1+s_2r^{-\alpha}} - \frac{1 - \mathbb{E}[1(\Gamma_{c,i} \geq \theta_c)]}{1+s_1r^{-\alpha}})r\mathrm{d}r\Big) \\
&= \exp\left(-\pi\lambda p_c(\theta_c, \lambda, \alpha)\int_{r_0^2}^\infty\frac{1}{1+s_2^{-1}r^{\alpha/2}}\mathrm{d}r\right. \\
&\qquad \left.- \pi\lambda(1 - p_c(\theta_c, \lambda, \alpha))\int_{r_0^2}^\infty\frac{1}{1+s_1^{-1}r^{\alpha/2}}\mathrm{d}r\right).
\end{aligned}$$

where (a) follows from the same hypothesis used in the proof of Theorem 1.

## References


[1] K. Feng and M. Haenggi, "A location-dependent base station cooperation scheme for cellular networks," *IEEE Trans. Commun.*, vol. 67, no. 9, pp. 6415–6426, Sep. 2019.

[2] T. D. Novlan, R. K. Ganti, A. Ghosh, and J. G. Andrews, "Analytical evaluation of fractional frequency reuse for ofdma cellular networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 12, pp. 4294–4305, Dec. 2011.

[3] P. Mankar, G. Das, and S. Pathak, "Load-aware performance analysis of cell center/edge users in random hetnets," *IEEE Trans. Veh. Technol.*, vol. 67, no. 3, pp. 2476–2490, Mar. 2018.

[4] C. Skouroumounis, C. Psomas, and I. Krikidis, "Heterogeneous fd-mm-wave cellular networks with cell center/edge users," *IEEE Trans. Commun.*, vol. 67, no. 1, pp. 791–806, Jan. 2019.

[5] J. Yoon and G. Hwang, "Distance-based inter-cell interference coordination in small cell networks: stochastic geometry modeling and analysis," *IEEE Trans. Wireless Commun*, vol. 17, no. 6, pp. 4089–4103, Jun. 2018.

[6] H. Wang, K. Huang, and T. A. Tsiftsis, "Base station cooperation in millimeter wave cellular networks: Performance enhancement of cell-edge users," *IEEE Trans. Commun.*, vol. 66, no. 11, pp. 5124–5139, Nov. 2018.

[7] H. Zhuang and T. Ohtsuki, "A model based on poisson point process for analyzing mimo heterogeneous networks utilizing fractional frequency reuse," *IEEE Trans. Wireless Commun.*, vol. 13, no. 12, pp. 6839–6850, Dec. 2014.

[8] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Tran. Commun.*, vol. 59, no. 11, pp. 3122–3134, Nov. 2011.

[9] S. N. Chiu, D. Stoyan, W. S. Kendall, and J. Mecke, *Stochastic geometry and its applications*. John Wiley & Sons, 2013.