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# Optimal parameter estimation in a landslide motion model using the adjoint method

Mohit Mishra<sup>1</sup>, Gildas Besançon<sup>1</sup>, Guillaume Chambon<sup>2</sup>, and Laurent Baillet<sup>3</sup>

**Abstract**—This work proposes an optimal approach for parameter estimation in a landslide motion, based on the so-called adjoint method. The system is described by an extended sliding-consolidation model composed of an ordinary differential equation and 1D parabolic partial differential equation that represents landslide motion and pore pressure evolution respectively. The key feature of this model is pore pressure feedback, which regulates landslide motion and leads to coupling between both differential equations. Parameters to be estimated include the friction and dilatancy angle of the material. The objective functional for the optimal estimation is composed of: i) a cost function defined as the least square error between measurements and related simulated values, and ii) a product of Lagrange variables and system dynamics. A variational approach is applied to get the gradients of the cost functional with respect to parameters to be estimated and adjoint model. The cost functional is optimized, employing the steepest descent method to estimate parameters. Finally, the presented optimal estimation method is validated on a simulated test case.

## I. INTRODUCTION

Rapid urbanization [1] and climate change [2] have increased the frequency of occurrence of landslides. Which, in return, can have severe socio-economic consequences such as substantial cost in life losses, infrastructure, economy, and ecosystem of the region. Traditional strategies in landslide risk management are mostly oriented at avoiding building infrastructure at an exposed zone based on landslide hazard maps or stabilizing unstable slopes (landslide geometry corrections, water draining) or installing protecting structures [3]. However, infrastructure is still being developed either on or near massive landslides due to a lack of risk awareness. In such cases, slope stabilization is unaffordable, while moving the population to stable areas can pose considerable societal problems. Under these circumstances, implementation of an Early Warning System (EWS) can help to take timely actions to reduce life and economic losses in advance of hazardous events [4]. According to the United Nation's International Strategy for Disaster Reduction (ISDR) checklist [5], one of the principal element of EWS is a monitoring and warning service. An essential function of this service is to assess the

current status of the environment and establish the trends in environmental parameters to generate accurate warnings.

Some of the landslide EWS makes use of a rainfall threshold approach to examine the relationship between the movement and the triggering precipitation. The threshold is defined as a critical value, above which the probability of landslide occurrence is high. In most cases, the thresholds are defined on statistical and experience bases, neglecting physical criteria [6]. Another widely used and dependable approach of landslide forecasting considers the use of monitoring displacement (or velocity), pore pressure, and rain for continuously active landslides. Such approaches generate warnings based on a change in slope displacement rates over time, e.g., inverse velocity criteria [7].

As a consequence, many developments are proposed in the literature for slope displacement prediction models. The sliding-consolidation model [8] proposes a single event behavior of flow slides in loose, cohesionless materials. This model was later revisited incorporating viscous force [3], [9], [10] and termed as a viscoplastic sliding-consolidation model. The extended sliding-consolidation model [11] demonstrates diverse rates of landslide due to mechanical feedback. Apart from the models mentioned above, many more hydrological, hydrogeological, meteorological, and geotechnical models [12] are investigated. These dynamical models are sensitive to the parameters of the system. For known geometry and material properties of the landslide, equations governing the dynamics of the landslide can be solved to predict the displacement pattern knowing rainfall input. However, in fact, not all the parameter values are known.

This paper presents optimal parameter estimation in an extended sliding-consolidation model (coupled ODE-PDE system) of a landslide using the adjoint method. The adjoint method scheme demonstrated its effectiveness in many studies and applications, for instance, air traffic flow management [13], space shuttle reentry problem [14], state and parameter estimation in switched 1D hyperbolic PDEs [15], traffic flow [16], and overland flow [17]. Some authors [18], [19], [20], [21] studied a similar approach to estimate the Manning roughness coefficient or to stabilize and control the water level in an open channel flow.

The contents of the paper are as follows: Section 2 defines a landslide model depicting landslide behavior and the problem statement, while Section 3 presents the proposed method to solve it. In Section 4, the simulation results demonstrate the effectiveness of the solution. Finally, Section 5 provides a conclusion and discusses future directions of the work.

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## II. PROBLEM FORMULATION

### A. Extended sliding-consolidation model

In the extended sliding-consolidation model of a landslide [11], a slide block is assumed to be placed on an inclined surface, as shown in Fig. 1. The model proposes a mechanism of opposition to slide block downslope movement by basal Coulomb friction and regulation through basal pore fluid pressure feedback. The model assumes two components of total basal pore pressure: i) imposed pore pressure  $p_i$  due to rain infiltration and ii) development of excess pore pressure  $p_e$  in response to the contraction or dilation of the basal shear zone. The motion of the slide block and excess pore pressure evolution are described by Eq. (1) and (2) respectively.

#### Momentum equation

$$\begin{aligned} \frac{d^2 u_x}{dt^2} &= \frac{dv}{dt} = g \cos \psi [\sin(\theta - \psi) - \cos(\theta - \psi) \tan \phi] \\ &+ \frac{\cos^2 \psi \tan \phi}{\rho Z} [p_i(0, t) + p_e(0, t)] \quad \text{i.e.} \quad (1) \\ \dot{v} &= f(\phi, \psi, p_e(0, t), p_i(0, t)), \quad v(0) = v_0 \end{aligned}$$

#### Excess pore pressure diffusion equation

$$\begin{aligned} \frac{\partial p_e(z, t)}{\partial t} &= D \frac{\partial^2 p_e(z, t)}{\partial z^2} \\ \frac{\partial p_e(0, t)}{\partial z} &= \frac{\rho_w g \psi}{K} v, \quad (2) \\ p_e(Z, t) &= 0, \quad p_e(z, 0) = p_{e_0} \end{aligned}$$

where  $u_x(t)$  and  $v(t)$  denote the displacement and velocity of the slide block respectively,  $p_i(0, t)$  is the imposed pore pressure at the slide block base,  $p_e(z, t)$  is the excess pore pressure distribution,  $(z, t) \in [0, Z] \times [0, T]$  with  $Z$  as a spatial domain length (slide block thickness), and  $T$  is the length of time horizon. A coordinate  $z$  translates with the base of the slide block such that with dilation or contraction of shear zone the base of the slide block is always located at  $z = 0$ .  $\phi$  is the friction angle characterizing the mechanical strength of the material,  $\psi$  is the dilatancy angle representing volume change of the material when they are subjected to deformation,  $\rho$  is the soil density,  $\rho_w$  is the pore water density,  $D$  is the diffusion coefficient ( $D > 0$ ),  $K$  is the hydraulic conductivity,  $g$  is the acceleration due to gravity and  $\theta$  is the sliding angle. In addition,  $v_0$  and  $p_{e_0}$  are initial values (assumed to be known) of the  $v$  and  $p_e$  respectively. Finally,  $f$  is the function characterizing the right hand side of Eq. (1), depends on  $\phi$ ,  $\psi$ ,  $p_e(0, t)$ , and  $p_i(0, t)$ .

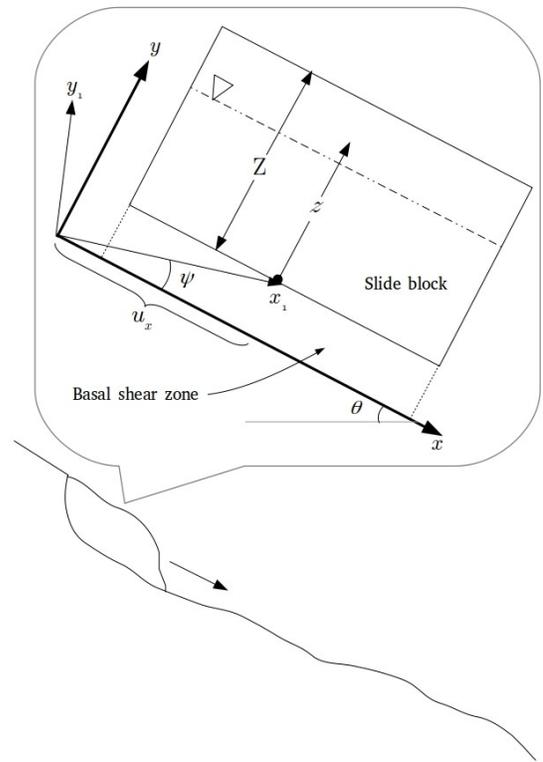


Fig. 1. The coordinate systems, geometric variables and material property of the slide block

### B. Optimal estimation problem

On the basis of the formed model, the main goal in this paper is to estimate friction and dilatancy angle ( $\phi$  &  $\psi$ ) of the material in a landslide from a measured velocity  $v_{mea}(t)$  and known imposed pore pressure evolution  $p_i(0, t)$ . Technically, we are interested in minimizing the cost function  $J(\phi, \psi)$  defined as the least square error between velocity measurement and simulated velocity profile in (3).

$$\begin{aligned} J(\phi, \psi) &= \frac{\epsilon_1}{2} \|\phi - \phi_G\|^2 + \frac{\epsilon_2}{2} \|\psi - \psi_G\|^2 \\ &+ \frac{\epsilon_3}{2} \int_0^T [v(t) - v_{mea}(t)]^2 dt \quad (3) \end{aligned}$$

where  $\epsilon_1$ ,  $\epsilon_2$  and  $\epsilon_3$  are weighting factors to calibrate the estimated and guessed parameter values, and  $T$  is the simulation time. The first guessed values of parameters  $\phi_G$  and  $\psi_G$  are introduced to improve the convergence of the optimization problem, which are chosen in a reasonable range around the expected real ones.

For cost function  $J$  in (3), continuous velocity measurement is required, which is sometimes not feasible. Instead, in many cases velocity measurements are only available at particular sampling times (e.g., on an hourly basis) i.e., the observation process is realized at some discrete points ( $t_k$ ) on a time domain. In such a scenario, the cost function is formulated as

$$\begin{aligned} J(\phi, \psi) &= \frac{\epsilon_1}{2} \|\phi - \phi_G\|^2 + \frac{\epsilon_2}{2} \|\psi - \psi_G\|^2 \\ &+ \frac{\epsilon_3}{2} \sum_{k=1}^N \left( \int_0^T \delta_A(t - t_k) v(t) dt - v_{mea}(t_k) \right)^2 \quad (4) \end{aligned}$$

where  $N$  is the the number of observation values of  $v_{mea}(t_k)$  and  $\delta_A(t-t_k)$  is an approximate Dirac-Delta function defined as a Gaussian function with a very small variance  $\sigma^2$ ,

$$\delta_A(t-t_k) = e^{-\frac{(t-t_k)^2}{\sigma^2}}.$$

This Gaussian approximation guarantees the smoothness of the observation function. In this paper, we thus consider the minimization of the cost function (4).

### III. SOLUTION METHOD

#### A. Cost functional

From the defined problem statement, the optimal values of  $\phi$  and  $\psi$  must minimize the cost function (4) subject to the dynamics (1)-(2) as constraints. To solve this constrained optimization problem, let us consider the Lagrange multipliers  $\lambda(t)$  and  $\Gamma(z,t)$  combining both system equations and cost function into a new cost functional  $L$

$$L(v, p_e, \phi, \psi) = J + \int_0^T \lambda(t) [\dot{v} - f(\psi, \phi, p_e(0,t), p_i(0,t))] dt + \int_0^T \int_0^Z \Gamma(z,t) \left( \frac{\partial p_e}{\partial t} - D \frac{\partial^2 p_e}{\partial z^2} \right) dz dt. \quad (5)$$

Using integration by parts, the cost functional can be redefined as

$$\begin{aligned} L(v, p_e, \phi, \psi) &= \frac{\varepsilon_1}{2} \|\phi - \phi_G\|^2 + \frac{\varepsilon_2}{2} \|\psi - \psi_G\|^2 \\ &+ \frac{\varepsilon_3}{2} \sum_{k=1}^N \left( \int_0^T \delta_A(t-t_k) v(t) dt - v_{mea}(t_k) \right)^2 \\ &+ [\lambda v]_0^T - \int_0^T v \dot{\lambda} dt - \int_0^T \lambda(t) f(\phi, \psi, p_e(0,t), p_i(0,t)) dt \\ &+ \int_0^Z [\Gamma p_e]_0^T dz - \int_0^T \int_0^Z p_e \left( \frac{\partial \Gamma}{\partial t} + D \frac{\partial^2 \Gamma}{\partial z^2} \right) dz dt \\ &+ D \int_0^T \left[ \frac{\partial \Gamma}{\partial z} p_e \right]_0^Z dt - D \int_0^T \Gamma(Z,t) \frac{\partial p_e(Z,t)}{\partial z} dt \\ &+ \frac{D \rho_w g \Psi}{K} \int_0^T \Gamma(0,t) v(t) dt \end{aligned} \quad (6)$$

#### B. Adjoint-based approach

Based on the adjoint approach, the first derivatives of  $L$  with respect to  $v$  and  $p_e$  are set to zero, the gradients of the cost functional with respect to  $\phi$  and  $\psi$  are also computed.

- Variation of  $L$  w.r.t.  $v$ :

$$\begin{aligned} L_v &= \varepsilon_3 \sum_{k=1}^N \left( \int_0^T \delta_A(t-t_k) v(t) dt - v_{mea}(t_k) \right) \cdot \int_0^T \delta_A(t-t_k) dt \delta v \\ &- \int_0^T \dot{\lambda} dt \delta v + \lambda(T) \delta v(T) - \lambda(0) \delta v(0) \\ &+ \frac{D \rho_w g \Psi}{K} \int_0^T \Gamma(0,t) dt \delta v \end{aligned}$$

(Here,  $\delta v(0) = 0$  since the initial value  $v_0$  is fixed)

$$L_v = 0 \Leftrightarrow \begin{cases} \dot{\lambda} = \varepsilon_3 \sum_{k=1}^N \delta_A(t-t_k) \left[ \int_0^T \delta_A(t-t_k) v(t) dt - v_{mea}(t_k) \right] \\ \quad + \frac{D \rho_w g \Psi}{K} \Gamma(0,t) \\ \lambda(T) = 0 \end{cases} \quad (7)$$

- Variation of  $L$  w.r.t.  $p_e$ :

$$\begin{aligned} L_{p_e} &= - \int_0^T \lambda(t) f_{p_e(0,t)}(\phi, \psi, p_e(0,t), p_i(0,t)) dt \delta p_e(0,t) \\ &+ \int_0^Z \Gamma(z,T) dz \delta p_e(z,T) - \int_0^Z \Gamma(z,0) dz \delta p_e(z,0) \\ &+ D \int_0^T \frac{\partial \Gamma(Z,t)}{\partial z} dt \delta p_e(Z,t) - D \int_0^T \frac{\partial \Gamma(0,t)}{\partial z} dt \delta p_e(0,t) \\ &- \int_0^T \int_0^Z \left( \frac{\partial \Gamma}{\partial t} + D \frac{\partial^2 \Gamma}{\partial z^2} \right) dz dt \delta p_e \\ &- D \int_0^T \Gamma(Z,t) dt \delta \frac{\partial p_e(Z,t)}{\partial z} \end{aligned}$$

( $\delta p_e(z,0) = \delta p_e(Z,t) = 0$  because  $p_e(z,0)$  and  $p_e(Z,t)$  are fixed)

$$L_{p_e} = 0 \Leftrightarrow \begin{cases} \frac{\partial \Gamma}{\partial t} = -D \frac{\partial^2 \Gamma}{\partial z^2} \\ \frac{\partial \Gamma(0,t)}{\partial z} = -\frac{1}{D} \lambda(t) f_{p_e(0,t)} \\ \Gamma(Z,t) = 0 \\ \Gamma(z,T) = 0 \end{cases} \quad (8)$$

Equations (7) and (8) are the adjoint system equations where  $f_{p_e(0,t)}$  is the differentiation of  $f(\phi, \psi, p_e(0,t), p_i(0,t))$  w.r.t.  $p_e(0,t)$ .

- Gradient of  $L$  w.r.t.  $\phi$ :

$$L_\phi = \frac{\partial L}{\partial \phi} = \varepsilon_1 (\phi - \phi_G) - \int_0^T \lambda(t) f_\phi(t) dt \quad (9)$$

where  $f_\phi(t)$  is the differentiation of  $f(\phi, \psi, p_e(0,t), p_i(0,t))$  w.r.t.  $\phi$ .

- Gradient of  $L$  w.r.t.  $\psi$ :

$$\begin{aligned} L_\psi &= \frac{\partial L}{\partial \psi} = \varepsilon_2 (\psi - \psi_G) - \int_0^T \lambda(t) f_\psi(t) dt \\ &+ \frac{D \rho_w g \Psi}{K} \int_0^T \Gamma(0,t) v(t) dt \end{aligned} \quad (10)$$

where  $f_\psi(t)$  is the differentiation of  $f(\phi, \psi, p_e(0,t), p_i(0,t))$  w.r.t.  $\psi$ .

The gradients (9)-(10) describe sensitivity of the cost function (4) to variation in parameters  $(\phi, \psi)$  under the constraints of system dynamics (1)-(2).

#### C. Steepest descent method

We solve the optimization problem with a steepest descent method. The gradients  $L_\phi$  and  $L_\psi$  give the descent directions to estimate optimal parameter values  $(\phi^*, \psi^*)$ . We choose constant step sizes  $\gamma_\phi$  and  $\gamma_\psi$ , which minimize the cost functional in the descent direction. We use Algorithm 1 below to solve the optimization problem. The algorithm stops when the norm of the gradient is smaller than the chosen tolerance  $\xi_\phi$  and  $\xi_\psi$ . Notice that computing gradients require solving system equations (1)-(2) along with the adjoint system (7)-(8).

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**Algorithm 1:** Optimal parameter estimation
 

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**Input:** Initial values,  $v(0)$  &  $p_e(z,0)$   
 Imposed pore pressure time series,  $p_i(0,t)$   
 Measured velocity profile,  $v_{mea}(t_k)$   
 Guessed parameter values,  $\phi_G$  &  $\psi_G$   
 Set initial parameter values,  $\phi_0$  &  $\psi_0$   
 Step sizes,  $\gamma_\phi$  &  $\gamma_\psi$   
 The gradient tolerances,  $\xi_\phi$  &  $\xi_\psi$   
 Stop\_flag = false  
 Iteration index  $k=1$   
 Set  $\phi^k = \phi_0$  and  $\psi^k = \psi_0$

**Output:**  $\phi^*$ ,  $\psi^*$

**while** Stop\_flag = false **do**  
 Simulate system equations (1)-(2) with  $v(0)$ ,  
 $p_e(z,0)$ ,  $\phi^k$ , &  $\psi^k$ ;  
 Simulate the adjoint system equation (7)-(8)  
 (backward in time);  
 Compute gradients  $L_\phi^k$ ,  $L_\psi^k$  using (9)-(10);  
**if**  $|L_\phi| \leq \xi_\phi$  &  $|L_\psi| \leq \xi_\psi$  **then**  
 | Stop\_flag = true;  
**else**  
 |  $\phi^{k+1} = \phi^k - \gamma_\phi L_\phi^k$ ;  
 |  $\psi^{k+1} = \psi^k - \gamma_\psi L_\psi^k$ ;  
 |  $k = k + 1$ ;  
**end**  
 Display  $\phi$ ,  $\psi$  and  $J$   
**end**  
 Return  $\phi^*$ ,  $\psi^*$

---

#### IV. SIMULATION RESULTS

##### A. Synthetic Data

To validate the effectiveness of our approach, a measured velocity profile  $v^{mea}(t)$  is generated synthetically by solving the system equations (1)-(2). The parameter values and initial values used for the simulation are summarized in Table I. The numerical solution is here obtained with a stepwise analytical method for solving (1) and a Crank-Nicolson method for solving (2). In the simulations, imposed pore pressure is assumed to be sinusoidal in time, representing rainfall variations. The value of imposed pore pressure oscillates around  $p_{crit}$  given as

$$p_{crit} = \frac{g \cos \psi [\cos(\theta - \psi) \tan \phi - \sin(\theta - \psi)]}{\cos^2 \psi \tan \phi / \rho Z},$$

which corresponds to the value of pore pressure above which slide block starts to accelerate. At instances, when imposed pressure is less than or equal to  $p_{crit}$  the slide block is at rest, i.e.,  $v_0 = 0$ , and as excess pore pressure generates in response to the motion of the slide block  $p_{e0} = 0$ . Simulated synthetic velocity measurement (with noises) and pore pressure profiles are shown in Fig. 2 and Fig. 3 respectively. The study is carried out for different noise levels in a measurement. White Gaussian noises are added to simulated velocity profile such that signal to noise ratios (SNR) are 10db and 20db respectively.

TABLE I  
PARAMETER VALUES

Parameters	Value	Unit
Initial velocity, $v_0$	0	m/s
Initial excess pore pressure distribution, $p_{e0}$	0	Pa
Simulation time, $T$	2000	s
Time step, $\Delta t$	0.01	s
Space step, $\Delta z$	0.0066	m
Diffusion coefficient, $D$	$3 \times 10^{-3}$	$m^2/s$
Acceleration due to gravity, $g$	9.8	$m/s^2$
Slide block thickness, $Z$	0.65	m
Hydraulic conductivity, $K$	$2 \times 10^{-5}$	m/s
Plane inclination angle, $\theta$	31	deg
Slide block mass density, $\rho$	2000	$kg/m^3$
Pore water density, $\rho_w$	1000	$kg/m^3$
<b>Friction angle, <math>\phi</math></b>	<b>35</b>	<b>deg</b>
<b>Dilatancy angle, <math>\psi</math></b>	<b>6</b>	<b>deg</b>

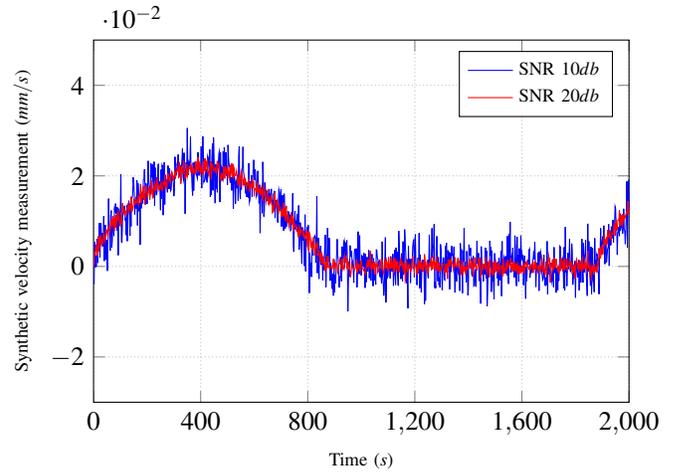


Fig. 2. Synthetic velocity measurement ( $v_{mea}$ )

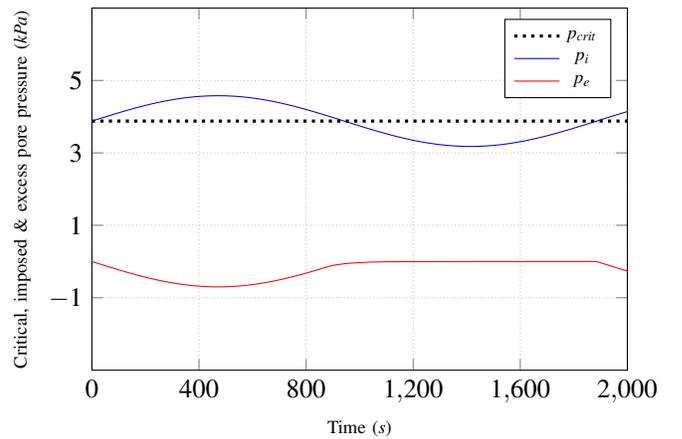


Fig. 3. Critical ( $p_{crit}$ ), imposed ( $p_i$ ), and excess pore pressure ( $p_e$ )

##### B. Parameter estimation

The simulation for Parameter estimation is performed, following Algorithm 1, with the parameter values given

in Table II. Though the generated synthetic measurement is continuous, for the optimization problem only  $N = 101$  observation values are taken into consideration i.e., at each  $t_k = \{0, 20, 40, \dots, 2000\} \forall k \in (0, N)$ . For the sake of simplicity, all the velocity measurements are considered in  $mm/sec$  without any loss of generality.

TABLE II  
PARAMETER VALUES FOR THE ALGORITHM

Parameters	Value	Unit
Number of observation values, $N$	101	-
Guessed friction angle, $\phi_G$	31	deg
Guessed dilatancy angle, $\psi_G$	4	deg
Initial friction angle, $\phi_0$	25, 29, 32	deg
Initial dilatancy angle, $\psi_0$	3, 4, 5	deg
Step size, $\gamma_\phi$	$2.5 \times 10^{-4}$	-
Step size, $\gamma_\psi$	$7 \times 10^{-5}$	-
Weighting factor, $\epsilon_1$	$2.5 \times 10^{-3}$	-
Weighting factor, $\epsilon_2$	$5 \times 10^{-3}$	-
Weighting factor, $\epsilon_3$	30	-
Stop condition1, $\xi_\phi$	$10^{-2}$	-
Stop condition2, $\xi_\psi$	$10^{-2}$	-

A convergence of the parameter estimates for two different noise levels and distinct initial parameter values can be seen in Fig. 4, Fig. 5, Fig. 6 and Fig. 7. We also observe a decrease in cost function and gradients in Fig. 8, and Fig. 9. For all these simulations step sizes ( $\gamma_\phi, \gamma_\psi$ ) and tolerance for gradients ( $\xi_\phi, \xi_\psi$ ) are kept the same. From the simulation results, the following observations can be made: i) for similar initial parameter values, the algorithm takes few more iterations to estimate parameters in case of a higher noise level in measurements, with some less accuracy (e.g., for  $\phi_0 = 29$ , with 20 dB SNR measurement estimated  $\phi$  is 34.95 after  $k=128$  iterations, while with 10 dB SNR measurement estimated  $\phi$  is 34.90 after 130 iterations) and ii) for same measurements but initial parameter values farther from actual ones, the algorithm requires few more iterations to estimate parameters (e.g., for 10 dB SNR measurement, parameter estimation took  $k=126$  iterations with  $\psi_0 = 5$ , whereas 130 iterations with  $\psi_0 = 3$ ).

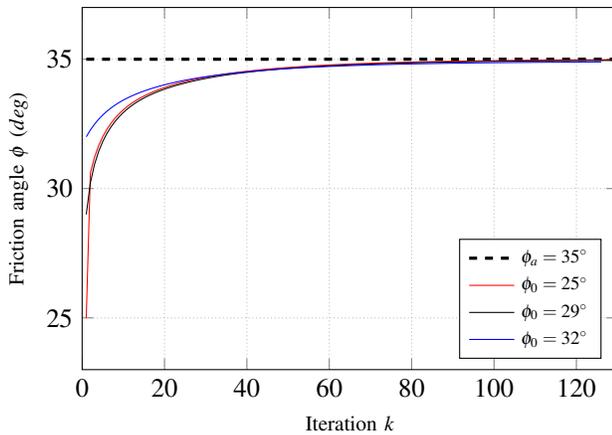


Fig. 4. Evolution of the parameter estimate ( $\phi$ ) for synthetic velocity measurement with SNR 20dB

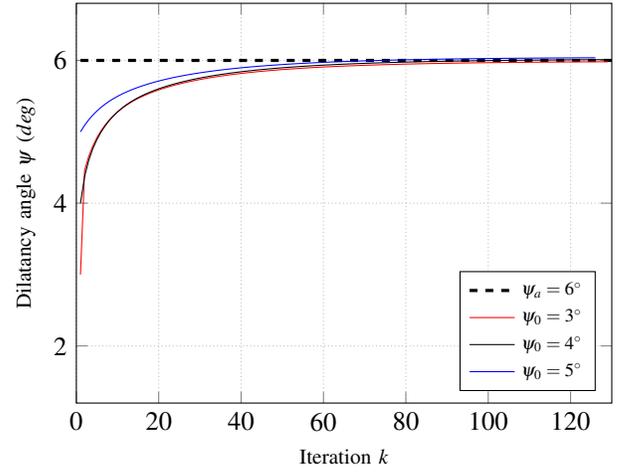


Fig. 5. Evolution of the parameter estimate ( $\psi$ ) for synthetic velocity measurement with SNR 20dB

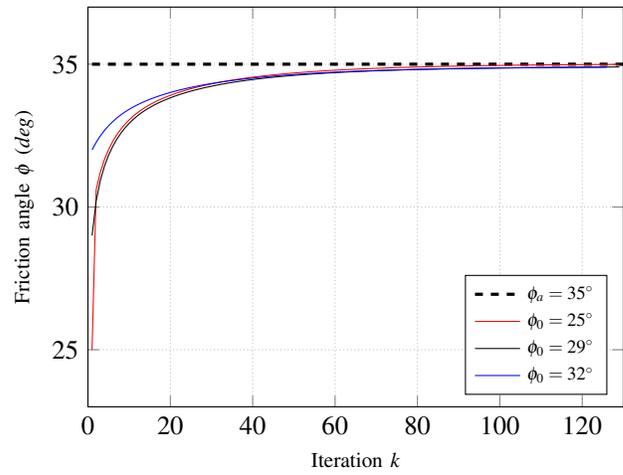


Fig. 6. Evolution of the parameter estimate ( $\phi$ ) for synthetic velocity measurement with SNR 10dB

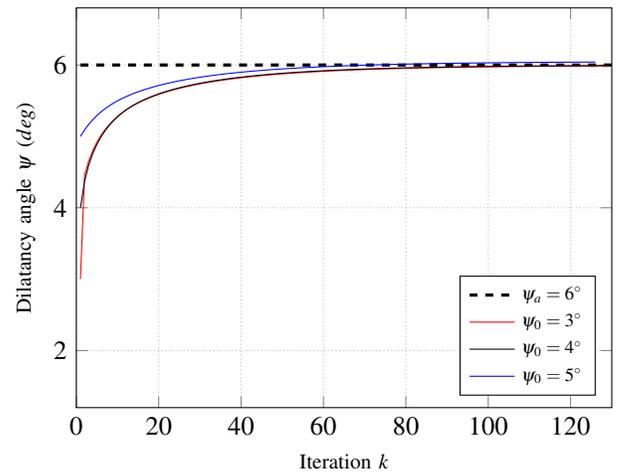


Fig. 7. Evolution of the parameter estimate ( $\psi$ ) for synthetic velocity measurement with SNR 10dB

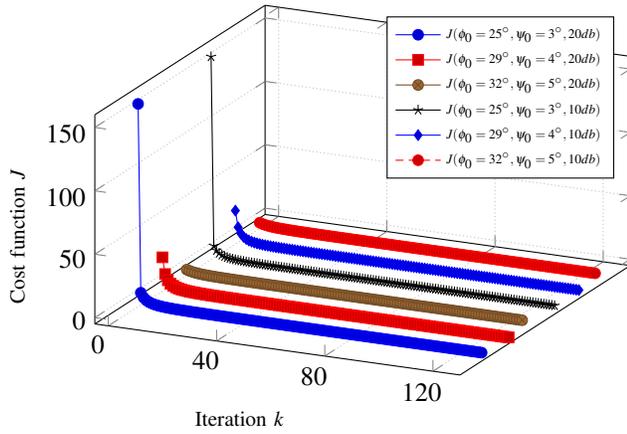


Fig. 8. Evolution of cost function  $J$

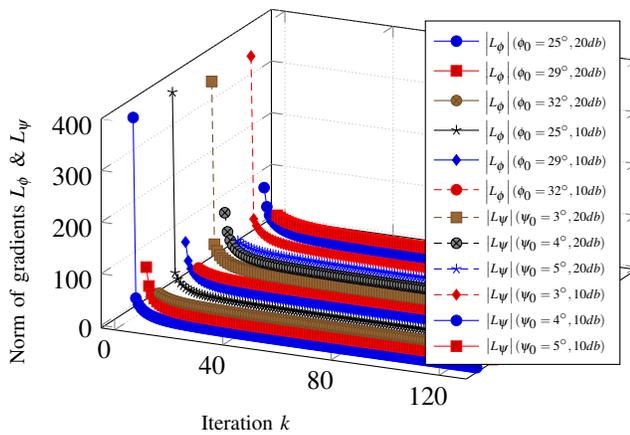


Fig. 9. Evolution of norm of gradients  $L_\phi$  &  $L_\psi$

## V. CONCLUSION AND FUTURE WORK

An optimal approach for parameter estimation in a landslide motion based on the adjoint method and the steepest descent approach has been proposed and validated in this paper. Firstly, an extended sliding-consolidation model of a landslide has been presented, which is an ODE-PDE coupled system. Secondly, the parameter estimation problem has been formulated as an optimization problem using Lagrange multiplier approach. Then the adjoint method has been introduced to obtain gradients of the cost functional and the adjoint equations. These gradients are then utilized as descent directions for the steepest descent method to get optimal parameter values. Lastly, the proposed solution method has been validated for synthetically generated noisy data. The optimal values of friction and dilatancy angle of the material have finally been well estimated.

Based on this result, a future direction for work will be to include estimation of the initial states of the system along with the parameters of interest for synthetically generated data and actual field measurements to relax the assumption on imposed pore pressure. Finally, this approach could be extended to more complex landslide models.

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