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Novel Computing Technique in Combinatorics

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Abstract: The growing complexity of mathematical modelling and its application demands the simplicity of numerical equations and combinatorial techniques for solving the scientific problems facing today. In this connection, the paper introduces a novel computing technique in the area of combinatorics.

Keywords: computing model, combinatorics, counting technique, binomial coefficient

I. Introduction

Combinatorics is a collection of various counting techniques or methods and models and has many applications in science, technology, and management. In the research paper, novel counting techniques in the field of combinatorics are introduced that are useful for scientific researchers who are solving scientific problems and meeting today's challenges.

II. New Combinatorial Techniques

New combinatorial expression is

$$V_r^n = \frac{(r+1)(r+2)(r+3)\cdots(r+n-1)(r+n)}{n!} \quad (n, r \in N, n \geq 1, \& r \geq 0),$$

where $N = \{0, 1, 2, 3, 4, 5, \dots\}$ is the set of natural numbers including the element 0.

This kind of combinatorial expressions has already been used in the previous papers [1,2] as binomial coefficients.

Some results and its proofs of the combinatorial expression are provided below.

Result 1: $V_0^1 = V_0^n = 1$

$$\text{Proof. } V_0^1 = \frac{(0+1)}{1!} = 1 \quad (i)$$

$$V_0^n = \frac{(0+1)(0+2)(0+3)\cdots(0+n)}{n!} = \frac{n!}{n!} = 1 \quad (ii)$$

From (i) and (ii), the result 1 is true.

Result 2: $V_r^{n+1} - V_r^n = V_{r-1}^n$

$$\text{Proof. } V_r^n = \frac{(r+1)(r+2)(r+3)\cdots(r+n)}{n!}$$

$$V_r^{n+1} = \frac{(r+1)(r+2)(r+3)\cdots(r+n)(r+n+1)}{(n+1)!}$$

$$V_r^{n+1} - V_r^n = \frac{(r+1)(r+2)(r+3)\cdots(r+n)}{n!} \left[\frac{r+n+1}{n+1} - 1 \right]$$

$$V_r^{n+1} - V_r^n = \frac{r(r+1)(r+2)(r+3)\cdots(r+n)}{n!} = V_{r-1}^n \quad (\text{iii})$$

It is understood from (iii) that the result 2 is true.

Result 3: $1 + V_1^1 + V_1^2 + V_1^3 \cdots V_1^n = V_2^n$

Proof. $V_2^n = \frac{(2+1)(2+2)(2+3)\cdots(2+n-1)(2+n)}{n!} = \frac{(n+1)(n+2)}{2!} \quad (\text{iv})$

$$1 + V_1^1 + V_1^2 + V_1^3 \cdots V_1^n = 1 + 2 + 3 + \cdots + n + 1 = \frac{(n+1)(n+2)}{2!} \quad (\text{v})$$

From (iv) and (v), the result 3 is true.

Result 4: $V_r^n = V_n^r$ ($n, r \geq 1$ & $n, r \in N$)

Proof. $V_r^n = V_n^r$ implies $\frac{(r+1)(r+2)\cdots(r+n)}{n!} = \frac{(n+1)(n+2)\cdots(n+r)}{r!}$

Assume that $r = n + m$ ($m \in N$ & $m \geq 1$). Let us show that $V_{n+m}^n = V_n^{n+m}$.

$$V_{n+m}^n = \frac{(n+m+1)(n+m+2)\cdots(n+m+n)}{n!} = \frac{(n+1)(n+2)\cdots(n+m+n)}{(n+m)!} \quad (\text{vi})$$

$$V_n^{n+m} = \frac{(n+1)(n+2)\cdots(n+n)(n+n+1)(n+n+2)\cdots(n+n+m)}{(n+m)!} \quad (\text{vii})$$

From (vi) and (vii), $V_{n+m}^n = V_n^{n+m}$ is true.

Assume that $r = n - m$ ($n > m$). Let us show that $V_{n-m}^n = V_n^{n-m}$.

$$V_{n-m}^n = \frac{(n-m+1)(n-m+2)\cdots(n-m+n)}{n!} = \frac{(n+1)(n+2)\cdots(n+n-m)}{(n-m)!} \quad (\text{viii})$$

$$V_n^{n-m} = \frac{(n+1)(n+2)\cdots(n+n-m)}{(n-m)!} \quad (\text{ix})$$

From (viii) and (ix), $V_{n-m}^n = V_n^{n-m}$ is true.

If $r = n$, $V_r^n = V_n^r$ is obviously true for $r = n$.

Hence, the result 4 is true.

Result 5: $V_n^n = 2V_{n-1}^n$

Proof.
$$V_n^n = \frac{(n+1)(n+2)\cdots(n+n-1)2n}{(n-1)!n} = \frac{2(n+1)(n+2)\cdots(n+n-1)}{(n-1)!} = 2V_{n-1}^n$$

Hence, the result 5 is true.

Result 6: $V_0^n + V_1^n + V_2^n + V_3^n \cdots + V_{r-1}^n + V_r^n = V_r^{n+1}$

Proof. This result is proved by mathematical induction.

Basis. Let $r = 1$. $V_0^n + V_1^n = V_1^{n+1}$ implies $n + 2 = n + 2$.

Inductive hypothesis.

Let us assume that $V_0^n + V_1^n + V_2^n + \cdots + V_{k-1}^n = V_{k-1}^{n+1}$ is true for $r = k - 1$.

Inductive step. We must show that the inductive hypothesis is true for $r = k$.

$V_0^n + V_1^n + \cdots + V_{k-1}^n + V_k^n = V_k^{n+1}$ implies $V_0^n + V_1^n + \cdots + V_{k-1}^n = V_k^{n+1} - V_k^n = V_{k-1}^{n+1}$.

Hence, it is proved.

Result 7: $\frac{(n+r)!}{n!r!} = V_r^n \quad (n, r \geq 1 \ \& \ n, r \in \mathbf{N})$

Proof.
$$\frac{1.2.3 \cdots r(r+1)(r+2)\cdots(r+n)}{n!r!} = \frac{r!(r+1)(r+2)\cdots(r+n)}{n!r!} = V_r^n$$

Hence, the result 7 is true.

III. Conclusion

In the research paper, novel counting techniques in the field of combinatorics have been introduced that are useful for scientific researchers who are solving scientific problems and meeting today's challenges.

IV. References

- [1]. Annamalai C (2019) "A Model of Iterative Computations for Recursive Summability", Tamsui Oxford Journal of Information and Mathematical Sciences – Airiti Library, Vol 33(1), pp 75-77.
- [2]. Annamalai C, Srivastava H M, and Mishra V N (2020) "Recursive Computations and Differential and Integral Equations for Summability of Binomial Coefficients with Combinatorial Expressions", International Journal of Scientific Research in Mechanical and Materials Engineering, Vol 4(1), pp 1-5.