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▶ To cite this version:

Giuseppe Rosi, Nicolas Auffray, Christelle Combescure. On the failure of classic elasticity in predicting elastic wave propagation in gyroid lattices for very long wavelengths. Symmetry, 2020. hal-02800649v1

HAL Id: hal-02800649 https://hal.science/hal-02800649v1

Submitted on 5 Jun 2020 (v1), last revised 23 Jun 2020 (v2)

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Article

On the failure of classic elasticity in predicting elastic wave propagation in gyroid lattices for very long wavelengths

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Version June 5, 2020 submitted to Symmetry

- Abstract: In this work we investigate the properties of elastic waves propagating in gyroid lattices.
- ² First, we rigorously characterize the lattice from the point of view of crystallography. Second, we use
- ³ Bloch-Floquet analysis to compute the dispersion relations for elastic waves. The results for very long
- wavelengths are then compared to those given by classic elasticity for a cubic material. A discrepancy
- is found in terms of the polarization of waves and it is related to the noncentrosymmetry of the
- ⁶ gyroid. The gyroid lattice results to be acoustically active, meaning that transverse waves exhibit a
- ⁷ circular polarization when they propagate along an axis of rotational symmetry. This phenomenon is
- present even for very long wavelengths and it is not captured by classic elasticity.
- Keywords: elastic waves, noncentrosymmetry, gyroid, crystallography, chirality

10 1. Introduction

Architectured materials are those that possess an inner geometry [1]. This multi-scale spatial 11 arrangement of the constitutive materials allows for achieving mechanical properties that are not 12 present in the bulk material itself [2]. Although this appears to be an engineering-based approach to 13 materials design, it should be noted that this strategy is, in fact, central in nature where biomaterials 14 must perform many functions from a small and limited set of elementary chemical elements [3,4]. 15 Therefore, to enhance some target properties, regular patterns often emerge. The best-known example 16 is the honeycomb, where bees need to maximise the volume of the cells while minimizing the quantity 17 of matter (wax) used [5]. Another example is the iridescent color of the wings of some butterflies. This 18 phenomenon is due to the non-centrosymmetric mesostructure of the material constituting the wings 19 which acts a photonic crystal [6,7]. 20 The study of elastic waves propagating in architectured materials is of particular interest since 21 unconventional effects due to the local organisation of the matter can emerge at the macroscale. In 22

order to study these phenomena adequately, two points of view can be adopted. Either to describe
all the details of the architecture, or to consider an effective continuum as replacement. The first
option is very general since no particular modeling assumptions are involved. However, since the
inner geometry of the material has to be explicitly described and meshed, the numerical cost is often
prohibitive for actual applications. Moreover, the computed solution often contains many unnecessary
details for practical use. The second option, which is based on elastodynamic homogenization [8–10]
amounts to substitute the initial heterogeneous material by an equivalent homogeneous continuum.

- This equivalence is only valid under specific assumptions on the range of variation of some intrinsic
- I his equivalence is only valid under specific assumptions on the range of variation of some intrinsic parameters, and hence more restrictive than the first approach. However, within the validity domain of

the method, the physics, up to a certain order¹, is correctly described. This results in a fairly important reduction in the computational cost, which is very interesting for optimizing an architectured material,

since in that case the numerical model has to be computed many times along the process. For infinite Periodic Architectured Materials (PAM), which are the subject of this paper, the 35 condition under which the complete wave problem can be substituted by an effective one relies on the 36 ratio (η) between the size of the periodic unit cell (*L*) and the wavelength (λ) of the mechanical field. 37 When this ratio approaches zero the classical Long-Wavelength (LW) approximation is obtained and, 38 provided that the frequency ω is also low (Low-Frequency (LF) approximation), the heterogeneous material can be replaced by a classical effective continuum. This situation, which has been well 40 investigated, is completely contained in what is called LF-LW elastodynamics homogenization [8–10]. 41 When the equivalent medium is a classical continuum (Cauchy continuum), the effective behavior 42 is non-sensitive to certain features of the inner geometry such as noncentrosymmetry, chirality or 43 n > 4-fold axis of rotational anisotropy [11–13]. Now, when the scale separation ratio η is small, but not vanishingly small, elastic waves 45 propagating through the matter interact with the inner architecture. In this situation, several 46 propagation quantities, such as the phase and group velocities, become frequency dependent and the 47 wave propagation is *dispersive* [14]. Non standard dependencies on the architecture, that were left over 48 in LF-LW approximation, may thus appear. These situations, which are outside the frame of standard 49 elastodynamic homogenization, can nevertheless be modeled if the Cauchy equivalent continuum is 50 replaced by a generalized continuum [15–17]. In this work we focus on bulk propagation, however it 51 is important to notice that effects near boundaries, such as surface waves [18,19], are also of particular 52 interest. 53 Wave propagation in non-centrosymmetric or chiral materials, the two concepts being distinct², 54 has been a subject of interest among physicists for centuries, mainly in the field of optics and

electromagnetism. The first experiments showing the interaction of light with chiral molecules 56 like sugar goes back to the beginning of the 19th century [20]. The effect which is associated to 57 electromagnetic waves propagating in non-centrosymmetric crystals is the rotation of the plane of 58 polarization when the wave propagates along an optical axis, i.e. an axis of rotational symmetry. The 59 rotation is due to the decomposition of the linearly polarised transverse wave into two circularly polarised waves with opposite handedness and different phase velocities [21]. This phenomenon is 61 known under the name of "optical activity". The analogue of this effect can be observed for elastic 62 waves, and it is known as "acoustical activity" [21]. It is interesting to remark that optically active 63 crystals are also found to be acoustically active and that, as it will be shown in this paper, this effect 64 can occur also in the LF-LW regime. 65

Recently, the interest in investigating the properties of materials based on chiral and
 non-centrosymmetric architectures has grown. To this end it is important to point out that chirality
 and noncentrosymmetry are not equivalent, and that their impact on the physics of the problem can be
 different.

The set of transformations that let the unit cell of an architectured material invariant constitutes its symmetry group. The material is said *chiral* if its symmetry group contains only rotations, and it is

⁷² said to be centrosymmetric when it contains the inversion [22]. It is important to observe that in a 2D

⁷³ space the inversion is a rotation (preserving the material orientation) while, in 3D, it is a transformation
 ⁷⁴ reversing the material orientation. Since the nature of the inversion depends on the dimension of the

⁷⁵ space, the implication between chirality and centrosymmetry are not the same in 2D and 3D. In 2D,

¹ An effective theory is a reduced model obtained by filtering the actual physics so as to retain, in the continuum formulation, only the most prominent effects. Depending on the targeted applications, the effective model can be of different degrees of richness.

² see Appendix A for a dictionary of point groups and their associated properties

the chirality and centrosymmetry are independent [15], while in 3D chiral materials are necessarily 76

non-centrosymmetric [23]. 77

Several works can be found in the literature that investigate 2D chiral elasticity, and focus on their 78

unusual mechanical properties such as negative Poisson ratio [24,25]. Concerning wave propagation, 79

these architectures have been extensively studied [26–28] and the need for a generalized continuum 80

theory in order to capture the onset of dispersion and anisotropy at higher frequencies has been 81 pointed out [15,16]. In all these cases, the unit cells under investigation are chiral and centrosymmetric. 82

It is worth noting that such a combination is possible uniquely in 2D. When moving to 3D, the picture

becomes more complex, and due to habits coming from the 2D situation, an ambiguity between the two 84

definitions can be usually found in the literature. For instance, the well-known and studied in-plane 85

hexachiral and tetrachiral patterns [29,30], are no more chiral once extruded in 3D. 86

Concerning wave propagation in non-centrosymmetric materials, if the phenomenon can be 8 studied in 2D [15], the effects become even more interesting in 3D as polarization of waves are then 88

richer. As a consequence, interest in 3D non-centrosymmetric metamaterials have recently emerged for 89

their strech-twist coupling [31] or their acoustical activity [32]. The effects related to size dependent 90

properties and characteristic lengths are also exploited and investigated in [33] and a micropolar 91

generalized model is used to investigate acoustical activity in [34]. 92

In the present work, the features of elastic wave propagation in non-centrosymmetric architectured 93 materials are investigated. Among them, gyroid materials are probably the most commonly used.

In electromagnetics they are widely studied as metamaterials [6], in acoustics as phononic crystals 95

[35], and in biomechanics as bone substitutes [36,37]. In this paper, we will highlight a particular 96

situation for which the solution predicted by classic continuum mechanics is wrong even for very 97

long wavelengths. It is important to note that the sensitivity of the mechanical behavior to the lack of 98

centrosymmetry can also manifest in statics [38–40]. 99

The paper is organized as follows: in section 2 the gyroid lattice is described. In section 3 the 100 Bloch-Floquet analysis is introduced along with some necessary definitions for polarization studies. 101 Dispersion analysis is performed and discussed in section 4. Section 5 compares the results from 102 section 4 to those obtained in the LW-LF approximation. Finally, some conclusions are drawn in 103 section 6.

104

Notations 105

Throughout this paper, the Euclidean space \mathcal{E}^3 is equipped with a rectangular Cartesian coordinate system with origin O and an orthonormal basis $\mathcal{B} = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$. Upon the choice of a reference point O in \mathcal{E}^3 , the Euclidean space and its underlying vector space E^3 can be considered as coincident. As a consequence, points will be designated by their vector positions with respect to O. For the sake of simplicity, E^3 will, from now on, simply be denoted E. In the following, r will designate the position vector of a point P, and, with respect to \mathcal{B} ,

$\mathbf{r} = x\mathbf{e}_1 + y\mathbf{e}_2 + z\mathbf{e}_3.$

When needed, Einstein summation convention is used, i.e., when an index appears twice in an 106

expression, it implies summation of that term over all the values of the index. The dot operator (\cdot) 107

stands for the scalar product, the \wedge for the cross product and δ_{ij} is the Kronecker delta. 108

Moreover, the following convention is retained: 109

 Blackboard fonts will denote tensor spaces: T; 110

Tensors of order > 1 will be denoted using uppercase Roman Bold fonts: T; 111

Vectors will be denoted by lowercase Roman Bold fonts: t. 112

The orthogonal group in \mathbb{R}^3 is defined as $O(3) = \{\mathbf{Q} \in GL(3) | \mathbf{Q}^T = \mathbf{Q}^{-1}\}$, in which GL(3) denotes 113 the set of invertible transformations acting on \mathbb{R}^3 . 114

115 2. The gyroid lattice

The gyroid is a triply periodic minimal surface introduced for the first time in [41,42]. Since it is a minimal surface, it has a zero mean curvature, meaning that every point on the surface is a saddle point with equal and opposite principal curvatures [43]. It is periodic with respect to three orthogonal space vectors and is chiral, meaning that the surface only possesses rotation symmetry elements or, equivalently, that it does not possess any symmetry plane nor symmetry center [44].

121 2.1. Parametrization of the gyroid lattice

The gyroid's morphology is usually described using a level surface given by the following equation:

$$\phi(x, y, z; a, b) = 0 \tag{1}$$

with *a* and *b* real parameters and *x*, *y*, *z* coordinates of the position vector **r**. In this paper, we will focus on a particular gyroid defined by :

$$\phi(x, y, z; a, b) := \sin 2\pi a x \cos 2\pi a y + \sin 2\pi a y \cos 2\pi a z + \sin 2\pi a z \cos 2\pi a x - b, \tag{2}$$

that exists only in the range $|b| < \sqrt{2}$ [44]. Indeed, beyond this peculiar value of *b*, it is possible to show that the surface described by ϕ presents discontinuities located at the borders of the fundamental (or asymmetrical) unit cell. A proof of this geometric constraint is provided in Appendix C. Due to its chiral nature, the gyroid surface exists in two enantiomorphic forms : dextrogyre and levogyre. The surface described by the implicit equation Equation 1 will be, arbitrarily, chosen to be the dextrogyre form. The levogyre form of the implicit equation is easily obtained by applying, for instance, the transformation $x \rightarrow -x$ in the Equation 1. From the definition of the surface, one can then obtain a volume by defining the presence of matter for points satisfying the following inequality:

$$\phi(x, y, z; a, b) > 0. \tag{3}$$

The parameter a controls the spatial period while the parameter b controls the porosity . Examples of unit cells of such solids obtained with different values of b are plotted in Figure 1.



Figure 1. The unit cells of gyroid lattices obtained for different values of the parameter b. Despite what the angle of view may suggest, all these structures are simply connected.

The relationship between the porosity p and the parameter b is not analytical but can be estimated numerically. As can be observed in Figure 2 this relationship is almost linear and for porosities between 0.2 and 0.8, the following linearized formula can be used to estimate the porosity:

p = -0.325b + 0.5.



Figure 2. The relationship between the parameter *b* and the porosity, dashed line represents a linear approximation of real porosity plotted in plain line. The evaluation of the porosity is obtained numerically.

124

125 2.2. Symmetry properties

Thanks to its triply periodic nature, the gyroid structure can be considered as a crystal [35] with 126 Body Centred Cubic (BCC) Bravais lattice and point group \mathcal{O} , using group theoretic notation [45], or 127 432 using Hermann-Maugin notation. It should be noted that this cubic group only contains rotations³, 128 the geometry of the gyroid is hence chiral. The symmetry of the spatial structure is described by 129 the space group, which details how transformations from the Bravais lattice and the point group are 130 combined in the actual crystal. The space group (SG) of the gyroid crystal is, using Hermann-Maugin 131 notation, $I4_132$ (space group #214 in the International Tables of Crystallography [46]), where the I 132 stands for body-centered (BC), meaning that the conventional unit cell defined in crystallography is 133 not primitive, but body-centered (more details provided in subsection 2.3). This space group contains 134 screw axes and, as such, is not symmorphic⁴. 135

If C stands for the "crystal" structure, and \star for the group action as defined in Appendix section B

$$\forall \mathbf{r} \in \mathcal{C}, \ \forall g \in \mathcal{S}G, \quad \mathbf{r}' = g \star \mathbf{r} \in \mathcal{C}.$$

From the generating transformations defined in Appendix B and using the equation of the gyroid surface (c.f. Equation 1) it is straightforward to verify that

$$\forall g \in \mathcal{S}G, \quad \phi(g \star \mathbf{r}) = \phi(\mathbf{r}).$$

136 2.3. Unit cell

Due to the periodicity of the geometry, the study of the gyroid structure can be restricted to a *unit cell*

$$\forall \mathbf{r} \in \mathbf{E}, \exists (\mathbf{r}_0, \mathbf{t}) \in (\mathcal{T}, \mathcal{R}), \quad \mathbf{r} = \mathbf{r}_0 + \mathbf{t}$$
(4)

³ To be more specific there are 3 different cubic point groups: $\mathcal{O}, \mathcal{O}^-$ and $\mathcal{O} \oplus \mathbb{Z}_2^c$. The first one just contains rotations, the group is hence chiral and non-centrosymetric. The second, \mathcal{O}^- , possesses symmetry planes but not the inversion, the group is achiral and non-centrosymetric. The last group $\mathcal{O} \oplus \mathbb{Z}_2^c$ is centrosymmetric hence achiral. Some details are provided in the Appendix A, and more information can be found in [23].

⁴ A space group is called symmorphic if, apart from the lattice translations, all generating symmetry operations leave one common point fixed. Permitted as generators are thus only the point-group operations: rotations, reflections, inversions and rotoinversions. The symmorphic space groups may be easily identified because their Hermann-Mauguin symbol does not indicate a glide or screw operation.



Figure 3. The conventional unit cell. The lattice vectors are indicated in blue.

where \mathcal{T} designates a unit cell and \mathcal{R} a periodicity lattice. It can be convenient to chose a reference unit cell from which other unit cells will be defined using lattice vectors \mathbf{a}_i and a triplet (n^1, n^2, n^3) combined as follow: $\mathbf{t} = n^1 \mathbf{a}_1 + n^2 \mathbf{a}_2 + n^3 \mathbf{a}_3$, $n_i \in \mathbb{Z}$. The triplet (0, 0, 0) is then associated to the reference unit cell. Once a lattice basis chosen, the considered unit cell is defined as follows,

$$\mathcal{T}_{C} = \{\mathbf{r}_{0} \in E | \mathbf{r}_{0} = r\mathbf{a}_{1} + s\mathbf{a}_{2} + t\mathbf{a}_{3}, (r, s, t) \in [0, 1]^{3}\}$$

and the associated periodicity lattice is given by:

$$\mathcal{R}_{\mathcal{C}} = \{\mathbf{t} \in \mathrm{E} | \mathbf{t} = n^1 \mathbf{a}_1 + n^2 \mathbf{a}_2 + n^3 \mathbf{a}_3, n_i \in \mathbb{Z}\}$$

These geometrical sets can have been described using lattice vectors \mathbf{a}_i , gathered into a basis as $\mathcal{B}' = {\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3}$. Note that they can as well be described with respect to the basis $\mathcal{B} = {\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3}$ of E.

Among all the possible unit cells, some are special and have been given a standard name in the crystallography community: the *conventional unit cell* (CUC) and *primitive unit cell* (PUC).

The *conventional unit cell* (CUC) of the a BCC lattice is depicted in Figure 3. It is defined as the smallest cell having its edges along the symmetry directions of the Bravais lattice. Notice that, for body centered (BC) lattice, this unit cell is not minimal and a so-called *primitive unit cell* (PUC) can be considered instead. For a continuous structure tilling of the space, the primitive unit cell is defined as the smallest tile that generates the whole tiling using only translations. As such the primitive unit cell is a *fundamental domain* with respect to translational symmetries only.

148 2.3.1. BCC Conventional Unit Cell

For a BCC lattice, the conventional unit cell is defined as depicted in Figure 3. As its faces are perpendicular to Bravais lattice directions, despite its non minimality, this unit cell is easy to use for numerical computations. In this case, the conventional lattice vectors \mathbf{a}_i , are chosen such that $\mathbf{a}_i \wedge \mathbf{e}_i = \mathbf{0}$.

153 2.3.2. BCC Primitive Unit Cell

For a BCC lattice, two possible primitive unit cells (PUC) are represented in Figure 4. The primitive lattice vectors \mathbf{a}_i are not unique and the ones for the PUC depicted in Figure 4a) are defined as:

$$\mathbf{b}_1 = \frac{a}{2}(\mathbf{e}_1 + \mathbf{e}_2 - \mathbf{e}_3), \quad \mathbf{b}_2 = \frac{a}{2}(-\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3), \quad \mathbf{b}_3 = \frac{a}{2}(\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3),$$

while the ones presented in Figure 4b) are defined as:

$$\mathbf{b}'_1 = a\mathbf{e}_1, \quad \mathbf{b}'_2 = a\mathbf{e}_2, \quad \mathbf{b}'_3 = \frac{a}{2}(\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3).$$



Figure 4. Two examples of primitive unit cells. The lattice vectors are indicated in red

They form $\mathcal{P} = {\mathbf{b}_i}_{1 \le i \le 3}$ and $\mathcal{P}' = {\mathbf{b}'_i}_{1 \le i \le 3}$ two other bases of , which metric tensors are given by:

$$g(\mathcal{P}) = (\mathbf{b}_i \cdot \mathbf{b}_j) = \frac{a^2}{4} \begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix}, \qquad g(\mathcal{P}') = (\mathbf{b}'_i \cdot \mathbf{b}'_j) = \frac{a^2}{4} \begin{pmatrix} 4 & 0 & 2 \\ 0 & 4 & 2 \\ 2 & 2 & 3 \end{pmatrix}$$

Being defined by a more symmetrical set of vectors \mathcal{P} , only the first primitive unit cell will be considered here after.

156 2.4. Reciprocal basis and Brillouin Zone

The vector space dual to E is symbolized by E^* , and is formally defined as the space of linear forms on E,

$$\forall l \in E^{\star}, \ \forall \mathbf{u} \in E, \ l(\mathbf{u}) = \alpha \in \mathbb{R}.$$

Upon the choice of a scalar product the two spaces can be identified

$$(\forall l \in E^{\star}, \exists \mathbf{v} \in E), \forall \mathbf{u} \in E, l(\mathbf{u}) = \mathbf{v} \cdot \mathbf{u} = \alpha \in \mathbb{R}$$

and from a basis of E a basis of E^{*} can be constructed. In the field of physics, E^{*} corresponds to the space of wavevectors, and a generic element of E^{*} is denoted by **k**. For our applications, it is fundamental to introduce the reciprocal lattice \mathcal{R}^* of \mathcal{R} :

$$\mathcal{R}^{\star} = \{ \boldsymbol{\xi} \in \mathrm{E}^{\star} | \boldsymbol{\xi} = \xi_1 \mathbf{a}^{\star 1} + \xi_2 \mathbf{a}^{\star 2} + \xi_3 \mathbf{a}^{\star 3}, \ \xi_i \in \mathbb{Z} \}.$$

The vectors $(\mathbf{a}^{\star 1}, \mathbf{a}^{\star 2}, \mathbf{a}^{\star 3})$ constitute the lattice basis \mathcal{B}^{\star} of \mathcal{R}^{\star} and verify

$$\mathbf{a}^{\star i} \cdot \mathbf{a}_{j} = \delta_{ij}, \quad \text{where} \quad \delta_{ij} = \begin{cases} 1 & \text{if} \quad i = j, \\ 0 & \text{if} \quad i \neq j. \end{cases}$$
(5)

 \mathcal{B}^{\star} can be computed from any lattice basis $(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)$ of \mathcal{R} according to

$$\mathbf{a}^{\star 1} = \frac{\mathbf{a}_2 \wedge \mathbf{a}_3}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \wedge \mathbf{a}_3)}, \qquad \mathbf{a}^{\star 2} = \frac{\mathbf{a}_3 \wedge \mathbf{a}_1}{\mathbf{a}_2 \cdot (\mathbf{a}_3 \wedge \mathbf{a}_1)}, \qquad \mathbf{a}^{\star 3} = \frac{\mathbf{a}_1 \wedge \mathbf{a}_2}{\mathbf{a}_3 \cdot (\mathbf{a}_1 \wedge \mathbf{a}_2)}.$$
 (6)

Due to the following property,

$$\forall (\mathbf{t}, \boldsymbol{\xi}) \in \mathcal{R} \times \mathcal{R}^{\star}, \quad \mathbf{e}^{2\pi i \boldsymbol{\xi} \cdot \mathbf{t}} = 1 \tag{7}$$

vectors of the reciprocal lattice are the supports of \mathcal{R} -periodic functions on E since,

$$\forall (\mathbf{r}_0, \mathbf{t}) \in \mathcal{T} \times \mathcal{R}, \quad f(\mathbf{r}_0 + \mathbf{t}) = \sum_{\boldsymbol{\xi} \in \mathcal{R}^*} \lambda_{\boldsymbol{\xi}} e^{2\pi i \boldsymbol{\xi} \cdot (\mathbf{r}_0 + \mathbf{t})} = \sum_{\boldsymbol{\xi} \in \mathcal{R}^*} \lambda_{\boldsymbol{\xi}} e^{2\pi i \boldsymbol{\xi} \cdot \mathbf{r}_0} e^{2\pi i \boldsymbol{\xi} \cdot \mathbf{t}} = f(\mathbf{r}_0).$$

In addition, use will be made of the *First Brillouin Zone* (FBZ) \mathcal{T}^* of the reciprocal lattice \mathcal{R}^* defined as:

$$\mathcal{T}^{\star} := \{ \mathbf{k} \in \mathbf{E}^{\star} | \forall \boldsymbol{\xi} \in \mathcal{R}^{\star} - 0, \| \mathbf{k} \| < \| \mathbf{k} - \boldsymbol{\xi} \| \}$$
(8)

Using the reciprocal lattice \mathcal{R}^* and the FBZ \mathcal{T}^* , any wavevector **k** can be expressed as

$$\forall \mathbf{k} \in \mathrm{E}^{\star}, \exists (\mathbf{k}_0, \boldsymbol{\xi}) \in (\mathcal{T}^{\star}, \mathcal{R}^{\star}), \quad \mathbf{k} = \mathbf{k}_0 + \boldsymbol{\xi}.$$

¹⁵⁷ We can geometrically interpret \mathcal{T}^* as the set of wavevectors **k** which are closer to the null ¹⁵⁸ wavevector than to any other wavevector $\boldsymbol{\xi}$ of the reciprocal lattice \mathcal{R}^* . It is the *Wigner-Seitz* cell of the ¹⁵⁹ reciprocal lattice, this cell is uniquely defined and independent of the choice of \mathcal{T} . Similarly to the ¹⁶⁰ primitive unit cell in the direct space, the FBZ is a fundamental domain with respect to translations.

Physically, the wavelength λ is defined as the inverse of the wavenumber, which is the norm of the wavevector : $\lambda = 1/||\mathbf{k}||$. Then, wavevectors belonging to \mathcal{T}^* have wavelengths that are greater than the wavelength of the periodicity lattice. When $||\mathbf{k}|| \rightarrow 0$ the wavelength becomes infinite, sollicitations varying with almost null wavenumber are said to be *scale separated* with respect to the periodicity lattice. This is usually the regime in which the LW approximation of elastodynamics homogenization holds.

The FBZ can be further reduced if we consider also the symmetry operations of the point group. The 167 result is an Irreducible Brillouin Zone (IBZ), that is delimited by points of high symmetry, summarized 168 for the considered gyroid lattice in Table 1. In this table, the high symmetry points are given in 169 the non-orthogonal reciprocal lattice basis \mathcal{P}^{\star} , dual to the primitive lattice basis \mathcal{P} as well as in the 170 orthonormal lattice basis $\mathcal B$ which coincides with its dual in the reciprocal space. The path obtained 171 connecting these high symmetry points along the edges of the IBZ is often used to characterize the 172 photonic and phononic properties of the lattice [6,26]. However, it has been pointed out that this 173 choice is not always reliable as some relevant information, e.g. about band gaps, could be missing [47]. 174 175

In this paper we consider the basis \mathcal{P} , that corresponds to the one depicted in Figure 4a). The reciprocal lattice is itself a Bravais lattice, and in the case of BCC lattice, it is a Face Centered Cubic (FCC) lattice. Using Equation 6, the reciprocal basis \mathcal{P}^* is equal to

$$\mathbf{a}^{\star 1} = \frac{1}{a}(\mathbf{e}_1 + \mathbf{e}_2), \quad \mathbf{a}^{\star 2} = \frac{1}{a}(\mathbf{e}_2 + \mathbf{e}_3), \quad \mathbf{a}^{\star 3} = \frac{1}{a}(\mathbf{e}_1 + \mathbf{e}_3).$$

The metric tensor of \mathcal{P}^{\star} is the inverse of the one of \mathcal{P}

$$g(\mathcal{P}^{\star}) = g(\mathcal{P})^{-1} = \frac{1}{a^2} \begin{pmatrix} 2 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 2 \end{pmatrix}$$

Table 1. The high symmetry points of the gyroid lattice. The group notations are detailed in Appendix section A.

Symmetry point	Coordinates w.r.t. \mathcal{P}^* (k_1, k_2, k_3)	Coordinates w.r.t. \mathcal{B} (x_1, x_2, x_3)	Point Group (Math.)	Point Group (H-M)	Illustration of the first Brillouin zone
					Н
Г	(0, 0, 0)	(0, 0, 0)	\mathcal{O}	432	
Н	$\left(-\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$	$(0, 0, \frac{1}{a})$	\mathcal{O}	432	N
Р	$(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$	$(\frac{1}{2a}, \frac{1}{2a}, \frac{1}{2a})$	D_3	32	r VP
Ν	$(0, \frac{1}{2}, 0)$	$(0, \frac{1}{2a}, \frac{1}{2a})$	D_2	222	

176 3. Analysis tools

In the previous section we characterized the lattice from the point of view of crystallography. In this next section we will use these results to compute the elastodynamic response of the lattice. The objective of the present section is to provide the analysis tools to be used to perform the computation and to interpret the results.

3.1. Bloch-Floquet analysis

Since the material is periodic, the dispersion diagram will be computed using Bloch-Floquet analysis [48]. The elastodynamics equation for the periodic continuum reads

$$\operatorname{div}\left[\mathbb{C}(\mathbf{r}):(\mathbf{u}(\mathbf{r})\otimes\underline{\nabla})\right] = \rho(\mathbf{r})\ddot{\mathbf{u}}(\mathbf{r}) \tag{9}$$

where $\rho(\mathbf{r})$ is the \mathcal{R} -periodic mass density and $\mathbb{C}(\mathbf{r})$ is the \mathcal{R} -periodic fourth-order elasticity tensor. As we saw in subsection 2.3, each cell of the assembly can be identified by the triplet (n^1, n^2, n^3) , where the triplet (0, 0, 0) is conventionally assigned to the reference unit cell. The position of a point \mathbf{r} of the (n^1, n^2, n^3) -cell is obtained from the position of a point in the reference unit cell \mathbf{r}_0 by Equation 4 where $\mathbf{t} = n^p \mathbf{a}_p$. Being \mathcal{R} -periodic, $\rho(\mathbf{r})$ and $\mathbb{C}(\mathbf{r})$ verify:

$$\forall (\mathbf{r}_0, \mathbf{t}) \in \mathcal{T} \times \mathcal{R}, \ \rho(\mathbf{r}_0 + \mathbf{t}) = \rho(\mathbf{r}_0), \ \mathbb{C}(\mathbf{r}_0 + \mathbf{t}) = \mathbb{C}(\mathbf{r}_0)$$

Thanks to the Floquet-Bloch theorem [48], elementary solutions to the Equation 9 over C can be searched for in the form of Bloch-waves :

$$\mathbf{u}_{\mathbf{k}}(\mathbf{r}_0) = \mathbf{U}_{\mathbf{k}}(\mathbf{r}_0) e^{2\pi i (ft - \mathbf{k} \cdot \mathbf{r}_0)}, \quad \mathbf{U}_{\mathbf{k}} \in \mathbb{C}^3,$$
(10)

where U_k is the complex polarisation vector which is \mathcal{R} -periodic in space and constant in time, f is the frequency of the Bloch-wave and \mathbf{k} its wavevector⁵. U_k describes the movement of matter as the wave propagates. In the case of an homogeneous material the polarization vector becomes constant in space and the classical plane wave solution is retrieved. Since the displacement field in Equation 10 is complex valued, its real part should be computed in order to retrieve the physical solution.

From its definition as a Bloch-wave, the displacement at a point **r** image of the $\mathbf{r}_0 \in \mathcal{T}$ by a translation $\mathbf{t} \in \mathcal{R}$ has the following expression

$$\mathbf{u}_{\mathbf{k}}(\mathbf{r}) = \mathbf{u}_{\mathbf{k}}(\mathbf{r}_0 + \mathbf{t}) = \mathbf{U}_{\mathbf{k}}(\mathbf{r}_0 + \mathbf{t})e^{2\pi i(ft - \mathbf{k} \cdot (\mathbf{r}_0 + \mathbf{t}))} = \mathbf{u}_{\mathbf{k}}(\mathbf{r}_0)e^{-2\pi i\mathbf{k} \cdot \mathbf{t}}$$
(11)

The physical meaning is that the displacement vector at two homologous points⁶ only differs by a
 phase factor.

Additionally, the Bloch-wave expression in Equation 10 has the interesting property to be also \mathcal{R}^* -periodic. Indeed, due to its \mathcal{R} -periodicity, $\mathbf{U}_{\mathbf{k}}(\mathbf{r}_0)$ can be decomposed as a Fourier series, leading to the equivalent expression for $\mathbf{u}_{\mathbf{k}}(\mathbf{r}_0)$:

$$\mathbf{u}_{\mathbf{k}}(\mathbf{r}_{0}) = \sum_{\boldsymbol{\xi} \in \mathcal{R}^{\star}} \widetilde{\mathbf{U}}_{\mathbf{k} + \boldsymbol{\xi}} e^{2\pi i (ft - (\mathbf{k} + \boldsymbol{\xi}) \cdot \mathbf{r}_{0})}$$

where $\tilde{\mathbf{U}}_{\mathbf{k}+\boldsymbol{\xi}}$ stands for the Fourier coefficients of the series expansion. Using this particular form, the \mathcal{R}^* -periodicity of the Bloch-waves is easily proven using the change of variable $\boldsymbol{\xi} = \boldsymbol{\xi} + \boldsymbol{\xi}^*$:

⁵ It is important to remark that wavevectors **k** follow the so-called "crystallographer's definition" which consists in dropping the often seen 2π coefficient. This implies, for instance, that the norm $\|\mathbf{k}\|$, i.e. the wavenumber, is directly the inverse of the wavelength λ , which is more convenient for physical interpretation of results.

⁶ Two points \mathbf{r}_1 , \mathbf{r}_2 and are said homologous if $\mathbf{r}_1 - \mathbf{r}_2 \in \mathcal{R}$.

$$\mathbf{u}_{\mathbf{k}+\boldsymbol{\xi}^{\star}}(\mathbf{r}_{0}) = \sum_{\boldsymbol{\xi}\in\mathcal{R}^{\star}} \widetilde{\mathbf{U}}_{\mathbf{k}+\boldsymbol{\xi}^{\star}+\boldsymbol{\xi}} e^{2\pi i \left(ft-(\mathbf{k}+\boldsymbol{\xi}^{\star}+\boldsymbol{\xi})\cdot\mathbf{r}_{0}\right)} = \sum_{\boldsymbol{\tilde{\xi}}\in\mathcal{R}^{\star}} \widetilde{\mathbf{U}}_{\mathbf{k}+\boldsymbol{\tilde{\xi}}} e^{2\pi i \left(ft-(\mathbf{k}+\boldsymbol{\tilde{\xi}})\cdot\mathbf{r}_{0}\right)} = \mathbf{u}_{\mathbf{k}}(\mathbf{r}_{0})$$

The main consequence of this property is that the characterization of the elastodynamics behavior of a periodic material does not require to investigate the mechanical response to all the $\mathbf{k} \in \mathbf{E}^*$ but can be

periodic material does not require to investigate the mechanical response to all the $\mathbf{k} \in E^*$ but can be restricted to the study of $\mathbf{k} \in \mathcal{T}^*$ via \mathcal{R}^* -periodicity of the wavevector, where \mathcal{T}^* corresponds to the First Brillouin Zone (FBZ), as introduced in 2.3.

¹⁹⁵ 3.2. Polarization of waves in homogeneous materials

Before presenting the results, it is useful to recall some definitions concerning the polarization of elastic waves in homogeneous materials. Let's take us back to the Bloch-wave ansatz introduced in Equation 10, since the material is now considered homogeneous, the polarization vector $\mathbf{U}_{\mathbf{k}}$ is constant both in space and time. In the most general case, the complex polarization vector $\mathbf{U}_{\mathbf{k}}$, that will be denoted **U** from now on for the sake of simplicity, can be decomposed in its real and imaginary parts as follows:

$$\mathbf{U} = \mathbf{U}^{\mathbb{R}} + i\mathbf{U}^{\mathbb{C}}.$$

An interpretation of this decomposition, and of its consequences on wave propagation, can be obtained by considering the real part of Equation 10:

$$\hat{\mathbf{u}} = \operatorname{Re}(\mathbf{u}) = \mathbf{U}^{\mathbb{R}} \cos\left(2\pi (ft - \mathbf{k} \cdot \mathbf{r})\right) - \mathbf{U}^{\mathbb{C}} \sin\left(2\pi (ft - \mathbf{k} \cdot \mathbf{r})\right).$$

Since the vectors $\mathbf{U}^{\mathbb{R}}$ and $\mathbf{U}^{\mathbb{C}}$ are independent, the polarization of the displacement can be very rich. Its precise nature is directly related to conditions on $\mathbf{U}^{\mathbb{R}}$ and $\mathbf{U}^{\mathbb{C}}$, as summarized in Table 2. It is important to remark that different conventions are used to define the handedness of circularly polarized waves. In this paper, we will consider that a wave is right handed if it follows the curl of the fingers of a right hand whose thumb is directed towards the wave propagation, away from the source. In the table, the unit normal vector defining the direction of propagation is defined by $\mathbf{n} = \frac{\mathbf{k}}{\|\mathbf{k}\|}$.

polarization Condition Transverse polarization $\mathbf{U}\wedge\mathbf{n}=\mathbf{0}$ Longitudinal polarization $\mathbf{U} \cdot \mathbf{n} = 1$ $\mathbf{U}^{\mathbb{R}}\wedge\mathbf{U}^{\mathbb{C}}=\mathbf{0}$ Linear polarization $\tilde{U}{}^{\mathbb{R}}\wedge U^{\mathbb{C}}\neq 0$ Elliptic polarization $\mathbf{U}\cdot\mathbf{U}=\mathbf{0}$ Circular polarization $\mathbf{n} \cdot (\mathbf{U}^{\mathbb{R}} \wedge \mathbf{U}^{\mathbb{C}}) < 0$ ↓ Right handedness $\mathbf{n} \cdot (\mathbf{U}^{\mathbb{R}} \wedge \mathbf{U}^{\mathbb{C}}) > 0$ Left handedness

Table 2. The polarizations of plane waves and conditions on the complex amplitude.

Thus, a complex polarization vector can lead to a phase shift between the components of the displacement vector $\hat{\mathbf{u}}$, and thus to a polarization that is other than linear, see Figure 5 for illustration of a linear and two circular polarizations with opposite handedness.

²⁰⁵ 4. Dispersion analysis using Finite Elements Analysis (FEA)

In order to investigate the ultrasonic properties of the gyroid lattice, and given the periodicity of the architecture as described in the previous sections, an approach based on Bloch-Floquet analysis will be followed. For the sake of simplicity, the conventional unit cell depicted in Figure 6 is used to define the numerical model. For the investigation of the elastodynamic properties of the gyroid crystal, the wavevector will be restricted to the boundaries of the Irreducible Brillouin Zone (IBZ), as depicted in red in Figure 7a. The high symmetry points of this IBZ are defined in Table 1. The



Figure 5. Some examples of polarizations: a) linear, b) Circular right handed, c) Circular left handed.

model has been implemented using the commercial software Comsol Multiphysics and considering titanium as constitutive material, the parameters of which are displayed in Table 3. The mesh of the unit cell is presented in Figure 6, and it consists of 66,232 tetrahedral elements. Lagrange quadratic elements are used, for a total of 329,277 degrees of freedom. Periodic Bloch-Floquet conditions are implemented by imposing them as displacement conditions at the boundaries, following Equation 11. Then, the wavenumber in k is imposed and the corresponding frequencies are retrieved by solving the corresponding eigenvalue problem. The computation of each wavenumber took an average of 109 seconds on a workstation equipped with an Intel(R) Xeon(R) CPU E5-1650 v2 at 3.50 GHz using six

220 COTES.



Figure 6. The meshed unit cell used in simulations

Table 3. The	parameters used	in the n	umerical s	imulations,	correspondin	ng to	bulk tit	anium
--------------	-----------------	----------	------------	-------------	--------------	-------	----------	-------

Mass Density [kg/m ²]	Young Modulus [GPa]	Poisson Ratio [1]	Porosity [1]	Unit cell size [mm]
$ ho_b$	E _b	ν_b	р	а
4506	115.7	0.321	0.7	1

The results of the dispersion analysis are depicted in Figure 7(a). It can be observed, qualitatively, that these results are similar to those obtained for electromagnetic waves in [6] (see Figure 8). In particular, the behavior of the acoustic branches, i.e. those branches starting from the origin Γ , corresponding to transverse waves (gray lines in Figure 7(a)) is remarkably similar.

Since the objective of the paper is to investigate the behavior of the lattice within the LW-LF approximation given by classic continuum mechanics, the phase velocities and polarization of waves have been computed for large values of the wavelength with respect to the size of the unit cell – in this case $||\mathbf{k}|| = 37 \text{ m}^{-1}$, that corresponds to a wavelength close to 26 times the size of the unit cell. For each mode, the polarization vector has been estimated by computing the average of the complex displacement of the eigen-mode over the unit cell, and the results are listed in Table 4. We will now analyze the results along the following directions of propagation, also depicted in Figure 7(b):



Figure 7. The dispersion diagram of the Gyroid lattice computed along the boundaries of the Irreducible Brillouin Zone (IBZ) and directions of propagation with respect to the unit cell. (a) The dispersion relation of the Gyroid lattice computed along the boundaries of the Irreducible Brillouin Zone (IBZ). (b) The direction of propagation with respect to the unit cell."



Figure 8. The photonic band diagram of a single gyroid photonic crystal. Reproduced with permission from [6].

- [001]: this direction is going from the center of the fundamental cell to the middle of a face. It corresponds to an axis of rotation of order 4 (rotations of $\pi/2$ rad);
- [011]: this direction is going from the center of the fundamental cell to the middle of an edge. It corresponds to an axis of rotation of order 2 (rotations of π rad);
- [111] : this direction is going from the center of the fundamental cell to a vertex. It corresponds to an axis of rotation of order 3 (rotations of $2\pi/3$ rad).
- Using the conditions listed in Table 2, we have characterized the polarization for each of the above propagation directions. The results are summarized in Table 4.

Along the direction [001] a longitudinal wave propagating at 3018 m/s can be observed. The two transverse waves have eigenmodes with complex amplitude and propagate with the same phase velocity of 1932 m/s. These complex amplitudes correspond to two circularly polarized transverse waves, with opposite handedness. In the direction [011], a longitudinal wave propagating at 4595 m/s can be observed. One transverse wave is linearly polarized in direction [100], and propagates with a phase velocity of 2728 m/s. The last solution corresponds again to a transverse wave, linearly polarized along (0, 1, -1), with velocity 2136 m/s. Finally, direction [111] has a linearly polarized Version June 5, 2020 submitted to Symmetry



- longitudinal wave at 5757 m/s, and two circularly polarized waves with opposite handedness and 247 propagating at the same velocity 2881 m/s. 248
- In summary, circularly polarized waves exist only if the direction of propagation is along a 249 rotation axis of symmetry of order greater than 2.



Figure 10.

250

Furthermore, the overall dispersion is normal for direction [001], i.e. phase velocity decreases 251 when increasing the frequency, and *anomalous* for direction [001] (see [49] for the definition). 252

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lable 4. The	phase velocit	y ana po	larization i	in the very	long wave	length ap	proximation.

Direction	Phase velocity [m/s]	Polarization (x_1, x_2, x_3)	Type of wave
[001]	3018 1932 1932	$(0,0,1) \\ (i,1,0) \\ (1,-i,0)$	Longitudinal Circular R () Circular L ()
[011]	4595 2728 2136	$(0,1,1) \\ (1,0,0) \\ (0,1,-1)$	Longitudinal Transverse Transverse
[111]	5757 2881 2881	$\begin{array}{c}(1,1,1)\\(-1+i0.58,1+i0.55,-i1.13)\\(-1-i1.68,-1+i1.72,2-i0.03)\end{array}$	Longitudinal Circular R () Circular L ()

²⁵³ 5. Long-Wavelength and Low-Frequency approximation and classic elasticity

In this last section we will introduce and identify the equivalent homogenized model in the framework of classic linear elasticity. This equivalent homogenized model is characterized by a couple of effective tensors ρ^{H} and \mathbb{C}^{H} in such a way that the displacement field **v** is solution of the following problem:

div
$$\left[\mathbb{C}^{H}:(\mathbf{v}(\mathbf{r})\otimes\underline{\nabla})\right] = \rho^{H}\ddot{\mathbf{v}}(\mathbf{r})$$
 (12)

where **v** verifies $\langle \mathbf{u} \rangle = \mathbf{v}$, $\langle . \rangle$ denotes the spatial average operator over \mathcal{T} and **u** is the displacement field solution to the heterogeneous problem Equation 9, as done for instance in [10].

Since the effective continuum is homogeneous, we consider a plane wave solution with $\mathbf{k} = f/c \mathbf{n}$ where *c* is the phase velocity of the wave and **n** a unitary vector. The substitution of this wave solution and of a linear elastic constitutive law into Equation 12 leads to following equation

$$\mathbf{\Gamma} \cdot \mathbf{U} = \rho c^2 \mathbf{U} \tag{13}$$

where $\Gamma = \mathbf{n} \cdot \mathbb{C}^H \cdot \mathbf{n}$ is the Christoffel, or acoustic, tensor. The solution of the eigenvalue problem stated in Equation 13 for a given direction \mathbf{n} gives the phase velocities and polarizations of waves in the effective continuum.

In classical elasticity, a material with cubic symmetry is defined by three independent material constants. Using Mandel notation [53], the corresponding elastic tensor for a material having its symmetry axis parallel to is \mathcal{B} reads:

$[\widetilde{C}^H] =$	c_{11}	c_{12}	c_{12}	0	0	0	
	<i>c</i> ₁₂	c_{11}	c_{12}	0	0	0	
	<i>c</i> ₁₂	c_{12}	c_{11}	0	0	0	
	0	0	0	c_{44}	0	0	•
	0	0	0	0	c_{44}	0	
	0	0	0	0	0	C44_	в

²⁵⁹ It is worth noting that classical elasticity is insensitive to the lack of centrosymmetry [54]. The symmetry

class of the elasticity tensor in the cubic system is $\mathcal{O}\oplus Z_2^c$ meaning that even if the material symmetry

²⁶¹ group of the unit cell does not contain the inversion, the symmetry group of elasticity tensor will ²⁶² inherit it.

In the case of cubic materials, the solutions of Equation 13 listed in Table 5 are directly obtained. Using the phase velocities computed from the Floquet-Bloch analysis, parameters c_{11} , c_{12} , c_{44} and ρ_s

Direction	Phase velocities [m/s]	Polarization	Type of wave
	$\sqrt{\frac{c_{11}}{ ho}} \approx 3018$	(0,0,1)	Longitudinal
[001]	$\sqrt{\frac{c_{44}}{ ho}} \approx 1932$	(0, 1, 0)	Transverse
	$\sqrt{\frac{c_{44}}{ ho}} \approx 1932$	(1,0,0)	Transverse
	$\sqrt{\frac{c_{11}+c_{12}+2c_{44}}{ ho}} pprox 4595$	(0,1,1)	Longitudinal
[011]	$\sqrt{rac{2c_{44}}{ ho}}pprox 2728$	(1, 0, 0)	Transverse
	$\sqrt{\frac{c_{11}-c_{12}}{ ho}} \approx 2136$	(0, 1, -1)	Transverse
	$\sqrt{rac{c_{11}+2c_{12}+4c_{44}}{ ho}} pprox 5757$	(1,1,1)	Longitudinal
[111]	$\sqrt{rac{c_{11}-c_{12}+c_{44}}{ ho}}pprox 2881$	(-1, 1, 0)	Transverse
	$\sqrt{rac{c_{11}-c_{12}+c_{44}}{ ho}}pprox 2881$	(-1, -1, 2)	Transverse

Table 5. The phase velocity and polarization in classic elasticity.

	Elastic coefficients		Mass density
c ₁₁ [GPa]	c ₁₂ [GPa]	c ₄₄ [GPa]	$ ho [kg/m^3]$
12.32	6.126	5.040	1351.8

Table 6. The material properties in the very long wavelength approximation.

can be identified and the homogeneous equivalent properties listed in Table 6 are then deduced. It
can be noticed that, as presented in Table 5, since we considered the propagation along the rotational
axes of symmetry, for each direction we observe a purely longitudinal wave and two purely transverse
waves.

We now move on to comparing phase velocities and polarizations obtained from the Floquet-Bloch 268 analysis with the ones forecast by the Long-Wavelength and Low-Frequency approximation using classical elasticity. We start with direction [001]. As already mentioned, this direction corresponds to a 270 rotational axis of symmetry of order 4. In this case, as the elasticity tensor is non sensitive to chirality, 271 the symmetry group of the physical phenomenon⁷ is conjugate to $D_4 \oplus Z_2^c$. Indeed, as the acoustic 272 tensor defined in Equation 13 is a second-order tensor, the Hermann theorem of Crystal physics [11] 273 predicts its behavior as transversely isotropic, i.e. $O(2) \oplus Z_2^c$. As a consequence Γ must have an eigenspace of dimension 2. . Moving to the results presented in Table 5, we can see that the classic 275 theory indeed predicts one faster longitudinal wave and two slower transverse waves propagating 276 with the same phase velocity. 277

Similar results are obtained for propagation along [111], which corresponds to rotational axis of 278 symmetry of order 3. In this case the symmetry group of the physical phenomenon is conjugate to 279 $D_3 \oplus Z_2^c$, and thus again transverse isotropy is imposed to the acoustic tensor. Finally, the direction 280 of propagation [011] is along to a rotation axis of symmetry of order 2, the physical point group is 281 thus conjugate to $D_2 \oplus Z_2^c$. Here, the symmetry class of the acoustic tensor is $D_2 \oplus Z_2^c$, and all the 282 eigenspaces are unidimensional. In this last case, as this kind of symmetry can be seen by second order 283 tensors, the results from FEA on the heterogeneous material and classic elasticity are in agreement in 284 the LF-LW regime. 285

In this section we have shown that some discrepancies can be observed when using an overall homogeneous continuum of Cauchy type. Classical elasticity (as opposed to generalized elasticity) is not rich enough to capture certain specific physical phenomena related to the symmetries of the material. In particular, if phase velocities are correctly estimated the polarizations are incorrectly predicted. Moreover, as it is well known, the onset of dispersion when frequency or wavenumber increase cannot be described in the classic Cauchy model.

292 6. Conclusions

In this work we have shown that a classical continuum model cannot capture the correct behavior of elastic waves propagating in gyroid lattices. This is due to the fact that the classic continuum mechanics is insensitive to the lack of centrosymmetry of the architectured material. However, it is a well established belief that the effects of noncentrosymmetry are only related to waves having a wavelength which compares to the size of the microstructure. Here we demonstrate that the solution given by the classical theory fails to predict the correct response, even in the Long Wavelength - Low Frequency domain.

In order to capture the onset of this unconventional behavior, called acoustical activity, the elastic continuum model needs to be enriched. Different strategies of enrichment are possible. In particular, the use of strain-gradient elasticity will be investigate in a forthcoming study.

⁷ The symmetry group of the physical phenomenon is the intersection of the symmetry group of the constitutive equations and the symmetry group of the mechanical solicitation.

303 Author Contributions:

- ³⁰⁴ Funding: The authors acknowledge the support of the French Agence Nationale de la Recherche (ANR), under
- grant ANR-19-CE08-0005 (project MaxOasis). This work was partially funded by CNRS/IRP Coss&Vita between
 Fédération Francilienne de Mécanique (F2M, CNRS FR2609) and M&MoCS

307 Acknowledgments:

308 Abbreviations

³⁰⁹ The following abbreviations are used in this manuscript:

- 310
- BCC Body Centered Cubic
- BZ Brillouin Zone
- IBZ Irreducible Brillouin Zone
- 311 FEA Finite Elements Analysis
 - FCC Face Centered Cubic
 - LF Low Frequency
 - LW Long Wavelength

312 Appendix A Dictionary

- To obtain the normal forms for the different classes the generators provided in the following table
- have been used :

Group	Generators
Z ₂	$\mathbf{P}_{\mathbf{e}_3}$
Zn	$\mathbf{R}\left(\mathbf{e}_{3};\frac{2\pi}{n}\right)$
D _n	$\mathbf{R}\left(\mathbf{e}_{3};\frac{2\pi}{n} ight)$, $\mathbf{R}(\mathbf{e}_{1};\pi)$
$Z^{2n'}, n \ge 2$	$-\mathbf{R}\left(\mathbf{e}_{3};\frac{\pi}{n}\right)$
$D^h_{2n} \ n \geq 2$	$-\mathbf{R}\left(\mathbf{e}_{3};\frac{\pi}{n}\right)$, $\mathbf{R}(\mathbf{e}_{1},\pi)$
D_n^v	$\mathbf{R}\left(\mathbf{e}_{3};\frac{2\pi}{n}\right)$, $\mathbf{P}_{\mathbf{e}_{1}}$
\mathcal{T}	R (e ₃ ; π), R (e ₁ ; π), R (e ₁ + e ₂ + e ₃ ; $\frac{2\pi}{3}$)
0	R (e ₃ ; $\frac{\pi}{2}$), R (e ₁ ; π), R (e ₁ + e ₂ + e ₃ ; $\frac{2\pi}{3}$)
\mathcal{O}^-	$-\mathbf{R}(\mathbf{e}_{3};\frac{\pi}{2}), \ \mathbf{P}_{\mathbf{e}_{2}-\mathbf{e}_{3}}$

Table A1. The set of group generators used to construct matrix representation for each symmetry class.

315 Type I subgroups

System	Hermann-Maugin	Schonflies	Group	Nature
Triclinic	1	Z ₁	1	ĪCP
Monoclinic	2	C ₂	Z ₂	ĪCP
Orthotropic	222	D ₂	D ₂	ĪCP
Trigonal	3	C ₃	Z ₃	ĪCP
Trigonal	32	D ₃	D ₃	ĪCP
Tetragonal	4	C ₄	Z ₄	ĪCP
Tetragonal	422	D ₄	D ₄	ĪCP
Hexagonal	6	C ₆	Z ₆	ĪCP
Hexagonal	622	D ₆	D ₆	ĪCP
	~	C∞	SO(2)	ĪCP
	∞2	D_{∞}	O(2)	ĪCP
Cubic	23	Т	\mathcal{T}	ĪCP
Cubic	432	0	0	ĪCP
	532	I	I	ĪCP
	000		SO(3)	ĪCP

Table A2. Dictionary between different group notations for Type I subgroups. The last column indicates the nature of the group: C = Chiral, P=Polar, I = Centrosymetric, and overline indicates that the property is missing.

316 Type II subgroups

System	Hermann-Maugin	Schonflies	Group	Nature
Triclinic	Ī	C _i	Z_2^c	ICP
Monoclinic	2/ <i>m</i>	C _{2h}	$Z_2 \oplus Z_2^c$	ICP
Orthotropic	mmm	D _{2h}	$D_2 \oplus Z_2^c$	ICP
Trigona	3	S ₆ , Z _{3i}	$Z_3 \oplus Z_2^c$	ICP
Trigonal	3m	D _{3d}	$D_3\oplus Z_2^c$	ICP
Tetragonal	4/ <i>m</i>	C _{4h}	$Z_4 \oplus Z_2^c$	ICP
Tetragonal	4/ <i>mmm</i>	D _{4h}	$D_4\oplus Z_2^c$	ICP
Hexagonal	6/ <i>m</i>	C _{6h}	$Z_6 \oplus Z_2^c$	ICP
Hexagonal	6/ <i>mmm</i>	D _{6h}	$D_6 \oplus Z_2^c$	ICP
	∞/m	$C_{\infty h}$	$SO(2) \oplus Z_2^c$	ICP
	∞/mm	$D_{\infty h}$	$O(2) \oplus Z_2^c$	ICP
Cubic	m3	T_h	$\mathcal{T}\oplus Z_2^c$	ICP
Cubic	m3m	O _h	$\mathcal{O}\oplus Z^c_2$	ICP
	$5\overline{3}m$	I_h	$\mathcal{I}\oplus Z^c_2$	ICP
	$\infty/m\infty/m$		O(3)	

Table A3. Dictionary between different group notations for Type II subgroups. The last column indicates the nature of the group: C = Chiral, P=Polar, I = Centrosymetric, and overline indicates that the property is missing.

317 Type III subgroups

System	Hermann-Maugin	Schonflies	Group	Nature
Monocinic	т	Cs	Z_2^-	ĪĊP
Orthotropic	2 <i>mm</i>	C _{2v}	$\overline{D_2^v}$	ĪĊP
Trigonal	3 <i>m</i>	C _{3v}	D_3^v	ĪĊP
Tetragonal	$\bar{4}$	S ₄	Z_4^-	ICP
Tetragonal	4mm	C _{4v}	D_4^v	ĪĊP
Tetragonal	$\bar{4}2m$	D _{2d}	D_4^h	ĪCP
Hexagonal	ō	C _{3h}	Z_6^-	ĪCP
Hexagonal	6 <i>mm</i>	C _{6v}	D_6^v	ĪCP
Hexagonal	<u></u> 62 <i>m</i>	D _{3h}	D_6^h	ĪCP
Cubic	4 3m	T _d	\mathcal{O}^{-}	ĪCP
	∞m	$C_{\infty v}$	O(2) ⁻	ĪCP

Table A4. Dictionary between different group notations for Type III subgroups. The last column indicates the nature of the group: C = Chiral, P=Polar, I = Centrosymetric, and overline indicates that the property is missing.

318 Appendix B Generators of space group #214

Consider the affine space \mathcal{E}^3 , the vector space \mathbb{R}^3 acts on \mathcal{E}^3 by translations. The affine group $Aff(\mathcal{E}^3)$ of \mathcal{E}^3 , which is the set of all affine invertible transformations is constructed as the semidirect product of \mathbb{R}^3 by GL(3), the general linear group of \mathbb{R}^3

$$\operatorname{Aff}(\mathcal{E}^3) = \operatorname{GL}(\mathbb{R}^3) \rtimes \mathbb{R}^3$$

as such, an affine transformation is given by a pair $(\mathbf{Q}, \mathbf{v}) \in GL(\mathbb{R}^3) \times \mathbb{R}^3$. Composition of transformations follows from the construction of $Aff(\mathcal{E}^3)$ as a semi-direct product, to be explicit:

$$(\mathbf{Q}_2, \mathbf{v}_2) \odot (\mathbf{Q}_1, \mathbf{v}_1) = (\mathbf{Q}_2 \mathbf{Q}_1, \mathbf{Q}_2 \mathbf{v}_1 + \mathbf{v}_2)$$

Elements of Aff(\mathcal{E}^3) can be nicely represented by(4*x*4) block matrices:

$$\left(\begin{array}{c|c} \mathbf{Q} & \mathbf{v} \\ \hline \mathbf{0} & \mathbf{1} \end{array}\right)$$

the internal law in Aff(\mathcal{E}^3) following the matrix product in $\mathbb{M}_{4,4}$.

For our needs, we are interested not in the full affine group but in the group of isometries of \mathcal{E}^3 , this group $\text{Euc}(\mathcal{E}^3)$ is a subgroup of $\text{Aff}(\mathcal{E}^3)$ and defined as the semi direct product of the orthogonal group and the spatial translation of \mathbb{R}^3

$$\operatorname{Euc}(\mathcal{E}^3) = \operatorname{O}(\mathbb{R}^3) \rtimes \mathbb{R}^3$$

Space groups can be considered as discrete subgroups of $\text{Euc}(\mathcal{E}^3)$.

The generators of the space group $I4_132$ (No. 214) are given in the following table in various notations [55]:

Seitz	Math	Matrices in conventional basis ${\cal B}$
$\{2_{001} ^{1/2} \ 0 \ ^{1/2}\}$	$\left(\mathbf{R}(\pi,\mathbf{e}_3); \frac{1}{2}(\mathbf{e}_1+\mathbf{e}_3)\right)$	$\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}_{\mathcal{B}}$
$\{2_{010} 0 ^{1/2} ^{1/2}\}$	$\left(\mathbf{R}(\pi,\mathbf{e}_2);\frac{1}{2}(\mathbf{e}_2+\mathbf{e}_3)\right)$	$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}_{\mathcal{B}}$
$\{3^+_{111} 0\}$	$\left(\mathbf{R}\left(\frac{2\pi}{3};\mathbf{e}_{1}+\mathbf{e}_{2}+\mathbf{e}_{3}\right),\underline{0}\right)$	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{\mathcal{B}}$
$\{2_{110} ^{3/4}, \frac{1}{4}, \frac{1}{4}\}$	$\left(\mathbf{R}(\pi, \mathbf{e}_1 + \mathbf{e}_2); \frac{1}{4}(3\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3)\right)$	$\begin{bmatrix} 0 & 1 & 0 & 3/4 \\ 1 & 0 & 0 & 1/4 \\ 0 & 0 & -1 & 1/4 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{\mathcal{B}}$
$\{1 ^{1/2} {}^{1/2} {}^{1/2}\}$	$\left(\mathrm{Id}; \ \frac{1}{2}(\mathbf{e}_1 + \mathbf{e}_2 + \mathbf{e}_3) \right)$	$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}_{\mathcal{B}}$

Table A5. Generators of the Group *I*4₁32 (No. 214)

323 Appendix C Proof

The gyroid lattice is defined from an implicit equation (Equation 1) that creates a periodic surface. For a given value of parameter $b (=\sqrt{2})$, this surface is found to become singular, thus creating an unrealistic discontinuous solid. This section presents an explanation for the admissible variation range of gyroid parameter $:|b| < \sqrt{2}$.

328

Let us first restrict the variation range of variables x, y, z in Equation 2 to [0, 1/2] in order to work in the fundamental domain of function ϕ . The gyroid lattice restricted to this domain is presented in Figure A1 a).

³³² The fundamental domain of the gyroid is invariant with respect to the following symmetry operations:

- Rotation of angle $2\pi/3$ along the axis defined by equations y = z = x, plotted in plain line in Figure A1 and corresponding to the transformation $(x, y, z) \rightarrow (y, z, x)$. The directing vector of this axis is (1, 1, 1) in orthonormal basis \mathcal{B} and it passes through point (0, 0, 0).

- Three rotations of angle π about the three axes defined by equations $\{y = 1/4 - x, z = 1/8\}$, $\{z = 1/4 - x, y = 1/8\}$ and $\{z = 1/4 - y, x = 1/8\}$ and plotted in dashed lines in Figure A1. These axes correspond to transformations $(x, y, z) \rightarrow (1/4 - y, 1/4 - x, 1/4 - z)$, $(x, y, z) \rightarrow (1/4 - z, 1/4 - y, 1/4 - x), (x, y, z) \rightarrow (1/4 - z, 1/4 - y)$, respectively. The



Figure A1. (a) The gyroid restricted to its fundamental domain along with symmetry axes C₃ (plain) and C₂ (dashed); (b) Evolution of parameter *b* as a function of *x* position along \mathbf{e}_1 axis, dashed line corresponds to $b = \sqrt{2}$

directing vector of these axes are, in orthonormal basis \mathcal{B} , (1, -1, 0), (1, 0, -1) and (0, 1, -1) and they pass through points (0, 1/4, 1/8), (0, 1/8, 1/4) and (1/8, 0, 1/4), respectively.

It is trivial to see that these transformations leave Equation 2 unchanged thus defining symmetry operations. As a consequence, the point symmetry group of the fundamental domain of the gyroid is conjugated to D₃. For the sake of simplicity, we will only consider generating operations of $2\pi/3$ rotation about (*x*, *x*, *x*) axis and π rotation about (*x*, 1/4 – *x*, 1/8) axis, denoted C₃ and C₂ in the following, respectively ; the two other π rotations being generated by combination of these two generators.

348

If the gyroid surface intersects one of the rotation axes non-orthogonally, then the surface automatically becomes degenerate. The expression of the normal director to the gyroid surface at its intersection point with generating symmetry axes and the equation defining this intersection point are summarized in the following Table A6:

Sym. axis & directing vector	normal to the gyroid surface	intersection point
$C_3 (1, 1, 1)_{\mathcal{B}}$	$\left(\cos^2 2\pi x - \sin^2 2\pi x\right)(1,1,1)_{\mathcal{B}}$	(x, x, x) with $3\cos 2\pi x \sin 2\pi x = b$
$C_2 (1, -1, 0)_{\mathcal{B}}$	$\sin 2\pi x \left(\sqrt{2}/2 - \cos 2\pi x\right) (1, -1, 0)_{\mathcal{B}}$	$(x, -x, 0)$ with $\sqrt{2}\cos 2\pi x + \sin^2 2\pi x = b$

Table A6. The expression of the normal director to the gyroid surface at the intersection point with its symmetry axes and expression of this intersection point.

353

Note that the normal director to the gyroid surface depends on variable x which, itself, is determined by parameter b through the non-linear equation defining the intersection point at which the normal director is computed.

One can easily check that the normal directors are generically colinear with directing vectors of C₃ and C₂ operations. However, for given values of variable *x* (or equivalently of parameter *b*), the normal to the gyroid surface is null, thus leading to singularity of the gyroid surface. These values are x = 0 and thus $b = \sqrt{2}$ – leading to singularity of the gyroid surface at its intersection with axis C₂ and x = 1/8 - and thus b = 3/2 - leading to singularity of the gyroid surface at its intersection with both axes. Finally, the equation defining the intersection point between the gyroid surface and the C₂ axis (see Figure A1 b) depends on *x* and parameter $b : \sqrt{2} \cos x + \sin x^2 - b = 0$. By plotting this equation considering *b* as a function of *x*, we can see that there are two intersection points between the gyroid surface and the C₂ axis for values of *b* over $\sqrt{2}$ thus showing that the gyroid surface forms a closed

³⁶⁷ domain in these directions leading to an unrealistic discontinuous solid.

368 References

- Schaedler, T.A.; Carter, W.B. Architected Cellular Materials. *Annual Review of Materials Research* 2016, 46, 187–210. doi:10.1146/annurev-matsci-070115-031624.
- Ashby, M.; Bréchet, Y. Designing hybrid materials. *Acta materialia* **2003**, *51*, 5801–5821.
- 372 3. Fratzl, P.; Weinkamer, R. Nature's hierarchical materials. *Progress in materials Science* 2007, 52, 1263–1334.
- 4. Estrin, Y.; Bréchet, Y.; Dunlop, J.; Fratzl, P. Architectured Materials in Nature and Engineering; Springer, 2019.
- 374
 5.
 Hales, T.C.
 The Honeycomb Conjecture.
 Discrete & Computational Geometry 2001, 25, 1 22.

 375
 doi:10.1007/s004540010071.
- Bolan, J.A.; Wilts, B.D.; Vignolini, S.; Baumberg, J.J.; Steiner, U.; Wilkinson, T.D. Optical Properties of Gyroid Structured Materials: From Photonic Crystals to Metamaterials. *Advanced Optical Materials* 2015, 3, 12 32. doi:10.1002/adom.201400333.
- Wilts, B.D.; Michielsen, K.; Raedt, H.D.; Stavenga, D.G. Iridescence and spectral filtering of the
 gyroid-type photonic crystals in Parides sesostris wing scales. *Interface Focus* 2012, 2, 681 687.
 doi:10.1098/rsfs.2011.0082.
- Boutin, C.; Auriault, J. Rayleigh scattering in elastic composite materials. *International Journal of Engineering* Science 1993, 31, 1669–1689.
- Parnell, W.; Abrahams, I. Homogenization for wave propagation in periodic fibre-reinforced media with
 complex microstructure. i—theory. *Journal of the Mechanics and Physics of Solids* 2008, *56*, 2521–2540.
- 10. Nassar, H.; He, Q.C.; Auffray, N. Willis elastodynamic homogenization theory revisited for periodic media.
 Journal of the Mechanics and Physics of Solids 2015, 77, 158–178.
- 11. Hermann, C. Tensoren und Kristallsymmetrie. *Zs. Kristallogr* **1934**.
- Olive, M.; Auffray, N. Symmetry classes for even-order tensors. *Mathematics and Mechanics of Complex Systems* 2013, 1, 177 210. doi:10.2140/memocs.2013.1.177.
- Olive, M.; Auffray, N. Symmetry classes for odd-order tensors. ZAMM Journal of Applied
 Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik 2014, 94, 421 447.
 doi:10.1002/zamm.201200225.
- ³⁹⁴ 14. DiVincenzo, D.P. Dispersive corrections to continuum elastic theory in cubic crystals. *Physical Review B* ³⁹⁵ **1986**, 34, 5450 5465. doi:10.1103/physrevb.34.5450.
- Auffray, N.; Dirrenberger, J.; Rosi, G. A complete description of bi-dimensional anisotropic strain-gradient
 elasticity. *International Journal of Solids and Structures* 2015, 69-70, 195 206. doi:10.1016/j.ijsolstr.2015.04.036.
- Rosi, G.; Auffray, N. Anisotropic and dispersive wave propagation within strain-gradient framework.
 Wave Motion 2016, 63 IS -, 120 134.
- Eremeyev, V.A. On the material symmetry group for micromorphic media with applications to granular
 materials. *Mechanics Research Communications* 2018, *94*, 8–12. doi:10.1016/j.mechrescom.2018.08.017.
- Eremeyev, V.A.; Rosi, G.; Naili, S. Comparison of anti-plane surface waves in strain-gradient materials
 and materials with surface stresses. *Mathematics and Mechanics of Solids* 2018, 24, 108128651876996.
 doi:10.1177/1081286518769960.
- Eremeyev, V.A.; Rosi, G.; Naili, S. Transverse surface waves on a cylindrical surface with coating.
 International Journal of Engineering Science 2019, p. 103188. doi:10.1016/j.ijengsci.2019.103188.
- Arago, F. Mémoire sur une modification remarquable qu'éprouvent les rayons lumineux dans leur passage à travers
 certains corps diaphanes et sur quelques autres nouveaux phénomènes d'optique; Vol. 1, Institut National de
 France, 1811; pp. 93–134.
- Portigal, D.L.; Burstein, E. Acoustical Activity and Other First-Order Spatial Dispersion Effects in Crystals.
 Physical Review 1968, 170, 673 678. doi:10.1103/physrev.170.673.
- 412 22. Sivardière, J. Description de la symétrie; EDP, 2004.

- Auffray, N.; He, Q.C.; Le Quang, H. Complete symmetry classification and compact matrix representations
 for 3D strain gradient elasticity. *International Journal of Solids and Structures* 2019, 159, 197–210.
- Prall, D.; Lakes, R.S. Properties of a chiral honeycomb with a Poisson's ratio of—1. *International journal of mechanical sciences* 1997.
- Lakes, R. Elastic and viscoelastic behavior of chiral materials. *International journal of mechanical sciences* 2001, 43, 1579 1589. doi:10.1016/s0020-7403(00)00100-4.
- Spadoni, A.; Ruzzene, M.; Gonella, S.; Scarpa, F. Phononic properties of hexagonal chiral lattices. *Wave Motion* 2009, 46, 435 450. doi:10.1016/j.wavemoti.2009.04.002.
- Liu, X.N.; Hu, G.K.; Sun, C.T.; Huang, G.L. Wave propagation characterization and design of
 two-dimensional elastic chiral metacomposite. *Journal of Sound and Vibration* 2011, 330, 2536–2553.
 doi:10.1016/j.jsv.2010.12.014.
- Liu, X.N.; Huang, G.L.; Hu, G.K. Chiral effect in plane isotropic micropolar elasticity and its
 application to chiral lattices. *Journal of the Mechanics and Physics of Solids* 2012, 60, 1907 1921.
 doi:10.1016/j.jmps.2012.06.008.
- ⁴²⁷ 29. Dirrenberger, J.; Forest, S.; Jeulin, D. Effective elastic properties of auxetic microstructures: anisotropy and
 ⁴²⁸ structural applications. *International Journal of Mechanics and Materials in Design* 2013, 9, 21–33.
- Bacigalupo, A.; Gambarotta, L. Homogenization of periodic hexa-and tetrachiral cellular solids. *Composite Structures* 2014, 116, 461–476.
- Fernandez-Corbaton, I.; Rockstuhl, C.; Ziemke, P.; Gumbsch, P.; Albiez, A.; Schwaiger, R.; Frenzel, T.;
 Kadic, M.; Wegener, M. New Twists of 3D Chiral Metamaterials. *Advanced materials (Deerfield Beach, Fla.)* 2019, 31, e1807742. doi:10.1002/adma.201807742.
- Chen, Y.; Frenzel, T.; Guenneau, S.; Kadic, M.; Wegener, M. Mapping acoustical activity in 3D chiral
 mechanical metamaterials onto micropolar continuum elasticity. *Journal of the Mechanics and Physics of Solids* 2020, p. 103877. doi:10.1016/j.jmps.2020.103877.
- 437 33. Ziemke, P.; Frenzel, T.; Wegener, M.; Gumbsch, P. Tailoring the characteristic length scale of 3D chiral
 438 mechanical metamaterials. *Extreme Mechanics Letters* 2019, 32, 100553. doi:10.1016/j.eml.2019.100553.
- Chen, Y.; Frenzel, T.; Guenneau, S.; Kadic, M.; Wegener, M. Mapping acoustical activity in 3D chiral
 mechanical metamaterials onto micropolar continuum elasticity. *Journal of the Mechanics and Physics of Solids* 2020, p. 103877. doi:10.1016/j.jmps.2020.103877.
- Chen, Y.; Yao, H.; Wang, L. Acoustic band gaps of three-dimensional periodic polymer cellular solids with
 cubic symmetry. *Journal of Applied Physics* 2013, *114*, 043521. doi:10.1063/1.4817168.
- 36. Rammohan, A.V.; Lee, T.; Tan, V.B.C. A Novel Morphological Model of Trabecular Bone Based on the
 Gyroid. *International Journal of Applied Mechanics* 2015, 07, 1550048 18. doi:10.1142/s1758825115500489.
- Ma, S.; Tang, Q.; Feng, Q.; Song, J.; Han, X.; Guo, F. Mechanical behaviours and mass transport properties
 of bone-mimicking scaffolds consisted of gyroid structures manufactured using selective laser melting.
- Journal of the Mechanical Behavior of Biomedical Materials **2019**, 93, 158 169. doi:10.1016/j.jmbbm.2019.01.023.
- Poncelet, M.; Somera, A.; Morel, C.; Jailin, C.; Auffray, N. An experimental evidence of the failure of Cauchy
 elasticity for the overall modeling of a non-centro-symmetric lattice under static loading. *International Journal of Solids and Structures* 2018, 147, 223–237. doi:10.1016/j.ijsolstr.2018.05.028.
- Abdoul-Anziz, H.; Seppecher, P.; Bellis, C. Homogenization of frame lattices leading to second gradient
 models coupling classical strain and strain-gradient terms. *Mathematics and Mechanics of Solids* 2019,
 24, 3976–3999.
- 455 40. Yvonnet, J.; Auffray, N.; Monchiet, V. Computational second-order homogenization of materials with 456 effective anisotropic strain-gradient behavior. *International Journal of Solids and Structures* **2020**.
- 457 41. Schoen, A.H. Infinite periodic minimal surfaces without self-intersections. *Nasa Technical Notes* 1970, *TN* 458 D-5541.
- 459 42. Schoen, A.H. Reflections concerning triply-periodic minimal surfaces. *Interface Focus* 2012, 2, 658–668.
 doi:10.1098/rsfs.2012.0023.
- 461 43. Dacorogna, B. Introduction to the Calculus of Variations; World Scientific Publishing Company, 2014.
- 462 44. Wohlgemuth, M.; Yufa, N.; Hoffman, J.; Thomas, E.L. Triply periodic bicontinuous cubic microdomain
 463 morphologies by symmetries. *Macromolecules* 2001, 34, 6083 6089. doi:10.1021/ma0019499.
- 404 45. Golubitsky, M.; Stewart, I.; Schaeffer, D.G. Singularities and Groups in Bifurcation Theory; Vol. 69, Applied
- 465 *Mathematical Sciences*, Springer New York: New York, NY, 1988. doi:10.1007/978-1-4612-4574-2.

- 466 46. Hahn, T.; Shmueli, U.; Wilson, A.J.C.; Prince, E. *International tables for crystallography*; D. Reidel Publishing
 467 Company, 2005.
- 468 47. Craster, R.V.; Antonakakis, T.; Makwana, M.; Guenneau, S. Dangers of using the edges of the Brillouin
 469 zone. *Physical Review B* 2012, *86*, 115130. doi:10.1103/physrevb.86.115130.
- 470 48. Gazalet, J.; Dupont, S.; Kastelik, J.C.; Rolland, Q.; Djafari-Rouhani, B. A tutorial survey on waves
 471 propagating in periodic media: Electronic, photonic and phononic crystals. Perception of the Bloch
 472 theorem in both real and Fourier domains. *Wave Motion* 2013, *50*, 619–654.
- 473 49. Achenbach, J. *Wave Propagation in Elastic Solids*; Elsevier, Elsevier, 1984.
- Zakharenko, A.A. On cubic crystal anisotropy for waves with Rayleigh-wave polarization. *Nondestructive Testing and Evaluation* 2006, 21, 61–77. doi:10.1080/10589750600779704.
- ⁴⁷⁶ 51. Rosi, G.; Nguyen, V.H.; Naili, S. Surface waves at the interface between an inviscid fluid and a dipolar
 ⁴⁷⁷ gradient solid. *Wave Motion* 2015, *53*, 51–65. doi:10.1016/j.wavemoti.2014.11.004.
- 478 52. Gourgiotis, P.A.; Georgiadis, H.G.; Neocleous, I. On the reflection of waves in half-spaces of
 479 microstructured materials governed by dipolar gradient elasticity. *Wave Motion* 2013, 50, 437–455.
 480 doi:10.1016/j.wavemoti.2012.10.004.
- 481 53. Mandel, J. Généralisation de la théorie de plasticité de WT Koiter. *International Journal of Solids and* 482 structures 1965, 1, 273–295.
- Forte, S.; Vianello, M. Symmetry classes for elasticity tensors. J. Elast. 1996, 43, 81–108.
 doi:10.1007/BF00042505.
- 485 55. Aroyo, M.I.; Perez-Mato, J.M.; Capillas, C.; Kroumova, E.; Ivantchev, S.; Madariaga, G.and K irov, A.;
- 486 Wondratschek, H. Bilbao Crystallographic Server: I. Databases and crystallographic computing programs. 7. Zeitschrift für Kristallographic-Crustalling Materials 2006, 221, 15–27
- *Zeitschrift für Kristallographie-Crystalline Materials* **2006**, 221, 15–27.
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