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# Impact-Aware Task-Space Quadratic-Programming Control

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SAGE

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## Abstract

Robots usually establish contacts at rigid surfaces with near-zero relative velocities. Otherwise, impact-induced energy propagates in the robot's linkage and may cause irreversible damage to the hardware. Moreover, abrupt changes in task-space contact velocity and peak impact forces also result in abrupt changes in robot joint velocities and torques; which can compromise controllers' stability especially for those based on smooth models. In reality, several tasks would require establishing contact with moderately high velocity. We propose to enhance task-space multi-objective controllers formulated as a quadratic program (QP) to be resilient to frictional impacts in three dimensions. We devise new constraints and reformulate the usual ones to be robust to the abrupt joint state changes mentioned earlier. The impact event becomes a controlled process once the optimal control search space is aware of: (1) the hardware-affordable impact bounds and (2) analytically-computed feasible set (polyhedra) that constrain post-impact critical states. Prior to and nearby the targeted contact spot, we assume, at each control cycle, that the impact will occur at the next iteration. This somewhat one-step preview makes our controller robust to impact time and location. To assess our approach, we experimented its resilience to moderate impacts with the Panda manipulator and achieved swift grabbing tasks with the HRP-4 humanoid robot.

## Keywords

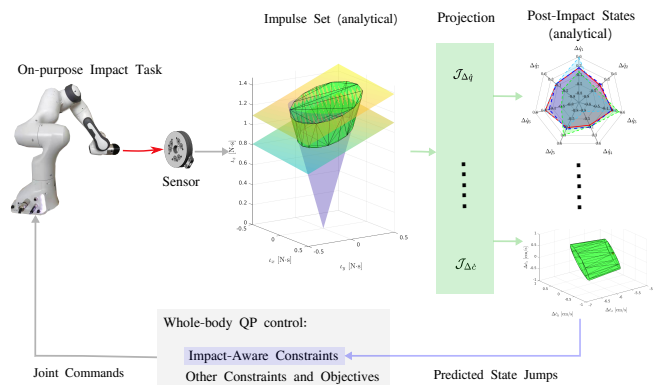
Impact-aware task-space control, on-purpose impact tasks, optimization-based control.

## 1 Introduction

When a robot collides – intentionally or inadvertently – with a rigid surface with a fairly high relative speed, the induced forces are impulsive and the contact state is uncertain. The shock propagates through the robot linkage into the joints and can severely damage some parts of the hardware, e.g., the harmonic gears and torque sensors (if any).

A common remedy is to carefully plan contact transitions with near-to-zero relative speed. However, this strategy can not achieve specific tasks such as walking or jumping humanoids, hammering, and swift grabbing. For such tasks, the following robotic issues shall be improved concurrently: (i) design impact-resilient hardware (this is not tackled in this paper); and (ii) devise robust control strategies that switch the robot equations of motions and subsequent controllers following a transition policy called reset maps. The switching often requires a precise impact model and knowledge of additional parameters that depend on the environment and the robot, e.g., the impact localization on the robot (and on the environment surface), the contact normal, and the exact impact time. Acquiring these parameters *in-situ*, instantaneously, and reliably is not always possible in practice.

An impact event is instantaneous and too short for a robot to react effectively. The impact duration (i.e., time interval) depends on the particular contact properties and the robot controller. In our recent studies Wang et al. (2022b), an impact lasted about 20 ms. This is the reason why, any impact-friendly control strategy shall act *a priori* and *a posteriori*.



**Figure 1.** The impact-aware QP regulates the contact velocity in a modified search space to ensure that the post-impact state jumps are hardware-affordable.

To tackle on-purpose impact tasks safely, we challenge the possibility of building on our existing task-space control

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framework\* instead of devising an entirely new control scheme. There are many reasons for this choice: (i) the framework has been proved to be efficient in handling complex industrial scenarios, e.g. (Kheddar et al. 2019) and multi-robot control (Bouyarmane et al. 2019); (ii) to our best knowledge, on-purpose impact objectives using the task-space quadratic programming (QP) formalism was not explicitly addressed; (iii) there is a relatively large community using it beyond the authors' circle. Our guiding quest is to try as much as possible to enhance task-space QP control to deal with impact without considerably changing its structure. In other words: can we envision handling impact tasks simply by adding or reformulating task objectives and constraints without introducing new decision variables? As we will see later, the answer to this latter question is *yes, to some extent*.

Formulating hardware impact limits<sup>†</sup> as additional QP constraints, is straightforward but insufficient. The main problem is dealing with unexpected state jumps that may damage the robot. Unexpected because, in practice, even if impact is planned to occur, there will be uncertainties on both contact time and location. The feedback velocity and force jump compromise the constraints' feasibility. Consequently, we reformulate the usual constraints to account for such discontinuities. Then the QP is impact-aware and robust to state jumps by modifying the search space according to a one-step-ahead prediction of nearby intended (i.e., expected) impacts. As a result, the controller updates the optimized – and hence feasible – impact velocity reference in every control cycle. If the impact happened, the robot would fulfill both the hardware resilience and task-dictated constraints bounds.

We summarize our main contributions as follows:

- C.1 bound the *post-impact states* (Sec. 4) with analytically-constructed convex sets (half-space represented polyhedra) assuming the impact is frictional in three dimensions;
- C.2 bound a generic post-impact robot state with a closed-form *impact-aware template constraint*, (Sec. 5.1);
- C.3 assess our *impact-aware control design* (Sec. 5.3) through experiments on the Panda manipulator and the humanoid robot HRP-4 (Sec. 6).

This work builds on top of our initial concept proposed in simulation for fixed-base robots in Wang and Kheddar (2019) and the preliminary extension to the floating-base robot in Wang et al. (2019), and finally on our recent experimental modeling work in Wang et al. (2022a,b).

## 2 Related Work

Second order equations of motion, which are commonly used for robot control and planning, cannot effectively capture the impact dynamics. Thus, the robotics community has developed dedicated models to predict the impact-induced state jumps. These models are derived from non-smooth mechanics theory, see e.g., Stronge (2000). Sec. 2.1 summarizes main models used in robotics starting from the seminal paper by Zheng and Hemami (1985) in the 1980s. Given the impact mechanics, Sec. 2.2 briefly reports on

mitigating impacts by few ideas in absorbing shock through hardware design and a large part of control strategies in various robotic impact tasks.

### 2.1 Impact modeling

Because robots are commonly made from structures and links that are rigid; and because of the real-time constraints in robot control and planning, impulsive motions are modeled based on rigid-body dynamics in contrast to the computationally demanding approaches such as stress wave or finite element deformation models applied by Pashah et al. (2008).

Recently, Wang et al. (2022a) found that for kinematic-controlled robots<sup>‡</sup>, it is more accurate to formulate the momentum-conservation equations in the task space and assume the robot to behave like a composite-rigid body during the impact event. Formulations relying on joint-space generalized momentum, e.g., Zheng and Hemami (1985); Lankarani (2000); Khulief (2013), do not account for high-stiffness joints and apply for compliant robots, e.g., pure-torque controlled robots or partially actuated pendulums.

In the impulse space, the momentum conservation defines the *plane of compression*, and Coulomb's friction law defines the *friction cone*; see Stronge (2000). The intersection points of them conservatively bound all the candidate impulses of an impact event.

When the impulse is two-dimensional, post-impact contact modes are limited, i.e., bounce, sliding, reverse sliding, and sticking. Checking the intersection points against the candidate contact modes leads to an analytical impact model. The state-of-the-art impact mechanics Jia et al. (2019); Lankarani (2000); Khulief (2013); Wang and Mason (1992) refer to this strategy as Routh's graphical approach Routh et al. (1955). When legged robots undergo multiple impacts, post-impact velocity and sliding direction are ambiguous Remy (2017). Halm and Posa (2019, 2021) recently extended Routh's impact model for multiple planar impacts by reformulating it as a differential inclusion. The novel formulation guarantees the solution's existence. Casting the impact model as a linear complementarity problem, Halm and Posa (2021) developed a probabilistically-complete algorithm for approximating the post-impact velocity set.

However, many tasks can not escape from their full three-dimensional space dimension. Moving from 2D to 3D, makes it impossible to determine the post-impact tangential velocity and impulse without numerical integrations, as observed by Stronge (2000) and Jia and Wang (2017).

We are interested in predicting the worst-case impulse to fulfill the hardware-affordable resilience bounds. In this paper, we analytically construct an impulse set in the three-dimensional space, whose interior covers all the candidate impulses that fulfill Coulomb's friction cone and task-space momentum conservation. Optimization-based controllers and in particular task-space formulated as a QP,

\*mc\_rtc [https://github.com/jrl-umi3218/mc\\_rtc](https://github.com/jrl-umi3218/mc_rtc)

<sup>†</sup> Very few robot providers disclose the hardware's impact-resilience bounds.

<sup>‡</sup> Most commercially-available robots are joint velocity (or position) controlled; we refer to them as kinematic-controlled robots.

can integrate the half-space-represented impulse set as part of the constraints set.

## 2.2 Handling impacts in robotics

Shock-absorbing mechanisms can contribute to increase the robot resilience to impact and ease the control aspects, e.g., by providing more time to the controller to react to impacts. For example, [Singh and Featherstone \(2020\)](#) redesigned the quadruped leg’s mechanical structure to cancel the shock propagating to the floating base. [Battaglia et al. \(2009\)](#) devised a soft cover which thickness and material depends on required maximum pre-impact velocity and sensing plus field-bus transmission latencies. [Dehio et al. \(2022\)](#) attached soft pads on end-effectors, this allows the implementation of an impact-aware preview controller for dual-arm fast grabbing of objects. In the bipedal walking domain [De Magistris et al. \(2017\)](#); [Pajon et al. \(2017\)](#) mounted soft soles on humanoid feet for both leveraging impacts and casting irregularities of terrains, which allowed walking on gravels. Since hardware is not the focus of this study, note that there is an active research perspective that we do not cover on hybrid hardware/control codesign to increase the robots’ resilience to impacts.

There are control strategies to avoid impacts. [Pagilla and Yu \(2001\)](#) modified the reference trajectories for zero reference velocity along the contact surface normal. The control design based on Zero-tilting Moment Point (ZMP) establishes contacts with near-zero velocity (e.g., [Kajita et al. \(2010\)](#)). However, [Grizzle et al. \(2014\)](#) summarized that the impact-less reference trajectories are challenging to generate and inefficient to execute. Hypothetically, if we can leave aside the impact dynamics and assume that the associated state jumps remain hardware-affordable, [Bombile and Billard \(2022\)](#) proposed a sequence of time-invariant dynamical systems in a single control framework to continuously control the reach, grab, and toss motion.

Impact is a prominent part of robotic legged locomotion tasks. The robot dynamics varies according to the contact event’s type and order. Complementarity problems uniformly represent a large range of (combinatorial) conditions that would require modeling using discontinuous functions, e.g., [Stewart \(2000\)](#). Combining the robot equations of motion and the complementarity conditions, the *complementarity dynamical system* (CDS) summarized by [Hurmuzlu et al. \(2004\)](#) offers a general conceptual tool to describe a walking robot’s dynamics, see also [Brogliato \(2003\)](#). [Posa et al. \(2014\)](#) applied the complementarity Lagrangian models for trajectory optimization. However, according to [Grizzle et al. \(2014\)](#), CDS has no effective control design.

To avoid discontinuous task-space tracking errors, [Yang and Posa \(2021\)](#) projected the tracking objectives to a subspace invariant to the impact event. The impact-invariant subspace is a generalization of a preliminary observation by [Gong and Grizzle \(2020\)](#), i.e., the angular momentum about the impact application point is invariant to the impacts.

If the null-space of the impulse-to-joint-velocity mapping is not empty, [Yang and Posa \(2021\)](#) recently proposed to avoid discontinuous tracking errors by employing impact-invariant joint velocities in the vicinity of the impact event.

[Hu et al. \(2007\)](#); [Stanisic and Fernández \(2012\)](#); [Heck et al. \(2016\)](#) model the dynamic contact transition with a mass-spring-damper approach. Yet, the continuity of the model cannot capture the non-smooth nature of impacts, thus failing to predict state jumps that could damage the hardware.

On the other hand, joint trajectory planners that are aware of the predicted state jumps can find the feasible impact motion. [Konno et al. \(2011\)](#) programmed a Fujitsu HOAP-2 robot to break wooden boards with a Karate chop. The impact motion consists of three-phase trajectories solved from nonlinear optimization. [Konno et al. \(2011\)](#) guaranteed standing stability by restricting the ZMP state jump. [Rijnen et al. \(2017\)](#) simulated an iCub robot impacting a wall employing two sets of pre-planned *reference spreading* trajectories (the first is pre-impact, the second is post impact) generated from an off-line task-space QP controller. Another controller switch robustly from the pre-impact trajectory to the post-impact one upon impact detection. Recently, [van Steen et al. \(2022\)](#) extended the *reference spreading control* to task-space trajectories. [Konno et al. \(2011\)](#); [Rijnen et al. \(2017\)](#); [Shield et al. \(2022\)](#); [van Steen et al. \(2022\)](#) share the same impulse prediction following [Zheng and Hemami \(1985\)](#).

Off-line trajectory planning is computationally expensive and non-reactive. Tracking the trajectories through contact-rich dynamics raises additional issues, e.g., impact timing, and invariant contact sequencing, see [Johnson et al. \(2016\)](#). Thus, the final state variation is discontinuous due to the perturbations. [Pace and Burden \(2017\)](#) summarized additional *admissibility condition* ensures continuous trajectory outcomes are piecewise-differentiable with respect to initial conditions. To prove the tracking error is infinitesimally contractive for mechanical systems, [Burden et al. \(2018\)](#) summarized conditions on the post-impact discretized system dynamics based on the stability index originally provided by [Aizerman and Gantmacher \(1958\)](#). Whereas, [Burden et al. \(2018\)](#) have not validated the contraction conditions by real robot experiments.

Within the scope of this paper, we focus on fulfilling the hardware-affordable resilience bounds before and right after a single impact event rather than generating periodic impacts. Hence, control design tools aiming for cyclic behaviors, e.g., Poincaré-map-dependent controllers ([Grizzle et al. 2014](#)) or *reference spreading control* [Rijnen et al. \(2017\)](#), do not straightforwardly apply to our aim.

More than a decade ago, [Zhang et al. \(2004\)](#) and ([Abe et al. 2007](#)) proposed task-space quadratic-programing (QP) based controllers for redundant manipulators and computer graphics. Task-space QP formulations are implemented for controlling redundant robots in recent years ([Wang et al. 2014](#); [Righetti et al. 2013](#); [Kuindersma et al. 2016](#); [Liu et al. 2016](#); [Bouyarmene and Kheddar 2018](#)). Like trajectory planners, the proposed impact-aware constraints embed the mapping between the impact-induced state jumps and the pre-impact contact velocities. The constraints modify the QP search space such that the optimized solution conforms to hardware limitations and post-impact feasibility.

### 3 Impact-Unaware QP-Control

In this section, we recall the task-space QP control formalism following [Bouyarmane et al. \(2019\)](#) and highlights impact-induced problems. Sec. 3.1 presents the equations of motion; Sec. 3.2 and 3.3 detail the impact-affected constraints for joint and centroidal spaces; Sec. 3.4 summarizes the QP controller without impacts.

#### 3.1 Equations of motion

Given  $n$  actuated joints, a robot has  $(n + 6)$  degrees of freedom (DOF); additional 6 DOF concern, if any, the floating-base configuration in  $SE(3)$ . Assuming  $m_1$  sustained contacts, the equations of motion write:

$$M(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = B\boldsymbol{\tau} + J^\top \mathbf{W}, \quad (1)$$

where  $M(\mathbf{q}) \in \mathbb{R}^{(n+6) \times (n+6)}$  is the generalized inertia matrix,  $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{(n+6)}$  gathers both Coriolis and gravitation vectors. We drop the dependency on  $\mathbf{q}$  and  $\dot{\mathbf{q}}$  in the rest of the paper for simplicity.  $B \in \mathbb{R}^{(n+6) \times n}$  is the selection matrix for the actuated joints;  $\boldsymbol{\tau} \in \mathbb{R}^n$  is the joint torques vector. Further,  $J \in \mathbb{R}^{6m_1 \times (n+6)}$  and  $\mathbf{W} \in \mathbb{R}^{6m_1 \times 1}$  vertically stacks  $m_1$  sustained contacts' Jacobians and wrenches, respectively.

#### 3.2 Joint space constraints

The equations of motion (1) fulfill the joint torque limits  $\boldsymbol{\tau} \leq \boldsymbol{\tau}_{t_k} \leq \bar{\boldsymbol{\tau}}$  by:

$$\begin{bmatrix} \mathbf{1} \\ -\mathbf{1} \end{bmatrix} M\ddot{\mathbf{q}}_{t_k} \leq \begin{bmatrix} B\bar{\boldsymbol{\tau}} \\ -B\boldsymbol{\tau} \end{bmatrix} + \begin{bmatrix} \mathbf{1} \\ -\mathbf{1} \end{bmatrix} \left( J^\top \mathbf{f}_{t_{k-1}} - \mathbf{N}_{t_{k-1}} \right). \quad (2)$$

Other constraints, such as collision avoidance, Cartesian space position constraints, actuated joint position or velocity limits... do not express directly in the robot state acceleration. We can rewrite them as a function of the QP decision variable  $\ddot{\mathbf{q}}_{t_k}$  by numerical derivation scheme, see examples in [Djeha et al. \(2020\)](#) for a closed-loop implementation.

#### 3.3 Centroidal space constraints

In the humanoid case (that we use later in our experiments), [Sugihara \(2009\)](#) showed that the horizontal COM velocity  $\dot{\mathbf{c}}_{x,y} \in \mathbb{R}^2$  must remain within a convex 2D polygon  $\mathcal{S}_{\dot{\mathbf{c}}}$ , i.e., the *capture region*, to keep the balance.  $\mathcal{S}_{\dot{\mathbf{c}}}$  depends on the sustained contacts and the COM position  $\mathbf{c}_{x,y} \in \mathbb{R}^2$ . Employing proper matrix  $\mathcal{G}_{\dot{\mathbf{c}}_{x,y}}$  and vector  $\mathbf{h}_{\dot{\mathbf{c}}_{x,y}}$ , the half-plane represented constraint  $\dot{\mathbf{c}}_{x,y} \in \mathcal{S}_{\dot{\mathbf{c}}}$  writes:

$$\mathcal{G}_{\dot{\mathbf{c}}_{x,y}} \dot{\mathbf{c}}_{x,y} \leq \mathbf{h}_{\dot{\mathbf{c}}_{x,y}}. \quad (3)$$

Due to the kinematic and actuation limits, the robot controller shall minimize its angular momentum  $\mathcal{L}_c \in \mathbb{R}^3$  ([Lee and Goswami 2012](#); [Wiedebach et al. 2016](#)). Suppose the angular momentum bounds  $\mathcal{L}_c \leq \bar{\mathcal{L}}_c$ , we take the angular part  $A_{\omega G}(\mathbf{q}_{t_k}) \in \mathbb{R}^{3 \times (n+6)}$  from the centroidal momentum matrix  $A_G(\mathbf{q})(\mathbf{q}_{t_k})$  proposed by [Orin et al. \(2013\)](#) to formulate:

$$A_{\omega G}(\mathbf{q}_{t_k}) \dot{\mathbf{q}} \leq \bar{\mathcal{L}}_c. \quad (4)$$

#### 3.4 Impact-unaware whole-body QP controller

The optimization-based whole-body controller prioritizes multiple task objectives while imposing strict constraints:

$$\min_{\boldsymbol{\nu}} \sum_{i \in \mathcal{X}_o} w_i \|e_i(\boldsymbol{\nu})\|^2$$

s.t. **Joint space constraints:**

Equations of motion and joint torque limits: (2),

Joint position, velocity, acceleration limits,

**Centroidal space constraints:**

COM velocity constraint: (3),

Angular momentum constraint: (4),

Other constraints, e.g., collision avoidance, vision... (5)

where  $\sum_{i \in \mathcal{X}_o} w_i \|e_i(\boldsymbol{\nu})\|^2$  scalarizes multiple task objectives included in a set  $\mathcal{X}_o$ ,  $w_i$  weights the  $i$ -th task and  $e_i(\boldsymbol{\nu})$  denotes the  $i$ -th task error.  $e_i(\boldsymbol{\nu})$  is linear in terms of the QP decision variables  $\boldsymbol{\nu}$ , see the details in [Bouyarmane et al. \(2019\)](#). The optimization variables are the generalized joint accelerations  $\ddot{\mathbf{q}}_{t_k}$  and the generating vectors of the contact wrench cone  $\mathbf{f}_\lambda(t_k)$ , i.e.,  $\boldsymbol{\nu}_{t_k} := \{\ddot{\mathbf{q}}_{t_k}, \mathbf{f}_\lambda(t_k)\}$ . The QP updates the robot state and sensory feedback data in each control cycle.

When it occurs, impact instantaneously change the contact velocity. As a consequence, all the constraints that express in terms of velocity, e.g., joint velocity bounds, angular momentum, COM velocity... will have their value changed with a substantially high increment. As a result, the QP search space might shrink instantly to an empty set, and hence the QP becomes infeasible for the next control iteration.

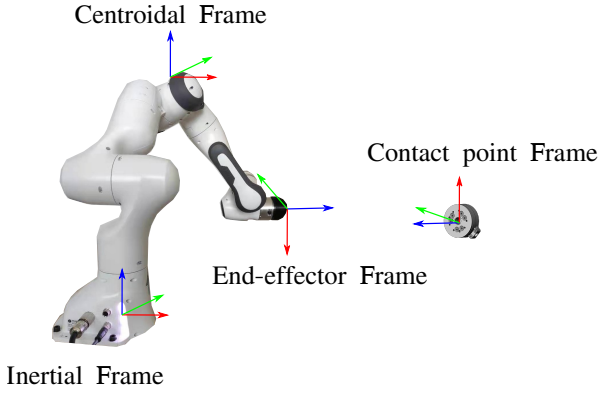
Our main idea, is to prepare such constraints to handle and be robust to such an abrupt change of the velocity values. If possible, without changing much the QP control formalism and framework. The controller will then have the dual objective to meet at best desired possible pre-impact speeds while keeping the QP controller feasible for the next iteration. That is to say, make the QP aware of the possible change in the tasks and constraints due to post-impact speed. We therefore explicitly need models that constrain the post-impact state with an analytically-computed polyhedra in Sec. 4 and formulate impact-aware constraints in Sec. 5.

## 4 Modeling 3D Frictional Impact

Sec. 4.1 presents the impulse set that fulfills the Coulomb's friction cone and the task-space momentum conservation. This set can be integrated as an additional constraint to the QP. To numerically represent the impulse set, it's common to discretize the Coulomb's friction cone as a convex cone. Projecting the cone's vertices result onto different spaces, we constrain other quantities: (1) contact velocities in Sec. 4.2, (2) joint velocities in Sec. 4.4.1, and (3) COM velocities and angular momentum in Sec. 4.5. Utilizing heuristics, we also constrain the peak impulsive force in Sec. 4.3 and the joint torque jump in 4.4.2.

We use the coordinate frames illustrated in Fig. 2 to define and exploit the impact model. Sec. 4.6 introduces the (1) impulse-to-velocity mapping in task space, and (2) the

transforms that can uniformly represent the velocities and impulses in the same frame.



**Figure 2.** The red, green and blue color indicates the  $x$ ,  $y$ , and  $z$  axis respectively. The inertial frame  $\mathcal{F}_O$  and the centroidal frame  $\mathcal{F}_c$  share the same orientation. When the robot impacts, the force torque sensor, the end-effector frame  $\mathcal{F}_e$ , and the contact point frame  $\mathcal{F}_p$  share the same translation  $\mathbf{p}_{Op} \in \mathbb{R}^3$  with different orientations.

The whole-body QP can seamlessly integrate the polyhedra to enable impact awareness since all the polyhedra are linear with respect to the pre-impact contact velocity, and hence to the QP optimization variables (joint accelerations).

#### 4.1 Feasible impulse set

We assume the followings are true during the impact event:

A.1 The impact force is dominant compared to other forces. Consequently, generalized forces such as the centrifugal forces or motor command torques are negligible (Stronge 2000, Chapter 8.1.1).

A.2 The impulse  $\boldsymbol{\iota} \in \mathbb{R}^3$  fulfills Coulomb's friction cone (Stronge 2000, Eq. 4.8), Jia and Wang (2017):

$$\sqrt{\iota_x^2 + \iota_y^2} \leq \mu \iota_z. \quad (6)$$

A.3 The robot is kinematic-controlled with high gains such that the robot behaves like a composite-rigid body Wang et al. (2022a,b).

A.4 The coefficient of restitution  $c_r \in \mathbb{R}^+$  is configuration-dependent Stoianovici and Hurmuzlu (1996); Wang et al. (2022b). We can not apply the material-dependent  $c_r$  for a kinematic-controlled robot (Wang et al. 2022b, Fig. 5).

A.5 The contact area is tiny compared to the robot dimensions such that a 3D point contact model is appropriate Chatterjee and Ruina (1998).

A.6 The impact induces significant impulsive forces and negligible moments Stronge (2000); Chatterjee and Ruina (1998).

A.7 The impacting limb has a minimum of three degrees of freedom. The joint configuration is not singular.

The feasible impulse set is the intersection of:

1. the discretized Coulomb's friction cone, see Sec. 4.1.1;

2. the two *planes of restitution* given the coefficient of restitution  $c_r \in [c_r, \bar{c}_r]$ , see Sec. 4.1.3.

We analytically represent the impulse set as a polyhedron, whose interior points are candidate impulses fulfilling assumptions A.1–A.7.

**4.1.1 Coulomb's friction cone** The impulse fulfills the Coulomb's law of friction Jia et al. (2019); Stronge (2000). We discretize the friction cone (in A.2) with  $N_\mu$  vertices:

$$\mathcal{X}_\mu := \{\boldsymbol{\iota} \in \mathbb{R}^3, \boldsymbol{\lambda}_\mu \geq \mathbf{0} : \boldsymbol{\iota} = \mathbf{K}_\mu \boldsymbol{\lambda}_\mu\}, \quad (7)$$

where the matrix  $\mathbf{K}_\mu \in \mathbb{R}^{3 \times N_\mu}$  is constant and  $\boldsymbol{\lambda}_\mu \in \mathbb{R}^{N_\mu}$  are the impulse generators. Note that the same discretization applies to the static friction cone Abe et al. (2007); Bouyarmane et al. (2019).

**4.1.2 Plane of compression** During the impact event, the contact velocity  $\mathbf{v} \in \mathbb{R}^3$  is equal to the sum of the pre-impact velocity  $\mathbf{v}^- \in \mathbb{R}^3$  and the net contact velocity jump  $\Delta \mathbf{v} \in \mathbb{R}^3$ :  $\mathbf{v} = \mathbf{v}^- + \Delta \mathbf{v}$ . The inverse inertia matrix (IIM)  $\mathbf{W} \in \mathbb{R}^{3 \times 3}$  defines the task-space impulse-to-velocity mapping  $\Delta \mathbf{v} = \mathbf{W} \boldsymbol{\iota}$  (Stronge 2000, Chapter 4.1) (Jia and Wang 2017, Sec. 2.1). Thus, the contact velocity writes:

$$\mathbf{v} = \mathbf{v}^- + \Delta \mathbf{v} = \mathbf{v}^- + \mathbf{W} \boldsymbol{\iota}. \quad (8)$$

The impact event consists of the compression and the restitution phases. The compression ends when the normal contact velocity is zero, that is  $v_n = \hat{\mathbf{n}}^\top \mathbf{v} = 0$ , where  $\hat{\mathbf{n}} \in \mathbb{R}^3$  denotes the impact normal (often taken as the impacted surface normal). Equating the inner product of  $\hat{\mathbf{n}}$  and (8) to zero:

$$v_n = v_n^- + \hat{\mathbf{n}}^\top \mathbf{W} \boldsymbol{\iota} = 0,$$

we obtain the *plane of compression* in the 3D impulse space:

$$\mathcal{X}_{\text{comp}} := \{\boldsymbol{\iota} \in \mathbb{R}^3 : v_n^- + \hat{\mathbf{n}}^\top \mathbf{W} \boldsymbol{\iota} = 0\}. \quad (9)$$

**4.1.3 Plane of restitution** Given the upper-and-lower bounds of the coefficient of restitution  $c_r \in [c_r, \bar{c}_r]$ , we can define two constraining planes similar to (9):

$$-c_r v_n^- \leq v_n = v_n^- + \hat{\mathbf{n}}^\top \mathbf{W} \boldsymbol{\iota} \leq -\bar{c}_r v_n^-, \quad (10)$$

where the minus sign is due to  $v_n$  would increase from negative to positive at the end of restitution according to the impact mechanics frame definitions Stronge (2000). Thus, (10) defines the set of impulse:

$$\mathcal{X}_{\text{res}} := \{\boldsymbol{\iota} \in \mathbb{R}^3 : -c_r v_n^- \leq v_n^- + \hat{\mathbf{n}}^\top \mathbf{W} \boldsymbol{\iota} \leq -\bar{c}_r v_n^-\}. \quad (11)$$

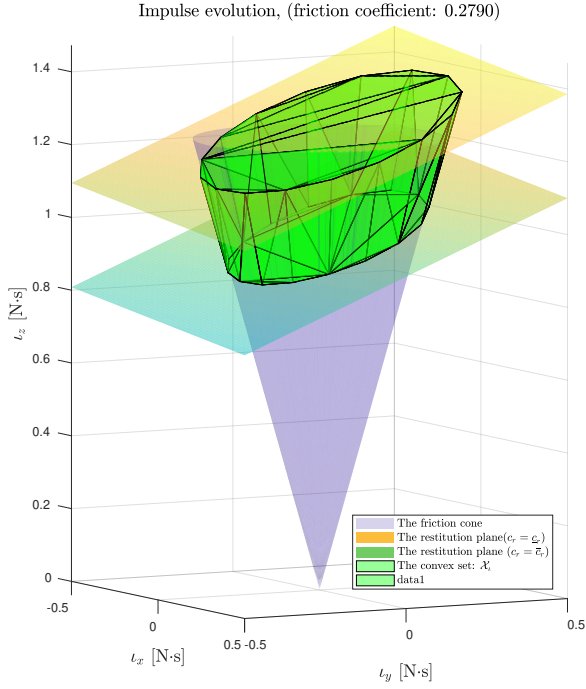
**4.1.4 Intersection** The intersection of the friction cone (7) and the two planes (10-11) defines the impulse polyhedron:

$$\mathcal{X}_\iota := \{\boldsymbol{\iota} \in \mathbb{R}^3 : \boldsymbol{\iota} \in \mathcal{X}_{\text{res}} \cap \mathcal{X}_\mu\}. \quad (12)$$

#### 4.2 Contact velocity

Given the impulse set and the impulse-to-velocity mapping  $\mathbf{W}$ , the contact velocity set is:

$$\mathcal{X}_v := \{\Delta \mathbf{v} \in \mathbb{R}^3 : \Delta \mathbf{v} = \mathbf{W} \boldsymbol{\iota}, \boldsymbol{\iota} \in \mathcal{X}_\iota\}. \quad (13)$$



**Figure 3.** We visualize the impulse set  $\mathcal{X}_l$  (12) when the Panda robot in Fig. 2 impacts a rigid surface with contact velocity  $v = [0.004, 0.054, -0.175]$  m/s (in the contact point frame).  $\mathcal{X}_l$  contains the interior of the intersection by the plane of restitutions(10-11) and the Coulomb's friction cone (7).

### 4.3 Contact force

We suppose that the peak contact force is bounded by:

$$\mathbf{f} \leq a \frac{\boldsymbol{\iota}}{\delta t},$$

where we estimate the impact duration  $\delta t$  and the positive scalar  $a > 0$  from experiment data. Given the impulse set (12), the peak contact force set is:

$$\mathcal{X}_f := \{\mathbf{f} \in \mathbb{R}^3 : \mathbf{f} \leq a \frac{\boldsymbol{\iota}}{\delta t}, \boldsymbol{\iota} \in \mathcal{X}_l\}. \quad (14)$$

### 4.4 Joint-space projections

**4.4.1 Joint velocities** According to the recently-proposed impulse-to-joint-velocity mapping by Wang et al. (2022b):

$$\Delta \dot{\mathbf{q}} = \mathbf{J}^\top (\mathbf{J} \mathbf{J}^\top)^{-1} \mathbf{W} \boldsymbol{\iota} = \arg \min_{\Delta \dot{\mathbf{q}}} \|\mathbf{J} \Delta \dot{\mathbf{q}} - \mathbf{W} \boldsymbol{\iota}\|^2,$$

the joint velocity jumps span the following set:

$$\mathcal{X}_{\Delta \dot{\mathbf{q}}} := \{\Delta \dot{\mathbf{q}} \in \mathbb{R}^D : \Delta \dot{\mathbf{q}} = \mathbf{J}^\top (\mathbf{J} \mathbf{J}^\top)^{-1} \mathbf{W} \boldsymbol{\iota}, \boldsymbol{\iota} \in \mathcal{X}_l\}. \quad (15)$$

**4.4.2 Impulsive joint torques** Given the peak contact force set (14) and the mapping  $\mathbf{J}^\top \Delta \mathbf{f}$ , the impulsive joint torque jumps are:

$$\mathcal{X}_{\Delta \boldsymbol{\gamma}} := \{\Delta \boldsymbol{\gamma} \in \mathbb{R}^D : \Delta \boldsymbol{\gamma} = \mathbf{J}^\top \mathbf{f}, \mathbf{f} \in \mathcal{X}_f\}. \quad (16)$$

### 4.5 Centroidal space projections

The high-stiffness controlled robots behave as a composite rigid body during the impact event Wang et al. (2022b). Thus, we have the COM velocity jumps  $\mathcal{X}_{\Delta \dot{\mathbf{c}}}$  and angular momentum jumps  $\Delta \mathcal{L}$ . We ease the reading by leaving the *impulse to COM velocity* mapping (21) and the *impulse to angular momentum* mapping (22) in Sec. 4.6.

**4.5.1 COM velocity** Given the impulse polyhedron (12) and the mapping (21), the COM velocity jump span the following set:

$$\mathcal{X}_{\Delta \dot{\mathbf{c}}} := \{\Delta \dot{\mathbf{c}} \in \mathbb{R}^3 : \begin{cases} \Delta \dot{\mathbf{c}} = \frac{1}{m} R_{cp} \boldsymbol{\iota}, \\ \boldsymbol{\iota} \in \mathcal{X}_l. \end{cases}\}, \quad (17)$$

where  $m \in \mathbb{R}^+$  denotes the robot mass and  $R_{cp}$  denotes the rotation from COM to the contact point.

**4.5.2 Angular momentum** Similar to the derivation of (17), we have the set of angular momentum jump given the mapping (22):

$$\mathcal{X}_{\Delta \mathcal{L}} := \{\Delta \mathcal{L} \in \mathbb{R}^3 : \Delta \mathcal{L} = \hat{\mathbf{p}}_{cp} R_{cp} \boldsymbol{\iota}, \text{ and } \boldsymbol{\iota} \in \mathcal{X}_l\}, \quad (18)$$

where  $\hat{\mathbf{p}}_{cp} \in \mathbb{R}^{3 \times 3}$  denotes the skew-symmetric matrix constructed with the translation:  $\mathbf{p}_{cp} = \mathbf{p} - \mathbf{c}$ .

## 4.6 Implementation details

Sec. 4.6.1 summarizes the coordinate frame definitions in line with the impact mechanics notation in Stronge (2000); Jia and Wang (2017). Sec. 4.6.2 presents the equivalent inertial properties at the end-effector.

**4.6.1 The coordinate frame specifications** We represent the contact velocity and impulse in the contact point frame  $\mathcal{F}_p$  shown in Fig. 2 such that the normal contact velocity is negative  $v_n^- < 0$  and the impulse is positive  $\iota_n > 0$ . Thus, we align the body velocity Jacobian  ${}^b J_{Oe}^b \in \mathbb{R}^{3 \times (n+6)}$  to frame  $\mathcal{F}_p$  as:

$${}^t J_{Oe}^p = R_{ep}^\top {}^t J_{Oe}^b. \quad (19)$$

The translation-dependent term disappears in (19) as we assume  $\mathcal{F}_p$  shares the same translation as  $\mathcal{F}_e$  when the impact occurs. The other transforms from  $\mathcal{F}_e$  to  $\mathcal{F}_p$  are:

- velocity:

$$Ad_{g_{ep}}^{-1} = \begin{bmatrix} R_{ep}^\top & -R_{ep}^\top \hat{\mathbf{p}}_{ep} \\ 0 & R_{ep}^\top \end{bmatrix} = \begin{bmatrix} R_{ep}^\top & 0 \\ 0 & R_{ep}^\top \end{bmatrix},$$

- wrench (or momentum):

$$Ad_{g_{pe}}^\top = \begin{bmatrix} R_{pe} & 0 \\ \hat{\mathbf{p}}_{pe} R_{pe} & R_{pe} \end{bmatrix} = \begin{bmatrix} R_{pe} & 0 \\ 0 & R_{pe} \end{bmatrix}.$$

We compute the relative orientation  $R_{ep}$  as  $R_{Oe}^\top R_{Op}$ , assuming the inertial frame contact point orientation  $R_{Op}$  and end-effector orientation  $R_{Oe}$  are known.

**4.6.2 The inverse inertia matrix** The end-effector velocity with respect to the inertial frame is the contact velocity. We represent it in the contact point frame  $\mathcal{F}_p$  as  $\mathbf{v}_{Op}^p \in \mathbb{R}^3$ . The inverse inertia matrix  $W \in \mathbb{R}^{3 \times 3}$  denotes the impulse-to-velocity mapping at the contact point. We derive  $W$  with the composite-rigid-body inertia  ${}^{crb} I \in \mathbb{R}^{6 \times 6}$  under assumption A.3. The momentum conservation in the centroidal frame  $\mathcal{F}_c$  involves negligible impulsive moment and other forces according to assumption A.6 and A.1:

$${}^{crb} I \Delta \mathbf{V}_{Oe}^b = Ad_{g_{ep}}^\top \begin{bmatrix} \boldsymbol{\iota} \\ 0 \end{bmatrix} = \begin{bmatrix} R_{cp} & 0 \\ \hat{\mathbf{p}}_{cp} R_{cp} & R_{cp} \end{bmatrix} \begin{bmatrix} \boldsymbol{\iota} \\ 0 \end{bmatrix}. \quad (20)$$

The adjoint map  $Ad_{g_{cp}}^{-1} \in \mathbb{R}^{6 \times 6}$  transforms the momentum (or wrench) from frame  $\mathcal{F}_p$  to  $\mathcal{F}_c$ . The body velocity  $\mathbf{V}_{O_c}^b \in \mathbb{R}^6$  is in line with the average velocity (Orin and Goswami 2008, Eq. 24). As  ${}^{crb}I$  is block-diagonal, the first three rows of (20) defines the the COM velocity jump:

$$\Delta \mathbf{v}_{O_c}^b = \Delta \dot{\mathbf{c}} = \frac{1}{m} R_{cp} \boldsymbol{\iota}, \quad (21)$$

and other rows of (20) defines the angular momentum jump:

$$\Delta \mathcal{L} = \hat{\mathbf{p}}_{cp} R_{cp} \boldsymbol{\iota}. \quad (22)$$

We compute the velocity jump  $\Delta \mathbf{V}_{O_p}^p \in \mathbb{R}^6$  by left multiplying  ${}^{crb}I^{-1}$  and the velocity transform  $Ad_{g_{cp}}^{-1}$  to (20):

$$\Delta \mathbf{V}_{O_p}^p = Ad_{g_{cp}}^{-1} \Delta \mathbf{V}_{O_c}^b = Ad_{g_{cp}}^{-1} {}^{crb}I^{-1} Ad_{g_{cp}}^{-1} \begin{bmatrix} \boldsymbol{\iota} \\ 0 \end{bmatrix}. \quad (23)$$

Thus, the inverse inertia matrix  $W$  is the  $3 \times 3$  upper-left corner of  $Ad_{g_{cp}}^{-1} {}^{crb}I^{-1} Ad_{g_{cp}}^{-1}$ :

$$\Delta \mathbf{v}_{O_p}^p = \underbrace{\left( \frac{\mathbb{1}_{3 \times 3}}{m} - R_{cp}^T \hat{\mathbf{p}}_{cp} \mathcal{I}^{-1} \hat{\mathbf{p}}_{cp} R_{cp} \right)}_W \boldsymbol{\iota}, \quad (24)$$

where  $\mathbb{1}_{3 \times 3} \in \mathbb{R}^{3 \times 3}$  is the identity matrix.

## 5 The Impact-Aware Control Design

In this section, Sec. 5.1 presents the impact-aware template for constraints; Sec. 5.2 substitutes the impulse polyhedra and robot states of interest into the template constraint.

Due to the modified search space, the impact-aware QP summarized in Sec. 5.3 is feasible (under our hypotheses and correct values of the bounds) if the impact occurs in the next control cycle, i.e., all the post-impact states fulfill hardware resilience bounds and the QP is robust to abrupt changes of the velocity.

### 5.1 Impact-aware constraints

We account for the post-impact state of constraints that write in a generic quantity  $\mathbf{d}$  as follows

$$D \mathbf{d}_{k+1}^+ \leq \bar{\mathbf{d}} \quad (25)$$

with the matrix  $D$  representing half-spaces and the vector  $\bar{\mathbf{d}}$  representing the upper and lower bounds. Substituting  $\mathbf{d}_{k+1}^- \approx \mathbf{d}_{t_k}$  and the prediction  $\mathbf{d}_{k+1}^+ = \mathbf{d}_{k+1}^- + \Delta \mathbf{d}_{t_{k+1}}$ , we obtain

$$D(\mathbf{d}_{t_k} + \Delta \mathbf{d}_{t_{k+1}}) \leq \bar{\mathbf{d}}. \quad (26)$$

We define a special Jacobian  $\mathcal{J}_{\Delta \mathbf{d}}$  as a mapping from the impulse to the state jump of different quantities:

$$\Delta \mathbf{d}_{t_{k+1}} = \mathcal{J}_{\Delta \mathbf{d}} \boldsymbol{\iota}. \quad (27)$$

The various special Jacobians are:

- joint velocities  $\mathcal{X}_{\Delta \dot{\mathbf{q}}}$  given (15):

$$\mathcal{J}_{\Delta \dot{\mathbf{q}}} = J^T (J J^T)^{-1} W;$$

- impulsive joint torques  $\mathcal{X}_{\Delta \boldsymbol{\gamma}}$  given (14) and (16):

$$\mathcal{J}_{\Delta \boldsymbol{\gamma}} = J^T a \frac{1}{\delta t};$$

- COM velocity  $\mathcal{X}_{\Delta \dot{\mathbf{c}}}$  given (17):

$$\mathcal{J}_{\Delta \dot{\mathbf{c}}} = \frac{1}{m} R_{cp};$$

- centroidal angular momentum  $\mathcal{X}_{\Delta \mathcal{L}}$  given (18):

$$\mathcal{J}_{\Delta \mathcal{L}} = \hat{\mathbf{p}}_{cp} R_{cp}.$$

Thus, we reformulate (26) by substituting the impact-induced jump  $\Delta \mathbf{d}_{t_{k+1}}$  (27):

$$D \mathcal{J}_{\Delta \mathbf{d}} \boldsymbol{\iota} \leq \bar{\mathbf{d}} - D \mathbf{d}_{t_k}. \quad (28)$$

Note that it is easy to adapt (28) to constrain the actuated joints  $\mathbf{q}$  only.

### 5.2 Constraining the post-impact states

The template constraint (28) adapts to different quantities by choosing the corresponding half-planes  $D$  and the bounds  $\bar{\mathbf{d}}$ . We enumerate the following:

Constraint 1: Joint velocity  $\mathbf{d} := \dot{\mathbf{q}}$  with  $D := [\mathbb{1}, -\mathbb{1}]^T$  and  $\bar{\mathbf{d}} := \dot{\bar{\mathbf{q}}}$  yields:

$$\begin{bmatrix} \mathbb{1} \\ -\mathbb{1} \end{bmatrix} \mathcal{J}_{\Delta \dot{\mathbf{q}}} \boldsymbol{\iota} \leq \begin{bmatrix} \dot{\bar{\mathbf{q}}} \\ -\dot{\bar{\mathbf{q}}} \end{bmatrix} - \begin{bmatrix} \mathbb{1} \\ -\mathbb{1} \end{bmatrix} \dot{\mathbf{q}}_{t_k}, \quad (29)$$

where  $\mathcal{J}_{\Delta \dot{\mathbf{q}}}$  only includes the actuated joint rows.

Constraint 2: Impulsive joint torque  $\mathbf{d} := \Delta \boldsymbol{\gamma}$  with  $D := [\mathbb{1}, -\mathbb{1}]^T$  and  $\bar{\mathbf{d}} := \underline{\Delta \boldsymbol{\gamma}}$  leads to:

$$\begin{bmatrix} \mathbb{1} \\ -\mathbb{1} \end{bmatrix} \mathcal{J}_{\Delta \boldsymbol{\gamma}} \boldsymbol{\iota} \leq \begin{bmatrix} \underline{\Delta \boldsymbol{\gamma}} \\ -\underline{\Delta \boldsymbol{\gamma}} \end{bmatrix}, \quad (30)$$

where  $\Delta \boldsymbol{\gamma}_{t_k} = 0$  since we assume the impact occurs at the time step  $k + 1$ .

**Remark 5.1.** *Impulses propagate through the mechanical linkages to reach each joint. Hence impact must be mitigated not to damage joint mechatronic implements (e.g., gear-box, torque sensor (if any), etc.), or mechanical linkages, or covers. The exact origin of the limitation is not essential because, in the end, one will take the conservative (min/max) allowed shock. The proposed QP controller embeds such constraints concurrently for any on-purpose impact. The stall torque limits are set for the robot equations of motion. The impact-aware constraint (30) sets a different tolerance of impulsive-torque limits. ■*

Constraint 3: Angular momentum  $\mathbf{d} := \mathcal{L}_c$  with  $D := \mathbb{1}$  and  $\bar{\mathbf{d}} := \underline{\mathcal{L}}_c$  yields:

$$\mathcal{J}_{\Delta \mathcal{L}_c} \boldsymbol{\iota} \leq \underline{\mathcal{L}}_c - \mathcal{L}_{ct_k}. \quad (31)$$

Constraint 4: Horizontal COM velocity  $\mathbf{d} := \dot{\mathbf{c}}_{x,y}$  with  $D := \mathcal{G}_{\dot{\mathbf{c}}_{x,y}}$  and  $\bar{\mathbf{d}} := \mathbf{h}_{\dot{\mathbf{c}}_{x,y}}$  generates:

$$\mathcal{G}_{\dot{\mathbf{c}}_{x,y}} \mathcal{J}_{\Delta \dot{\mathbf{c}}_{x,y}} \boldsymbol{\iota} \leq \mathbf{h}_{\dot{\mathbf{c}}_{x,y}} - \mathcal{G}_{\dot{\mathbf{c}}_{x,y}} \dot{\mathbf{c}}_{x,y}(t_k). \quad (32)$$



### 5.3 Impact-aware whole-body QP controller

To seamlessly integrate the impact-aware constraints (29-32) with respect to the impulse, we need to:

- 1 add the impulse generators  $\lambda_\mu \in \mathbb{R}^{N_\mu}$  to the QP decision variables  $\nu$ ;
- 2 rewrite the pre-impact normal contact velocity  $v_n^-$  by the current-step joint velocities  $\dot{\mathbf{q}}_{t_k}$  and accelerations  $\ddot{\mathbf{q}}_{t_{k+1}}$  (QP decision variable).

Assuming the impact occurs at the next control iteration, we approximate the pre-impact normal velocity as:

$$\begin{aligned} v_n^-(t_{k+1}) &= v_n(t_{k+1}) = J_{t_{k+1}} \dot{\mathbf{q}}_{t_{k+1}} \\ &= (J_{t_k} + \dot{J}_{t_k} \Delta t + \ddot{J}_{t_k} \Delta t^2 + \dots)(\dot{\mathbf{q}}_{t_k} + \ddot{\mathbf{q}}_{t_k} \Delta t) \\ &= J_{t_k} \dot{\mathbf{q}}_{t_k} + J_{t_k} \ddot{\mathbf{q}}_{t_k} \Delta t + \underbrace{\dot{J}_{t_k} \dot{\mathbf{q}}_{t_k} \Delta t + \dot{J}_{t_k} \ddot{\mathbf{q}}_{t_k} \Delta t^2 + \dots}_{\approx 0}, \end{aligned}$$

where we read  $\dot{\mathbf{q}}_{t_k}$  from the current robot state,  $\Delta t$  denotes the sampling period, and we define the Jacobian as:  $J_{t_k} = \hat{\mathbf{n}}^\top J_{\mathcal{O}_e}^P$  given (19).

Substituting  $v_n^-(t_{k+1}) = J_{t_k} \dot{\mathbf{q}}_{t_k} + J_{t_k} \ddot{\mathbf{q}}_{t_k} \Delta t$  into (11), the following constraints specify the impulse polyhedron:

Friction cone: (7),

$$\lambda_\mu \geq \mathbf{0};$$

Planes of restitution: (11),

$$\begin{aligned} -(\bar{c}_r + 1)J_{t_k} \ddot{\mathbf{q}}_{t_k} \Delta t - \hat{\mathbf{z}}^\top W \mathbf{K}_\mu \lambda_\mu &\geq (\bar{c}_r + 1)J_{t_k} \dot{\mathbf{q}}_{t_k}, \\ -(\underline{c}_r + 1)J_{t_k} \ddot{\mathbf{q}}_{t_k} \Delta t - \hat{\mathbf{z}}^\top W \mathbf{K}_\mu \lambda_\mu &\leq (\underline{c}_r + 1)J_{t_k} \dot{\mathbf{q}}_{t_k}. \end{aligned} \quad (33)$$

Thus, we improve the baseline optimization-based controller (5) with the impulse polyhedron  $\mathcal{X}_\iota$  (33) and the impact-aware constraints (29-32) (substituting  $\iota = \mathbf{K}_\mu \lambda_\mu$  from (7)) as:

$$\min_{\nu, \lambda_\mu} \sum_{i \in \mathcal{X}_o} w_i \|e_i(\nu)\|^2$$

s.t. Impulse Polyhedron  $\mathcal{X}_\iota$  given by (33),

Joint space constraints:

Post-impact joint velocity: (29),

Post-impact impulsive joint torque: (30),

Joint position, velocity, and torque (2),

Centroidal space constraints:

Post-impact angular momentum: (31),

Post-impact COM velocity: (32),

Other constraints, e.g., collision avoidance: (26).

(34)

The improved formulation (34) shares the same objectives as the formulation (5) and modifies its search space such that all the post-impact states fulfill the corresponding bounds. Hence, (34) can decide on the highest feasible contact velocities if required.

**Remark 5.2.** We deactivate the impact-awareness upon the impact detection by switching the QP controller from (34)

to (5). Since the constraints associated with (34) are more conservative than (5), switching from (34) to (5) by full initialization does not lead to infeasible solutions.

Integrating (i.e. activating) impact-aware constraints by switching from (5) to (34) may lead to infeasible QP due to the actuation limits. Yet, this problem is common to any other usual inequality constraint that is triggered or inserted to the QP on the fly. Solutions exist for the case of one constraint, see e.g., Del Prete (2018); Djeha et al. (2020). Yet interaction among several constraints is still an open problem that we are currently investigating. ■

## 6 Experiments

We validated the impact-aware control design with the Panda manipulator. All the manipulator's post-impact states fulfilled the proposed impact model. The impact-aware QP (34) exploited the manipulator's resilience bounds (given by Franka Emika) to achieve the highest impact velocity; see the video<sup>§</sup>.

The impact model applies for Panda and humanoid robot HRP-4 since both are stiff kinematic-controlled robots. The impact-aware control enabled the HRP-4 to grab boxes with human-level swift motions. The impact-aware constraints guaranteed that the robot motion was impact-friendly. The detailed experiment description and the highlights are:

*Experiment 1* The Panda manipulator impacted an ATI sensor covered by a rigid planar surface, see Fig. 2. Upon the impact detection, the robot immediately pulled back the end-effector. We repeated the experiment for 10 times to remove the random effects and ensure consistent observations.

(H-1.1) All the impulses (measured by integrating the ATI force readings) fulfilled the impulse polyhedron proposed in Sec. 4.

(H-1.2) The impact-aware joint constraints are fulfilled.

(H-1.3) We empirically concluded when the impulsive joint torque constraint (30) is active, the post-impact joint velocities are still far away from the bounds.

(H-1.4) Given a high reference normal contact velocity, i.e., 0.50 m/s, the QP autonomously solved the highest impact-aware feasible contact velocity in real-time.

*Experiment 2* The HRP-4 robot swiftly grabbed two different boxes, see the snapshot in Fig. 9. The two arm's impacts are synchronized and symmetric.

(H-2.1) Grabbing the box swiftly without stopping for establishing contacts.

(H-2.2) The high contact velocities, i.e., 0.25 m/s, were well-tracked.

### 6.1 Panda Manipulator Experiments

Sec. 6.1.1 details the steps to measure the post-impact data; Sec. 6.1.2 customizes the impact-aware QP for the Panda manipulator; and Sec. 6.1.3 analyzes the results.

<sup>§</sup><https://seafiler.lirmm.fr/f/f037a114d58246cf8cf4/>

**6.1.1 Data acquisition** We applied the open-source dynamics model<sup>¶</sup> for the 7 DOF Panda manipulator from FrankaEmika. Note that there are alternative models available<sup>¶¶</sup>.

We mounted a 3D printed semi-spherical end-effector on the Panda robot to meet the point-contact assumption A.5, see Fig. 2. The friction coefficient is  $\mu = 0.279$ .

The impact event lasts for about 20 ms for the joint-velocity controlled Panda manipulator according to the recent experimental study by Wang et al. (2022a). Thus, we measured the sudden joint velocity changes in an interval (5 ms before and 15 ms after the impact) as the joint velocity jumps  $\Delta\dot{q}^\circ$ .

We sampled the ATI-mini45 sensor readings at 25000 Hz without low-pass filtering and integrate the force measurements to compute the impulse in three dimensions.

**Coefficient of restitution** We construct the two planes of restitution (11) by choosing the upper and lower bound of the coefficient of restitution  $c_r \in [c_r, \bar{c}_r]$ .

According to the observations from hundreds of impact experiments (Wang et al. 2022a, Sec. VII.C), the kinematic-controlled robot impact is almost inelastic (the coefficient of restitution is close to zero  $c_r \leq 0.15$ ) if the contact velocity is greater than 0.1 m/s. We choose the upper bound  $\bar{c}_r = 0.3$ , conservatively higher than 0.15, and the lower bound  $\bar{c}_r = 0.0$ , which corresponds to inelastic impact.

**Impact detection** The virtual force sensor of the Panda robot can not capture the impact dynamics. Timely impact detection is essential for accurately observing the post-impact states. We achieved 3–6 ms detection time by thresholding the joint torque error:

$$e_\tau = \sum_{i=5}^6 |\tau_i - \tau_i^*| \leq \tau_0,$$

where the motor torque  $\tau_i$  and its reference  $\tau_i^*$  update at 1 ms. The threshold  $\tau_0$  is 2.5 N·m.

**6.1.2 Controller formulation** A manipulator requires the following set of impact-aware constraints:

$$\begin{aligned} & \text{Impulse polyhedron } \mathcal{X}_\iota \text{ (33),} \\ & \text{Joint space constraints:} \\ & \text{Post-impact joint velocity: (29),} \\ & \text{Post-impact impulsive joint torque: (30).} \end{aligned} \quad (35)$$

Thus, we customized the impact-aware controller (34) for the Panda manipulator as:

$$\begin{aligned} \min_{\nu} \quad & \sum_{i \in \mathcal{X}_o} w_i \|e_i(\nu)\|^2 \\ \text{s.t.} \quad & \text{Manipulator Impact-awareness (35),} \\ & \text{Joint position, velocity, and torque (2).} \end{aligned} \quad (36)$$

The impulsive joint torque constraint (30) depends on the peak contact force. According to the contact force profiles (Wang et al. 2022a, Fig. 6), we conservatively choose the impact duration  $\delta t = 18$  ms and the positive scalar  $a = 3$  to construct the peak contact force set  $\mathcal{X}_f$  (14).

We assigned an unrealistically-high reference contact velocity, i.e.,  $v^{ref} = [0.50, 0.0, 0.0]$  m/s (represented in the contact point frame  $\mathcal{F}_p$  in Fig. 2). The other tasks include the end-effector orientation and posture tasks; see the detailed QP formulation by Bouyarmane et al. (2019).

### 6.1.3 Result analysis

**Fulfilling the Impulse Polyhedra** Fig. 4(a) shows that all the measured impulses (from 10 experiments marked by red crosses)  $\iota^\circ$  are within the (transparent-green) polyhedron-represented impulse set  $\mathcal{X}_\iota$  (12) (H-1.1). Substituting  $\iota^\circ$  into (14), (17), and (18), Fig. 4(b)-4(d) visualize that all the quantities are constrained by the corresponding polyhedron-represented sets  $\mathcal{X}_f$  (14),  $\mathcal{X}_{\Delta\dot{q}}$  (17),  $\mathcal{X}_{\Delta\mathcal{L}}$  (18).

**Fulfilling the impact aware constraints** According to the Panda robot descriptions, the joint velocity bounds are:

$$\bar{q} = \pm[2.175, 2.175, 2.175, 2.175, 2.61, 2.61, 2.61] \text{ rad/s} \quad (37)$$

We define the impulsive joint torque bounds:

$$\overline{\Delta\gamma} = \pm[87, 87, 87, 87, 12, 12, 12] \text{ N}\cdot\text{m}.$$

Substituting the measured impulse  $\iota^\circ$ , the predicted impulsive joint torque  $\Delta\gamma = J^\top a \frac{\iota^\circ}{\delta t}$  (16) fulfilled the set  $\mathcal{X}_{\Delta\tau}$  in Fig. 6.

Fig. 5 collects the mean of the measured joint velocity jumps  $\Delta\dot{q}^\circ$  from 10 experiments. All the measurements are within the set  $\mathcal{X}_{\Delta\dot{q}}$  (15) (H-1.2). The predicted joint velocity jumps are far from the bounds (37), i.e., the constraints (29) are not active (H-1.3).

**The contact velocities** The impact-aware QP (36) solves the desired joint commands in each control cycle. Since the reference is too high to be precisely tracked, (36) modified the maximum feasible contact velocity as shown in Fig. 7 (H-1.4).

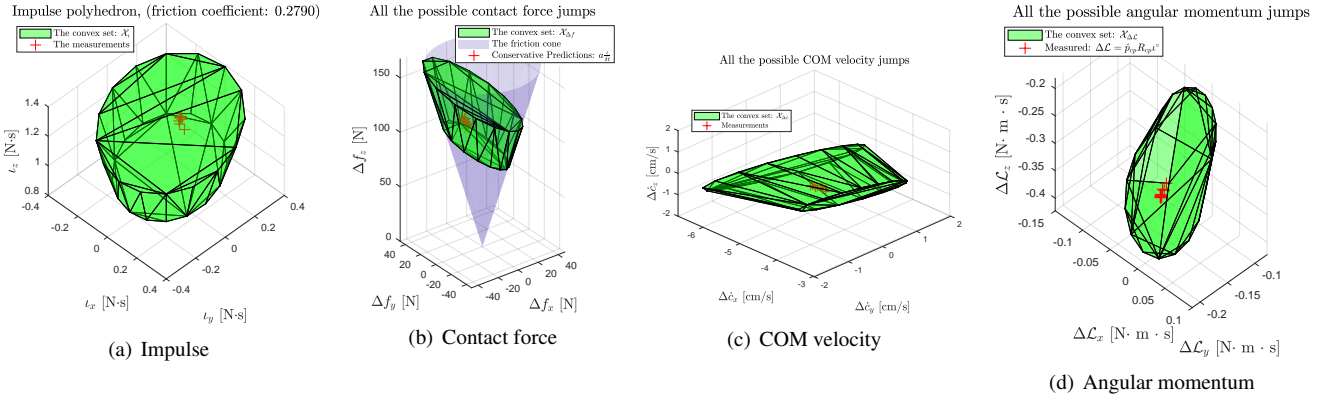
The light area in Fig. 7 (H-1.4) highlights the period during which the impact-aware constraints (35) modified the optimizer's search space. The active constraint was the impulsive joint torque constraint (30), i.e., the QP solution in Fig. 6 is close to the upper bound 12 N·m. Due to the conservative choice of parameters, the measured impulsive joint torque  $\Delta\gamma = J^\top a \frac{\iota^\circ}{\delta t}$  (16) of the sixth joint was lower than the corresponding QP solution.

## 6.2 Swift box-grabbing with the HRP-4 Robot

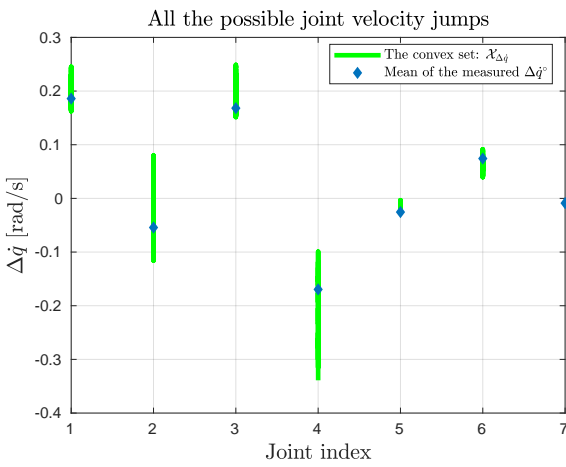
The HRP-4 robot swiftly grabbed a 1.08 kg box and another 0.38 kg box. The contact velocity was 0.25 m/s. The robot did not reduce speeds for establishing contacts in both cases. The impacts were synchronized for the two hands, and the contact locations were symmetric.

<sup>¶</sup>[github.com/jrl-umi3218/mc\\_panda/blob/master/data/urdf/panda\\_default.urdf](https://github.com/jrl-umi3218/mc_panda/blob/master/data/urdf/panda_default.urdf)

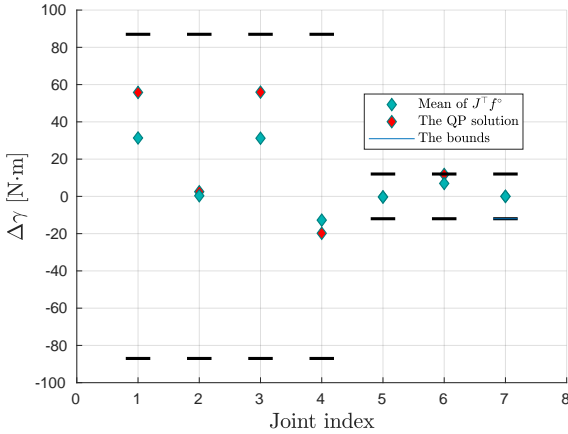
<sup>¶¶</sup>[github.com/marcocognetti/FrankaEmikaPandaDynModel](https://github.com/marcocognetti/FrankaEmikaPandaDynModel)  
<sup>\*\*</sup>[github.com/StanfordASL/PandaRobot.jl](https://github.com/StanfordASL/PandaRobot.jl)



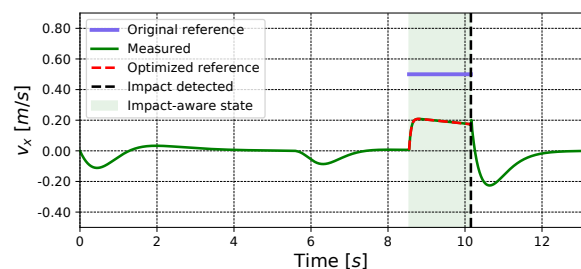
**Figure 4.** We visualize the polyhedra that constrain the impulse (12), the peak contact force (14), the COM velocity (17), and the angular momentum (18). All the measurements (red crosses) fulfilled the corresponding polyhedron-represented sets.



**Figure 5.** The measured and predicted joint velocity jumps.



**Figure 6.** The measured and predicted impulsive joint torque jumps. The inequality constraint (30) is active since the QP solution of joint 6 locates at the bound.



**Figure 7.** Optimized normal contact velocity  $v_x$ .

**6.2.1 Data acquisition** The HRP-4 is high-stiffness kinematic-controlled (joint-position), i.e., fulfilling assumption A.3. The HRP-4's geometric size is significantly higher than the contact area, i.e., fulfilling the point contact assumption A.5. Thus, we adopt the same impact model in *Experiment 1*. We bounded the coefficient of restitution by  $\bar{c}_r = 0.3$  and  $\bar{c}_r = 0.0$ . We mounted coarse material with friction coefficient  $\mu = 0.79$  on the two palms.

The HRP-4 has two ATI sensors at the wrists. We can timely and reliably detect the collisions by thresholding the force measurement at 20 N.

We measured the impulses in Fig. 4(a) by sampling the force torque sensors at 25000 Hz. On the other hand, the HRP-4's control system runs at 200 Hz. To study the effects of the low sampling rate on the impulse calculation, we compared the same impact experiments against different sampling rates. Fig. 10 collects four sets of contact force profiles corresponding to 25000 Hz, 1000 Hz, 500 Hz, and 200 Hz. As the mean impulses are similar, we concluded that it is legitimate to integrate force measurement sampled at 200 Hz to compute the impulses.

We placed two contact point frames at the two sides of the boxes. The frame axes are defined similar to Fig. 2 and in line with the impact mechanics definitions Stronge (2000). We visualize their orientations in Fig. 8. At the moment of the impact, the origin of the two contact point frames shared the same translation with the corresponding end-effector.

**6.2.2 Controller formulation** We formulated the impact-aware constraints (36) for the two arms independently. The impact prediction employed the centroidal momentum matrix considering the kinematic chain from the floating-base link to the end-effector. The chain<sup>††</sup> weighs 11.6284 kg.

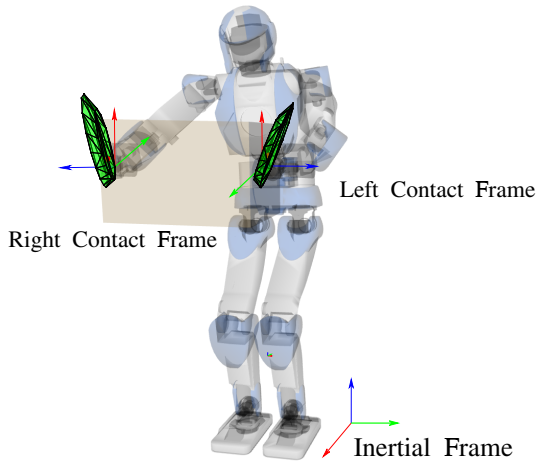
Given the inertial frame  $\mathcal{F}_O$ , the floating-base frame  $\mathcal{F}_B$ , and the end-effector frame  $\mathcal{F}_e$ , the end-effector's body velocity writes<sup>‡‡</sup>:

$$\mathbf{V}_{Oe}^b = \text{Ad}_{g_{Be}}^{-1} \mathbf{V}_{OB}^b + \mathbf{V}_{Be}^b,$$

where  $\mathbf{V}_{OB}^b \in \mathbb{R}^6$  parameterizes the floating-base velocity. Thus, the body velocity Jacobian  $\mathbf{J}_{Oe}^b$  for the kinematic chain

<sup>††</sup>The total weight of the HRP-4 robot is 34.6202 kg without the batteries.

<sup>‡‡</sup>Body velocity transform (Murray et al. 1994, Proposition 2.15).



**Figure 8.** We color  $x$ ,  $y$  and  $z$  axes with red, green and blue. The two green polyhedra are aligned to the two contact frames. They visualize the impulse sets  $\mathcal{X}_i$  for the two arms at the second grabbing. Note that their scale are not the same as robot geometry.

with a floating-base joint is:

$$J_{Oe}^b = [Ad_{gBe}^{-1}, J_{Be}^b]. \quad (38)$$

We align  $J_{Oe}^b$  to the contact point frame with (19) as before.

Reactively optimizing the following QP generates the impact-aware whole-body motion (prior to impact):

$$\begin{aligned} \min_{\nu} \quad & \sum_{i \in \mathcal{X}_o} w_i \|e_i(\nu)\|^2 \\ \text{s.t.} \quad & \text{Impact-awareness for the left arm (35),} \\ & \text{Impact-awareness for the right arm (35),} \\ & \text{Joint position, velocity, and torque (2),} \end{aligned} \quad (39)$$

where the impact-aware constraints (35) do not restrict the floating-base joint velocity or torque.

The QP (39) synchronized the two impacts to grab the box without exerting unnecessary rotating moments. The two arms followed pre-defined (off-line planned) trajectories considering the approximate location of the box. Upon the impact detection, the QP activated the admittance task to firmly grab the box and then tossed it to the ramp aside. Following another pre-defined trajectories, the HRP-4 robot repeated the grabbing and tossing motion for the second box before resuming the initial configuration. The impact-aware constraints were fulfilled for the impacts.

**6.2.3 Result analysis** The HRP-4 robot established the two contacts at 0.25 m/s, without slowing down or following pre-defined deceleration trajectories (H-2.1). We synchronized the two impacts (H-2.2) to grab the boxes.

We independently model the two synchronized impacts on the two plams, see Fig. 8. Given the ATI force sensor data, Fig. 11 shows that the measured impulses are within the predicted impulse sets regardless of different situations.

Fig. 12(a) and Fig. 12(b) plot the inertial frame end-effector velocities for the first and second grabbing. In both cases, the robot precisely tracked the reference contact velocities (instead of solving the maximum feasible contact velocity from a high reference in *Experiment 1*).

## 7 Conclusion and Future Work

Our aim was to enhance task-space QP controller to deal with intended impact tasks. To the authors best knowledge, our paper is the first to integrate to such controllers, frictional impacts in three dimensions.

We construct the impulse polyhedron to cover all the candidate solutions that fulfill Coulomb's friction law and task-space momentum conservation. Projecting the polyhedra (half-space representation) to joint or task space, the controller becomes aware of post-impact states. As we are interested in a conservative solution, the polyhedra can accommodate model uncertainties by decreasing the angle of the friction cone or the restitution coefficients' bounds.

The impact-aware constraints (convex polyhedra) modify the whole-body QP's search space according to the impact model and hardware-affordable resilience bounds. We assessed our approach with two robots: the HRP-4 humanoid and the Panda manipulator achieved high contact velocities without breaking hardware-affordable resilience bounds.

As future work, there are aspects that will be tackled from general task-space QP control perspective (robustness to interacting –i.e., potentially conflicting tasks, and robust activation on the fly of constraints whatever they are). We are already having very promising results by formulating the QP as an MPC governor.

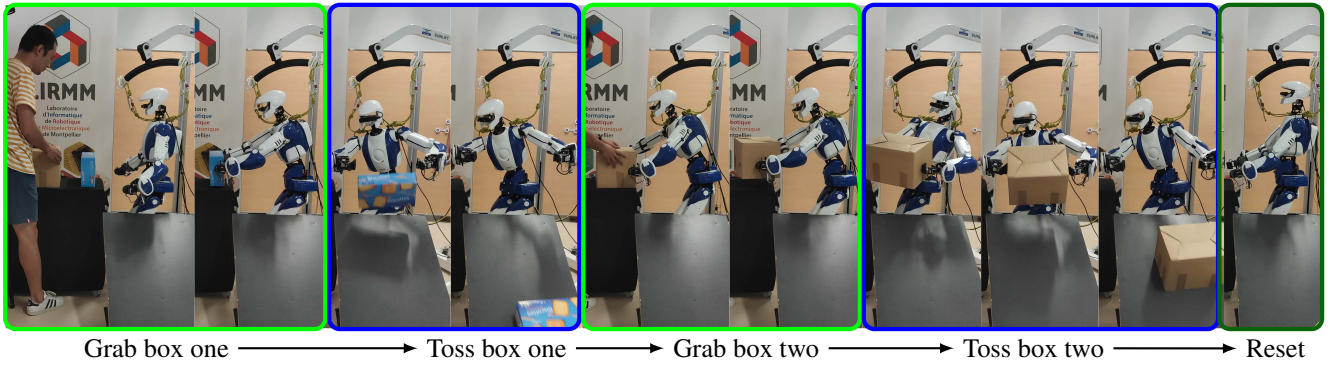
For aspects dealing more with impacts. We still have no solution if the critical bounds are not the correct ones (except being restrictive or conservative). There is also the need to devise a good shock propagation model for both fixed and floating based robots. We have also noticed that, for humanoid robot, the usual dynamic balance criteria have to be entirely revisited under external impacts. We are dedicating a separate paper for this topic. The humanoid experiments also revealed that handling multiple non-synchronous impacts on a moving object (floating box) is another shortcoming. This problem can be easily understood from a simple 2D toy example: grabbing fast a 2D floating box (i.e., a box that can move under external forces) with two points robots controlled under task-space QP that will not meet impact at the same time. In such a case, the mobility of the object in the opposite side to the first induced impact will generate a relative velocity higher than the one expected. We will examine carefully the multiple impacts under constraints on moving objects.

## Acknowledgment

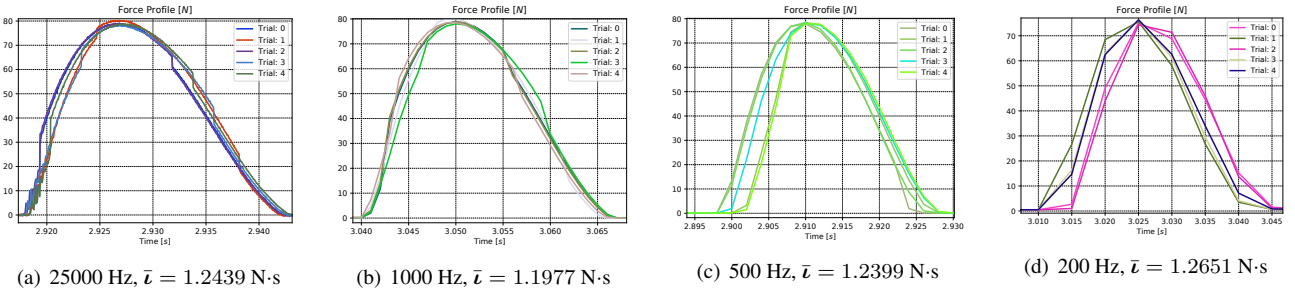
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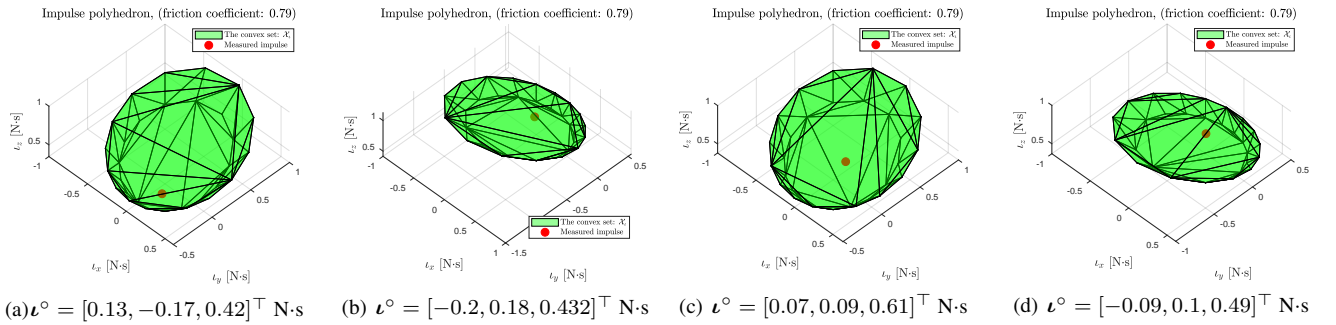
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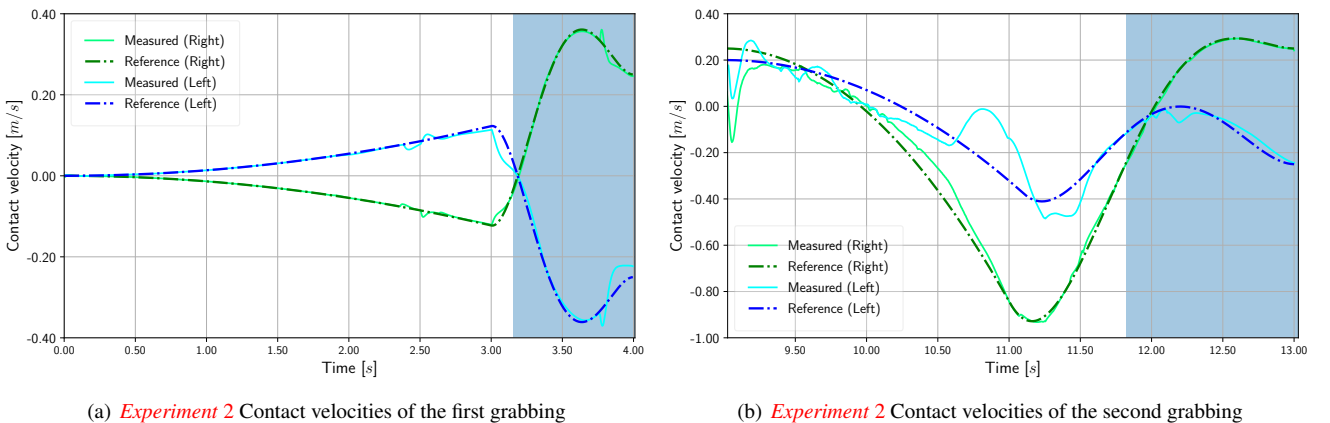
**Figure 9.** Snapshot of *Experiment 2*: the HRP-4 robot grabbed two boxes in a row with contact velocities 0.25 m/s. The green-highlighted *Grab box* states applied the impact-aware QP (34). During the blue-highlighted *Toss box* states, the robot grabbed the box with admittance control.



**Figure 10.** The measured force curves in Fig. 10(a)-10(d) are collected by repeating the same experiment: impacting the ATI force-torque sensor at 0.15 m/s by the Panda robot with the joint configuration shown in Fig. 2. The mean impulses  $\bar{\iota}$  (integrate force over time) are similar, despite the different sampling frequencies.



**Figure 11.** *Experiment 2* includes four impacts (two arms times two grabbing). The measured impulses are bounded by the impulse set  $\mathcal{X}$  in all the situations. Fig. 11(a) and 11(b) present the impulse sets of the left and right arms in the first grabbing. Similarly, Fig. 11(c) and 11(d) correspond to the second grabbing.



**Figure 12.** The robot accurately tracked the reference contact velocities in *Experiment 2*. The light-blue areas in Fig. 12(a), 12(b) indicate the period during which the controller activated the impact-aware constraints (35).

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