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Exploring the limits of a hybrid actuation system through Co-Design - Technical Report

Gianluigi Grandesso¹, Gabriel Bravo-Palacios², Patrick M. Wensing³, Marco Fontana⁴, Andrea Del Prete⁵

This document reports a miscellaneous collection of the main technical details of the work by *Grandesso et al.* presented in the paper "*Exploring the limits of a hybrid actuation system through Co-Design*".

- 1) In order to express integral variables, as in the case of the OCP cost function, it was not taken advantage of the *Integral()* component that PYOMO provides with the framework *pyomo.dae* because it is still under development. An implicit definition was used instead, that is to declare the consumed energy as a simple variable, *i.e.* using the *Var()* component, and its derivative, the total power, with *DerivativeVar()*. Then the expression of the total power is provided as constraint. In this way the total energy consumed by the system from time 0 to t_f is obtained interrogating PYOMO about the value of the energy variable at time t_f .
- 2) Due to the nonconvexity of the OCP in the case of the 2 DoF system, in many simulations IPOPT failed to find a local minimum. The capability of IPOPT to find a solution was enhanced and the time of the simulations was reduced by replacing the standard linear solver with the MA57 linear solver [1].
- 3) The two degrees of freedom $\theta_1(t)$ and $\theta_2(t)$, are respectively, the angle of the first link with respect to the horizontal and of the second link with respect to the first one. The hybrid actuation introduces four control variables represented by the torques $\tau_{se1}(t)$, $\tau_{se2}(t)$, $\tau_{gm1}(t)$ and $\tau_{gm2}(t)$ provided respectively by the two SEAs (characterised by gear ratios N_{se1} and N_{se2} , gearbox efficiencies η_{se1} and η_{se2} and spring stiffnesses K_{s1} and K_{s2}) and by the two GMs (with gear ratios N_{gm1} and N_{gm2} and gearbox efficiencies η_{gm1} and η_{gm2}). The gearbox efficiency was modeled as in the 1 DoF case. Mass, length and distance of center of mass from the joint have a subscript that specifies

which link they refers to. I_1 and I_2 are not only the rotational inertias of the links around the joints, namely I_{l1} and I_{l2} . The first includes also the inertia of the GM (motor and gearbox inertias) on the first joint and the presence of the SEA and the GM on the second joint. The second one takes into account also the inertia of the GM on the second joint. They are expressed as follows:

$$I_1 = I_{l1} + (m_{se2} + m_{gm2} + 2m_g)l_1^2 + (I_{gm1} + I_g)N_{gm1}^2 \quad (1)$$

$$I_2 = I_{l2} + (I_{gm2} + I_g)N_{gm2}^2 \quad (2)$$

Where m_{se2} and m_{gm2} are the masses of the SEA and GM motors on the second joint, while m_g and I_g are mass and inertia of the gearboxes. m_g and I_g were considered as constants and not dependent on the gear ratio because it was observed that their real values do not vary much for the selected range of gear ratios [2].

- 4) The accuracy of the model improved introducing a viscous friction term with constant friction coefficient β . It appears also in the SEA motors dynamics as showed in (3)-(4):

$$(I_{se1} + I_g) \ddot{\theta}_{se1}(t) = \tau_{se1} - \frac{K_{s1}}{\eta_{se1} N_{se1}} \left(\frac{\theta_{se1}}{N_{se1}} - \theta_1 \right) - \beta \dot{\theta}_{se1}(t) \quad (3)$$

$$(I_{se2} + I_g) \ddot{\theta}_{se2}(t) = \tau_{se2} - \frac{K_{s2}}{\eta_{se2} N_{se2}} \left(\frac{\theta_{se2}}{N_{se2}} - \theta_2 \right) - \beta \dot{\theta}_{se2}(t) \quad (4)$$

- 5) Remaining path constraints of the 1 DoF OCP:

$$\begin{aligned} -\tau_{max} &\leq \tau_{se} \leq \tau_{max} \\ -\tau_{max} &\leq \tau_{gm} \leq \tau_{max} \end{aligned} \quad (5)$$

Initial conditions of the 1 DoF OCP:

$$\theta(0) = \pi/2, \quad \dot{\theta}(t) = fA, \quad \Phi(0) = 0 \quad (6)$$

Periodicity conditions of the 1 DoF OCP:

$$\begin{aligned} \tau_{se}(0) &= \tau_{se}(t_f) \\ \tau_{gm}(0) &= \tau_{gm}(t_f) \\ \varepsilon_{se,k}(0) &= \varepsilon_{se,k}(t_f) & k = 1 \dots 2 \\ P_{se,k}(0) &= P_{se,k}(t_f) & k = 1 \dots 2 \end{aligned} \quad (7)$$

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6) Dynamics of the 2 DoF system:

$$\begin{aligned}
& (m_{l1} l_{cm1}^2 + m_{l2} (l_1^2 + l_{cm2}^2 + 2 l_1 l_{cm2} \cos(\theta_2))) + I_1 + I_2) \ddot{\theta}_1 + \\
& + (m_{l2} (l_{cm2}^2 + l_1 l_{cm2} \cos(\theta_2)) + I_2) \ddot{\theta}_2 - \\
& - (m_{l2} l_1 l_{cm2} \sin(\theta_2) \dot{\theta}_2^2 + 2 m_{l2} l_1 l_{cm2} \sin(\theta_2) \dot{\theta}_1 \dot{\theta}_2) + \\
& + g \cos(\theta_1) (m_{l1} l_{cm1} + m_{l2} l_1) + m_{l2} l_{cm2} g \cos(\theta_1 + \theta_2) - \\
& - K_{s1} \left(\frac{\theta_{se1}}{N_{se1}} - \theta_1 \right) - (\tau_{gm1} - \beta N_{gm1} \dot{\theta}_1) \eta_{gm1} N_{gm1} = 0
\end{aligned} \tag{8}$$

$$\begin{aligned}
& (m_{l2} l_{cm2}^2 + I_2) \ddot{\theta}_2 + (m_{l2} (l_{cm2}^2 + l_1 l_{cm2} \cos(\theta_2)) + I_2) \ddot{\theta}_1 + \\
& + (m_{l2} l_1 l_{cm2} \sin(\theta_2) \dot{\theta}_1^2) + (m_{l2} l_{cm2} g \cos(\theta_1 + \theta_2)) - \\
& - K_{s2} \left(\frac{\theta_{se2}}{N_{se2}} - \theta_2 \right) - (\tau_{gm2} - \beta N_{gm2} \dot{\theta}_2) \eta_{gm2} N_{gm2} = 0
\end{aligned} \tag{9}$$

Similarly to the 1 DoF model, to consider the case of only SEAs, the last terms in (8)-(9) involving the GMs should be neglected.

7) Path constraints of the 2 DoF OCP:

$$\begin{aligned}
& Eq.(1) - Eq.(4), Eq.(8), Eq.(9) \\
& \eta_k - N_k^{-0.0952} = 0 \quad k = 1 \dots 4 \\
& P_{se,k} - \tau_{se,k} \dot{\theta}_{se,k} - \frac{\tau_{se,k}^2}{K_{m_{se,k}}} = 0 \quad k = 1, 2 \\
& P_{gm,k} - \tau_{gm,k} \dot{\theta}_k N_{gm,k} - \frac{\tau_{gm,k}^2}{K_{m_{gm,k}}} = 0 \quad k = 1, 2 \\
& K_{m_{se,k}} - 0.0567 m_{se,k}^{1.8} = 0 \quad k = 1 \dots 2 \\
& K_{m_{gm,k}} - 0.0567 m_{gm,k}^{1.8} = 0 \quad k = 1 \dots 2 \\
& I_{se,k} - 2.85 \cdot 10^{-5} m_{se,k}^{1.72} = 0 \quad k = 1 \dots 2 \\
& I_{gm,k} - 2.85 \cdot 10^{-5} m_{gm,k}^{1.72} = 0 \quad k = 1 \dots 2
\end{aligned} \tag{10}$$

Boundary constraints of the 2 DoF OCP for the swing up task:

$$\begin{aligned}
\theta_1(0) &= -\theta_1(t_f) = -\pi/2 \\
\theta_2(0) &= \theta_2(t_f) = 0
\end{aligned} \tag{11}$$

Boundary constraints of the 2 DoF OCP for the pick-

and-place task:

$$\begin{aligned}
\theta_1(0) &\geq 0, \quad \Phi(0, \rho) = 0, \quad \theta_1(t_f) \geq 0 \\
l_1 \cos(\theta_1(0)) + l_2 \cos(\theta_1(0) + \theta_2(0)) &= \frac{l_1}{2} \\
l_1 \sin(\theta_1(0)) + l_2 \sin(\theta_1(0) + \theta_2(0)) &= 0 \\
l_1 \cos(\theta_1(t_f/2)) + l_2 \cos(\theta_1(t_f/2) + \theta_2(t_f/2)) &= \frac{3l_1}{2} \\
l_1 \sin(\theta_1(t_f/2)) + l_2 \sin(\theta_1(t_f/2) + \theta_2(t_f/2)) &= l_1 \\
l_1 \cos(\theta_1(t_f)) + l_2 \cos(\theta_1(t_f) + \theta_2(t_f)) &= \frac{l_1}{2} \\
l_1 \sin(\theta_1(t_f)) + l_2 \sin(\theta_1(t_f) + \theta_2(t_f)) &= 0 \\
\theta_{se1}(0) = \theta_{se1}(t_f), \quad \theta_{se2}(0) = \theta_{se2}(t_f)
\end{aligned} \tag{12}$$

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