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Positions of the secular resonances in the primordial Kuiper Belt disk

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Abstract

1 The dynamically hot and cold populations of the Kuiper Belt probably formed
2 from two distinct regions of the Solar System. The former originated from a
3 massive planetesimal disk extending from the primordial position of Neptune to
4 ~ 30 AU and the latter from a light extension of the planetesimal disk, prolong-
5 ing beyond 30 AU. Previous studies on the dynamical evolution of the primordial
6 cold population only accounted for the giant planets and did not consider its
7 evolution under the influence of the massive part of the planetesimal disk. The
8 latter affects it only indirectly through its interactions with the giant planets.
9 Our goal is to introduce the perturbation of the massive part of the planetes-
10 imal disk on the apsidal and nodal precessions of both the giant planets and
11 planetesimals, using the linear secular theory. We want to see how it affects the
12 positions of the secular resonances. In the first place, we look at the positions
13 of the secular resonances after the disappearance of the solar nebula, when the
14 giant planets were locked in a multiresonant configuration. Because of this mul-
15 tiresonant configuration, the linear secular theory allows us to compute only
16 the nodal part. The existence of a massive disk of planetesimals makes the f_5
17 frequency non-zero. We show that the associated secular resonance is located in
18 the region corresponding to the current cold Kuiper Belt in several multireso-
19 nant configurations of the giant planets. The efficiency of this secular resonance
20 in rising the inclinations of the objects depends on the misalignment between
21 the total angular momentum of the giant planets and the direction orthogonal
22 to the massive planetesimal disk. If both are aligned, the amplitude associated
23 to the f_5 frequency is null and the resonance has no effect. We illustrate this

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with simple numerical integrations, where the nodal precessions exerted by the massive disk is mimicked using fictitious forces. Then, we look at the positions of the apsidal and nodal secular resonances just before the instability between the giant planets. We find that taking into account the massive part of the disk only shifts the secular resonances by less than 1% compared with the case where we do not account for it and the f_5 secular resonance is located beyond 150 AU.

Keywords: celestial mechanics, Kuiper Belt, secular resonance

1. Introduction

The Kuiper Belt is the residual disk of a more massive primordial planetesimal disk. The Kuiper Belt objects, in their orbital structure, keep a trace of the past dynamical evolution of the giant planets. Studying their dynamics is therefore important to acquire a better understanding of the dynamical evolution of the giant planets during the life of the Solar System.

It has been shown that before the dissipation of the protoplanetary gas disk, the giant planets should have followed an inward migration until they got locked in a multiresonant configuration where each planet is in mean motion resonance (MMR) with the others (Masset and Snellgrove (2001), Morbidelli et al. (2007)), with a preference for the configuration where Jupiter and Saturn are in a 3:2 MMR because it allows their gaps in the protoplanetary gas disk to overlap. This overlap of the gaps is needed to explain why our solar system does not contain a hot Jupiter, because it refrains Jupiter from migrating close to the Sun.

From this multiresonant configuration, the giant planets, by interacting with the planetesimal disk, were driven in a migration phase until they reached their current positions and during which a dynamical instability among them occurred. This instability, which has been proposed in the framework of the Nice model (Tsiganis et al. (2005), Morbidelli et al. (2005), Gomes et al. (2005)) and the planetesimal-driven migration allow to reproduce many of the orbital features of the Kuiper Belt (Nesvorný, 2018).

In order to explain the difference in color, size distribution and binary fraction between the dynamically hot and the dynamically cold populations in the classical Kuiper Belt, it has been suggested that the hot population formed in

55 a region between Neptune and 30 AU, from a primordial massive planetesimal
 56 disk of mass in the range $\sim 10 - 60 M_{\oplus}$ (Gomes (2003), Levison et al. (2008),
 57 Nesvorný and Vokrouhlický (2016)). The outer edge of this disk is placed at
 58 30 AU because it allows to stop at this location the planetesimal-driven mi-
 59 gration of Neptune which corresponds to its current semi-major axis (Gomes
 60 et al., 2004). In other respects, the cold classical population is probably the
 61 remnant of a lighter planetesimal disk formed in-situ (Nesvorný, 2015b). Be-
 62 sides, numerical integrations that tried to reproduce the cold Kuiper Belt from
 63 a single massive disk extending from the primordial position of Neptune to 50
 64 AU did not succeed to reach a sufficient rate of mass depletion in the region of
 65 the current cold Kuiper Belt (Gomes et al., 2018). However, it is still unclear
 66 where the inner and the outer edge of this last one were located and why there
 67 is this sharp transition between the massive planetesimal disk and the light
 68 planetesimal disk.

69 In the rest of this paper, we will call the *massive disk* (MD) the part of the
 70 planetesimal disk between Neptune and 30 AU, and the *light disk* (LD) the part
 71 of the planetesimal disk beyond 30 AU.

72 The cold classical Kuiper Belt allows us to put strong constraints on the
 73 dynamical evolution of the giant planets and in particular Neptune (Batygin
 74 et al. (2011), Dawson and Murray-Clay (2012)), because the latter must migrate
 75 in such a way that the hot population forms from the MD while keeping the
 76 LD located in the region corresponding to the current classical Kuiper Belt
 77 cold enough by not exciting it. Given that the MD has a mass comparable to
 78 that of an ice giant planet, it can have a significant influence on the dynamics
 79 of an object in the LD and in particular, it will bring a perturbation on the
 80 apsidal and nodal precessions of the orbits of the planets and of the small bodies.
 81 This can have an important effect in determining the positions of the secular
 82 resonances. The latter occur where the frequency of the free apsidal or nodal
 83 precessions of a small body, depending on its semi-major axis, equals one of
 84 the eigenfrequencies of the apsidal or nodal precessions of the planetary system,
 85 respectively. Batygin et al. (2011) did numerical integrations where they follow
 86 the evolution of the MD and of the LD. In their paper, it is said that the
 87 presence of the massive Kuiper Belt enhanced the free precession of the Kuiper

88 Belt objects, so it suggests that the MD objects act on the LD objects. However,
 89 they did their numerical integrations by using the *Mercury6* integration software
 90 package. Yet, in the standard version of *Mercury6* the planetesimals of the disk
 91 do not interact with each other. Therefore, it is likely that in their numerical
 92 integrations the MD does not directly perturb the LD objects but that the latter
 93 see only the indirect effect through the action of the planets. On the other hand,
 94 [Dawson and Murray-Clay \(2012\)](#) analytically explored the excitation of the
 95 eccentricities of the LD objects caused by Neptune alone but where the other
 96 giant planets indirectly act on the LD objects through their influence on the
 97 apsidal precession of Neptune. However, they did not include the contributions
 98 of the other giant planets and of the MD to the apsidal precession of the LD
 99 objects. In their numerical study, where only Neptune is present, they included
 100 fictitious forces on the ice giant so as to reproduce its radial migration and its
 101 eccentricity damping under the effect of the interactions with the planetesimal
 102 disk as well as its apsidal precession. The MD itself is represented by a set
 103 of massless particles and does not act on the LD objects. [Nesvorný \(2015b\)](#)
 104 included fictitious forces on the giant planets so as to reproduce their radial
 105 migrations and their eccentricity and inclination damping but the MD is not
 106 present so it does not directly affect the apsidal and nodal precessions of the
 107 giant planets and of the LD objects

108 Thus, in this work we investigate the positions of the secular resonances by
 109 considering both the effects of the MD on the giant planets and on the LD
 110 objects. In order to do so, we represent the MD by an axisymmetric continuous
 111 thick disk and we take into account its contribution to the apsidal and nodal
 112 precession frequencies in the linear secular theory.

113 Studies of the positions of the secular resonances and of their effects in the
 114 Kuiper Belt under the perturbation of the gravitational potential of a disk have
 115 already been made by [Nagasawa and Ida \(2000\)](#) and [Li et al. \(2008\)](#). They
 116 showed that the locations of the secular resonances due to the giant planets are
 117 shifted when they account for the solar nebula. They modeled the latter based
 118 on the minimum mass solar nebula of [Hayashi \(1981\)](#). Then, the depletion of
 119 the nebula causes the sweeping of the secular resonances in the Kuiper Belt.
 120 With the current orbital architecture of the giant planets, they showed that if

121 the exponential depletion timescale is 10^7 yr, the sweeping secular resonances
 122 are efficient in exciting the eccentricities and inclinations of the Kuiper Belt
 123 objects at the observational level, provided that the inclination between the in-
 124 variant plane of the planets and the midplane of the solar nebula is high enough.
 125 Moreover, with a more compact orbital architecture of the giant planets, they
 126 showed that the exponential depletion timescale has to be 10^8 yr. However,
 127 the lifetime of gas nebulae around stars is of the order of $10^6 - 10^7$ yr ([Haisch](#)
 128 [et al. \(2001\)](#), [Mamajek \(2009\)](#)). Thus, with a compact orbital configuration,
 129 this mechanism cannot be sufficient to rise the eccentricities and inclinations
 130 of the Kuiper Belt objects. Besides, the mass distribution of the primordial
 131 planetesimal disk is not the same as the mass distribution of the solar nebula
 132 and the former is much less massive than the latter, while still as massive as the
 133 ice giant planets. Hence, we think that it is worthwhile to explore the positions
 134 of the secular resonances caused by the gravitational effects of the primordial
 135 planetesimal disk.

136 We present the method used in the next section. In section [3](#), we explore
 137 the positions of the secular resonances during the period when the giant planets
 138 were placed in a multiresonant configuration and the timescale needed to excite
 139 an object located in it. As it will be explained in the concerned section, we can
 140 only look at the nodal secular resonances in such multiresonant configurations.
 141 In section [4](#), we explore the positions of the secular resonances just before the
 142 dynamical instability. We conclude in section [5](#).

143 **2. Method**

144 We base our approach on the linear secular theory (see for example [Murray](#)
 145 [and Dermott \(2000\)](#), chapter 7, for details). Given the gravitational potential
 146 of a disk, the secular theory allows us to determine its contribution to the
 147 apsidal and nodal precessions of bodies. We model the MD as an axisymmetric
 148 continuous thick disk with a planar symmetry and centered on the Sun. We
 149 arbitrarily choose the reference frame such that its origin is also centered on the
 150 Sun and such that the plane (Oxy) corresponds to the plane of symmetry of the
 151 MD. Consequently, the MD has a radial density profile symmetric around the
 152 z -axis and a vertical density profile symmetric in regards to the plane $z = 0$. In

the following, the inclinations of the bodies are referred with respect to the latter plane. With the help of the numerical method developed by Fukushima (2016) we calculate the gravitational potential induced by the MD from its density profile. In the linear secular theory, the disturbing function developed to the second order in eccentricities and in inclinations is the gravitational potential averaged on one orbital period along a fixed ellipse. Therefore, in order to include the dynamical effect of the MD in the linear secular theory, we calculate its contribution to the second order disturbing functions of the giant planets and of a LD object, which is considered as a massless particle. For that, in the first place, for each of the giant planets, let us consider its total disturbing function $\mathcal{R}_j^{tot}$ where the subscript j denotes the j th giant planet, from the closest to the Sun to the outermost. We can write $\mathcal{R}_j^{tot} = \mathcal{R}_j^P + \mathcal{R}_j^{MD}$, where the superscript P denotes that this is the part of the disturbing function due to the giant planets only and the superscript MD denotes that this is the part of the disturbing function due to the MD. They are expressed as:

$$R_j^P = n_j a_j^2 \left(\frac{1}{2} A_{jj}^P e_j^2 + \sum_{k=1, k \neq j}^N A_{jk}^P e_j e_k \cos(\varpi_j - \varpi_k) + \frac{1}{2} B_{jj}^P I_j^2 + \sum_{k=1, k \neq j}^N B_{jk}^P I_j I_k \cos(\Omega_j - \Omega_k) \right), \quad (1)$$

$$R_j^{MD} = n_j a_j^2 \left(\frac{1}{2} A_{jj}^{MD} e_j^2 + \frac{1}{2} B_{jj}^{MD} I_j^2 \right), \quad (2)$$

where N is the number of giant planets, the coefficients $\{A_{jk}^P, 1 \leq j \leq N, 1 \leq k \leq N\}$ and $\{B_{jk}^P, 1 \leq j \leq N, 1 \leq k \leq N\}$ are frequencies associated to the apsidal and nodal precessions, respectively, of the giant planets without the MD and from which the eigenfrequencies g_i^P and f_i^P , respectively describing the apsidal and nodal evolutions of the planetary system, are determined. The coefficients A_{jj}^{MD} and B_{jj}^{MD} are frequencies associated to the apsidal and nodal precessions of the j th giant planet with the presence of the MD only. We can see that in equation 2, there are no coupling terms between the eccentricities of the giant planets and the "eccentricity" of the MD and between the inclinations of the giant planets and the "inclination" of the MD because of the axisymmetry and the planar symmetry of the MD. We can determine $\mathcal{R}_j^P$ analytically (see appendix A). To determine the MD contribution to the apsidal precession

frequency A_{jj}^{MD} , we calculate the gravitational potential numerically, and we averaged it on one period of the orbit of the planet, keeping its inclination fixed to zero, for values of the eccentricity starting from zero until $e_{max} = 0.002$ and by increasing the eccentricity of 0.0001 at each step. We obtain the averaged gravitational potential as a function of the eccentricity, corresponding to the disk contribution of the apsidal part of the disturbing function which is of the form $\mathcal{R}_j^{MD} = \frac{1}{2}n_j a_j^2 A_{jj}^{MD} e_j^2$ (see appendix A, figure 6, top panel). Second, we use the same method to determine the MD contribution to the nodal precession frequency B_{jj}^{MD} . We fix the eccentricity equal to zero and starting with an inclination equal to zero, we increase it with a step of 0.0001 rad until it reaches the value $I_{max} = 0.002$ rad. At each step, we calculate the gravitational potential averaged on one period of its orbit. We obtain the disk contribution to the nodal part of the disturbing function which is of the form $\mathcal{R}_j^{MD} = \frac{1}{2}n_j a_j^2 B_{jj}^{MD} I_j^2$ (see appendix A, figure 6, bottom panel). Then, using a linear regression we obtain the values of the apsidal precession frequency A_{jj}^{MD} and of the nodal precession frequency B_{jj}^{MD} . By using the frequencies $A_{jj}^{tot} = A_{jj}^P + A_{jj}^{MD}$ and $B_{jj}^{tot} = B_{jj}^P + B_{jj}^{MD}$, we can determine the eigenfrequencies g_i^{tot} and f_i^{tot} respectively describing the nodal and apsidal evolutions of the giant planets which account for the effect of the MD. To calculate the MD contribution to the free frequencies $A^{tot} = A^P + A^{MD}$ and $B^{tot} = B^P + B^{MD}$ of the apsidal and nodal free oscillations of a LD object, we can operate in the same way as previously for the giant planets. The apsidal and nodal secular resonances are localized at semi-major axes where $A^{tot} = g_i^{tot}$ and $B^{tot} = f_i^{tot}$ respectively.

3. Positions of the nodal secular resonances in the primordial orbital configuration of the giant planets

3.1. Model

In the first part, we look at the positions of the secular resonances in the initial conditions of the models proposed by [Deienno et al. \(2017\)](#). They studied the dynamical evolution of the giant planets, interacting with the MD, from several multiresonant configurations. In their models, there are initially Jupiter, Saturn, Uranus and Neptune as well as an extra ice giant. This additional planet is ejected during the dynamical instability. They found that only four

214 multiresonant configurations can lead to the current orbital architecture of the
 215 outer solar system. These configurations are the following: (a) 3:2, 3:2, 4:3,
 216 4:3; (b) 3:2, 3:2, 3:2, 3:2; (c) 3:2, 3:2, 2:1, 3:2; (d) 3:2, 3:2, 2:1, 2:1, with the
 217 semi-major axis of Jupiter initially at ~ 5.4 AU. We use those four multiresonant
 218 configurations. We place the semi-major axis of Jupiter at 5.4 AU, the inner
 219 edge of the MD at 1 AU from Neptune and its outer edge at 30 AU from the Sun.
 220 The radial density profile of the MD is of the form $\Sigma(r) = \Sigma_0 \frac{r_0}{r}$. Its vertical
 221 density profile follows a Gaussian law with $\sigma_z = r \tan(\sigma_i)$ where $\sigma_i = 2^\circ$. We
 222 choose to fix the mass of the extra ice giant to $15 M_\oplus$ and we place it between
 223 Saturn and Uranus. The giant planets being in a chain of first order resonances,
 224 in their disturbing functions the apsidal and nodal parts of the resonant part
 225 contain terms of orders $\mathcal{O}(e)$ and $\mathcal{O}(eI^2)$ respectively whereas in the secular
 226 part, the apsidal and nodal terms are of orders $\mathcal{O}(e^2)$ and $\mathcal{O}(I^2)$. Thus, for
 227 the apsidal part, the resonant effects are stronger than the secular ones so the
 228 computation of the apsidal eigenfrequencies of the planetary system by using the
 229 linear secular theory would be wrong and it would not give the correct positions
 230 of the secular resonances. Hence, we cannot use the secular theory to look at
 231 the positions of the apsidal secular resonances. For the nodal part, the secular
 232 effects are more important than the resonant ones, so we are allowed to use the
 233 secular theory. However, for the configurations (c) and (d), some giant planets
 234 have a ratio between their semi-major axes corresponding to a 3:1 MMR. In
 235 such a resonance, the nodal part of the resonant part of the disturbing function
 236 is of order $\mathcal{O}(I^2)$ and it is as high as in the secular part. Depending on the
 237 libration or circulation of the resonant angles involved in this resonance, it can
 238 have the effect of slightly moving the positions of the secular resonances from
 239 the original ones.

240 3.2. Positions of the nodal secular resonances

241 In the secular theory, if we do not consider the MD but only the five giant
 242 planets, one of the eigenfrequencies f_i associated to the inclinations and to the
 243 nodal precessions is null. The reason is that the choice of the reference plane
 244 being arbitrary, the inclination of a planet in regards to the reference plane
 245 evolves as a function of its inclination with respect to the other planets. In
 246 other words, there is an invariant plane, corresponding to the plane orthogonal

247 to the total angular momentum vector of the planetary system (hereafter called
 248 the planetary plane), that does not precess because of the invariance of the
 249 total angular momentum. The frequency in question (usually denoted f_5 for
 250 the Solar System) is associated to the precession rate of the planetary plane
 251 relative to the arbitrary reference plane, and therefore it is zero. Now, if we
 252 add the MD, the total angular momentum of the giant planets is not conserved
 253 anymore and the planetary plane precesses with the frequency f_5 which is no
 254 longer equal to zero. However, the MD being static in our model, if we put the
 255 constraint that the orthogonal direction to the plane of symmetry of the MD is
 256 aligned with the total angular momentum of the planets, the amplitude of the
 257 oscillation corresponding to the f_5 frequency is null. Nevertheless, if the MD is
 258 in such a way that its orthogonal direction is misaligned with the total angular
 259 momentum of the planets, the amplitude associated to the f_5 frequency is not
 260 null anymore.

261 Figure 1 shows the positions of the nodal secular resonances for the orbital
 262 configuration 3:2, 3:2, 3:2, 3:2. The frequencies are negative because the nodal
 263 precessions are retrograde. We arbitrarily choose the subscripts of the eigen-
 264 frequencies according to their values: the lower is the absolute value of the
 265 frequency the higher is the subscript, except for the f_5 frequency corresponding
 266 to the frequency of the precession of the planetary plane of the giant planets.
 267 We represent three different cases: (a) in the first case, only the giant planets
 268 are present (top panel). All the secular resonances are located below 20 AU.
 269 (b) In the second case, we add the MD, with a mass $M_{disk} = 40M_{\oplus}$, in the
 270 model but it affects only the giant planets. It has for effect to increase the
 271 absolute values of the eigenfrequencies but the nodal free frequency of the LD
 272 object remains the same as in the case where only the giant planets are present
 273 (bottom panel, the horizontal lines represent the values of the eigenfrequencies
 274 of the planetary system affected by the MD and the dashed curve represents the
 275 nodal free frequency of the LD object not affected by the MD). It corresponds
 276 to the case of numerical integrations where the MD is represented by a set of
 277 massive planetesimals acting on the planets but not on the other planetesimals.
 278 The f_6 , f_7 , f_8 and f_9 secular resonances are still located below 20 AU. However,
 279 the secular resonance associated to the f_5 frequency, which is not null anymore,

280 is located in the LD near 33 AU. (c) Finally, in the third case, the MD also
 281 affects the LD objects and increases their nodal free frequencies (bottom panel,
 282 the full black curve represents the nodal free frequency of the LD object affected
 283 by the MD). The positions of the f_6 and f_7 secular resonances remain nearly
 284 the same as in the other two cases but the f_8 and f_9 secular resonances are
 285 pushed toward ~ 30 AU and the f_5 secular resonance is located near 44 AU
 286 which is inside the region of the current cold classical Kuiper Belt. This high-
 287 lights that the consideration of the direct effect of the MD on the LD objects
 288 shifts significantly the positions of three secular resonances. The fact that a
 289 secular resonance was primordially located in the region corresponding to the
 290 current classical Kuiper Belt, which is a stable region populated with objects
 291 whose orbits only have had a slight evolution, is important because it can let a
 292 dynamical signature which can be observable today. The migration of the giant
 293 planets, the evolution of the shape of the disk and its loss of mass, cause the
 294 migration of the secular resonances. If the excitation timescale of the f_5 secular
 295 resonance is short enough with respect to its migration timescale, the resonance
 296 can remove the cold population objects from the swept region by rising their
 297 inclinations to values higher than 5° , which roughly corresponds to the inclina-
 298 tion border beyond which the objects are not considered as cold anymore, and
 299 it creates a local population of objects with excited inclinations but with ec-
 300 centricities staying low, which are on stable orbits. Otherwise, if the excitation
 301 timescale of the f_5 secular resonance is too long compared with its migration
 302 timescale, its passage preserves the cold population while slightly exciting it. In
 303 this configuration, the f_8 and f_9 secular resonances are located just below 30
 304 AU whereas the 2:1 MMR with Neptune is located near 26 AU. Thus, the two
 305 secular resonances must excite the inclinations of the objects before the passage
 306 of the sweeping 2:1 MMR during the migration of Neptune. This could have
 307 affected the inclination distribution of the objects captured by the 2:1 MMR.

308 Hence, we will now look at the positions of the f_5 secular resonance in
 309 the different orbital configurations proposed by [Deienno et al. \(2017\)](#) and with
 310 several masses of the MD. The results are shown in table 1. The f_5 secular
 311 resonance is located in the current cold Kuiper Belt for the configuration 3:2,
 312 3:2, 4:3, 4:3 with $M_{disk} = 20M_\oplus$ and for the configuration 3:2, 3:2, 3:2 for

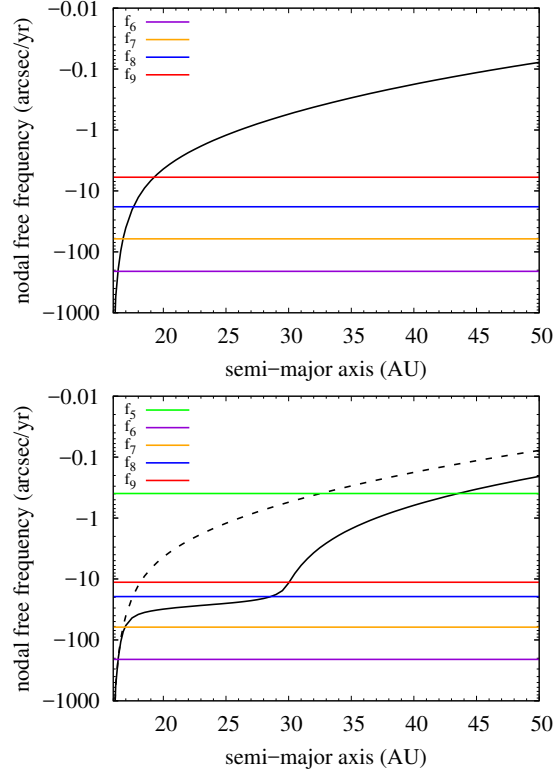


Figure 1: Nodal free frequency of a LD object as a function of its semi-major axis in the configuration 3:2, 3:2, 3:2, 3:2, in a model without the MD (top panel) and in a model with a MD of mass $M_{disk} = 40M_{\oplus}$ (bottom panel). The horizontal lines are the nodal eigenfrequencies. In the bottom panel, the dashed curve is the nodal free frequency in the case where the MD does not act on the LD object and the full black curve is the nodal free frequency but in the case where the MD acts on the LD object. The nodal secular resonances occur at the semi-major axes where the nodal free frequency crosses one of the nodal eigenfrequencies.

the three different masses of the disk. In the configuration 3:2, 3:2, 2:1, 3:2, with $M_{disk} = 40M_{\oplus}$ and $M_{disk} = 60M_{\oplus}$, the resonance is located at the outer border of the current cold Kuiper Belt, near the 2:1 MMR. With $M_{disk} = 20M_{\oplus}$, it is located in a region where an extension of the cold population beyond the 2:1 MMR has been discovered (Bannister et al., 2018). In the configuration 3:2, 3:2, 2:1, 2:1, the resonance is located in a region where cold Kuiper Belt objects have still not been observed so far. Finally, in the configuration 3:2, 3:2, 4:3, 4:3, with $M_{disk} = 40M_{\oplus}$ and $M_{disk} = 60M_{\oplus}$ the resonance is located in a region where the current cold Kuiper Belt does not exist anymore because the

Orbital configuration	$M_{disk} (M_{\oplus})$	Position of f_5 (AU)
3:2, 3:2, 4:3, 4:3	20	42.7
	40	41.0
	60	40.5
3:2, 3:2, 3:2, 3:2	20	45.0
	40	43.5
	60	43.3
3:2, 3:2, 2:1, 3:2	20	48.3
	40	47.5
	60	47.7
3:2, 3:2, 2:1, 2:1	20	55.1
	40	54.7
	60	54.7

Table 1: Position of the f_5 secular resonance for four orbital configurations and for several masses of the MD.

lifetime of an object on an orbit in that region is short, in particular because of the presence of the current g_8 secular resonance. The other secular resonances remain below 33 AU in all the configurations and never reach the region of the current classical Kuiper Belt.

3.3. Efficiency of the f_5 secular resonance

We performed simple numerical integrations in order to illustrate the efficiency of the f_5 secular resonance. We cannot include directly the massive disk and compute the forces it exerts on the bodies because it is computationally too heavy. Therefore, we only include the nodal precession it exerts. To do so, we proceed as follow: as a first step, for a given shape and mass of the disk, we use the method described in section 2 in order to sample the values of the nodal precession frequency as a function of the semi-major axis. We sample the frequency every 0.1 AU in a range from 1 AU to 60 AU. It is illustrated in figure 2. Then, we use a linear interpolation in order to have a continuous function of the frequency. In a second step, to allow the variation of the longitude of

the node of a body, we use the method of Lee and Peale (2002) implemented in the same way as Wolff et al. (2012) in the *Mercury6* N-body integrator package (Chambers, 1999), adding acceleration and velocity perturbation terms corresponding to the variation of the longitude of the node, as described in appendix B.

Here, the goal of our numerical integrations is not to include the effect of the f_5 secular resonance in a complete model of dynamical evolution of the giant planets and of the Kuiper Belt and then to compare the Kuiper Belt obtained with the observations. It would demand to construct a strong model where the evolution of the shape and of the mass of the disk is coherent with the migration of the giant planets. The construction of such a model is beyond the scope of this study and is left for future work. The aim of the present study is qualitative and is to illustrate how efficient the f_5 secular resonance can be in a numerical integration. Therefore, in the present model, the giant planets remain unrealistically at their initial semi-major axes during the whole numerical integration and the shape of the MD does not evolve but we allow its mass to decrease exponentially with time, simulating the migration of the f_5 secular resonance. We do not allow the giant planets to migrate because, as we can see in figure 2, there is a strong variation of the nodal precession frequency in the region of the MD. However, as Neptune follows its outward migration, the inner border of the MD moves back and Neptune never reaches the region where this strong variation takes place. So, if we allow Neptune to migrate but we do not adapt the shape of the disk to it, the nodal precession frequency of Neptune would be more enhanced than it is in a more realistic model. We do not explore the effect of the f_8 and f_9 secular resonances either because the decaying disk would cause them to move inward whereas the migration of the giant planets would make the resonances to migrate outward. Therefore, if the effect of the outward migration is stronger than that of the inward migration, a model that accounts for the two effects would see the outward migration of the resonances whereas in our model we only see an inward migration, thus the representation would be wrong.

We illustrate our numerical integrations with the example of the multi-resonant configuration 3:2, 3:2, 2:1, 3:2 which is evaluated as the most probable

370 by [Deienno et al. \(2017\)](#). We choose the initial mass of the massive disk to be
 371 $M_{disk} = 20M_{\oplus}$. We use fictitious forces of [Lee and Peale \(2002\)](#) to keep the
 372 semi-major axes of the giant planets in the multiresonant configuration and to
 373 damp their eccentricities. Given that the efficiency of the f_5 secular resonance
 374 depends on the inclination of the planetary plane with respect to the mean plane
 375 of the disk, we did numerical integrations with different initial inclinations of
 376 the planetary plane. We place the giant planets initially on coplanar orbits.
 377 We put massless particles from 45 AU to 55 AU, with one particle every 0.1
 378 AU. We generate the eccentricities and the inclinations of the particles with a
 379 Rayleigh distribution with a mode of 0.01 and 0.01 radians respectively. The
 380 other angles of the giant planets and of the particles are uniformly distributed
 381 over 360° . Given that in the linear secular theory, the precession frequency is
 382 linearly proportional to the mass of the disk, we can account for the depletion
 383 of the disk directly by modifying the precession frequency. We represent the
 384 depletion of the mass of the disk by an exponential decay with a timescale τ .
 385 In a first set of numerical integrations, we explore the effect of the inclination
 386 of the planetary plane. We choose a disk depletion timescale $\tau = 50$ Myr and
 387 we run three numerical integrations over 10 Myr with a timestep of 100 days
 388 and with three different initial inclinations of the planetary plane: 0° , 1° and
 389 2° . Figure 3 presents the results and shows that the higher the inclination of
 390 the planetary plane, the higher the efficiency of the f_5 secular resonance.

391 In a second set of numerical integrations, we fix the initial inclination of the
 392 planetary plane at 1° but we try three values of the depletion timescale: $\tau = 50$
 393 Myr, $\tau = 30$ Myr and $\tau = 10$ Myr. We run our numerical integrations for
 394 200 Myr. The time step is kept at 100 days. Figure 4 shows the results, with
 395 the semi-major axes averaged over the last 50 Myr and the proper inclinations.
 396 In the case $\tau = 50$ Myr the resonance sweeps from ~ 48 AU to ~ 55 AU in
 397 ~ 100 Myr. The rising of the proper inclinations after 54 AU is associated to a
 398 dephasing effect. In the case $\tau = 30$ Myr it sweeps in ~ 60 Myr and for $\tau = 10$
 399 Myr it sweeps in ~ 30 Myr.

400 We remind that those numerical integrations are not autocohrent and it
 401 would demand to do stronger effort in order to perform numerical integrations
 402 including the effect of the massive disk with the latter evolving with respect to

403 the migration of Neptune in a consistent way. Besides, there is an uncertainty
 404 of the time of the triggering of the instability. It has a consequence on the
 405 timescale during which the f_5 secular resonance is efficient. During the pre-
 406 instability period, the mass of the disk is slowly depleted and remains high
 407 enough, so the resonance slowly moves from its initial location. Then, during
 408 the instability, the mass of the disk is quickly depleted and the secular resonance
 409 migrates very rapidly toward higher semi-major axes.

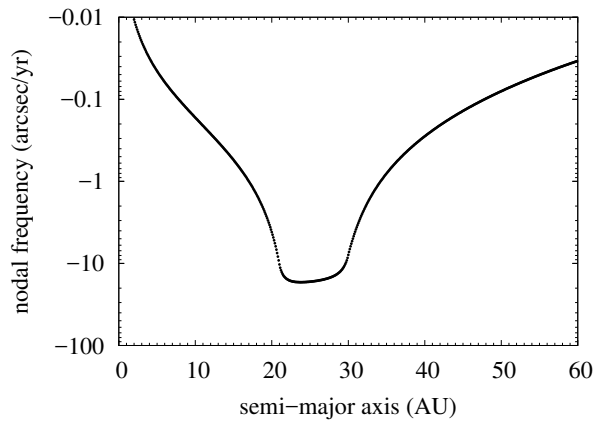


Figure 2: The frequency of the nodal precession caused by the MD as a function of the semi-major axis. The mass of the MD is $M_{disk} = 20M_{\oplus}$, its inner edge is at 21 AU and its outer edge at 30 AU.

410 **4. Positions of the secular resonances just before the planetary insta-** 411 **bility**

412 During the migration of the giant planets, they interact with the planetes-
 413 imals of the MD and some of the latter get ejected. Therefore, the MD is
 414 gradually depleted. The aim of this section is to check whether the shift of the
 415 secular resonances remains important or not and to check the position of the
 416 f_5 secular resonance, just before the planetary instability. For that purpose,
 417 we use the results of a numerical integration for a Neptune migration scenario
 418 developed by [Nesvorný \(2015a\)](#). It also includes the planetary instability, rep-
 419 resented via a jumping Neptune, which occurred when the ice giant is near 28
 420 AU in order to explain the kernel in the cold population ([Nesvorný, 2015b](#)).

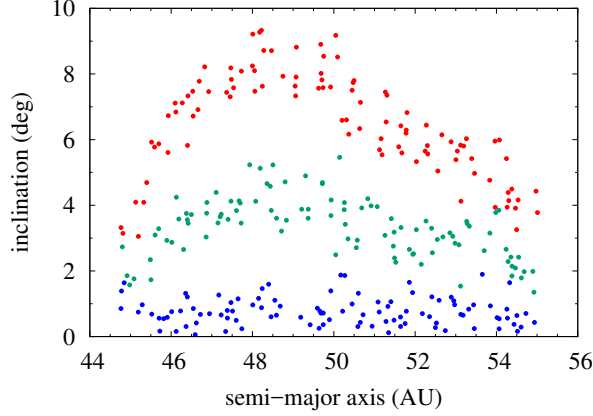


Figure 3: Inclinations of the particles after 10 Myr. The multiresonant configuration of the giant planets is 3:2, 3:2, 2:1, 3:2. The initial mass of the MD is $M_{disk} = 20M_{\oplus}$, its inner edge is at 21 AU, its outer edge at 30 AU and its depletion timescale is 50 Myr. The three colors of the particles represent the results of three different numerical integrations, for which the initial inclination of the planetary plane with respect to the mean plane of the disk is $i = 0^\circ$ (blue dots), $i = 1^\circ$ (green dots) and $i = 2^\circ$ (red dots).

421 The context of this numerical integration is different than that of numerical
422 integrations of [Deienno et al. \(2017\)](#). The aim of the study of [Nesvorný \(2015a\)](#)
423 was to reproduce the inclination distribution of the hot Kuiper Belt population
424 from a set of massless particles initially located between Neptune and 30 AU
425 with a slow radial migration of Neptune. Here, there are only the four giant
426 planets and they are not in a multiresonant configuration anymore. Thus, it is
427 possible to look at the positions of the apsidal secular resonances. The initial
428 positions of Jupiter, Saturn and Uranus correspond to their current ones and
429 Neptune was placed with an initial semi-major axis lower than 25 AU. Its radial
430 migration and its eccentricity and inclination damping were driven by fictitious
431 forces.

432 4.1. Model

433 From the orbital elements of the particles provided by the numerical inte-
434 gration of [Nesvorný \(2015a\)](#) we estimate the distribution of the MD at $t = 3$
435 Myr before the instability between the giant planets. Let (r, θ, z) be the helio-
436 centric coordinates. We assume that the MD distribution is axisymmetric and

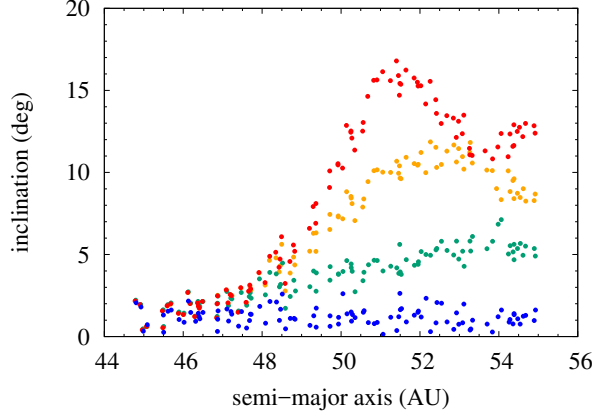


Figure 4: Proper inclinations of the particles after 200 Myr numerical integrations. The multiresonant configuration of the giant planets is 3:2, 3:2, 2:1, 3:2 and the initial inclination of the planetary plane with respect to the mean plane of the disk is $i = 1^\circ$. The initial mass of the MD is $M_{disk} = 20M_\oplus$, its inner edge is at 21 AU and its outer edge at 30 AU. The blue dots represent the results of the particles for a numerical integration where the nodal precession due to the MD is not included. The other colors represent the results for numerical integrations with different depletion timescales τ of the mass of the MD. The green dots are for $\tau = 10$ Myr, the orange dots for $\tau = 30$ Myr and the red dots for $\tau = 50$ Myr. The semi-major axes are averaged over the last 50 Myr of the numerical integrations.

uncoupled in r and z . We construct histograms of the r and z distributions from which we obtain the following density profiles:

$$\sigma_r(r) = \begin{cases} \frac{23.5}{2\pi} & \text{if } r \leq 28 \text{ AU}, \\ \frac{2961}{\pi r(r-19)} & \text{if } 28 \text{ AU} < r \leq 1000 \text{ AU}, \\ 0 & \text{if } r > 1000 \text{ AU}, \end{cases}$$

and

$$\lambda_z(z) = \begin{cases} \exp(-\frac{|z|}{10}) & \text{if } -100 \text{ AU} \leq z \leq 100 \text{ AU}, \\ 0 & \text{otherwise,} \end{cases}$$

where r and z are expressed in AU, $\sigma_r(r)$ is the radial surface density in kg.AU^{-2} and $\lambda_z(z)$ is the vertical linear density in kg.AU^{-1} . Here, those two last quan-

	Orbital elements	
	a (AU)	e
Jupiter	5.201731	0.01824838
Saturn	9.578610	0.05086522
Uranus	18.16524	0.03557539
Neptune	27.80457	0.01387243

Table 2: Semi-major axes and eccentricities of the giant planets 3 Myr before the planetary instability.

444 tities are not normalized. The density profile of the MD is:

$$445 \quad \rho(r, z) = M_{disk} \frac{\sigma_r(r) \lambda_z(z)}{\int_{r=0}^{r=R} \int_{z=z_{min}}^{z=z_{max}} \int_{\theta=0}^{\theta=2\pi} \sigma_r(r) \lambda_z(z) r \, dr \, dz \, d\theta}, \quad (3)$$

446 in $\text{kg} \cdot \text{AU}^{-3}$, where the denominator is used to normalize the function. The
447 mass of the MD is chosen with an initial value of $20 M_{\oplus}$, because the migration
448 timescale of Neptune in that numerical integration is typical of that with such
449 a mass. It has the value $5.9604 M_{\oplus}$ at the time $t = 3$ Myr before the planetary
450 instability.

451 4.2. Results

452 We apply the method described in section 2 to evaluate the apsidal and
453 nodal eigenfrequencies of the planetary system and the free frequencies of the
454 LD object.

455 Table 2 shows the semi-major axes and eccentricities of the giant planets 3
456 Myr before the planetary instability. The f_5 secular resonance is located beyond
457 150 AU, because the mass of the MD is low and the radial distance of the f_5
458 secular resonance increases as the disk loses mass.

459 Figure 5 presents the results that we obtained for the positions of the g_8
460 secular resonance, exhibited by the dashed vertical lines, according to several
461 perspectives. A^P (red curve) and g_8^P (red dashed horizontal line) are the fre-
462 quencies in the case where the apsidal precessions are only driven by the giant
463 planets, whereas A^{tot} (blue curve) and g_8^{tot} (blue dashed horizontal line) are the

464 frequencies in the case where the apsidal precessions are driven by the giant
 465 planets but also by the MD. The red dashed vertical line shows the position
 466 of the resonance in the case where the MD does not act neither on the giant
 467 planets nor the LD objects and then is located where $A^P = g_8^P$, at 37.21 AU.
 468 The purple dashed vertical line corresponds to the case where the MD acts on
 469 the giant planets but not on the LD objects and is located where $A^P = g_8^{tot}$,
 470 at 37.59 AU. Finally, the blue vertical line is the ideal case where the MD acts
 471 both on the giant planets and on the LD objects. It is located where $A^{tot} = g_8^{tot}$,
 472 at 37.44 AU. It makes a shift of 0.6% compared to the case where the MD is
 473 neglected. The other resonances are also shifted by less than 1%. We conclude
 474 that the shifts imposed by the MD are negligible.

475 We also check the positions of the secular resonances 50 Myr after the plane-
 476 tary instability. In this case, the mass of the disk is $2.9006 M_\oplus$, two times lower
 477 than at 3 Myr before the instability, but the peak of the mass distribution is
 478 located at ~ 30 AU, which is closer from the Kuiper Belt than in the previous
 479 case. However, we find that the f_5 secular resonance remains located beyond
 480 150 AU and the shift of the other secular resonances is even smaller than in the
 481 previous case.

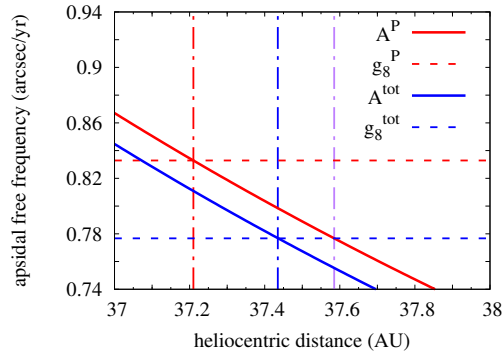


Figure 5: Position of the g_8 secular resonance in the case without the MD (red curve) and with it (blue curve), 3 Myr before the instability between the giant planets. Horizontal dashed lines represent the value of g_8 . We exhibit the intersection of the curves with the horizontal dashed lines by dashed vertical lines which is the position of the g_8 secular resonance. The purple dashed vertical line shows the position of g_8 in the case where the MD acts on the giant planets but not on the LD object.

482 5. Discussion and conclusion

483 Studies of the primordial dynamical evolution of the giant planets and of the
 484 primordial planetesimal disk have shown that the latter must have a sharp edge
 485 near 30 AU in its mass distribution. The part of the disk beyond this sharp edge
 486 is a light extension of the disk so as to make unefficient the planetesimal-driven
 487 migration of Neptune beyond this edge and to stop the ice giant at its current
 488 position. Besides, current models starting with initial conditions presenting a
 489 continuous mass distribution in the planetesimal disk between Neptune and 50
 490 AU do not succeed in sufficiently depleting the mass of the cold Kuiper Belt
 491 region. The cold Kuiper Belt population is supposed to be formed from this
 492 light extension.

493 That being said, dynamical studies of the objects of the light part of the disk
 494 have so far always been restricted to the dynamical evolution of those objects
 495 under the effect of the gravitational interactions with the giant planets only.
 496 However, given the important mass of the disk between Neptune and 30 AU
 497 ($\sim 10 - 60 M_{\oplus}$), it can have a non-negligible influence. This is what we wanted
 498 to check in this paper. We have called the *massive disk* (MD) the part of the
 499 planetesimal disk between Neptune and 30 AU, and the *light disk* (LD) the part
 500 of the planetesimal disk beyond 30 AU. We have studied the different positions
 501 that the secular resonances can have in three different situations: (a) a case
 502 where the MD is not included, (b) a case where the MD modifies the apsidal
 503 and nodal precessions of the giant planets but not those of the LD objects and
 504 (c) a case where the MD also acts on the apsidal and nodal precessions of the
 505 LD objects.

506 In section 3, we have investigated on the positions of the secular resonances
 507 once the solar nebula vanished and the giant planets were still locked in a
 508 multiresonant configuration. We could only look at the positions of the nodal
 509 secular resonances because the resonant dynamics is dominant in the evolution
 510 of the longitudes of the perihelion of the giant planets. We have found that the
 511 f_5 secular resonance, driven by the precession of the total angular momentum of
 512 the giant planets, is located in the region of the LD in several configurations. It
 513 can have an effect in exciting the inclinations of the LD objects. Consequently,
 514 it can create a population of objects with high inclinations but low eccentricities.

515 Its efficiency in rising their inclinations mainly depends on the angle between the
 516 planetary and the MD planes. While revising this manuscript we became aware
 517 of the work of [Toliou and Tsiganis \(2019\)](#), performed independently, that find
 518 results similar to ours on the existence and the role of a non-zero f_5 frequency
 519 during the disk phase.

520 In planetary formation models, the planets and the planetesimal disk formed
 521 from the same protoplanetary disk in the plane of the solar equator. In the cur-
 522 rent Solar System, we observe an inclination of $\sim 6^\circ$ between the solar equator
 523 and the invariant plane. It has been shown that the presence of a ninth planet
 524 located in the outer Solar System can explain this tilt ([Bailey et al. \(2016\)](#),
 525 [Gomes et al. \(2017\)](#) and [Lai \(2016\)](#)). Another possibility, as suggested by [Baty-
 526 gin \(2012\)](#), is that the tilt can be produced by the passage of a star in the birth
 527 cluster. The passage of a star can warp the protoplanetary disk, making its
 528 outer part inclined with respect to the inner part where the planets are formed.
 529 Moreover, [Izidoro et al. \(2015\)](#) have shown that during the accretion of Uranus
 530 and Neptune, ejections of planetary embryos caused by close encounters were
 531 common. As ejected objects take away with them a part of the total angular
 532 momentum, the ones remaining in the Solar System have an angular momentum
 533 different from the total one and that is not conserved. Therefore, the orienta-
 534 tion of the plane orthogonal to the angular momentum of the remaining objects
 535 varies with time, until the Solar System reaches its current architecture and the
 536 ejections of objects stopped, leaving what is currently called the invariant plane.
 537 All of these elements give weight to the conception that the invariant plane of
 538 the planets got misaligned with the plane of the disk.

539 In section 4, we have looked at the positions of the secular resonances during
 540 the migration of Neptune just before the instability between the giant planets
 541 occurred, when Neptune is near 28 AU, from data provided by the works of
 542 [Nesvorný \(2015a\)](#). We have found that during this phase of the planetary evo-
 543 lution, the mass of the MD is not high enough anymore and that consequently
 544 the positions of the secular resonances are only shifted by less than 1% with
 545 respect to the case where the MD is neglected. In this case, if numerical in-
 546 tegrations omit the effects of the MD, the evolution of the LD objects will be
 547 qualitatively the same as if they do not.

548

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659 **Appendix A**

660 In a planetary system with N giant planets, the secular part of the disturbing
661 functions R_j^P of the j th giant planet and R^P of a LD object are ([Murray and](#)

662 [Dermott, 2000](#)):

$$\begin{aligned}
R_j^P = & n_j a_j^2 \left(\frac{1}{2} A_{jj}^P e_j^2 + \sum_{k=1, k \neq j}^N A_{jk}^P e_j e_k \cos(\varpi_j - \varpi_k) \right. \\
& \left. + \frac{1}{2} B_{jj}^P I_j^2 + \sum_{k=1, k \neq j}^N B_{jk}^P I_j I_k \cos(\Omega_j - \Omega_k) \right), \tag{4}
\end{aligned}$$

$$\begin{aligned}
R^P = & n a^2 \left(\frac{1}{2} A^P e^2 + \sum_{j=1}^N A_j^P e e_j \cos(\varpi - \varpi_j) \right. \\
& \left. + \frac{1}{2} B^P I^2 + \sum_{j=1}^N B_j^P I I_j \cos(\Omega - \Omega_j) \right), \tag{5}
\end{aligned}$$

666 where the terms with the superscript P denotes that they are due to the plan-
667 etary system only and the variables with the subscript j and without it respec-
668 tively relate to the j th giant planet and to the LD object. n is the mean motion,
669 a the semi-major axis, e the eccentricity, I the inclination, ϖ the longitude of
670 the perihelion and Ω the longitude of the node. The coefficients A_{jj}^P , A_{jk}^P , B_{jj}^P ,
671 B_{jk}^P , A^P , A_j^P , B^P and B_j^P are frequencies, which depend on the masses and
672 semi-major axes of the bodies such that:

$$A_{jj}^P = \frac{1}{4} n_j \sum_{k=1, k \neq j}^N \frac{m_k}{m_c + m_j} \alpha_{jk} \bar{\alpha}_{jk} b_{3/2}^{(1)}(\alpha_{jk}), \tag{6}$$

$$A_{jk}^P = -\frac{1}{4} n_j \frac{m_k}{m_c + m_j} \alpha_{jk} \bar{\alpha}_{jk} b_{3/2}^{(2)}(\alpha_{jk}) \quad (k \neq j), \tag{7}$$

$$B_{jj}^P = -\frac{1}{4} n_j \sum_{k=1, k \neq j}^N \frac{m_k}{m_c + m_j} \alpha_{jk} \bar{\alpha}_{jk} b_{3/2}^{(1)}(\alpha_{jk}), \tag{8}$$

$$B_{jk}^P = \frac{1}{4} n_j \frac{m_k}{m_c + m_j} \alpha_{jk} \bar{\alpha}_{jk} b_{3/2}^{(1)}(\alpha_{jk}) \quad (k \neq j), \tag{9}$$

$$A^P = \frac{1}{4} n \sum_{j=1}^N \frac{m_j}{m_c} \alpha_j \bar{\alpha}_j b_{3/2}^{(1)}(\alpha_j), \tag{10}$$

$$A_j^P = -\frac{1}{4} n \frac{m_j}{m_c} \alpha_j \bar{\alpha}_j b_{3/2}^{(2)}(\alpha_j), \tag{11}$$

$$B^P = -\frac{1}{4} n \sum_{j=1}^N \frac{m_j}{m_c} \alpha_j \bar{\alpha}_j b_{3/2}^{(1)}(\alpha_j), \tag{12}$$

$$B_j^P = \frac{1}{4} n \frac{m_j}{m_c} \alpha_j \bar{\alpha}_j b_{3/2}^{(1)}(\alpha_j), \tag{13}$$

where m_c is the mass of the Sun, m_j is the mass of the j th giant planet,

$$\alpha_{jk} = \begin{cases} a_k/a_j & \text{if } a_j > a_k, \\ a_j/a_k & \text{if } a_j < a_k, \end{cases}$$

$$\bar{\alpha}_{jk} = \begin{cases} 1 & \text{if } a_j > a_k, \\ a_j/a_k & \text{if } a_j < a_k, \end{cases}$$

$$\alpha_j = \begin{cases} a_j/a & \text{if } a > a_j, \\ a/a_j & \text{if } a < a_j, \end{cases}$$

$$\bar{\alpha}_j = \begin{cases} 1 & \text{if } a > a_j, \\ a/a_j & \text{if } a < a_j, \end{cases}$$

and where

$$b_{3/2}^{(n)}(\alpha) = \frac{1}{\pi} \int_0^{2\pi} \frac{\cos n\psi \, d\psi}{(1 - 2\alpha \cos \psi + \alpha^2)^{\frac{3}{2}}}, \quad (14)$$

is called the Laplace coefficient.

We write $\mathbf{A}^P$ and $\mathbf{B}^P$ the matrices of dimension $N \times N$ containing the elements $\{A_{jk}^P, 1 \leq j \leq N, 1 \leq k \leq N\}$ and $\{B_{jk}^P, 1 \leq j \leq N, 1 \leq k \leq N\}$ respectively. By introducing $h^P \equiv e \sin \varpi$, $k^P \equiv e \cos \varpi$, $p^P \equiv I \sin \Omega$ and $q^P \equiv I \cos \Omega$, then by using the Lagrange planetary equations with the disturbing functions R_j^P and R^P , we obtain the solutions:

$$h^P = e_{free}^P \sin(A^P t + \beta^P) + h_0^P(t), \quad (15)$$

$$k^P = e_{free}^P \cos(A^P t + \beta^P) + k_0^P(t), \quad (16)$$

$$p^P = I_{free}^P \sin(B^P t + \gamma^P) + p_0^P(t), \quad (17)$$

$$q^P = I_{free}^P \cos(B^P t + \gamma^P) + q_0^P(t), \quad (18)$$

where e_{free}^P , I_{free}^P , β^P and γ^P are constants determined by the initial conditions. The first terms of the right members of the equations 15, 16, 17 and 18 correspond to the free oscillations while the terms $h_0^P(t)$, $k_0^P(t)$, $p_0^P(t)$ and $q_0^P(t)$

696 correspond to the forced oscillations:

$$h_0^P = - \sum_{i=1}^N \frac{\nu_i^P}{A^P - g_i^P} \sin(g_i^P t + \beta_i^P), \quad (19)$$

$$k_0^P = - \sum_{i=1}^N \frac{\nu_i^P}{A^P - g_i^P} \cos(g_i^P t + \beta_i^P), \quad (20)$$

$$p_0^P = - \sum_{i=1}^N \frac{\mu_i^P}{B^P - f_i^P} \sin(f_i^P t + \gamma_i^P), \quad (21)$$

$$q_0^P = - \sum_{i=1}^N \frac{\mu_i^P}{B^P - f_i^P} \cos(f_i^P t + \gamma_i^P), \quad (22)$$

698 where the frequencies g_i^P and f_i^P are the eigenvalues (which we will call eigen-
 699 frequencies in the following) of the matrices $\mathbf{A}^P$ and $\mathbf{B}^P$ respectively. We also
 700 have $\nu_i^P = \sum_{j=1}^N A_j^P e_{ji}^P$ and $\mu_i^P = \sum_{j=1}^N B_j^P I_{ji}^P$ where e_{ji}^P , I_{ji}^P , β_i^P and γ_i^P are
 701 constants depending on the eigenvectors associated to the eigenfrequencies and
 702 on the initial conditions of the giant planets. These equations bring out the ex-
 703 istence of resonances when the secular frequency A^P and B^P of the LD object,
 704 which depends on its semi-major axis, are equal to one of the eigenfrequencies
 705 g_i^P and f_i^P respectively. The resonances involving the frequencies A^P and g_i^P
 706 are called apsidal secular resonances whereas those involving the frequencies B^P
 707 and f_i^P are called nodal secular resonances.

708 Now, if we consider the disturbing function R_j^{MD} of the j th giant planet due
 709 to the MD and the disturbing function R^{MD} of the LD object due to the MD,
 710 we have:

$$R_j^{MD} = n_j a_j^2 \left(\frac{1}{2} A_{jj}^{MD} e_j^2 + \frac{1}{2} B_{jj}^{MD} I_j^2 \right), \quad (23)$$

$$R^{MD} = n a^2 \left(\frac{1}{2} A^{MD} e^2 + \frac{1}{2} B^{MD} I^2 \right), \quad (24)$$

714 where the terms with the superscript MD denotes that they are due to the MD.
 715 As described in section 2, we can determine R_j^{MD} and R^{MD} computationnally.
 716 For example, figure 6 shows the apsidal part (top panel) and the nodal part
 717 (bottom panel) of the disturbing function of Jupiter due to the MD as a function
 718 of the eccentricity and the inclination of the planet respectively.

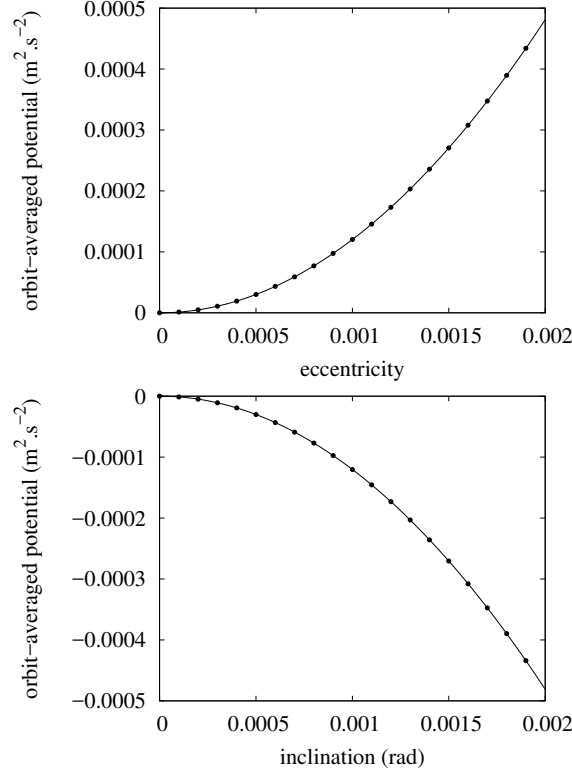


Figure 6: The disturbing function (or equivalently, the gravitational potential averaged on one orbital period along a fixed ellipse) of Jupiter (subscript $j = 1$) due to the MD as a function of the eccentricity (top panel) and of the inclination (bottom panel). The dots are the values for which we have computed the averaged gravitational potential. The curves are the best fit functions of the form $R_1 MD = \alpha e^2$ (top panel) and $R_1 MD = \beta I^2$ (bottom panel), where α and β are coefficients.

719 Appendix B

720 Here we describe the terms that we need to add to the user-defined accelera-
721 tion and to the user-defined velocity in order to allow the longitudes of the node
722 of the bodies to evolve as defined by the user in addition of the mutual interac-
723 tions between the bodies. The components of the position vector (x, y, z) and
724 the components of the velocity vector $(\dot{x}, \dot{y}, \dot{z})$ can be expressed as a function of
725 the osculating elements:

$$\begin{aligned}
x &= r \cos \Omega \cos(\omega + f) - r \cos i \sin \Omega \sin(\omega + f) \\
y &= r \sin \Omega \cos(\omega + f) + r \cos i \cos \Omega \sin(\omega + f) \\
z &= r \sin i \sin(\omega + f) \\
\dot{x} &= \cos \Omega \left(\dot{r} \cos(\omega + f) - r \dot{f} \sin(\omega + f) \right) \\
&\quad - \sin \Omega \left(\dot{r} \cos i \sin(\omega + f) + r \dot{f} \cos i \cos(\omega + f) \right) \\
\dot{y} &= \sin \Omega \left(\dot{r} \cos(\omega + f) - r \dot{f} \sin(\omega + f) \right) \\
&\quad + \cos \Omega \left(\dot{r} \cos i \sin(\omega + f) + r \dot{f} \cos i \cos(\omega + f) \right) \\
\dot{z} &= \dot{r} \sin i \sin(\omega + f) + r \dot{f} \sin i \cos(\omega + f),
\end{aligned} \tag{25}$$

where r is the heliocentric distance, f the true anomaly, i the inclination, ω the argument of the perihelion and Ω the longitude of the ascending node. Using the notations and the method of [Lee and Peale \(2002\)](#), we can obtain the terms needed to update the acceleration and the velocity:

$$\begin{aligned}
\left. \frac{dx}{dt} \right|_{\dot{\Omega}} &= \frac{\partial x}{\partial \Omega} \dot{\Omega} \\
&= (-r \sin \Omega \cos(\omega + f) - r \cos i \cos \Omega \sin(\omega + f)) \dot{\Omega} \\
&= -y \dot{\Omega}
\end{aligned} \tag{26}$$

$$\begin{aligned}
\left. \frac{dy}{dt} \right|_{\dot{\Omega}} &= \frac{\partial y}{\partial \Omega} \dot{\Omega} \\
&= (r \cos \Omega \cos(\omega + f) - r \cos i \sin \Omega \sin(\omega + f)) \dot{\Omega} \\
&= x \dot{\Omega}
\end{aligned} \tag{27}$$

$$\begin{aligned}
\left. \frac{dz}{dt} \right|_{\dot{\Omega}} &= \frac{\partial z}{\partial \Omega} \dot{\Omega} \\
&= 0
\end{aligned} \tag{28}$$

$$\begin{aligned}
\left. \frac{d\dot{x}}{dt} \right|_{\dot{\Omega}} &= \frac{\partial \dot{x}}{\partial \Omega} \dot{\Omega} \\
&= \left[-\sin \Omega \left(\dot{r} \cos(\omega + f) - r \dot{f} \sin(\omega + f) \right) \right. \\
&\quad \left. - \cos \Omega \left(\dot{r} \cos i \sin(\omega + f) + r \dot{f} \cos i \cos(\omega + f) \right) \right] \dot{\Omega} \\
&= -\dot{y} \dot{\Omega}
\end{aligned} \tag{29}$$

$$\begin{aligned}
\left. \frac{dy}{dt} \right|_{\dot{\Omega}} &= \frac{\partial \dot{y}}{\partial \Omega} \dot{\Omega} \\
&= \left[\cos \Omega \left(\dot{r} \cos(\omega + f) - r \dot{f} \sin(\omega + f) \right) \right. \\
&\quad \left. - \sin \Omega \left(\dot{r} \cos i \sin(\omega + f) + r \dot{f} \cos i \cos(\omega + f) \right) \right] \dot{\Omega} \\
&= \dot{x} \dot{\Omega}
\end{aligned} \tag{30}$$

$$\begin{aligned}
\left. \frac{dz}{dt} \right|_{\dot{\Omega}} &= \frac{\partial \dot{z}}{\partial \Omega} \dot{\Omega} \\
&= 0
\end{aligned} \tag{31}$$