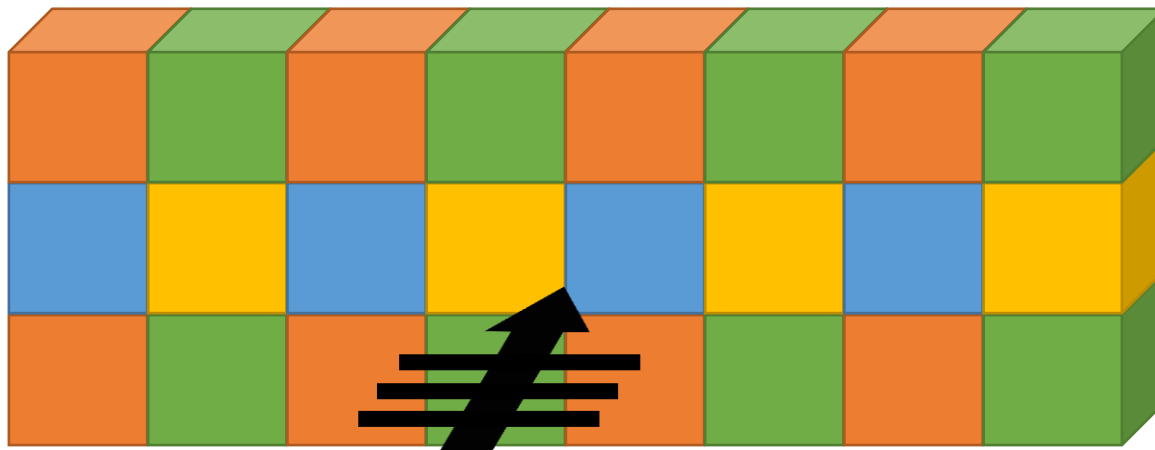


# Recent highlights on the parallel transfer matrix method (PTMM)



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## PRESENTATION OUTLINE

1. Series Transfer Matrix Method (STMM)
2. Parallel Transfer Matrix Method (PTMM)
3. PTMM versus Admittance Sum Method (ASM)
4. PTMM and lateral interactions
5. PTMM versus Mixing Rules Method (MRM)
6. PTMM and diffusion phenomenon
7. Homogeneity (pressure uniformity)
8. Conclusion

STMM

PTMM

PTMM Vs  
ASM

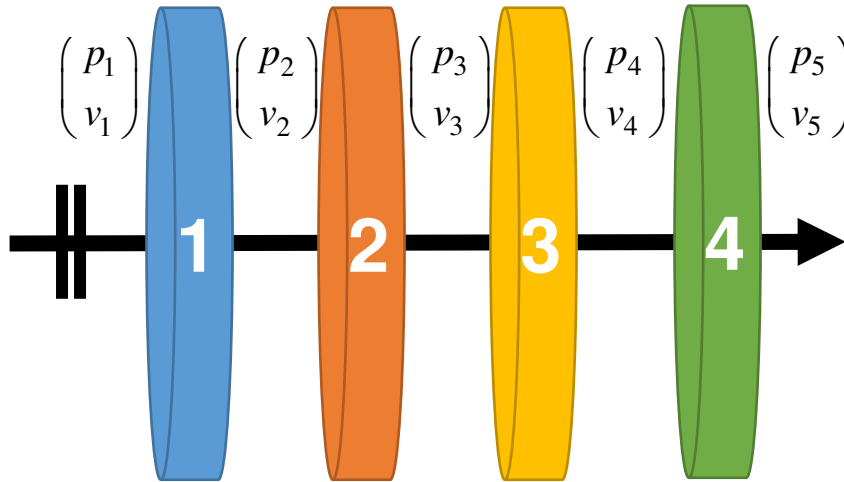
Lateral  
interaction

PTMM vs  
MRM

Diffusion

Homogeneity

Conclusion



$$\begin{pmatrix} p_1 \\ v_1 \end{pmatrix} = \mathbf{T}_1 \begin{pmatrix} p_2 \\ v_2 \end{pmatrix}$$

$$\begin{pmatrix} p_2 \\ v_2 \end{pmatrix} = \mathbf{T}_2 \begin{pmatrix} p_3 \\ v_3 \end{pmatrix}$$

$$\begin{pmatrix} p_3 \\ v_3 \end{pmatrix} = \mathbf{T}_3 \begin{pmatrix} p_4 \\ v_4 \end{pmatrix}$$

$$\begin{pmatrix} p_4 \\ v_4 \end{pmatrix} = \mathbf{T}_4 \begin{pmatrix} p_5 \\ v_5 \end{pmatrix}$$

$$\begin{pmatrix} p_1 \\ v_1 \end{pmatrix} = \mathbf{T}_1 \mathbf{T}_2 \mathbf{T}_3 \mathbf{T}_4 \begin{pmatrix} p_5 \\ v_5 \end{pmatrix} = \mathbf{T}_G \begin{pmatrix} p_5 \\ v_5 \end{pmatrix}$$

$\tilde{k}_i$  : Complex wave number of medium  $i$

$\tilde{Z}_i$  : Characteristic impedance of medium  $i$

$$\begin{pmatrix} p_i \\ v_i \end{pmatrix} = \begin{pmatrix} \cos \tilde{k}_i h_i & j \tilde{Z}_i \sin \tilde{k}_i h_i \\ j \frac{1}{\tilde{Z}_i} \sin \tilde{k}_i h_i & \cos \tilde{k}_i h_i \end{pmatrix} \begin{pmatrix} p_{i+1} \\ v_{i+1} \end{pmatrix} = \mathbf{T}_i \begin{pmatrix} p_{i+1} \\ v_{i+1} \end{pmatrix}$$

Absorption Coefficient  
Transmission Loss

STMM

**PTMM**

PTMM Vs  
ASM

Lateral  
interaction

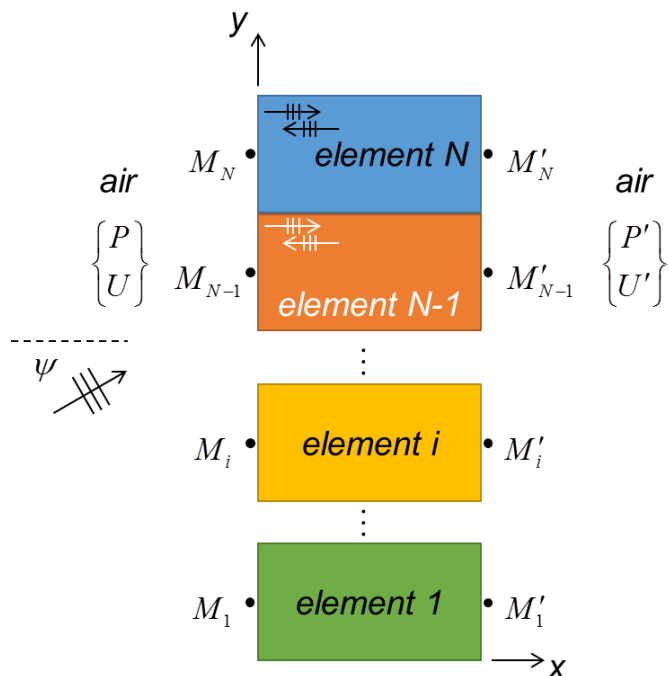
PTMM vs  
MRM

Diffusion

Homogeneity

Conclusion

# Patchwork



key assumptions :

- 1) only plane waves propagate upstream and downstream the construction;
- 2) only normal incidence plane waves propagate in the construction;
- 3) no exchange exists between adjacent parallel elements;
- 4) the wavelength is much larger than the periodic elementary patchwork;
- 5) each element is represented by a 2x2 transfer matrix.

STMM

**PTMM**

PTMM Vs  
ASM

Lateral  
interaction

PTMM vs  
MRM

Diffusion

Homogeneity

Conclusion

*Transfer matrix of each element i* **1**

$$\begin{Bmatrix} P_i \\ U_i \end{Bmatrix} = \begin{bmatrix} T_{i,11} & T_{i,12} \\ T_{i,21} & T_{i,22} \end{bmatrix} \begin{Bmatrix} P'_i \\ -U'_i \end{Bmatrix}$$



*Admittance matrix of each element i* **2**

$$\begin{Bmatrix} U_i \\ U'_i \end{Bmatrix} = \begin{bmatrix} y_{i,11} & y_{i,12} \\ y_{i,21} & y_{i,22} \end{bmatrix} \begin{Bmatrix} P_i \\ P'_i \end{Bmatrix}$$



**These conditions are the same as**

**the well known admittance sum method.**

**Connexion conditions** **3**

$$\begin{Bmatrix} U \\ U' \end{Bmatrix} = \sum \begin{Bmatrix} r_i U_i \\ r_i U'_i \end{Bmatrix}$$

Continuity of upstream flow

Continuity of downstream flow

$$\begin{Bmatrix} P \\ P' \end{Bmatrix} = \begin{Bmatrix} P_i \\ P'_i \end{Bmatrix}$$

Uniformity of upstream pressure

Uniformity of downstream pressure for open-end cells

**4**

$$\begin{Bmatrix} r_i U_i \\ r_i U'_i \end{Bmatrix} = r_i \begin{bmatrix} y_{i,11} & y_{i,12} \\ y_{i,21} & y_{i,22} \end{bmatrix} \begin{Bmatrix} P_i \\ P'_i \end{Bmatrix}$$



**Admittance matrix for the parallel assembly** **5**

$$\begin{Bmatrix} U \\ U' \end{Bmatrix} = \left( \sum r_i \begin{bmatrix} y_{i,11} & y_{i,12} \\ y_{i,21} & y_{i,22} \end{bmatrix} \right) \begin{Bmatrix} P \\ P' \end{Bmatrix}$$

STMM

**PTMM**

PTMM Vs  
ASM

Lateral  
interaction

PTMM vs  
MRM

Diffusion

Homogeneity

Conclusion

## 6 Transfer matrix for the parallel assembly with open-ended elements (*i*)

$$\begin{Bmatrix} P \\ U \end{Bmatrix} = \mathbf{T}_p \begin{Bmatrix} P' \\ U' \end{Bmatrix} = \frac{-1}{\sum r_i y_{i,21}} \begin{pmatrix} \sum r_i y_{i,22} & -1 \\ \sum r_i y_{i,22} \sum r_i y_{i,11} - \sum r_i y_{i,12} \sum r_i y_{i,21} & -\sum r_i y_{i,11} \end{pmatrix} \begin{Bmatrix} P' \\ U' \end{Bmatrix}$$

## 6 Transfer matrix for the parallel assembly with open-ended elements (*j*) and close-ended elements (*k*)

$$\mathbf{T}_p = \frac{-1}{\sum r_j y_{j,21}} \begin{pmatrix} \sum r_j y_{j,22} & -1 \\ \sum r_j y_{j,22} \left( \sum r_i y_{i,11} - \sum r_k \frac{y_{k,12} y_{k,21}}{y_{k,22}} \right) - \sum r_j y_{j,12} \sum r_j y_{j,21} & \sum r_k \frac{y_{k,12} y_{k,21}}{y_{k,22}} - \sum r_i y_{i,11} \end{pmatrix}$$

STMM

PTMM

**PTMM Vs ASM**

Lateral interaction

PTMM vs MRM

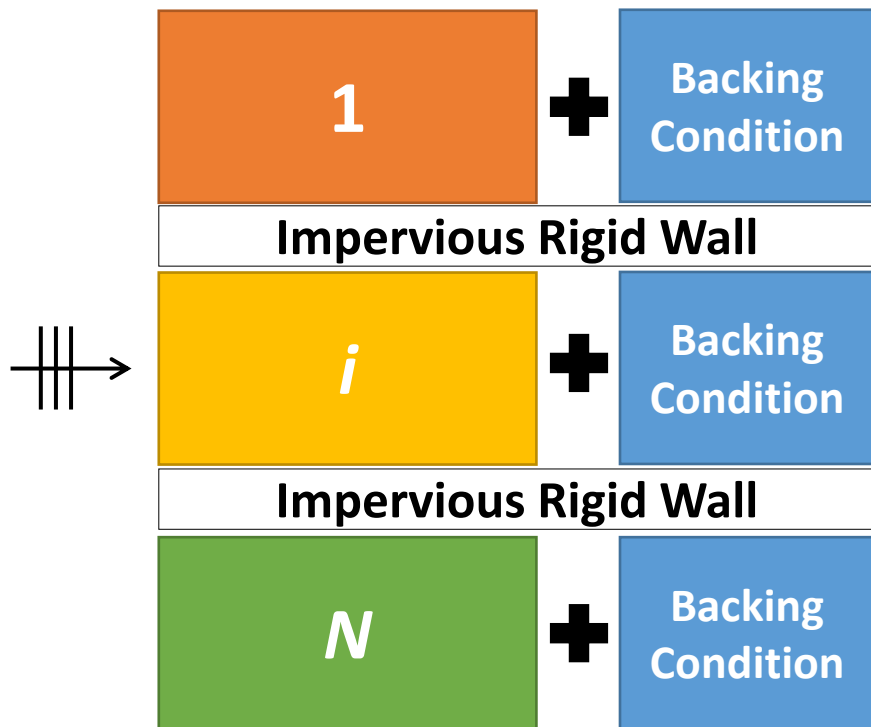
Diffusion

Homogeneity

Conclusion

**Admittance Sum Method**

**How does PTMM compare to ASM ?**



$$Y_G = \frac{1}{Z_s^G} = \sum_i r_i Y_i$$

*C. Zwicker and C. Kosten, Sound Absorbing Materials (Elsevier, New York, 1949), p. 161.*

Backing Condition	Close-ended	Anechoic	Air cavity of depth d
Admittance $Y_i$	$Y_i = \frac{T_{i,21}}{T_{i,11}}$	$Y_i = \frac{T_{i,21} + T_{i,22}/Z_0}{T_{i,11} + T_{i,12}/Z_0}$	$Y_i = \frac{T_{i,22} + jZ_0 \cot(k_0 d)T_{i,21}}{T_{i,12} + jZ_0 \cot(k_0 d)T_{i,11}}$

STMM

PTMM

**PTMM Vs ASM**

Lateral interaction

PTMM vs MRM

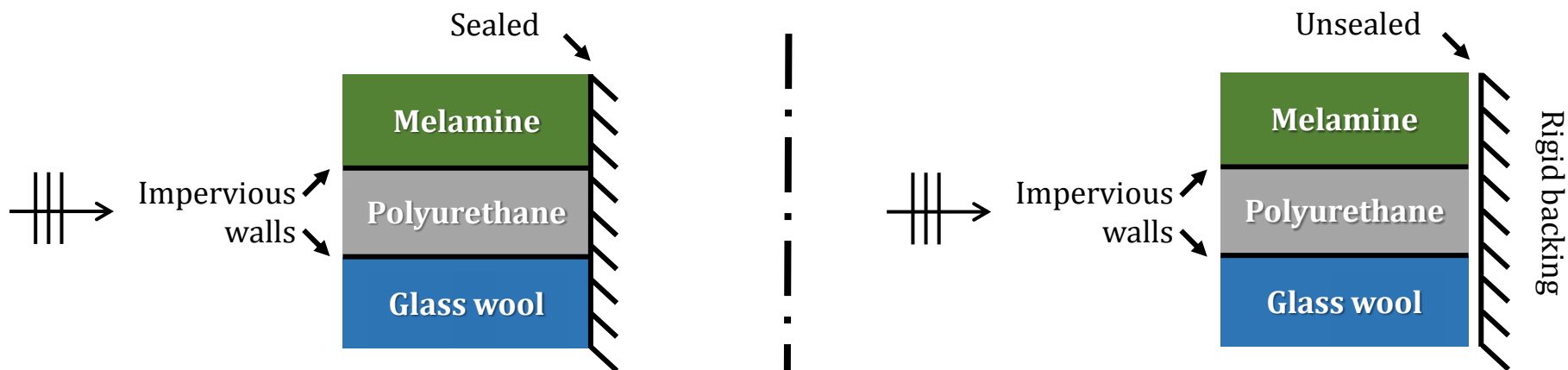
Diffusion

Homogeneity

Conclusion

**ASM**

$$\alpha \leftarrow Z_s \leftarrow \begin{cases} \frac{1}{\frac{1/3}{Z_s^{Melamine}} + \frac{1/3}{Z_s^{Polyurethane}} + \frac{1/3}{Z_s^{Glasswool}}} & \text{for the sealed case} \\ \frac{1}{\frac{1/3}{Z_s^{Melamine+air}} + \frac{1/3}{Z_s^{Polyurethane+air}} + \frac{1/3}{Z_s^{Glasswool+air}}} & \text{for the unsealed case} \end{cases}$$



**PTMM**

$$\alpha \leftarrow Z_s \leftarrow \begin{cases} TM_{PTMM}^{close-ended} & \text{for the sealed case} \\ TM_{PTMM}^{open-ended} TM_{air\ cavity} & \text{for the unsealed case} \end{cases}$$



STMM

PTMM

**PTMM Vs  
ASM**

Lateral  
interaction

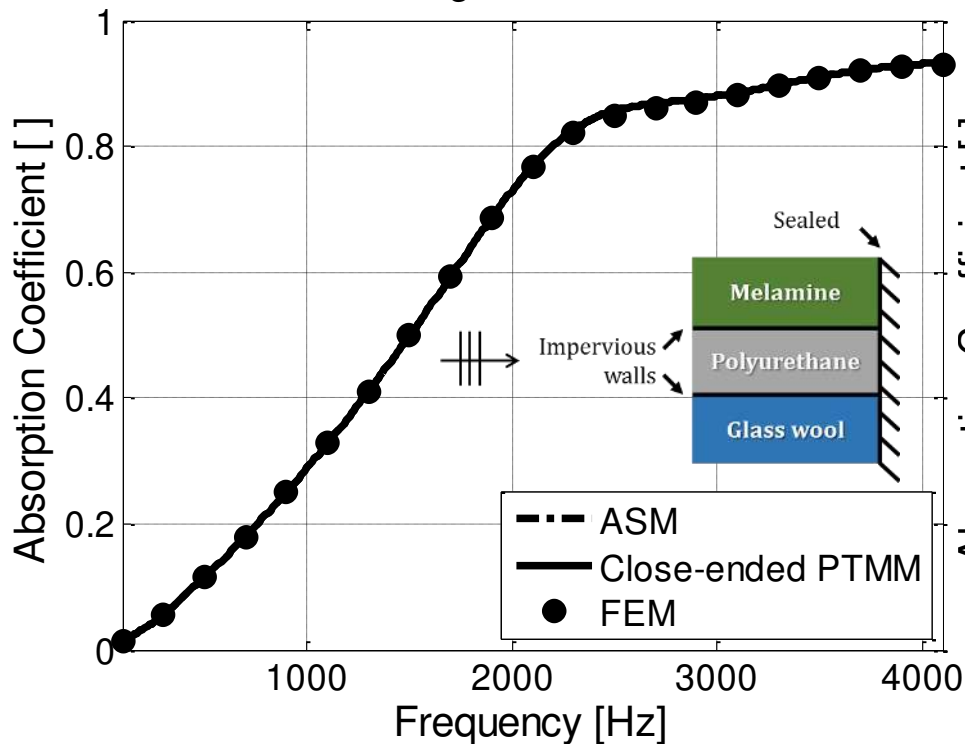
PTMM vs  
MRM

Diffusion

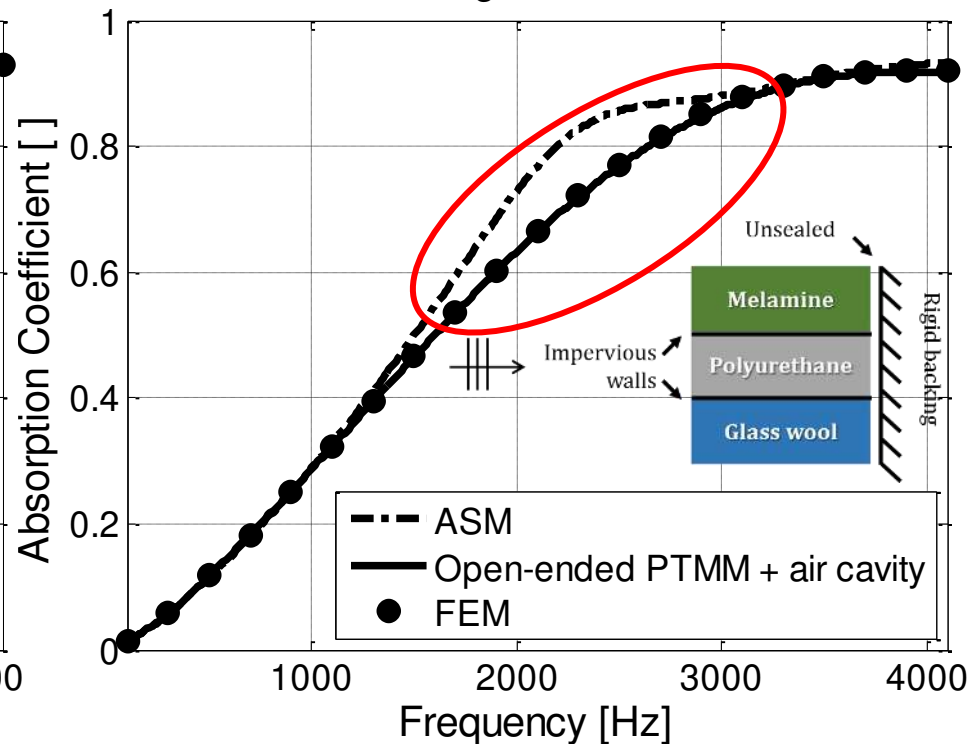
Homogeneity

Conclusion

Sealed rigid back condition



Unsealed rigid back condition



a slight acoustical leak in the parallel construction can change significantly the acoustic response.

STMM

PTMM

PTMM Vs  
ASM

Lateral  
interaction

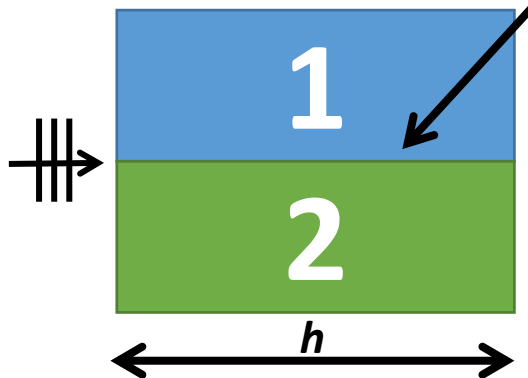
PTMM vs  
MRM

Diffusion

Homogeneity

Conclusion

A means to avoid the no exchange assumption  
No impervious interior wall is needed

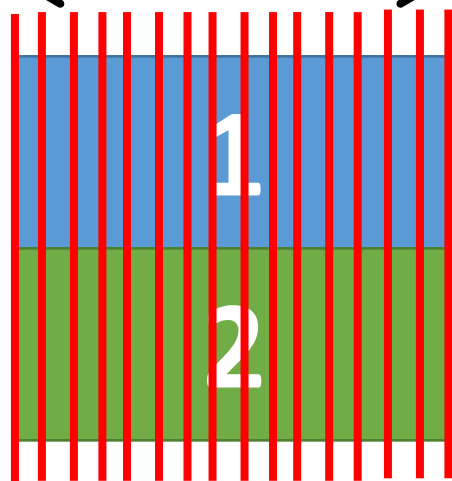


Virtually cut  
into  $n$  slices



the slice  $\delta h$  is expressed  
by a parallel matrix  $\delta T$  by  
classical PTMM

Back to the global patch :  
 $T = \delta T^n$



*As FEM, the number of slices can be chosen when convergence  
for absorption coefficient and transmission loss is obtained.*

Each red line corresponds to flow and pressure balances : the stack is homogenized.

STMM

PTMM

PTMM Vs  
ASM

Lateral  
interaction

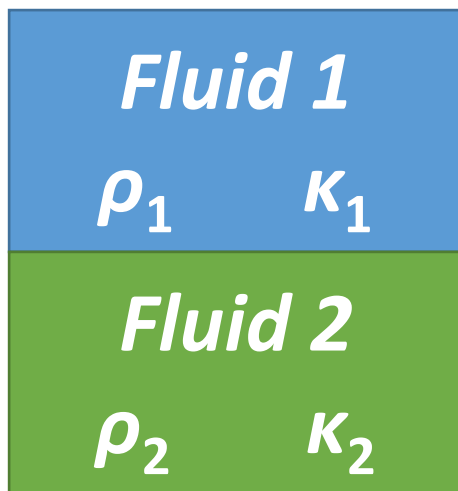
**PTMM vs  
MRM**

Diffusion

Homogeneity

Conclusion

Another method to homogenize ...



$$\rho_{eq} = \frac{1}{\frac{r_1}{\rho_1} + \frac{r_2}{\rho_2}}$$

$$K_{eq} = \frac{1}{\frac{r_1}{K_1} + \frac{r_2}{K_2}}$$

**Mixing Rules Method**

**How does PTMM compare to MRM ?**

generalized equation  
with  $i$  materials



$$\rho_{eq} = \left( \sum_i \frac{r_i}{\rho_i} \right)^{-1}$$

$$K_{eq} = \left( \sum_i \frac{r_i}{K_i} \right)^{-1}$$

STMM

PTMM

PTMM Vs  
ASM

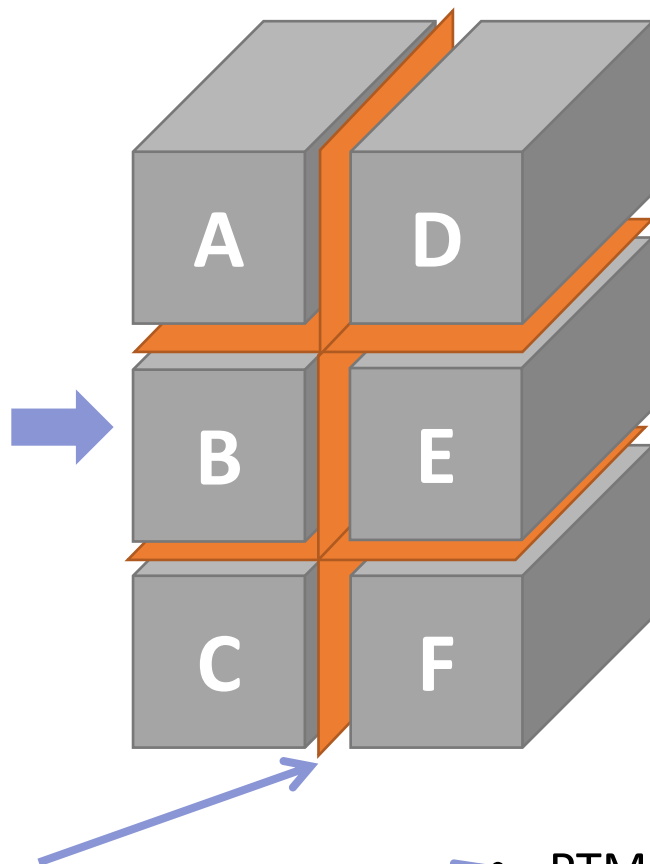
Lateral  
interaction

PTMM vs  
MRM

Diffusion

Homogeneity

Conclusion



2 FEM solutions:

- With interior walls
- Without interior walls

Compared with:

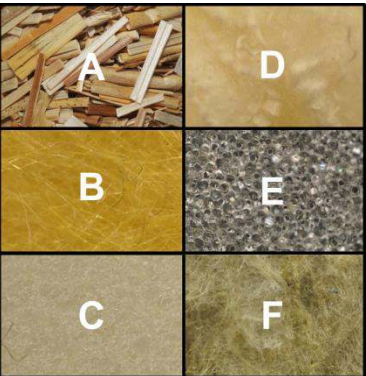
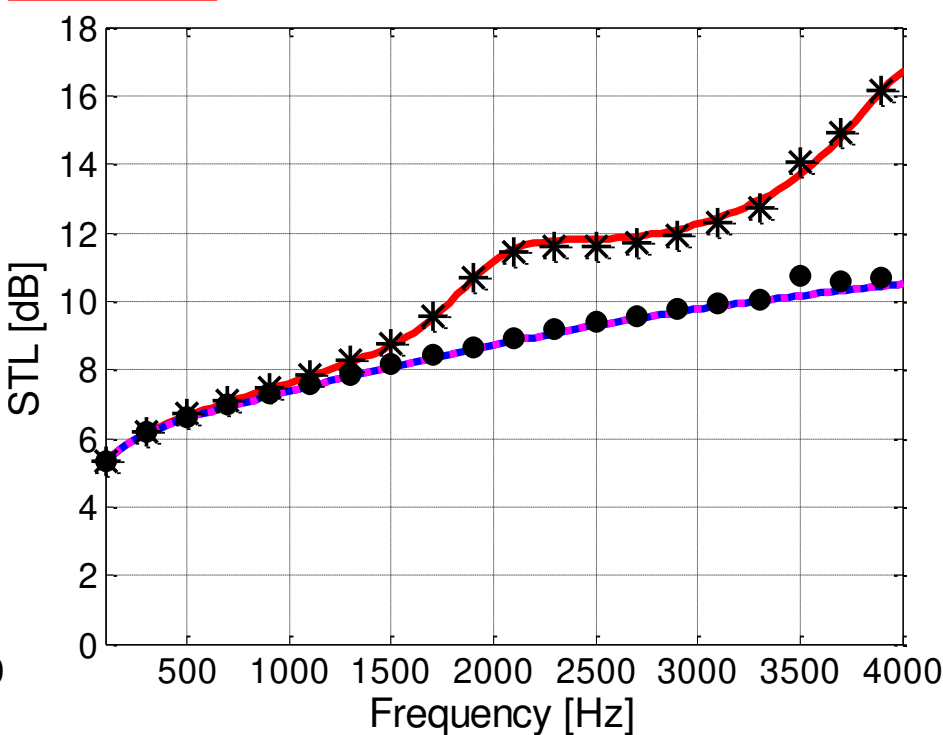
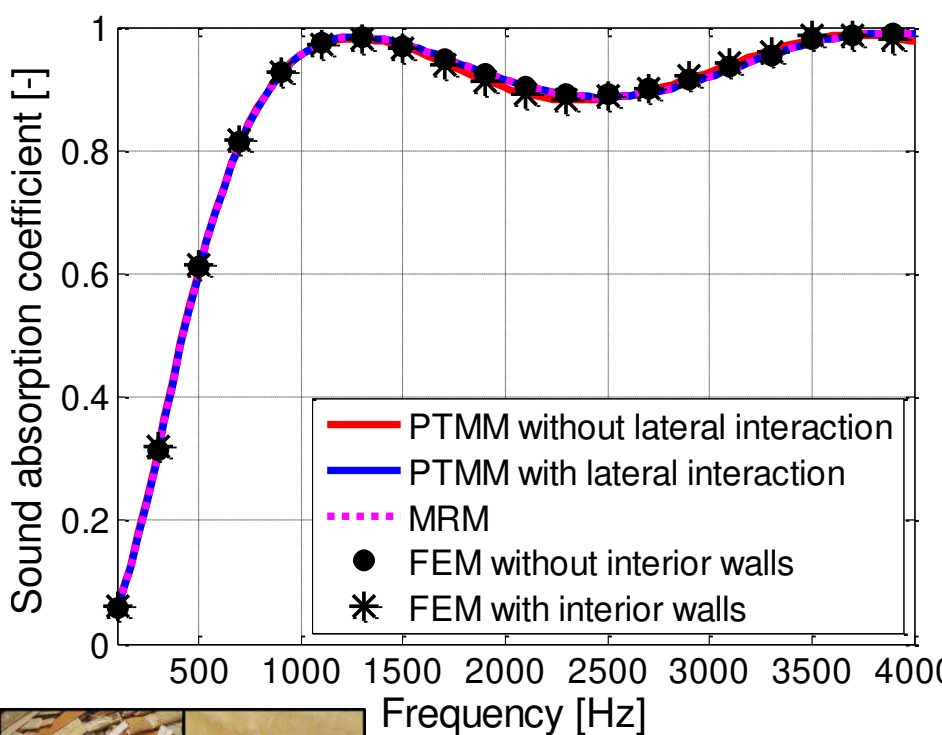
- PTMM
- MRM

With and without  
interior impervious walls

- PTMM without lateral interaction
- PTMM with lateral interaction

A: Switchgrass ; B: Glass wool ; C: Melamine foam ; D: Polyester fibre ; E: Polyurethane foam ; F: Rock wool

STMM    PTMM    PTMM Vs ASM    Lateral interaction    **PTMM vs MRM**    Diffusion    Homogeneity    Conclusion



**Be careful with the use of PTMM.**

**The question to ask:**  
**Does it exist impervious walls between each element ?**

A: Switchgrass ; B: Glass wool ; C: Melamine foam ; D: Polyester fibre ; E: Polyurethane foam ; F: Rock wool

STMM

PTMM

PTMM Vs  
ASM

Lateral  
interaction

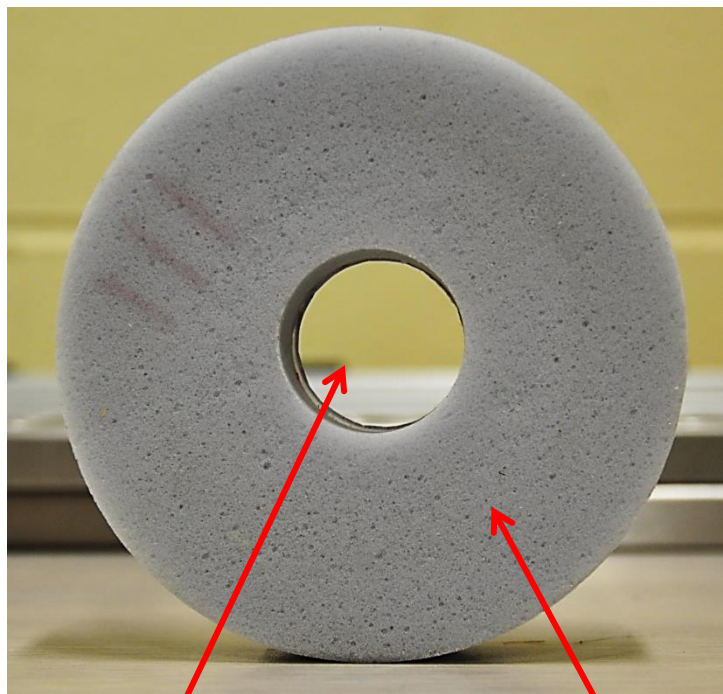
PTMM vs  
MRM

Diffusion

Homogeneity

Conclusion

## A double porosity medium



Macro porosity ( $p$ )      Micro porosity ( $m$ )

The acoustic model of this assembly depends on the contrast between the macroporous and the microporous phases.

For a low contrast → no diffusion exists and MRM is used.

For a high contrast → diffusion phenomenon appears and a modified version of MRM is used.

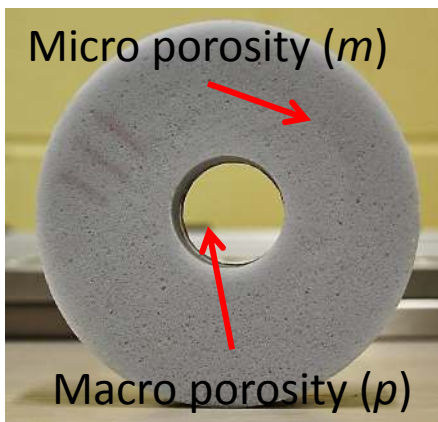
$$\kappa_{eq} = \left( \frac{r_p}{\kappa_p} + \frac{(1-r_p)F_d}{\kappa_m} \right)^{-1}$$

$$\rho_{eq} = \left( \frac{r_p}{\rho_p} + \frac{1-r_p}{\rho_m} \right)^{-1}$$

$F_d$  is a function which depends on the form of the bilayer, the contact shape between the two phases and their porous properties.

X. Olny and C. Boutin, "Acoustic wave propagation in double porosity media," J. Acoust. Soc. Am. 114, July 2003, pp 73-89

**How can diffusion phenomenon be taken into account with PTMM ?**



$$\kappa_{eq} = \left( \frac{r_p}{\kappa_p} + \frac{(1-r_p)F_d}{\kappa_m} \right)^{-1}$$

$$\rho_{eq} = \left( \frac{r_p}{\rho_p} + \frac{1-r_p}{\rho_m} \right)^{-1}$$

$$\kappa_{m,eq} = \frac{\kappa_m}{F_d}$$

$$T_m = \begin{bmatrix} \cos\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}}\right) & j\sqrt{\rho_m\kappa_m}\sin\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}}\right) \\ j\sin\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}}\right)/\sqrt{\rho_m\kappa_m} & \cos\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}}\right) \end{bmatrix} \rightarrow T_m^{diff} = \begin{bmatrix} \cos\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}F_d}\right) & j\sqrt{\frac{\rho_m\kappa_m}{F_d}}\sin\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}F_d}\right) \\ j\sin\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}F_d}\right)/\sqrt{\frac{\rho_m\kappa_m}{F_d}} & \cos\left(\omega\sqrt{\frac{\rho_m}{\kappa_m}F_d}\right) \end{bmatrix}$$

$$T_p = \begin{bmatrix} \cos\left(\omega\sqrt{\frac{\rho_p}{\kappa_p}}\right) & j\sqrt{\rho_p\kappa_p}\sin\left(\omega\sqrt{\frac{\rho_p}{\kappa_p}}\right) \\ j\sin\left(\omega\sqrt{\frac{\rho_p}{\kappa_p}}\right)/\sqrt{\rho_p\kappa_p} & \cos\left(\omega\sqrt{\frac{\rho_p}{\kappa_p}}\right) \end{bmatrix} \rightarrow \text{The macro porous transfer matrix is unchanged.}$$

STMM

PTMM

PTMM Vs  
ASM

Lateral  
interaction

PTMM vs  
MRM

Diffusion

Homogeneity

Conclusion

$$T_m^{diff} = \begin{bmatrix} \cos\left(\omega \sqrt{\frac{\rho_m}{\kappa_m}} F_d\right) & j \sqrt{\frac{\rho_m \kappa_m}{F_d}} \sin\left(\omega \sqrt{\frac{\rho_m}{\kappa_m}} F_d\right) \\ j \sin\left(\omega \sqrt{\frac{\rho_m}{\kappa_m}} F_d\right) / \sqrt{\frac{\rho_m \kappa_m}{F_d}} & \cos\left(\omega \sqrt{\frac{\rho_m}{\kappa_m}} F_d\right) \end{bmatrix}$$

$$T_p = \begin{bmatrix} \cos\left(\omega \sqrt{\frac{\rho_p}{\kappa_p}}\right) & j \sqrt{\rho_p \kappa_p} \sin\left(\omega \sqrt{\frac{\rho_p}{\kappa_p}}\right) \\ j \sin\left(\omega \sqrt{\frac{\rho_p}{\kappa_p}}\right) / \sqrt{\rho_p \kappa_p} & \cos\left(\omega \sqrt{\frac{\rho_p}{\kappa_p}}\right) \end{bmatrix}$$

Finally, both matrices are combined in parallel with PTMM.



STMM

PTMM

PTMM Vs  
ASM

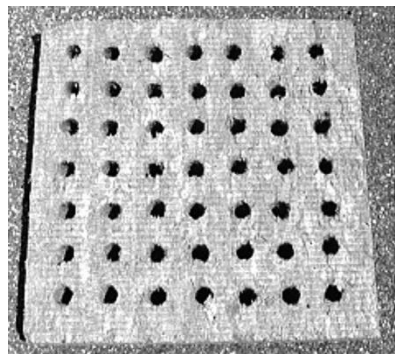
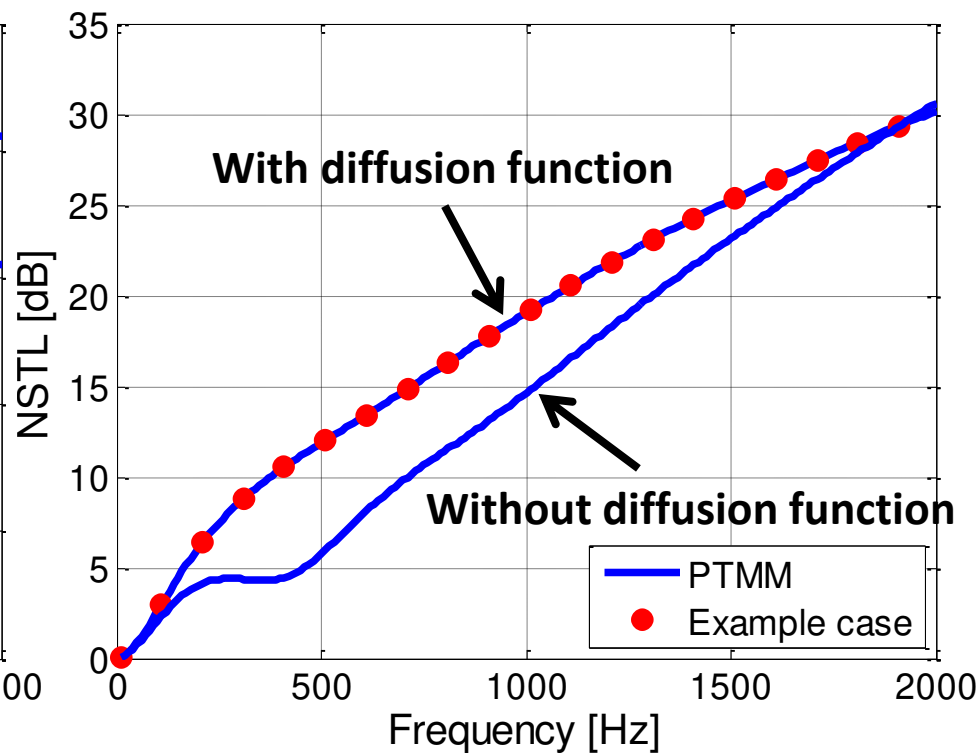
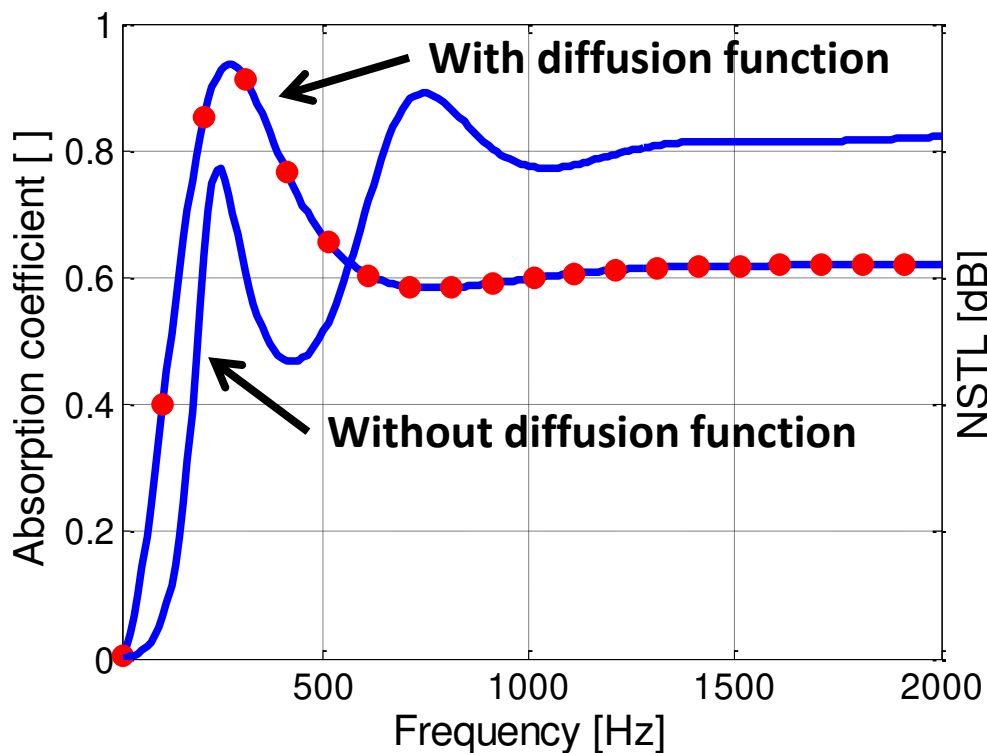
Lateral  
interaction

PTMM vs  
MRM

**Diffusion**

Homogeneity

Conclusion



Example case extracts from a paper of F. Sgard, X. Olny, N. Atalla and F. Castel.  
 "On the use of perforations to improve the sound absorption of porous materials",  
 Applied Acoustic (66) 2005, pp 625-651

STMM

PTMM

PTMM Vs  
ASM

Lateral  
interaction

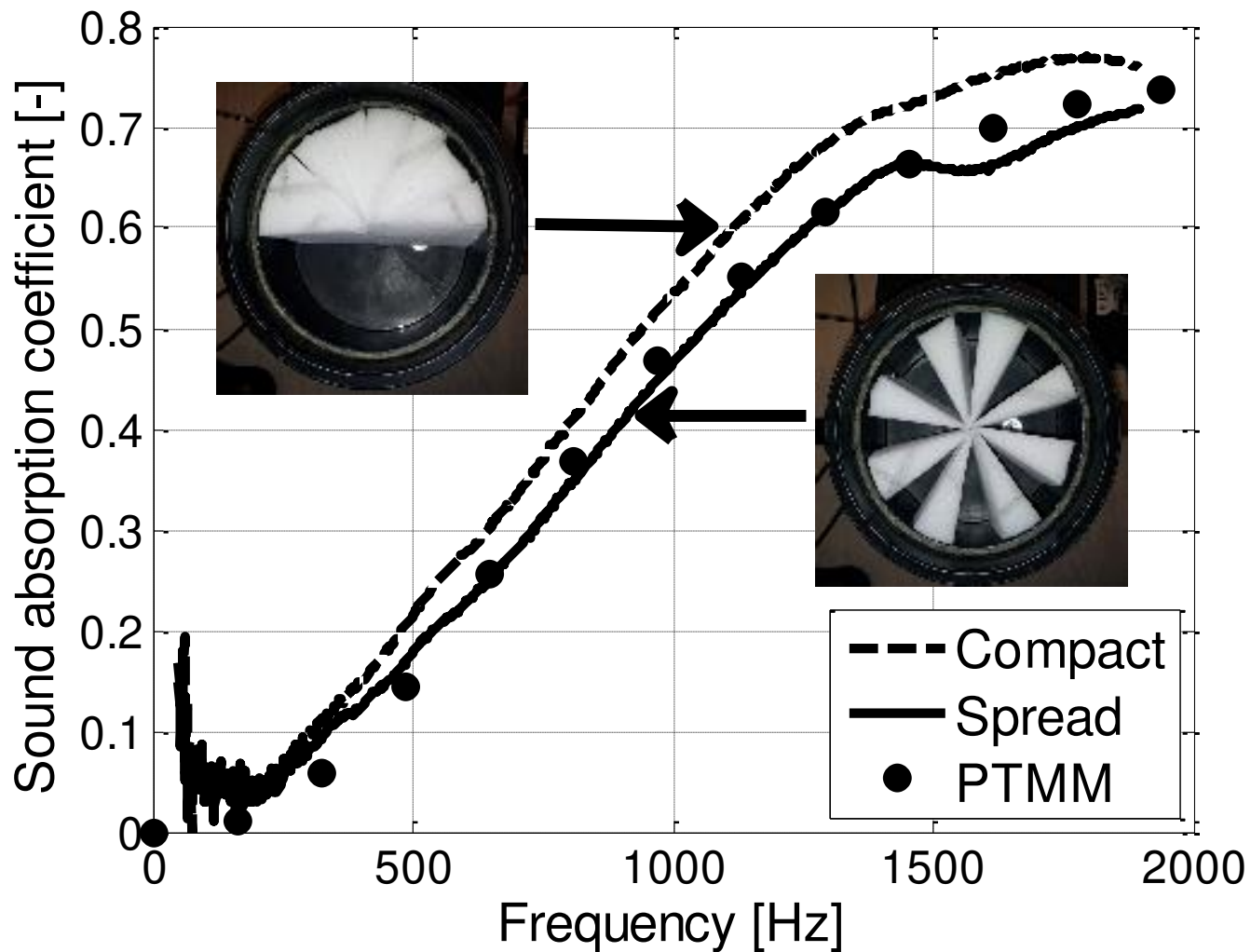
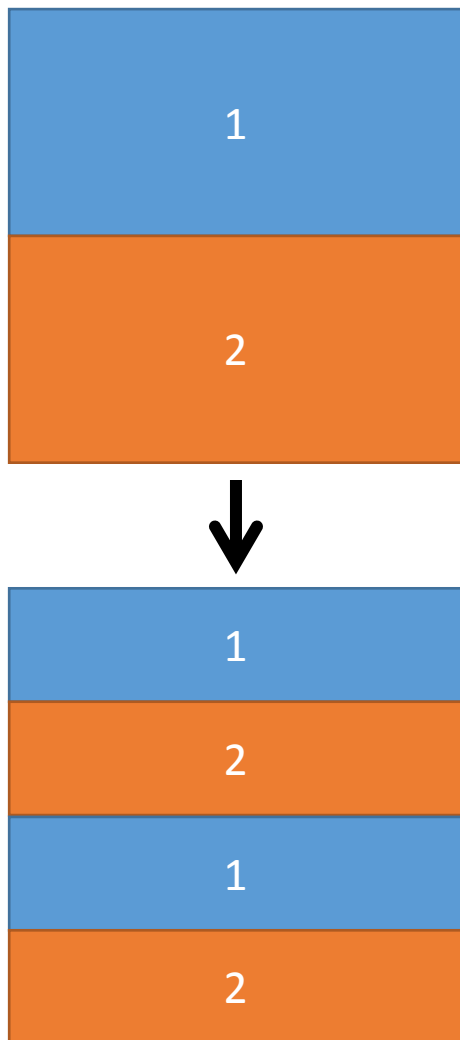
PTMM vs  
MRM

Diffusion

Homogeneity

Conclusion

**Pressure uniformity**



STMM

PTMM

PTMM Vs  
ASM

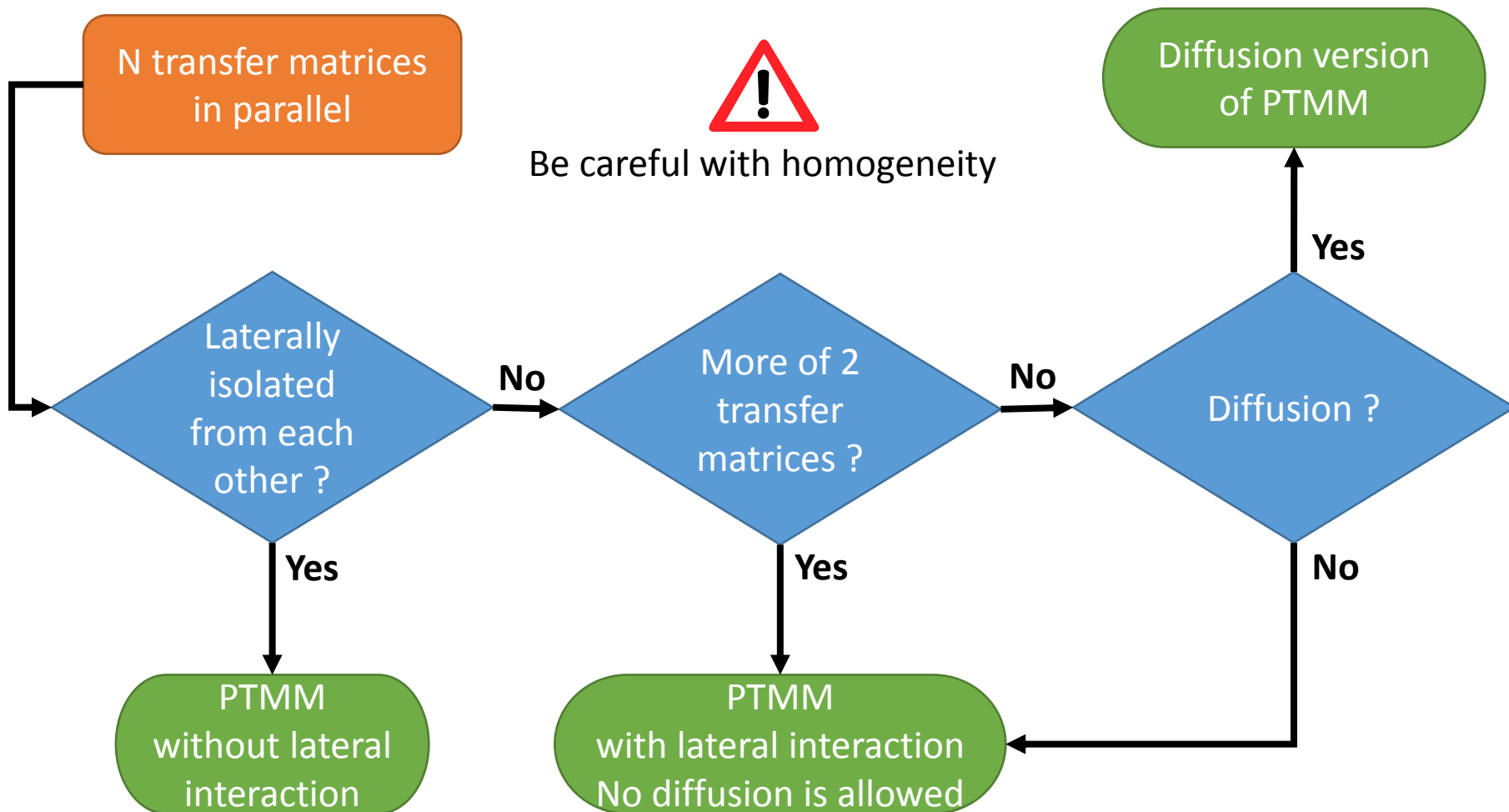
Lateral  
interaction

PTMM vs  
MRM

Diffusion

Homogeneity

Conclusion



STMM

PTMM

PTMM Vs  
ASM

Lateral  
interaction

PTMM vs  
MRM

Diffusion

Homogeneity

Conclusion

- Clarification on the use of PTMM has been given.
- The no exchange assumption can be avoid by using the discretized version of PTMM.
- Diffusion phenomenon can be taken into account by a modified transfer matrix and the discretized version of PTMM.
- The question of homogeneity has been raised:
  - To assure pressure uniformity as much as possible, the constituents have to be spread into the assembly.
- These discrepancies between experimental and theory could come from the uniform pressure assumption.
  - A new formulation without this assumption has to be written (Future Work).

STMM

PTMM

PTMM Vs  
ASM

Lateral  
interaction

PTMM vs  
MRM

Diffusion

Homogeneity

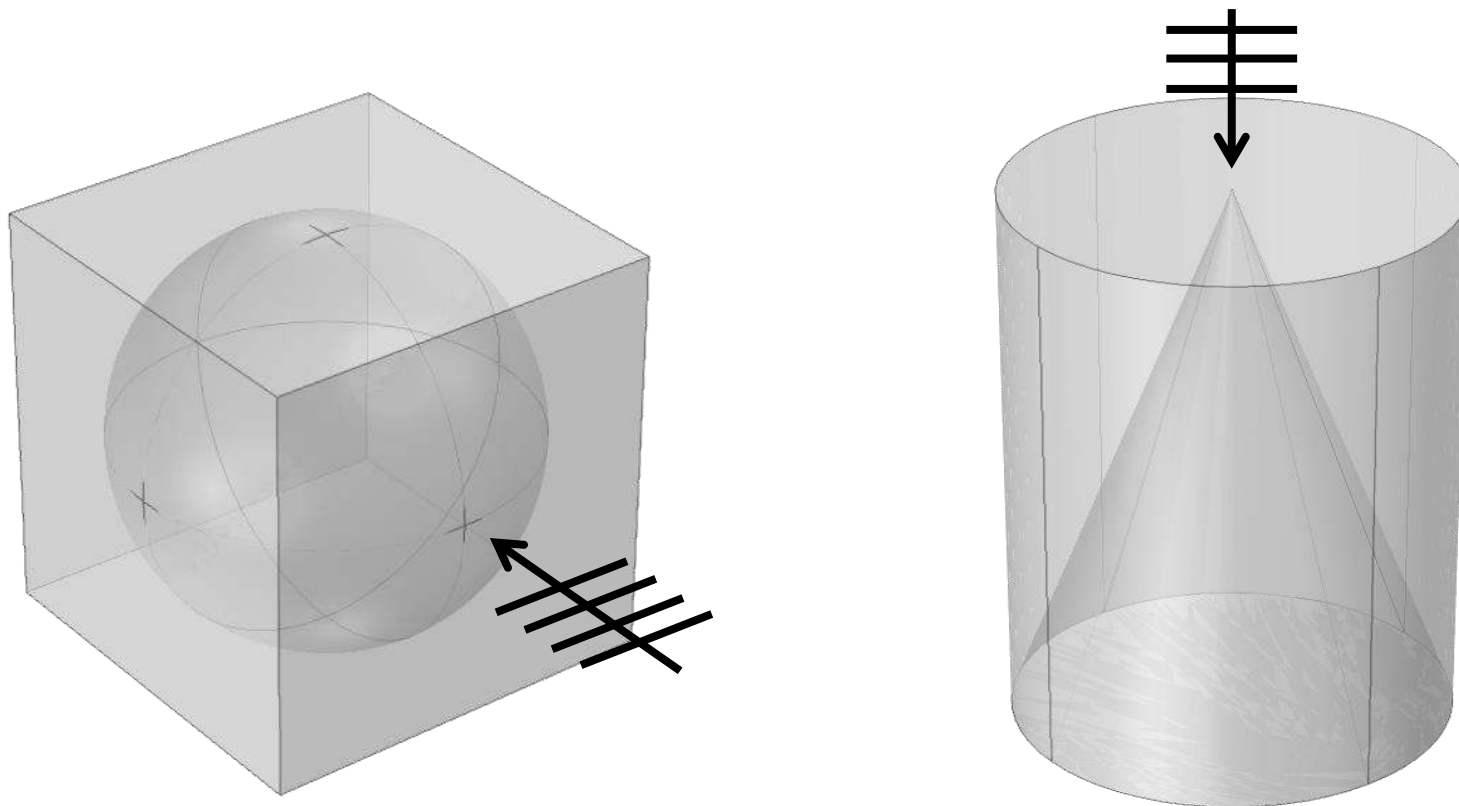
Conclusion

# Thank you!

# Questions ?

**If MRM and PTMM give the same results,  
what are the advantages to use PTMM rather than MRM ?**

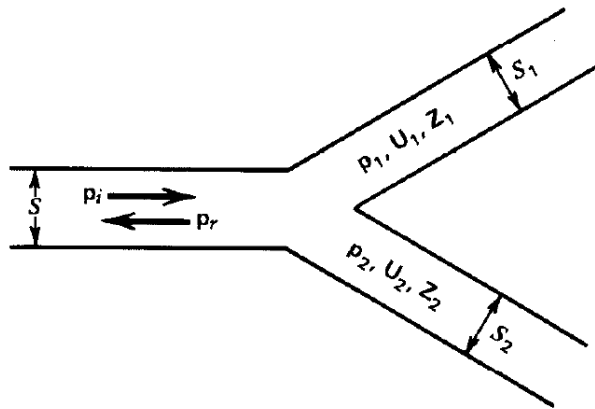
**Extra**



PTMM is used when the distribution of each material is not constant along the thickness of the construction. As PTMM is based on a discretization, only a distribution profile of each material is needed.

## pipe that branches into two pipes

Extra



**Figure 10.10.2** Conditions in the vicinity of a branch. The branches have cross-sectional areas  $S_1$  and  $S_2$  and input acoustic impedances  $z_1$  and  $z_2$ .

As a second example, consider a pipe that branches into two pipes each with arbitrary input impedance, as indicated in Fig. 10.10.2. If the junction is at  $x = 0$ , the pressures in the three pipes very close to  $x = 0$  are

$$\begin{aligned} p_i &= P_i e^{j\omega t} & p_r &= P_r e^{j\omega t} \\ p_1 &= Z_1 U_1 e^{j\omega t} & p_2 &= Z_2 U_2 e^{j\omega t} \end{aligned} \quad (10.10.9)$$

where  $Z_1$ ,  $Z_2$ , and  $U_1$ ,  $U_2$  are the input impedances and complex volume velocity amplitudes in the two branches. In the long wavelength approximation, continuity of pressure at the junction requires

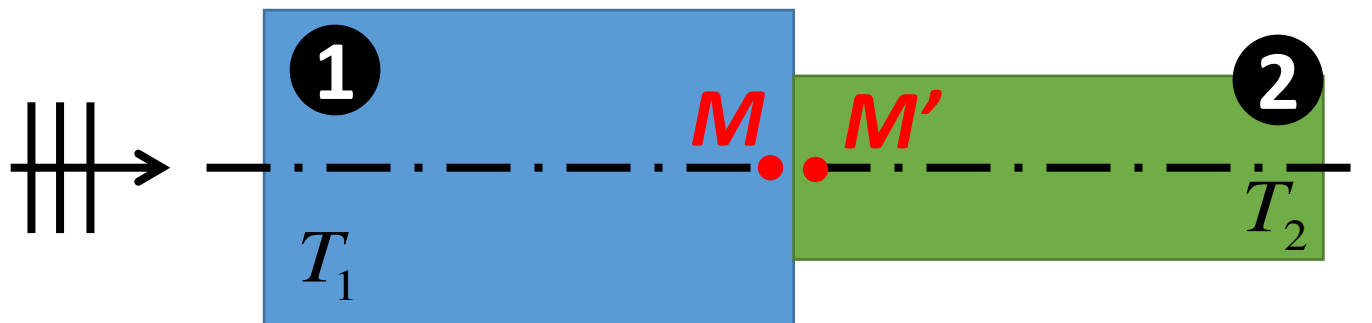
$$p_i + p_r = p_1 = p_2 \quad (10.10.10)$$

and continuity of volume velocity requires

$$U_i + U_r = U_1 + U_2 \quad (10.10.11)$$

**Cross Section Reduction – Expansion Chamber**

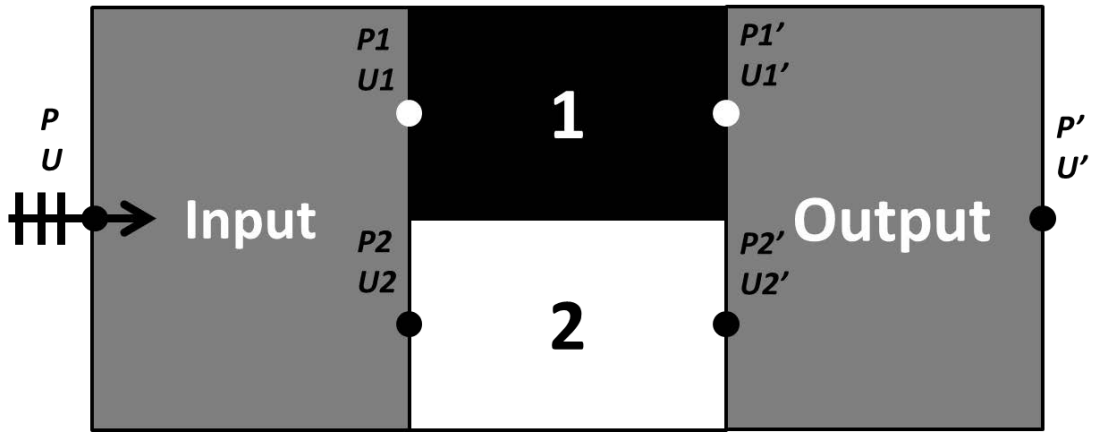
**Extra**



$$\begin{pmatrix} P(M) \\ U(M) \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & R \end{bmatrix} \begin{pmatrix} P(M') \\ U(M') \end{pmatrix}$$



**Future Work**



$$\begin{Bmatrix} P \\ U \end{Bmatrix} = T_{input} T_{assembly} T_{output} \begin{Bmatrix} P' \\ U' \end{Bmatrix}$$

No exchange assumption

Transfer matrix of the first element

$$\begin{Bmatrix} P \\ U \end{Bmatrix} = T_{input} \begin{Bmatrix} P_1 \\ U_1 \\ P_2 \\ U_2 \end{Bmatrix} \quad \left\| \right. \quad \begin{Bmatrix} P'_1 \\ U'_1 \\ P'_2 \\ U'_2 \end{Bmatrix} = T_{output} \begin{Bmatrix} P' \\ U' \end{Bmatrix} \quad \left\| \right. \quad \begin{Bmatrix} P_1 \\ U_1 \\ P_2 \\ U_2 \end{Bmatrix} = T_{assembly} \begin{Bmatrix} P'_1 \\ U'_1 \\ P'_2 \\ U'_2 \end{Bmatrix} \quad \text{with } T_{assembly} = \begin{bmatrix} T_{11,1} & T_{12,1} & 0 & 0 \\ T_{21,1} & T_{22,1} & 0 & 0 \\ 0 & 0 & T_{11,2} & T_{12,2} \\ 0 & 0 & T_{21,2} & T_{22,2} \end{bmatrix}$$

STMM

PTMM

PTMM Vs  
ASM

Lateral  
interaction

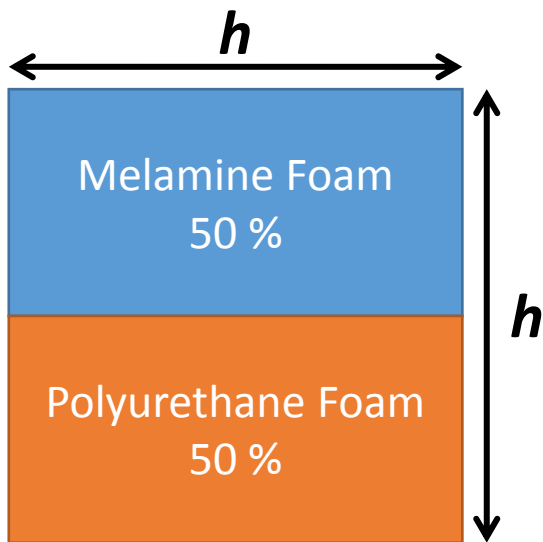
PTMM vs  
MRM

Diffusion

**Homogeneity**

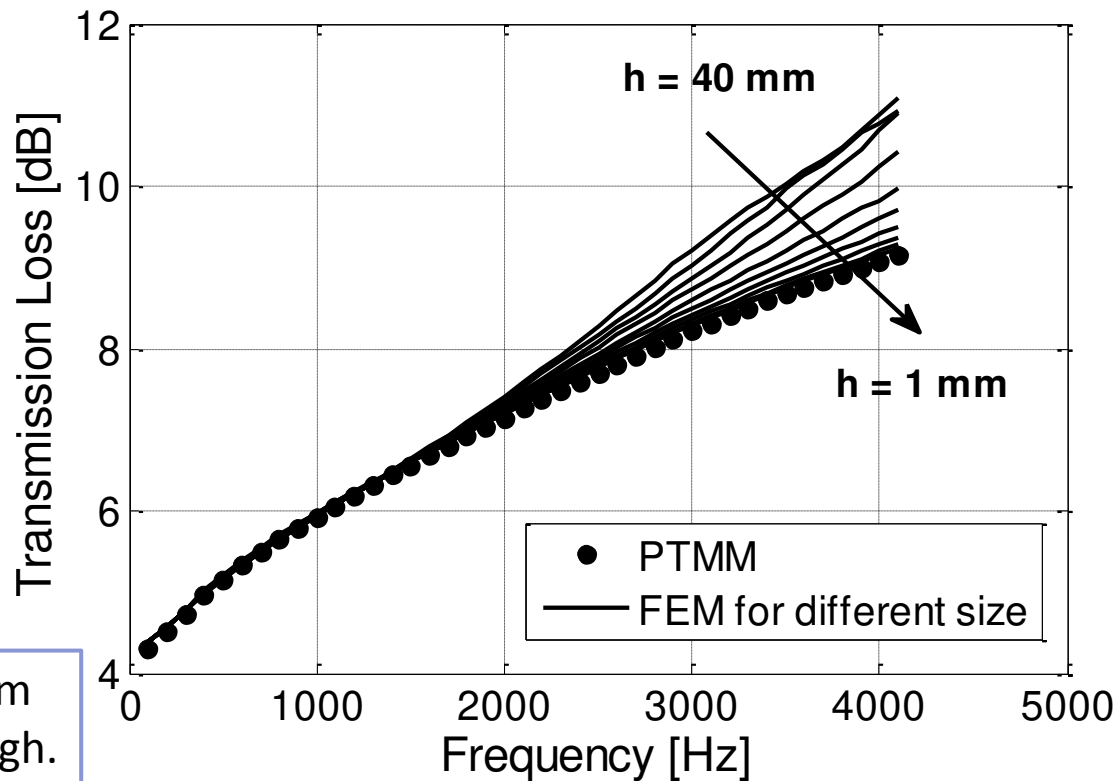
Conclusion

**Effect of the lateral size  $h$**



Thickness: 50 mm  
FEM analysis

The contrast between melamine foam and polyurethane foam is relatively high.



	Porosity	Air resistivity [N.s.m <sup>-4</sup> ]	Tortuosity	Viscous length [μm]	Thermal length [μm]
Melamine foam	0.999	9724	1	110	122
Polyurethane foam	0.958	11188	1.94	70	209

STMM

PTMM

PTMM Vs  
ASM

Lateral  
interaction

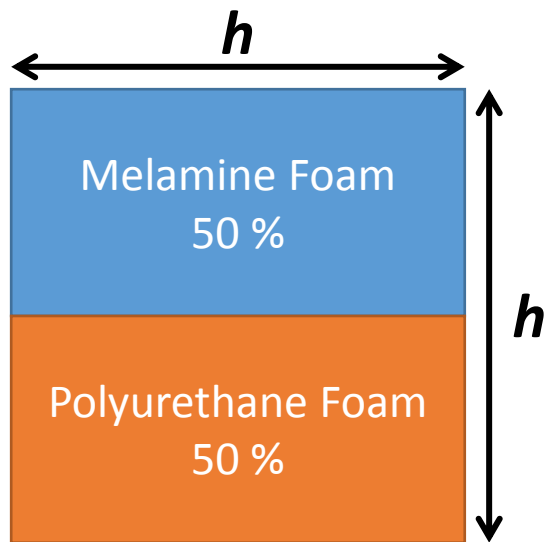
PTMM vs  
MRM

Diffusion

**Homogeneity**

Conclusion

**Effect of the lateral size  $h$**



Thickness: 50 mm  
FEM analysis

$$\varepsilon = \frac{|TL_{PTMM} - TL_{FEM}|}{TL_{FEM}} \cdot 100\%$$

