

Sound propagation in narrow tubes with periodically spaced lateral cavities

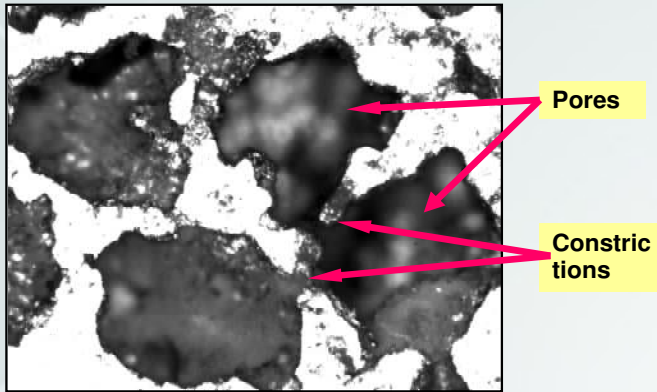
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Outline

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- 2. Structured (periodically distributed) dead-end pores
- 3. Models
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- 4. LF asymptotic behaviour – Long dead-ends
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1. Dead-end porosity materials (e.g. low porosity materials)



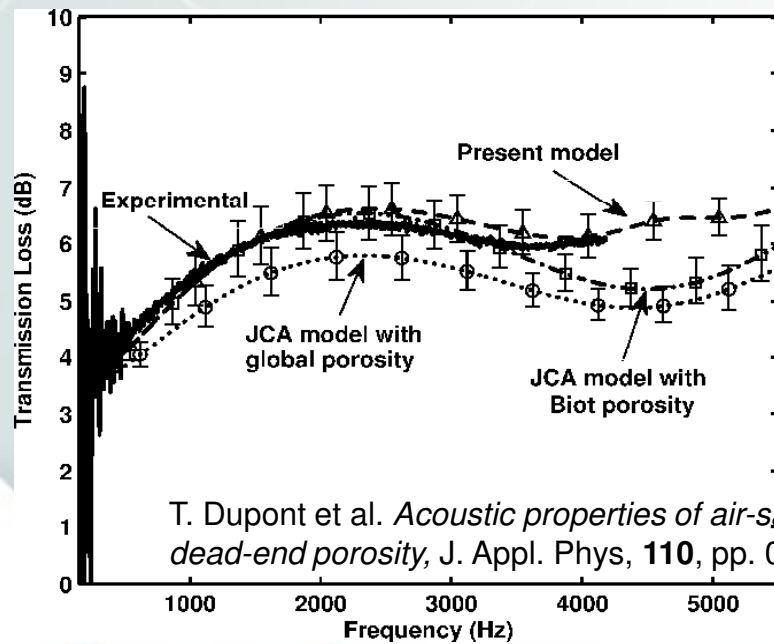
Metallic foam



Concrete with hemp particles



Sunflower crushed Stams

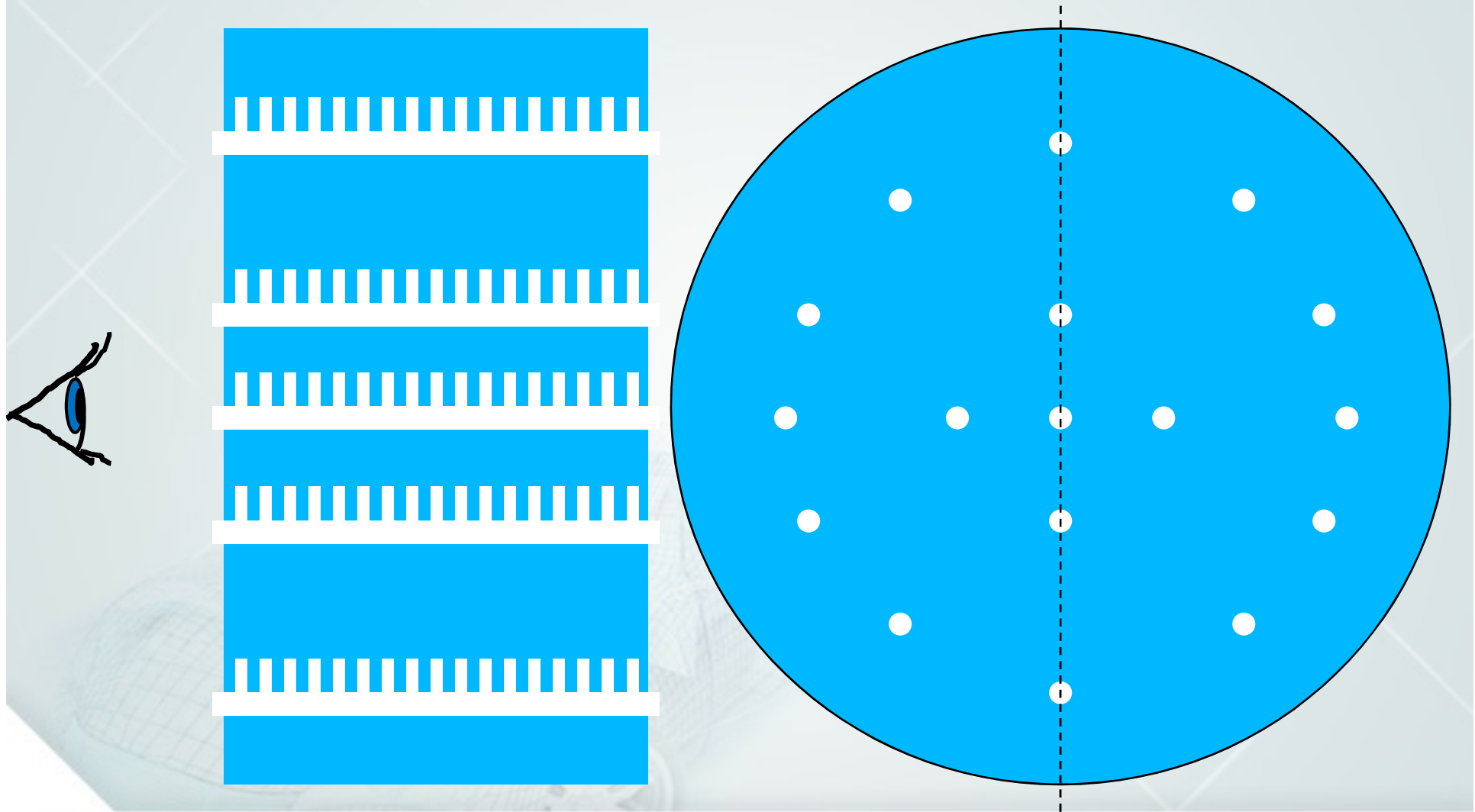


| Matériau | A (μm) | A' (μm) | α_∞ | σ (Pa.s/m ²) | ϕ (%) |
|----------|--------------------------|---------------------------|--------------------|------------------------------------|-----------------|
| A | 900 | 1000 | 1 | 1050 | 27.5 |
| B | 101 ± 4 | 352 ± 14 | 2.25 ± 0.05 | 19713 ± 300 | 64.5 ± 3 |

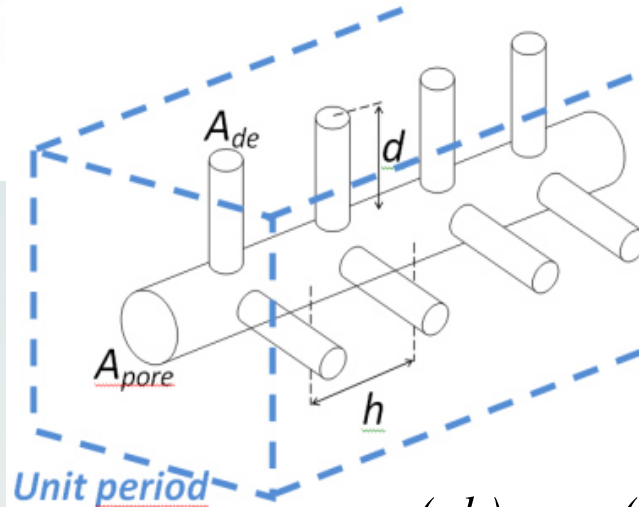
| Matériau | ϕ_B (%) | ϕ_{DE} (%) | l_{DE} (mm) |
|----------|-----------------|--------------------|--------------------------------|
| A | 14 | 13.5 | 25 |
| B | 55 ± 3.2 | 7.5 ± 3.2 | $\approx 7d_{cell}$ Fitting |

$$\bar{Y}(\phi) = \bar{Y}_2(\phi_B) + \bar{Y}_1(\phi_{DE})$$

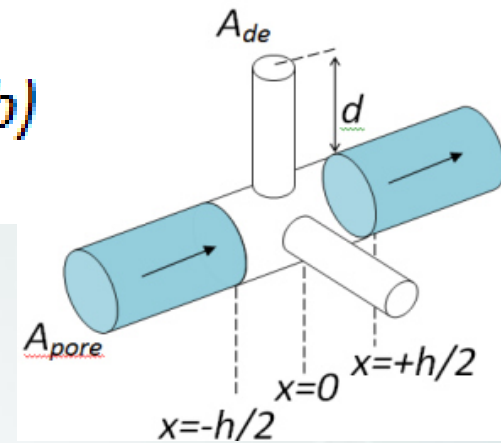
2. Structured (periodically distributed) dead-end pores



a)



b)



$$\cos(qh) = \cos(k_{pore}h) - \frac{X}{i} \sin(k_{pore}h)$$

$\lambda \gg$ pore diameters

→ Plane wave approximation, Fluid in the pore replaced by an “equivalent fluid”

HF - $\lambda < h$

- Periodic dispersion equation $q = q(\omega)$ → Study of stopbands and passbands
- TMM → Absorption coefficient and Transmission Loss

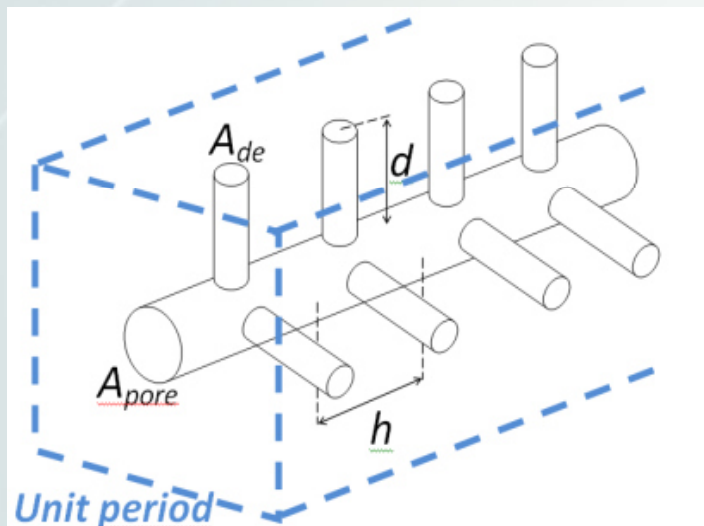
LF - $\lambda \gg h$

- « Wave slowing », « fluid softening », thermal diffusion effects between dead – end and main pores
- Low frequency absorption peaks

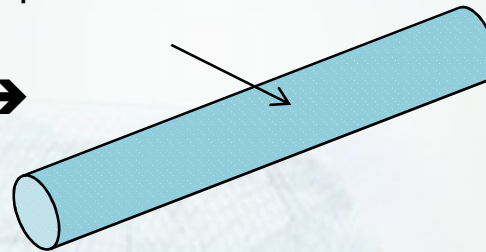
Structure \rightarrow pore filled with a fluid with still unknown effective properties.

“Thought experiment” :

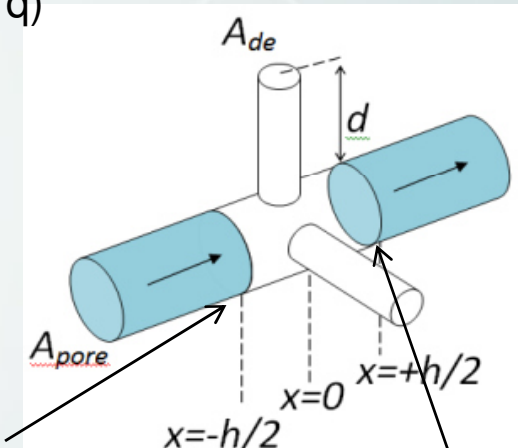
Unit cell inserted into this pore, **not disturbing the properties** of an effective fluid \rightarrow periodicity introduced



Equivalent fluid (wavenumber q)
verifying Bloch dispersion
equation



Zero reflexion
 r



Transmission coefficient
 $t = \exp(iqh)$

This approach allows TMM modeling \rightarrow absorption coefficient
Which is not possible directly from the dispersion equation

3. Models – Periodic dispersion equation

Periodic dispersion equation:

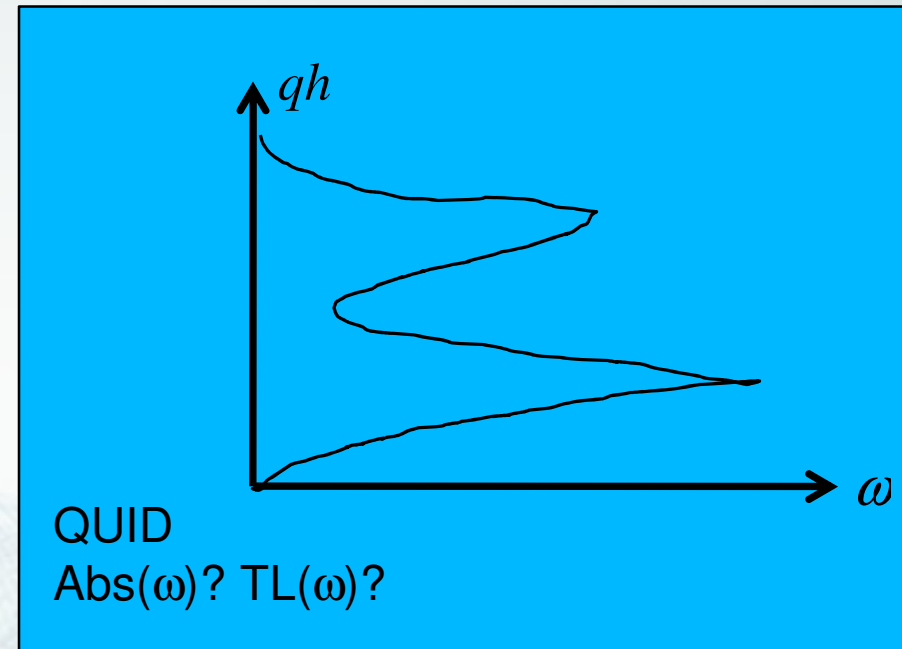
$$\cos(qh) = \cos(k_{pore}h) - \frac{X}{i} \sin(k_{pore}h)$$

$$X = -\frac{N}{2} \frac{A_{de}}{A_{pore}} \frac{1}{Z_{Sde}}$$

For a simple quarter wavelength cavity
(with $\exp(-i\omega t)$ convention):

$$Z_{Sde} = i \frac{Z_{de}}{Z_{pore}} \cotan(k_{de}d)$$

C. E. Bradley, *Time harmonic Bloch wave propagation in periodic waveguides. Part I. Theory*, J. Acoust. Soc. Am. **96**, pp. 1844-1853, 1994.



Including visco-thermal losses the following parameters are given by JCA model:

$$Z_{de}, Z_{pore}, k_{de}$$

D. L. Johnson, J. Koplik and R. Dashen, Theory of dynamic permeability and tortuosity in fluid-saturated porous media, J. Fluid. Mech., 176, pp 379-402, 1987.

Y. Champoux and J. F. Allard, Dynamic tortuosity and bulk modulus in air saturated porous media, J. Appl. Phys., 70, pp 1975, 1991

3. Models – Transfer matrix method

Transfer matrix method:

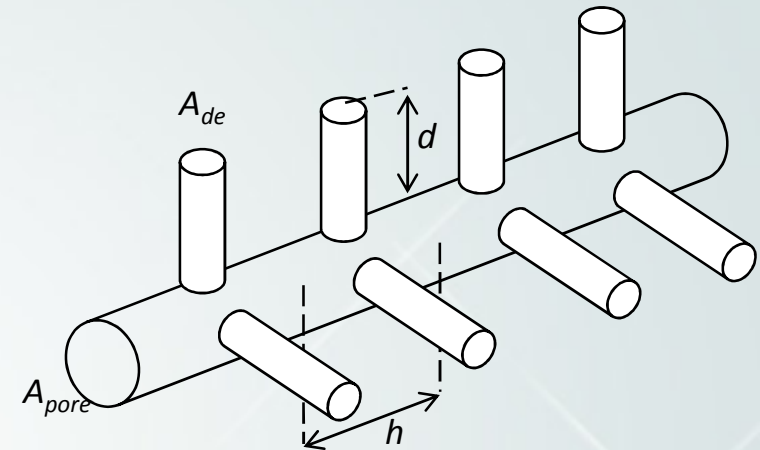
$y = \exp(i k_{pore} h)$ plane wave:

$$T_c = \begin{pmatrix} (1+X)y & X \\ -X & \frac{(1+X)}{y} \end{pmatrix} \quad M = (T_c)^n = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

$$T = \begin{pmatrix} \frac{1+\phi'}{2\phi} & \frac{1-\phi'}{2\phi} \\ -\frac{1-\phi'}{2\phi'} & \frac{1+\phi'}{2\phi'} \end{pmatrix} \quad \text{with} \quad \phi' = \phi \frac{Z_0}{Z_{pore}}$$

$$M' = M \times T = \begin{pmatrix} M'_{11} & M'_{12} \\ M'_{21} & M'_{22} \end{pmatrix}$$

$$R'_n = \frac{M'_{11} - M'_{21}}{M'_{22} - M'_{21}} \quad T_n = \frac{1}{M'_{22}}$$



Absorption coefficient

$$\alpha = 1 - |R'_N|^2$$

Transmission Loss

4. Models – Low frequency approximation – Long dead-ends

$$\cos(qh) = \cos(k_{pore}h) - \frac{X}{i} \sin(k_{pore}h) \quad \text{or TMM}$$

Low frequency approximation:

Long dead ends (λ can be of the order of dead-end length)

$$q = k_{pore} \sqrt{1 + \frac{2X}{ik_{pore}h}} = k_{pore} \sqrt{1 + \frac{NA_{de} Z_{pore} \tan(k_{de}d)}{A_{pore} Z_{de} k_{pore}h}}$$

$$\rho_e = \rho_{pore},$$

$$Z = \frac{Z_{pore}}{\sqrt{1 + \frac{2X}{ik_{pore}h}}} = \frac{Z_{pore}}{\sqrt{1 + \frac{NA_{de} Z_{pore} \tan(k_{de}d)}{A_{pore} Z_{de} k_{pore}h}}}$$

$$C_e = C_{pore} + C_{de} \frac{NA_{de} d}{A_{pore} h} \left(\frac{\tan(k_{de}d)}{k_{de}d} \right),$$

Quarter wavelength cavity
(Helmholtz resonator possible)

5. Models – LF approximation – Short dead-ends – Evidence of LF effects

$$\cos(qh) = \cos(k_{pore}h) - \frac{X}{i} \sin(k_{pore}h) \quad \text{or TMM}$$

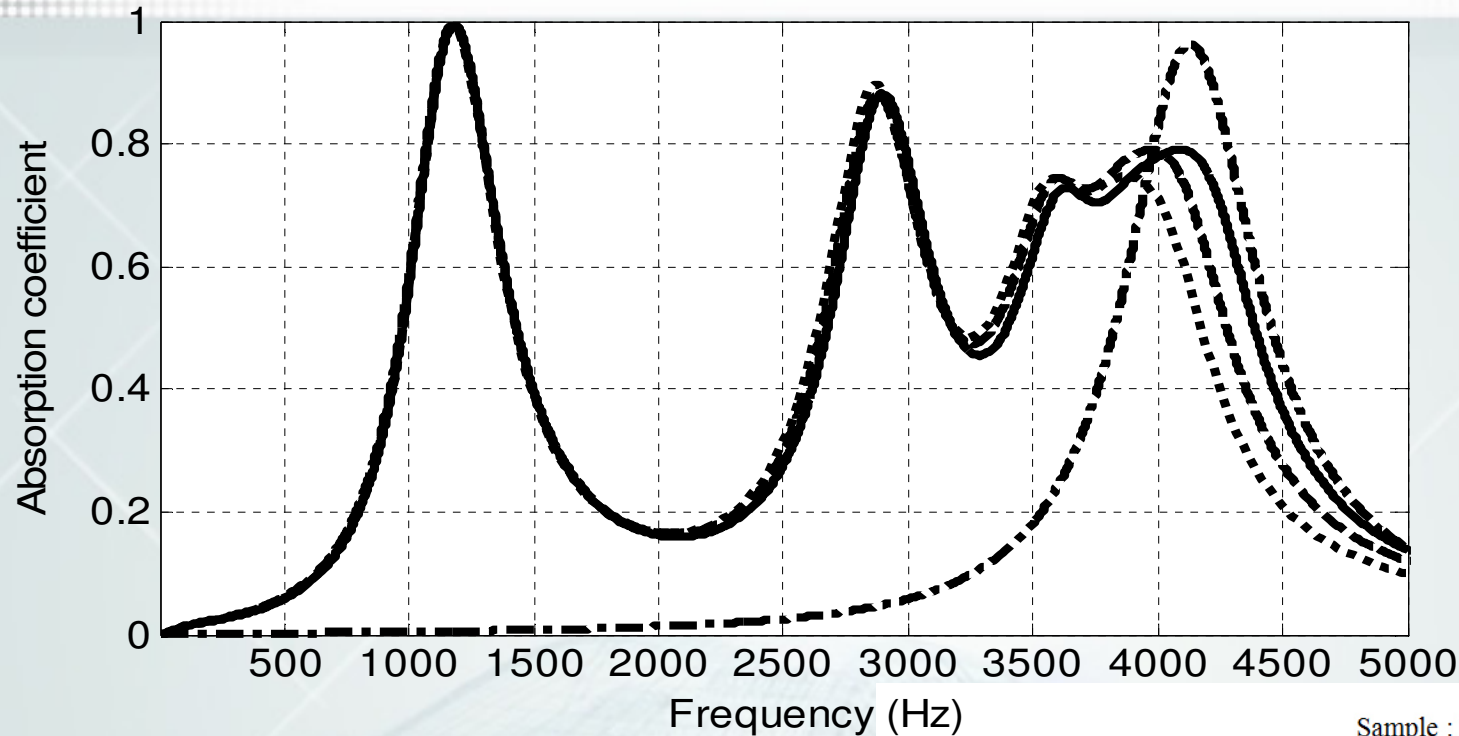
Short dead-ends:

$$\rho_s = \rho_{pore},$$

$$C = C_{pore} + C_{de} \frac{V_{de}}{V_{pore}},$$

- No noticeable viscous effects associated
- Thermal diffusion effect (radiator).
- Effective fluid “softening” (wave slowing)
- C increases with dead-end pore volume:
 - Increase N : Number of dead-ends per node
 - Increase h : Dead-end periodicity
 - Increase d_{de} : dead-end pore diameter
 - Increase l_{de} : Dead-end length
 - Deacrease d_{pore}

6. Models – Simulations and comparisons : Full TMM, LF, No Dead end, FEM



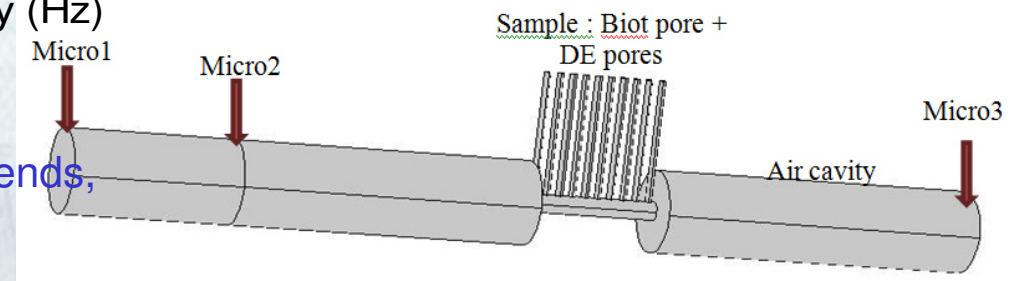
Absorption coefficient.

Dash-dotted line: Main pore with no dead-ends,

Dashed line: Full model (TMM)

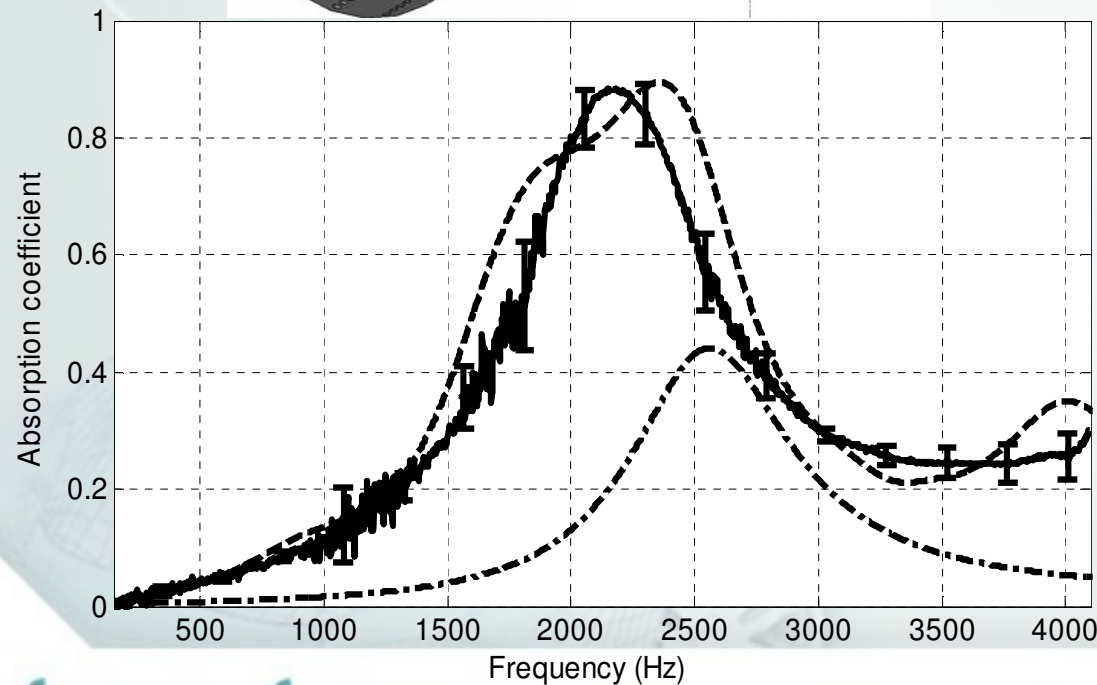
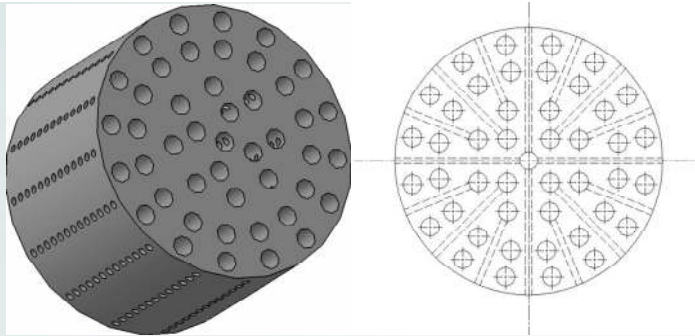
Solid line: Low frequency approximation,

Dotted line: FEM simulation



7. Experimental validation – example of design

| $\sigma_{pore,de}$ | $\kappa'_{pore,de}$ | $\alpha_{\infty,pore,de}$ | $\Lambda_{pore,de}$ | $\Lambda'_{pore,de}$ |
|----------------------------------|---------------------|---------------------------|---------------------|----------------------|
| $8\eta \backslash a_{pore,de}^2$ | $a_{pore,de}^2/8$ | 1 | $a_{pore,de}$ | $a_{pore,de}$ |



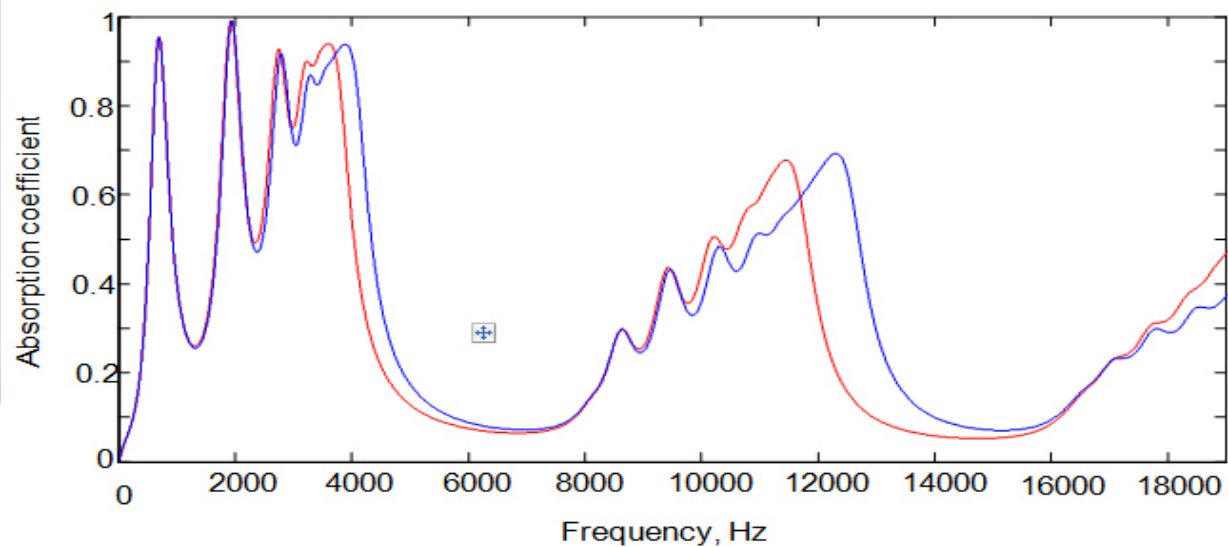
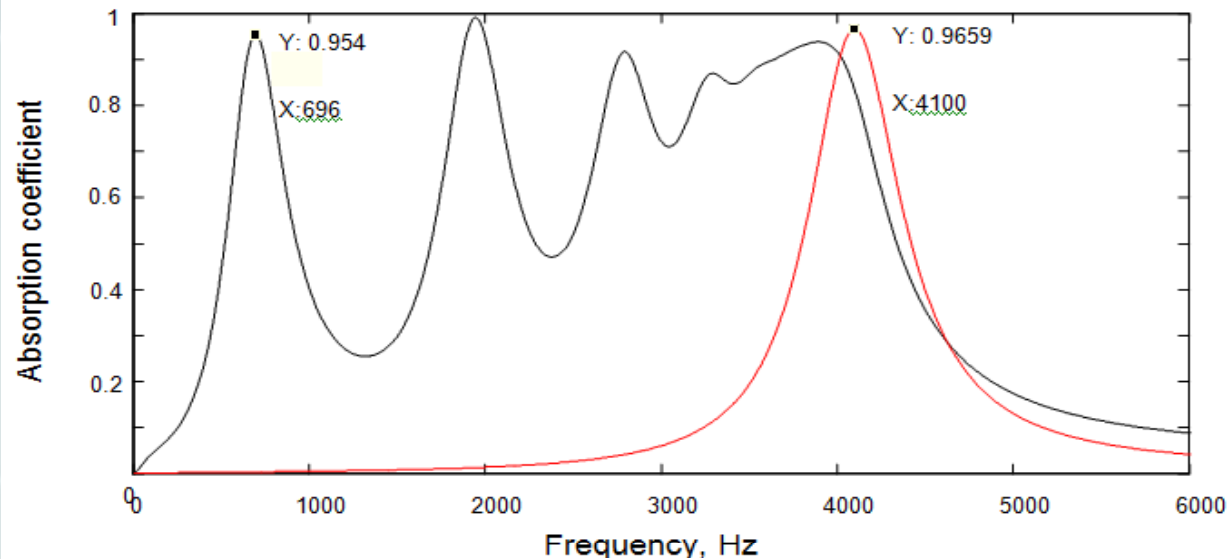
Equivalent fluid effective density and compressibility

$$\rho = \rho_0 \alpha_{\infty} \left(1 + \frac{\sigma}{-i\omega \alpha_{\infty} \rho_0} \sqrt{1 + \frac{-i\omega}{\omega_b}} \right)$$

$$C = \frac{1}{\rho_0 c^2} \left(\gamma - \frac{\gamma - 1}{1 + \frac{\eta}{-i\omega' \rho_0 \kappa'}} \sqrt{1 + \frac{-i\omega'}{\omega'_b}} \right)$$

8. Possible future designs

$$C = C_{pore} + C_{de} \frac{V_{de}}{V_{pore}},$$



Absorption coefficient

Black : LF Model

Red : No Dead-end

Blue : LF Model

Red : Full model (TMM)

$a_{pore} = 0.5 \text{ mm}$

$a_{de} = 0.5 \text{ mm}$

$l_{de} = 2 \text{ cm},$

$h = 2 \text{ mm},$

$N = 3,$

$L = 2 \text{ cm}$

perforation rate $\phi = 0.05.$

