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1 **Modelling the second wave of COVID-19 infections in France and Italy via a**
2 **Stochastic SEIR model**

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14 COVID-19 has forced quarantine measures in several countries across the world. These
15 measures have proven to be effective in significantly reducing the prevalence of the virus.
16 To date, no effective treatment or vaccine is available. In the effort of preserving both
17 public health as well as the economical and social textures, France and Italy governments
18 have partially released lockdown measures. Here we extrapolate the long-term behavior of
19 the epidemics in both countries using a Susceptible-Exposed-Infected-Recovered (SEIR)
20 model where parameters are stochastically perturbed to handle the uncertainty in the esti-
21 mates of COVID-19 prevalence. Our results suggest that uncertainties in both parameters
22 and initial conditions rapidly propagate in the model and can result in different outcomes
23 of the epidemics leading or not to a second wave of infections. Using actual knowledge,
24 asymptotic estimates of COVID-19 prevalence can fluctuate of order of ten millions units
25 in both countries.

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26 I. LEAD PARAGRAPH

27 **COVID-19 pandemic poses serious threats to public health as well as economic and so-**
28 **cial stability of many countries. A real time extrapolation of the evolution of COVID-19**
29 **epidemics is challenging both for the nonlinearities undermining the dynamics and the ig-**
30 **norance of the initial conditions, i.e., the number of actual infected individuals. Here we**
31 **focus on France and Italy, which have partially released initial lockdown measures. The**
32 **goal is to explore sensitivity of COVID-19 epidemic evolution to the release of lockdown**
33 **measures using dynamical (Susceptible-Exposed-Infected-Recovered) stochastic models. We**
34 **show that the large uncertainties arising from both poor data quality and inadequate estima-**
35 **tions of model parameters (incubation, infection and recovery rates) propagate to long term**
36 **extrapolations of infections counts. Nonetheless, distinct scenarios can be clearly identified,**
37 **showing either a second wave or a quasi-linear increase of total infections.**

38 II. INTRODUCTION

39 SARS-CoV-2 is a zoonotic virus of the coronavirus family¹ emerged in Wuhan (China) at the
40 end of 2019² and rapidly propagated across the world until it has been declared a pandemic by
41 the World Health Organization on March 11, 2020³. SARS-CoV-2 virus provokes an infectious
42 disease known as COVID-19 that has an incredibly large spectrum of symptoms or none depending
43 on the age, health status and the immune defenses of each individuals⁴. SARS-CoV-2 causes
44 potentially life-threatening form of pneumonia and/or cardiac injuries in a non-negligible patients
45 fraction^{5,6}.

46 To date, no treatment of vaccine is available for COVID-19⁷. Efforts to contain the virus and
47 to not overwhelm intensive care facilities are based on quarantine measures which have proven
48 very effective in several countries⁸⁻¹⁰. Despite this, lockdown measures entail enormous econom-
49 ical, social and psychological costs. Recent estimates of the International Monetary Fund recently
50 announced a global recession that will drag global GDP lower by 3% in 2020, although contin-
51 uously developing and changing as well as significantly depending country-by-country¹¹. More
52 than 20 million people have lost their job in United States¹² and a large percentage of Italians
53 have developed psychological disturbances such as insomnia or anxiety due to the strict lockdown
54 measures¹³. Those measures have been taken on the basis of epidemics models, which are fitted

55 on the available data¹⁴. In Italy, initial lockdown measures started on February 23rd for 11 mu-
56 nicipalities in both Lombardia and Veneto which were identified as the two main Italian clusters.
57 After the initial spread of the epidemics into different regions all Italian territory was placed into a
58 quarantine on March 9th, with total lockdown measures including all commercial activities (apart
59 supermarkets and pharmacies), non-essential businesses and industries, and severe restrictions to
60 transports and movements of people at regional, national, and extra-national levels¹⁵. People were
61 asked to stay at home or near for sporting activities and dog hygiene (within 200 m from home),
62 to reduce as much as possible their movements (only for food shopping and care reasons), and
63 smart-working was especially encouraged in both public and private administrations and compa-
64 nies. At the early stages of epidemics intensive cares were almost saturated with a peak of 4000
65 people on April 3rd and a peak of hospitalisations of 30000 on April 4th, significantly reducing
66 after these dates, reaching 1500 and 17000, respectively, at the beginning of phase 2 on May 4th,
67 and 750 and 1000 on May 18th when lockdown measures on commercial activities were relaxed.
68 These numbers, continuously declining during the next days and weeks, confirmed the benefit of
69 lockdown measures¹⁶.

70 Alarmed by the exponential growth of new infections and the saturation of the intensive care beds,
71 also France introduced strict lockdown measures on March 17th¹⁷. The French government re-
72 stricted travels to food shopping, care and work when teleworking was not possible, outings near
73 home for individual sporting activity and/or dog hygiene, and it imposed the closure of the Schen-
74 gen area borders as well as the postponement of the second round of municipal elections. The
75 number of patients in intensive care, like the number of hospitalisations overall peaked in early
76 April and then started to decline, showing the benefits of lockdown measures. On Monday, May
77 11th, France began a gradual easing of COVID-19 lockdown measures¹⁸. Trips of up to 100 kilo-
78 metres from home are allowed without justification, as will gatherings of up to 10 people. Longer
79 trips will still be allowed only for work or for compelling family reasons, as justified by a signed
80 form. Guiding the government's plans for easing the lockdown is the division of the country into
81 two zones, green and red, based on health indicators. Paris region (Ile de France), with about 12
82 millions inhabitants is flagged, to date, as an orange zone.

83 In both countries, the release of lockdown measures has been authorised by authorities after
84 consulting scientific committees which were monitoring the behavior of the curve of infections
85 using COVID-19 data. Those data are provided daily, following a request of the WHO. To date,
86 the WHO guidelines require countries to report, at each day t , the total number of infected patients

87 $I(t)$ as well as the number of deaths $D(t)$. Large uncertainties have been documented in the count
88 of $I(t)$ ¹⁹. Whereas in the early stage of the epidemic several countries tested asymptomatic indi-
89 viduals to track back the infection chain, recent policies to estimate $I(t)$ have changed. Most of
90 the western countries have previously tested only patients displaying severe SARS-CoV-2 symp-
91 toms²⁰. In an effort of tracking all the chain of infections, Italy and France are now testing all
92 individuals displaying COVID-19 symptoms and those who had strict contacts with infected indi-
93 viduals. The importance of tracking asymptomatic patients has been proven in a recent study²¹.
94 The authors have estimated that an enormous part of total infections were undocumented (80% to
95 90%) and that those undetected infections were the source for 79% of documented cases in China.
96 Tracking strategies have proven effective in supporting actions to reduce the rate of new infections,
97 without the need of lockdown measures, as in South Korea²².

98 The goal of this paper is to explore possible future epidemics scenarios of the long term behav-
99 ior of the COVID-19 epidemic²³ but taking into account the role of uncertainties in both the pa-
100 rameters value and the infection counts to investigate different outcomes of the epidemics leading
101 or not to a second wave of infections. To this purpose we use a stochastic Susceptible-Exposed-
102 Infected-Recovered (SEIR) model²⁴ which consist in a set of ordinary differential equations where
103 control parameters are time-dependent modelled via a stochastic process. This allows to mimic the
104 dependence on control parameters on some additional/external factors as super-spreaders²⁵ and the
105 enforcing/relaxing of confinement measures²⁴. As for the classical SEIR models²⁶ the population
106 is divided into four compartmental groups, i.e., Susceptible, Exposed, Infected, and Recovered
107 individuals. The stochastic SEIR model shows that long-term extrapolation is sensitive to both the
108 initial conditions and the value of control parameters²⁴, with asymptotic estimates fluctuating on
109 the order of ten millions units in both countries, leading or not a second wave of infections. This
110 sensitivity arising from both poor data quality and inadequate estimations of model parameters
111 has been also recently investigated by means of a statistical model based on a generalized logistic
112 distribution²⁷. The paper is organised as follows: in Section III we discuss the various sources
113 of data for COVID-19 and their shortcomings, and then we discuss in detail the SEIR model and
114 its statistical modelling. In Section IV we discuss the results focusing on the statistical sensitivity
115 of the modelling, and apply it to data from France and Italy. We finish, in Section V, with some
116 remarks and point out some limitations of our study.

117 III. DATA AND MODELLING

118 A. Data

119 This paper relies on data stored into the Visual Dashboard repository of the Johns Hopkins Uni-
120 versity Center for Systems Science and Engineering (JHU CSSE) supported by ESRI Living Atlas
121 Team and the Johns Hopkins University Applied Physics Lab (JHU APL). Data can be freely
122 accessed and downloaded at <https://systems.jhu.edu/research/public-health/ncov/>,
123 and refers to the confirmed cases by means of a laboratory test³. Nevertheless there are some
124 inconsistencies between countries due to different protocols in testing patients (suspected symp-
125 toms, tracing-back procedures, wide range tests)^{28,29}, as well as, to local management of health
126 infrastructures and institutions. As an example due to the regional-level system of Italian health-
127 care data are collected at a regional level and then reported to the National level via the Protezione
128 Civile transferring them to WHO. These processes could be affected by some inconsistencies and
129 delays³⁰, especially during the most critical phase of the epidemic diffusion that could introduce
130 errors and biases into the daily data. These incongruities mostly affected the period between Febru-
131 ary 23rd and March 10th, particularly regarding the counts of deaths due to a protocol change from
132 the Italian Ministry of Health³¹. A similar situation occurs in France where the initial testing strat-
133 egy was based only on detecting those individuals experiencing severe COVID19 symptoms³².
134 In the post lockdown phase, France has extended its testing capacity to asymptomatic individuals
135 who have been in contact with infected patients³³.

136 B. A Stochastic epidemiological Susceptible-Exposed-Infected-Recovered model

137 One of the most used epidemiological models is the so-called Susceptible-Exposed-Infected-
138 Recovered (SEIR) model belonging to the class of compartmental models²⁶. It assumes that the
139 total population N can be divided into four classes of individuals that are susceptible S , exposed
140 E , infected I , and recovered or dead R (assumed to be not susceptible to reinfection). The model
141 is based on the following assumptions:

- 142 1. the total population does not vary in time, e.g., $dN/dt = dS/dt + dE/dt + dI/dt + dR/dt =$
143 $0, \forall t \geq 0$;
- 144 2. susceptible individuals become infected that then can only recover or die, e.g., $S \rightarrow I \rightarrow R$;

- 145 3. exposed individuals E encountered an infected person but are not themselves infectious;
- 146 4. recovered or died individuals R are forever immune.

147 Thus, the model reads as

$$148 \quad \frac{dS}{dt} = -\lambda S(t)I(t), \quad (1)$$

$$149 \quad \frac{dE}{dt} = \lambda S(t)I(t) - \alpha E(t), \quad (2)$$

$$150 \quad \frac{dI}{dt} = \alpha E(t) - \gamma I(t), \quad (3)$$

$$151 \quad \frac{dR}{dt} = \gamma I(t), \quad (4)$$

152 where $\gamma > 0$ is the recovery/death rate, $\lambda = \lambda_0/S(0) > 0$ is the infection rate rescaled by the initial
 153 number of susceptible individuals $S(0)$, and α is the inverse of the incubation period. Its discrete
 154 version can be simply obtained via an Euler Scheme as

$$155 \quad S(t+1) = S(t) - \lambda S(t)I(t), \quad (5)$$

$$156 \quad E(t+1) = (1 - \alpha)E(t) + \lambda S(t)I(t), \quad (6)$$

$$157 \quad I(t+1) = (1 - \gamma)I(t) + \alpha E(t), \quad (7)$$

$$158 \quad R(t+1) = R(t) + \gamma I(t). \quad (8)$$

159 in which we fixed $dt = 1$ day that is the time resolution of COVID-19 counts. By means of γ and
 160 λ_0 the model also allows to derived the so-called R_0 parameter, e.g., $R_0 = \lambda_0/\gamma$, representing the
 161 average reproduction number of the virus. It is related to the number of cases that can potentially
 162 (on average) caused from an infected individual during its infectious period ($\tau_{inf} = \gamma^{-1}$). Early
 163 estimates in Wuhan³⁴ on January 2020 reported $R_0 = 2.68_{2.47}^{2.86}$ which lead to $\gamma = 0.37$ fixing $\lambda \simeq 1$
 164 as in³⁵ and a 95% confidence level range for the incubation period between 2 and 11 days³⁶.
 165 However, the R_0 parameter as well as models parameters λ , γ , and α can vary in time during
 166 the epidemics due to different factors as the possible presence of the so-called super-spreaders²⁵,
 167 intrinsic changes of the SARS-CoV-2 features, lockdown measures, asymptomatic individuals who
 168 are not tracked out, counting procedures and protocols, and so on³⁷.

169 To deal with uncertainties in long-term extrapolations and with the time-dependency of control
 170 parameters a stochastic approach could provide new insights in modeling epidemics³⁸⁻⁴⁰, espe-
 171 cially when epidemics show a wide range of spatial and temporal variability⁴¹⁻⁴³. However,

172 instead of investigating how to get a realistic behavior by stochastically perturbing control pa-
 173 rameters, here we investigate how uncertainties into the final counts $C(t)$ are controlled by model
 174 parameters²⁴. Thus, we use a stochastic version of the SEIR model in which the set of control
 175 parameters $\kappa \in \{\alpha, \lambda, \gamma\}$ is modelled via a stochastic process

$$176 \quad \kappa(t) = |\kappa_0 + \sigma \cdot \xi(t)|, \quad (9)$$

177 with being σ the intensity of the perturbation, $\xi(t)$ a random variable from a collection of
 178 normally distributed $N(0, 1)$ values at each time t , and since $\kappa(t) \geq 0$ we introduced the absolute
 179 value in Eq. (9). In this way we can introduce instantaneous daily discrete jumps (e.g., take
 180 into account daily uncertainties) in the control parameters to properly model detection errors on
 181 infection counts, appropriately described through a discrete process⁴⁴ than a continuous one⁴⁵.

182 IV. RESULTS

183 A. Model validation: first wave

184 We begin this section by validating the SEIR stochastic model on the first wave of infections.
 185 We have therefore to chose the initial conditions, and then introduce the lockdown measures in the
 186 parameters.

187 a. France

188 In France, the first documented case of COVID-19 infections goes back to December 27th, 2019.
 189 Doctors at a hospital in the northern suburbs of Paris retested samples from patients between De-
 190 cember 2nd, 2019, and January 16th, 2020. Of the 14 patient samples retested, one sample, from
 191 a 42-year-old man came back positive⁴⁶. As initial condition for the SEIR model, we therefore set
 192 $I(t = 1) = 1$ and $t = 1$ corresponds to December 27th, 2019. We then use $R_0 = 2.68_{2.47}^{2.86}$ which
 193 lead to $\gamma = 0.37$ fixing $\lambda_0 \simeq 1$. Strict lockdown measures are introduced at $t = 80$ (i.e., March
 194 17th, 2020). First wave modelling results are shown in Figure 1. Figure 1a) shows the modelled
 195 value of R_0 . During confinement, we let λ fluctuates by 20% of its value around 1/4. We base
 196 this new infection rate on the mobility data for France during confinement, which have shown a
 197 drop by $\sim 75\%$ according to the INSERM report #11⁴⁷. The resulting confinement $R_0 = 0.75_{0.5}^{1.0}$,
 198 a range of values compatible with that published by the Pasteur Institute⁴⁸. The cumulative num-
 199 ber of infections is shown in Figure 1b) and shows, on average, that 8 millions people have been

200 infected by SARS-CoV-2 in France. The uncertainty range is extremely large, according to the
201 error propagation given by the stochastic fluctuations of the parameters (see²⁴ for explanations). It
202 extends from few hundred thousands individuals up to 18 millions. The average is however close
203 to the value proposed by the authors in⁴⁹, who estimate a prevalence of $\sim 6\%$ of COVID-19 in
204 the French population. Another realistic feature of the model is the presence of an asymmetric
205 behavior of the right tail of daily infections distributions (Figure 1c) that has also been observed
206 in real COVID-19 published data⁵⁰.

207 *b. Italy*

208 For Italy, the first suspect COVID-19 case goes back to December 22nd, 2019, a 41-year-old
209 woman who could only be tested positive for SARS-CoV-2 antibodies in April 2020⁵¹. As initial
210 condition we therefore set $I(t = 1) = 1$ and $t = 1$ corresponds to December 22nd, 2019. As for
211 France we use $R_0 = 2.68_{2.47}^{2.86}$ leading to $\gamma = 0.37$ if fixing $\lambda_0 \simeq 1$. A first semi-lockdown was set
212 in Italy on March 9th, 2020 ($t = 78$) and enforced on March 22nd, 2020 ($t = 89$). To simulate
213 these two-steps lockdown we again base our reduction in R_0 on the mobility data for Italy which
214 show for the first part of the confinement a reduction of about 50 % and a similar reduction to
215 France (75%) for the strict lockdown phase. Figure 2 shows the results for the first wave by
216 letting $\lambda = 0.25 \pm \Delta\lambda$, where $\Delta\lambda$ represents a 20% fluctuations around the mean value, and by
217 fixing an initial condition on susceptible individuals $S(1) = 6.0 \cdot 10^7$ corresponding to the estimate
218 of the Italian population. A clear difference emerges with respect to the case of France in the
219 behavior of R_0 which shows an intermediate reduction near $t = 80$, corresponding to March 11th,
220 2020, to $R_0 = 1.4_{1.1}^{1.7}$ before reaching the final value of $R_0 = 0.7_{0.5}^{0.9}$. This sort of "step" into the
221 R_0 time behavior corresponds to the time interval between semi- and full-lockdown measures,
222 whose efficiency significantly increases after March 24th, 2020, also corresponding to the peak
223 value of infections. This is confirmed by looking at daily infections distributions (Figure 2c) that
224 shows a peak value near March 24th, 2020, also observed in real COVID-19 data²⁷. Finally, the
225 cumulative number of infections (Figure 2b) shows that, on average, almost 10 millions people
226 have been infected by SARS-CoV-2 in Italy, ranging between few hundred thousands up to 25
227 millions due to the the error propagation by the stochastic fluctuations of model parameters (see²⁴
228 for explanations). Nevertheless the wide range of uncertainty the average value is close to the
229 value estimated from a team of experts of the Imperial College London according to which the
230 9.6% of Italian population has been infected, with a 95% confidence level ranging between 3.2%
231 and 26%⁵². These estimates correspond to cumulative infections of ~ 6 millions, ranging from ~ 2

232 and ~ 16 millions, well in agreement with our model and other statistical estimates⁵³.

233 **B. Future epidemics scenarios**

234 After lockdown measures are released, for both countries, we model three different scenarios: a
235 first one where all restrictions are lifted (back to normality), a second one where strict measures are
236 taken (semi-lockdown) and a third one where the population remains mostly confined (lockdown).

237 *a. France*

238 Results for France are shown in Figure 3. Lockdown is released at $t = 136$, corresponding to
239 May 11th, 2020. The back to normality (red) scenario clearly shows a second wave of infections
240 peaking in summer (early July) and forcing group immunity in the French population. The semi-
241 lockdown (green) scenario, corresponding to a reduction of the mobility of about 50%, leads to
242 a second wave as intense as the first wave, but longer, at the end of August. As in the previous
243 scenario, the semi-lockdown scenario allows to reach a group immunity in France. A third lock-
244 down scenario is modelled (blue). This latter scenario simulates an $R_0 \simeq 1$, that can be achieved
245 by imposing strict distancing measures, contact tracking as well as reduction in mobility. It results
246 in a linear modest increase of the total number of infections that does not produce a proper wave of
247 infections. As in the first wave modelling, large uncertainties are also present in future scenarios
248 although the three distinct behaviors clearly appear.

249 *b. Italy*

250 Figure 4 shows the results for modeling future epidemic scenarios for Italy. The first relaxation
251 of lockdown measures started at $t = 131$, corresponding to May 4th, 2020, while strict measures
252 were finally released at $t = 146$, corresponding to May 18th, 2020. The back to normality (red)
253 scenario moves towards a second wave of infections whose peak occurs at $t = 193$, correspond-
254 ing to July 4th, 2020, exactly three months after initial lockdown measures were released (May
255 4th, 2020). This would lead the so-called herd immunity for the whole Italian population (see Fig-
256 ure 4b), with a peak of daily infections near 5 millions of people (Figure 4c), and R_0 re-approaching
257 the initial value ($R_0 = 2.68$). The semi-lockdown (green) scenario produces a second wave mostly
258 similar, in terms of intensity, as the first wave, but occurring at $t = 246$, e.g., August 26th, 2020.
259 This scenario will lead to 40 millions infected people, spanning between 25 and 55 millions, thus
260 producing a group immunity in Italy. A third scenario is modelled in which complete lockdown
261 measures are still considered (blue). This latter scenario leads to a more controlled evolution of

262 cumulative infections which still remain practically unchanged with respect to the first wave cu-
263 mulative number. It has been obtained by simulating an $R_0 \simeq 1$, resulting from strict distancing
264 measures and reduced mobility, and does not produce a proper wave of infections. However, all
265 scenarios are clearly characterized by a wide range of uncertainties, although producing three well
266 distinct behaviors in both cumulative and daily infections.

267 C. Phase Diagrams

268 In the previous section we have seen that increasing R_0 above 1 can or not produce a second
269 wave of infections and introduce also a time delay in the appearance of a second wave of infec-
270 tions. We now analyse this effect in a complete phase diagram fashion. Figures 5-6 show the
271 phase diagrams for France and for Italy, respectively. The diagrams are built in terms of ensemble
272 averages of number of infections per day $I(t)$ versus the average value of R_0 after the confine-
273 ment (panels a), and the errors (represented as standard deviation of the average $I(t)$ over the 30
274 realisations) are shown in panels b. First we note that despite some small differences in the de-
275 lay of the COVID-19 second wave of infections peak, the diagrams are very similar. In order to
276 avoid a second wave, R_0 could fluctuate on values even slightly larger than one. Furthermore, for
277 $1.5 < R_0 < 2$, the second wave is delayed in Autumn or Winter 2020/2021 months. The uncer-
278 tainty follows the same behavior as the average and it peaks when the number of daily infections
279 is maximum. This means that the ability to control the outcome of the epidemics is highly reduced
280 if R_0 is too high.

281 V. DISCUSSION

282 France and Italy have faced a long phase of lockdown with severe restrictions in mobility and
283 social contacts. They have managed to reduce the number of daily COVID-19 infections drasti-
284 cally and released almost simultaneously lockdown measures. This paper addresses the possible
285 future scenarios of COVID-19 infections in those countries by using one of the simplest possible
286 model capable to reproduce the first wave of infections and to take into account uncertainties,
287 namely a stochastic SEIR model with fluctuating parameters.

288

289 We have first verified that the model is capable to reproduce the behavior of the first wave of

290 infections and provide an estimate of COVID-19 prevalence that is coherent with clinical tests
291 and other studies. The introduction of stochasticity accounts for the large uncertainties in both
292 the initial conditions as well as the fluctuations in the basic reproduction number R_0 originating
293 from changes in virus characteristics, mobility or misapplication in confinement measures. 30
294 realisations of the model have been produced and they show very different COVID-19 prevalence
295 after the first wave. The range goes from thousands of infected to tens of millions of infections in
296 both countries. Average values are compatible with those found in other studies^{49,52}.

297

298 Then, we have modelled future epidemics scenarios by choosing specific fluctuating behaviors
299 for R_0 and performing again, 30 realisations of the stochastic SEIR model. Despite the very large
300 uncertainties, distinct scenarios clearly appear from the noise. In particular, they suggest that a
301 second wave can be avoided even with R_0 values slightly larger than one. This means that actual
302 distancing measures which include the use of surgical masks, the reduction in mobility and the
303 active contact tracking can be effective in avoiding a second peak of infections without the need
304 of imposing further strict lockdown measures. The analysis of phase diagrams show that there is
305 a sharp transition between observing or not a second wave of infections when the value of R_0 is
306 close to 1.5. Moreover, the models show that the higher R_0 , the lower the ability to control the
307 number of infections in the epidemics.

308 This model has also evident deficiencies in representing the COVID-19 infections. First of all,
309 the choice of the initial conditions is conditioned by our ignorance on the diffusion of the virus in
310 France and Italy in December 2019. Furthermore, we are unable to verify on an extensive dataset
311 the outcome of the first wave: on one side antibodies blood tests have still a lower reliability⁵⁴
312 and on the other they have not been applied on an extensive number of individuals to get reliable
313 estimates. On top of the data-driven limitations, we have those introduced by the use of compart-
314 ment models, as there are geographic, social and age differences in the spread of the COVID-19
315 disease in both countries¹⁸. Furthermore, we also assume that fluctuations on the parameters of
316 the SEIR model are Gaussian, although there are good reasons to think that they should be heavy
317 tailed distributions^{50,55}. We would like to remark however that, to overcome these limitations,
318 one would need to fit more complex models and introduce additional parameters which can, at the
319 present stage, barely inferred by the data.

320

321 Our choice to stick the stochastic SEIR model is indeed driven by its simplicity and the possi-

322 bility of modeling realistic the uncertainties with the stochastic fluctuations instead of adding new
323 parameters whose inference may affect the results. This study can be applied to other countries,
324 and this is why we publish along the code of our analysis alongside with the paper. To date, North-
325 ern Europe, UK, US and other American countries are still facing the first wave of infections, so
326 that future scenarios cannot be devised with the same clarity as those outlined in this study for
327 France and Italy.

328 **VI. ACKNOWLEDGMENTS**

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331 **VII. DATA AVAILABILITY**

332 The data that support the findings of this study are openly available in <https://systems.jhu.edu/research/public-health/ncov/>, maintained by Johns Hopkins University Center
333 for Systems Science.
334

335 **VIII. APPENDIX A: NUMERICAL CODE**

```
336 % This appendix contains the MATLAB code used to perform
337 % the analysis contained in the paper via a stochasitc
338 % SEIR model
339
340 %% PARAMETER DEFINITIONS
341 %tmax: number of day of integrations
342 tmax=500;
343 %nrel: number of realisations of the model
344 nrel=30;
345 %tconf: lockdown day
346 tconf=50
347 %tconf2: lockdown release
348 tconf2=100
```

```

349
350 %% LOOP ON DIFFERENT VALUES OF LAMBDA, INFECTION RATE
351 for la=1:50
352     lambdaconf=0.25;
353     lambdares=la.*0.02;
354
355     %% LOOP ON REALIZATIONS
356     for rel=1:nrel
357         S=zeros(1,tmax);
358         E=zeros(1,tmax);
359         I=zeros(1,tmax);
360         R=zeros(1,tmax);
361         C=zeros(1,tmax);
362         lambda=zeros(1,tmax);
363         %S Susceptible individuals (France population)
364         S(1)=67000000;
365         %I Infected individuals
366         I(1)=585;
367         % Recovered
368         R(1)=0;
369         % Inital time
370         T(1)=0;
371         % Cumulative infections
372         C(1)=0;
373         % alpha is the inverse of the incubation period (1/t_incubation)
374         alpha0=0.27;
375         % R0 is equal to 2.68
376         R0=2.68;
377         % gamma is the inverse of the mean infectious period
378         gamma0=lambda0./R0;
379         % uncertainty in gamma and lambda
380         coeff_gamma=0.5;

```

```

381     coeff_lambda=0.005;
382
383     %% LOOP ON TIME, INTEGRATION OF SEIR MODELS
384     for t=1:1:tmax
385         %gamma=1/Tr where Tr is the recovery time (2 weeks)
386         %Stochastic gamma
387         gamma=gamma0+gamma0./5*randn;
388         %Change lambda for confinement
389         if t==tconf
390             lambda0=lambdaconf;
391         end
392         if t==tconf2
393             lambda0=lambdares;
394         end
395         %Stochastic lambda
396         lambda(t+1)=(lambda0+lambda0./5*randn)./S(1);
397         %Stochastic alpha
398         alpha=alpha0+alpha0./5*randn;
399         %Computation of R0
400         R0(t+1)=lambda(t+1)./gamma0;
401         %Iteration of the model
402         T(t+1)=t;
403         S(t+1)=S(t)-(lambda(t+1)*S(t)*I(t));
404         E(t+1)=E(t)+(lambda(t+1)*S(t)*I(t))-alpha*E(t);
405         I(t+1)=I(t) +alpha*E(t) -gamma*I(t);
406         R(t+1)=R(t)+(gamma*I(t));
407         %cumulative infected
408         C(t+1)=gamma0.*sum(I);
409         %Variables for different realisations
410         Irel(rel,t+1)=I(t+1);
411         lambdarel(rel,t+1)=lambda(t+1);
412     end

```

```

413
414         end
415
416 %% AVERAGING OVER DIFFERENT REALIZATIONS
417 lambdamoy(la,:)=mean(lambdarel,1);
418 Imoy(la,:)=mean(Irel,1);
419 Istd(la,:)=std(Irel,1);
420 lambdavec(la)=lambdares;
421 R0moy(la,:)=lambdamoy(la,:)./gamma0.*S(1);
422
423 end
424

```

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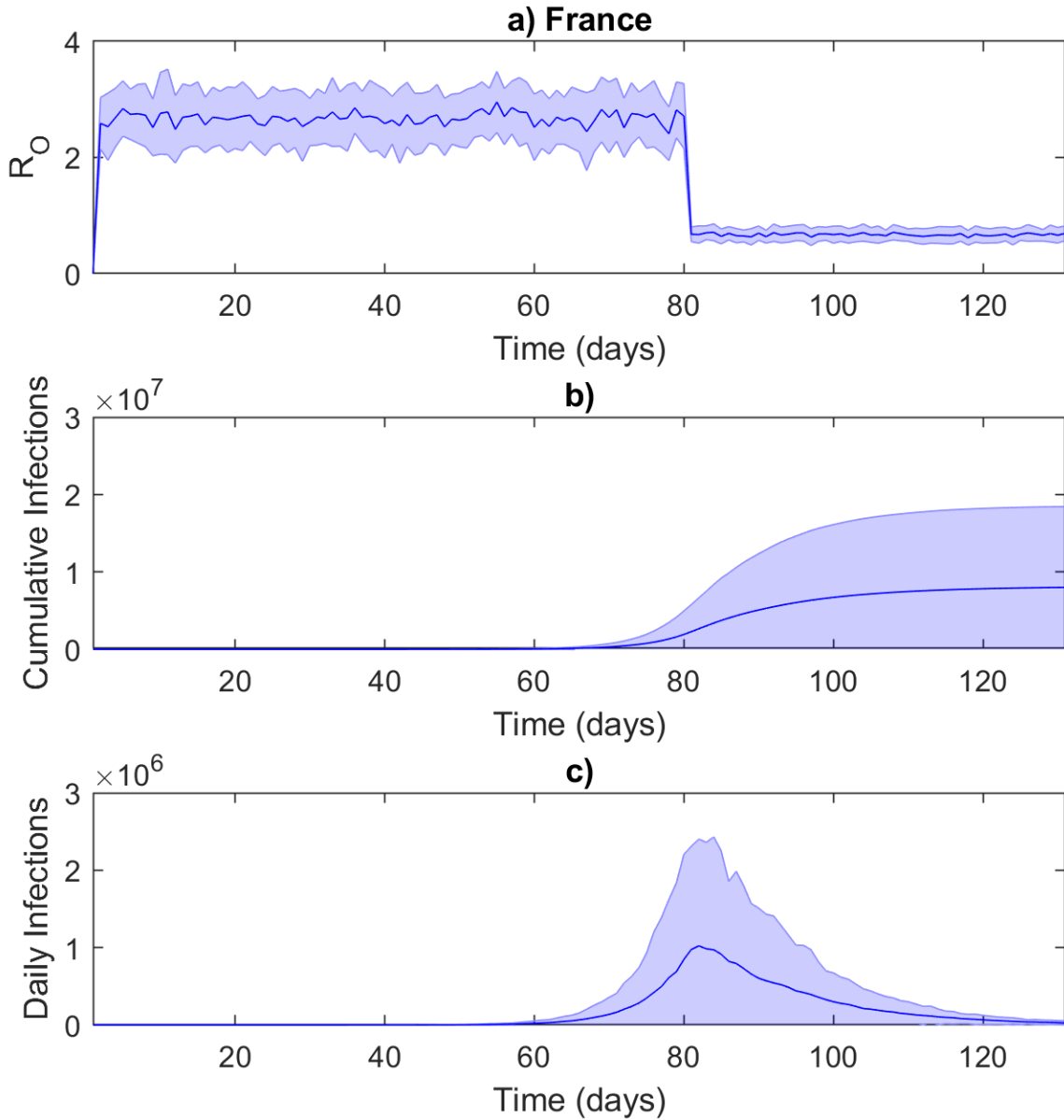


FIG. 1. Susceptible-Exposed-Infected-Recovered (SEIR) model of COVID-19 for France (Eqs 5-8) with $\lambda = 1./S(0)$, $\alpha = 0.27$, $\gamma = 0.37$. Initial conditions are set to $I(1) = 1$, $S(1) = 6.7 \cdot 10^7$, $E(1) = R(1) = 0$. $t = 1$ corresponds to Dec 27, 2019. Confinement is introduced at $t = 78$ (Mar 17, 2020). a) Time evolution for the basic reproduction number R_0 , b) Time evolution for the cumulative number of infections $C(t)$, c) Time evolution for the daily infected individuals $I(t)$. Solid line shows the average for 30 realisation of the SEIR stochastic models, shading extends to 3 standard deviations of the mean.

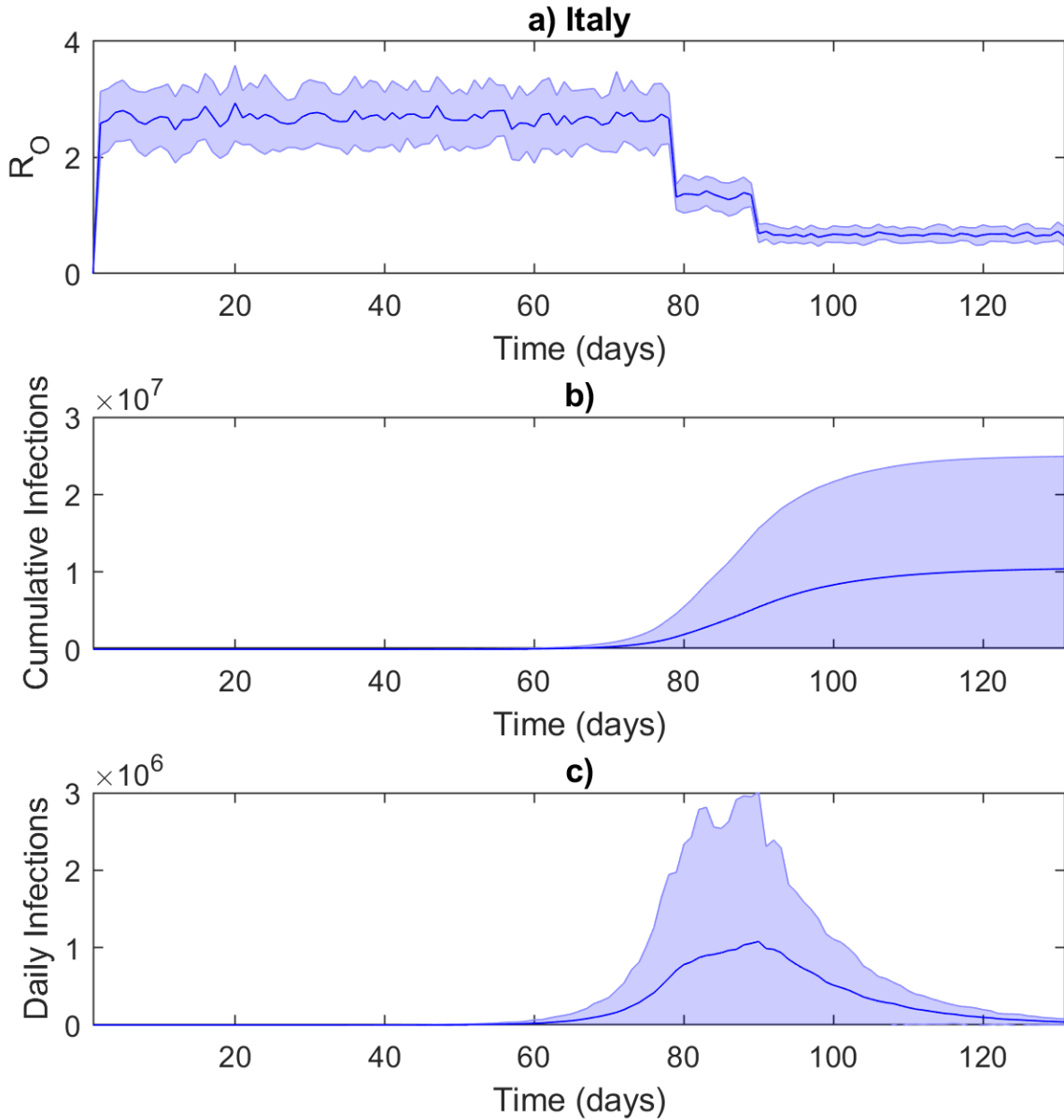


FIG. 2. Susceptible-Exposed-Infected-Recovered (SEIR) model of COVID-19 for Italy (Eqs 5-8) with $\lambda = 1./S(0)$, $\alpha = 0.27$, $\gamma = 0.37$. Initial conditions are set to $I(1) = 1$, $S(1) = 6.0 \cdot 10^7$, $E(1) = R(1) = 0$. $t = 1$ corresponds to Dec 22, 2019. First confinement measures are introduced at $t = 78$ (Mar 9, 2020) and enforced at $t = 89$ (Mar 22, 2020). a) Time evolution for the basic reproduction number R_0 , b) Time evolution for the cumulative number of infections $C(t)$, c) Time evolution for the daily infected individuals $I(t)$. Solid line shows the average for 30 realisation of the SEIR stochastic models, shading extends to 3 standard deviations of the mean.

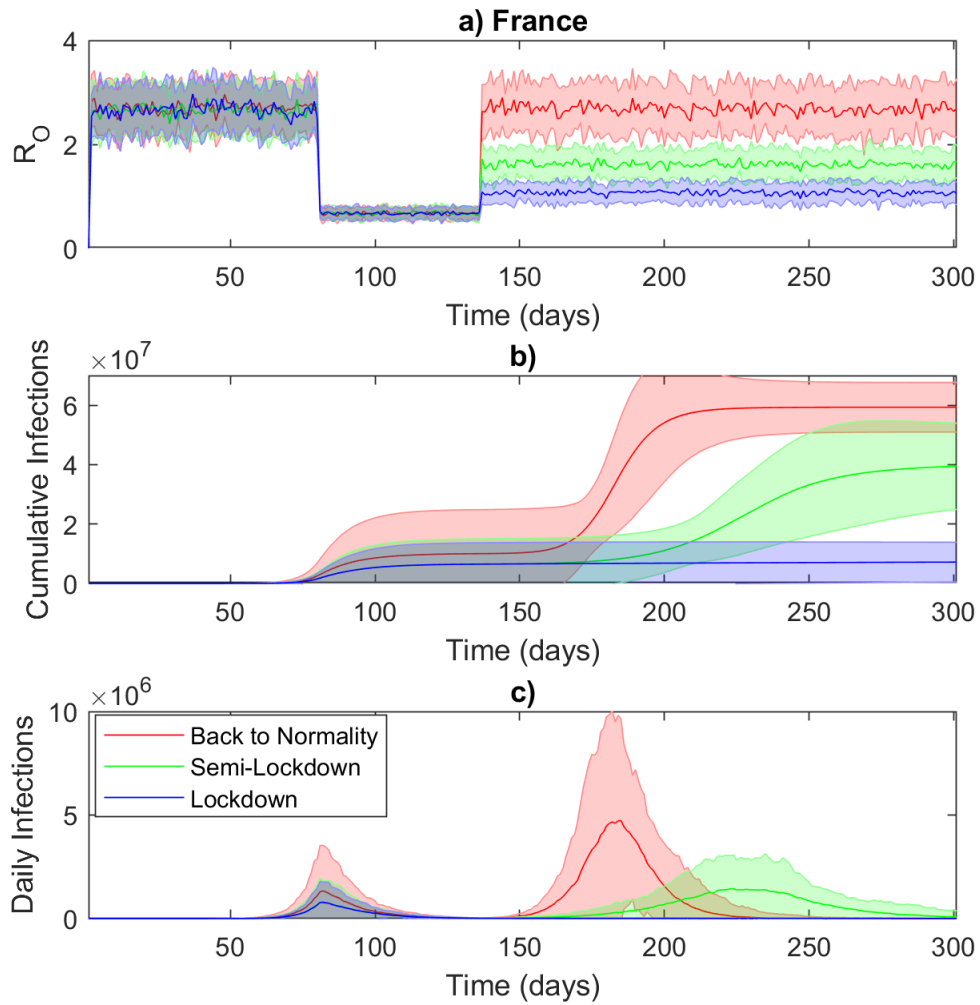


FIG. 3. Susceptible-Exposed-Infected-Recovered (SEIR) model of COVID-19 for the second wave in France. Initial conditions are set as in Figure 1. After the confinement is released ($t = 136$, May 11, 2020) three scenarios are modelled: back to normality (red), semi-lockdown (green), lockdown (blue). a) Time evolution for the basic reproduction number R_0 , b) Time evolution for the cumulative number of infections $C(t)$, c) Time evolution for the daily infected individuals $I(t)$. Solid line shows the average for 30 realisations of the SEIR stochastic models, shading extends to 3 standard deviations of the mean.

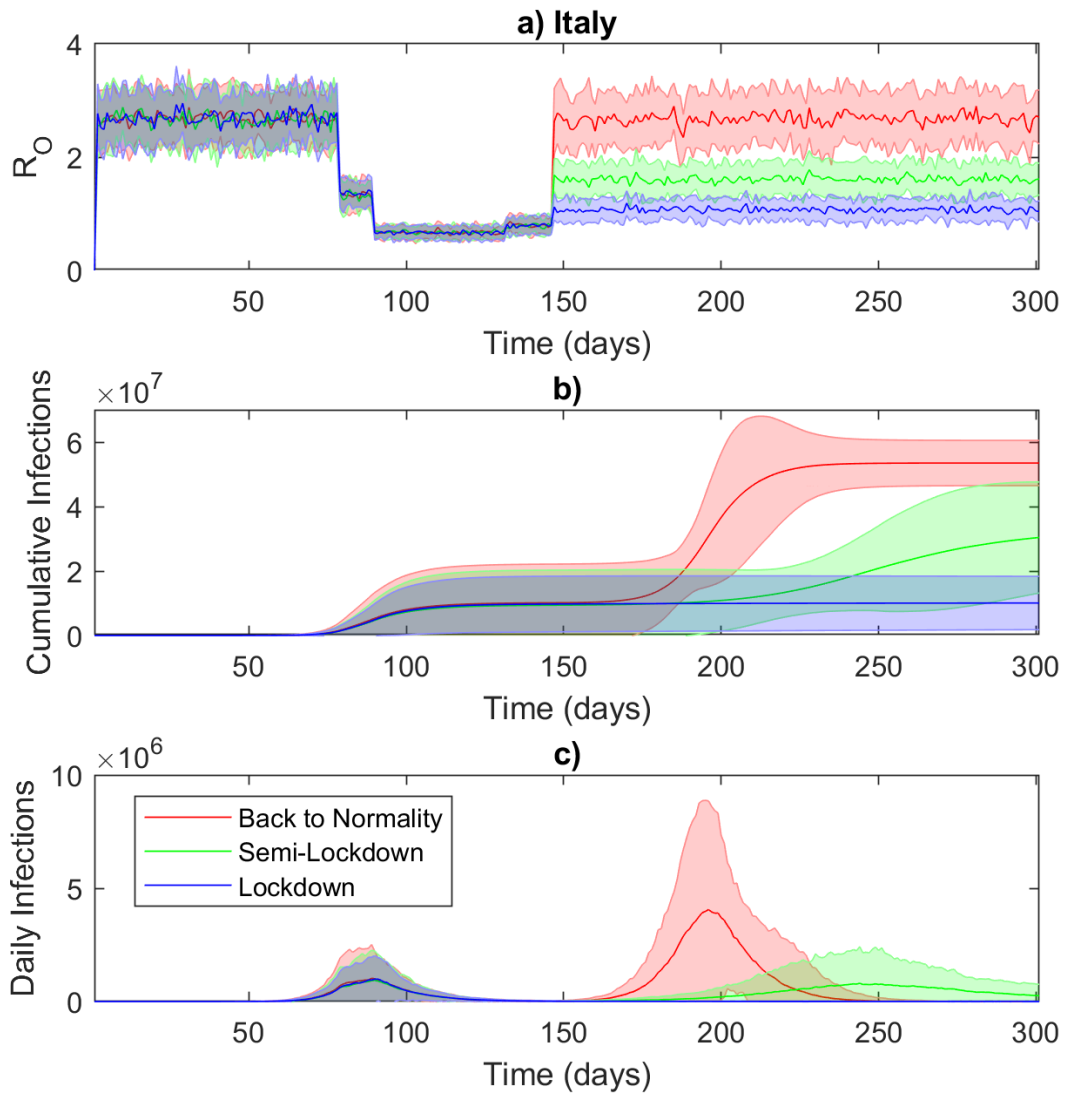


FIG. 4. Susceptible-Exposed-Infected-Recovered (SEIR) model of COVID-19 for the second wave in Italy. Initial conditions are set as in Figure 2. After the confinement is released ($t = 131$, May 4, 2020 and $t = 146$, May 18, 2020) three scenarios are modelled: back to normality (red), semi-lockdown (green), lockdown (blue). a) Time evolution for the basic reproduction number R_0 , b) Time evolution for the cumulative number of infections $C(t)$, c) Time evolution for the daily infected individuals $I(t)$. Solid line shows the average for 30 realisations of the SEIR stochastic models, shading extends to 3 standard deviations of the mean.

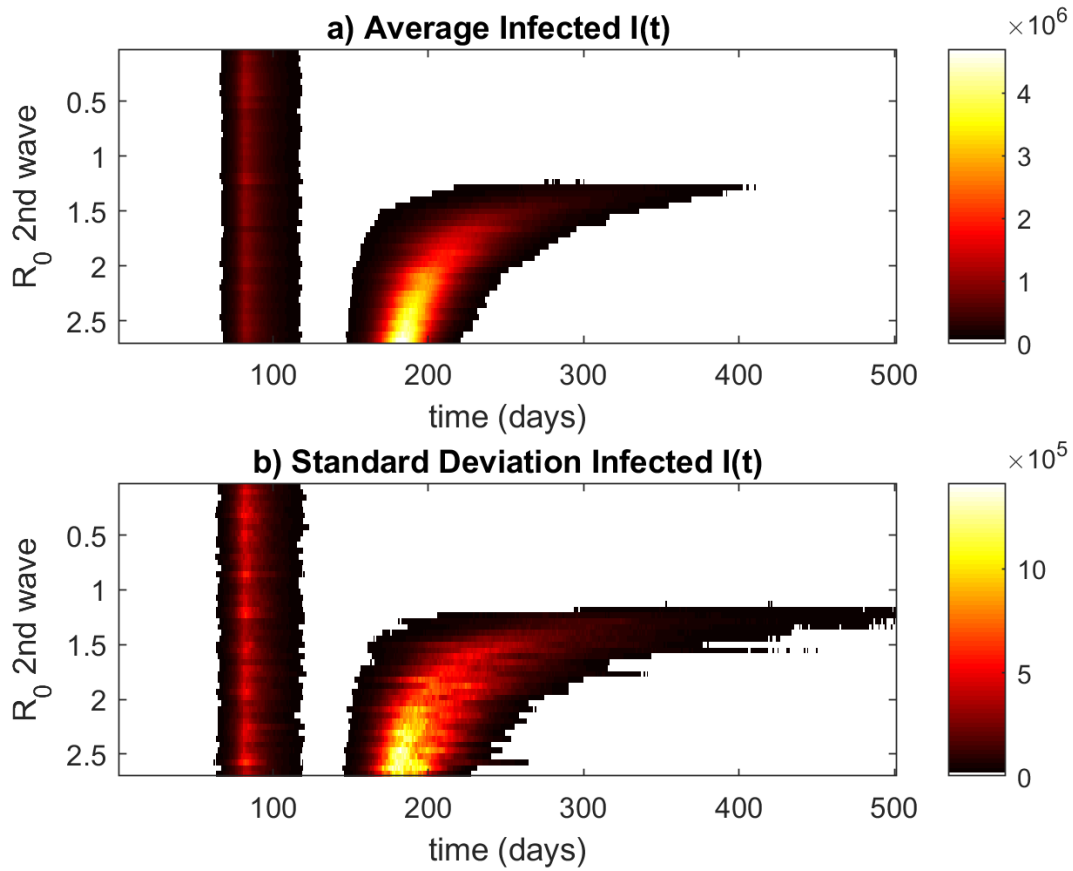


FIG. 5. Phase diagram for the Susceptible-Exposed-Infected-Recovered (SEIR) model of COVID-19 for the second wave in France. Initial conditions are set as in Figure 1. After the confinement is released ($t = 136$, May 11, 2020) all possible R_0 modelled. a) Average of daily infected individuals $I(t)$. b) Standard deviation of daily infected individuals. Diagrams are obtained using 30 realisations of the SEIR models.

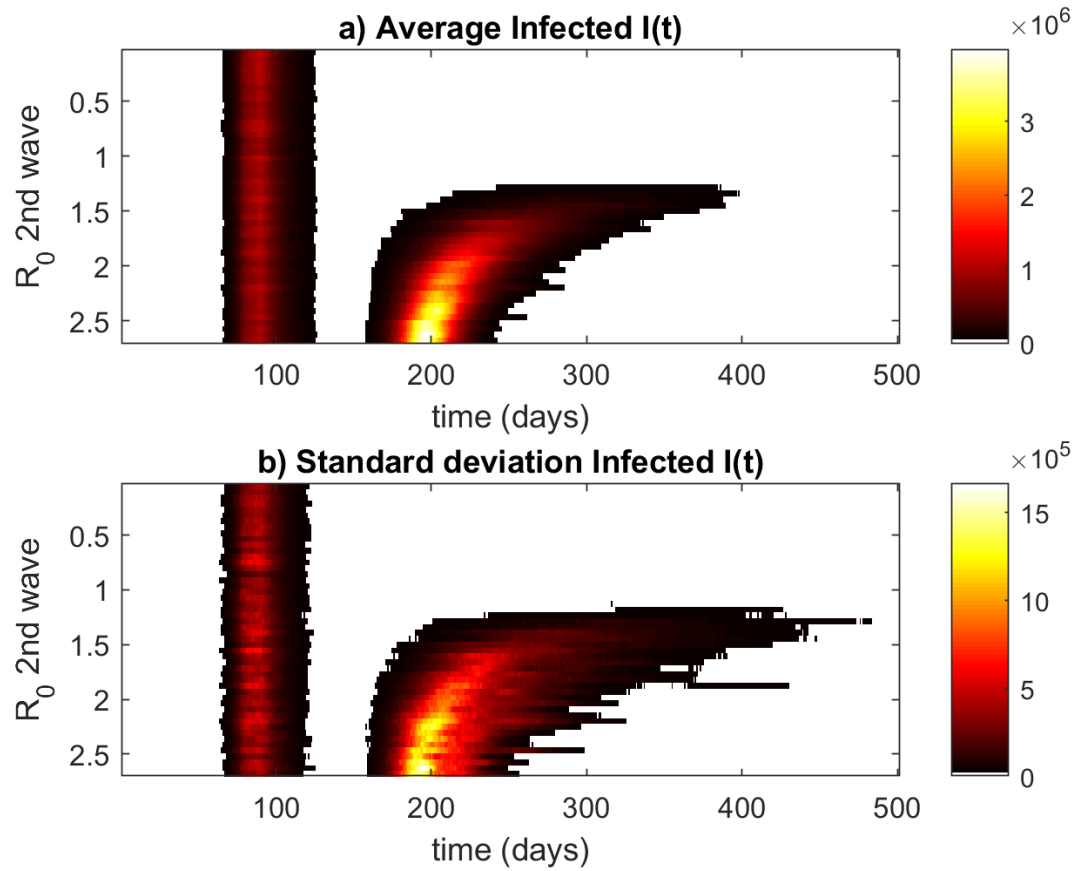


FIG. 6. Phase diagram for the Susceptible-Exposed-Infected-Recovered (SEIR) model of COVID-19 for the second wave in Italy. Initial conditions are set as in Figure 2. After the confinement is released ($t = 131$, May 4, 2020 and then $t = 146$ May 18, 2020) all possible R_0 modelled. a) Average of daily infected individuals $I(t)$. b) Standard deviation of daily infected individuals. Diagrams are obtained using 30 realisations of the SEIR models.