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Abstract. Heat transfer in turbulent forced convection of power-law fluids, in heated horizontal pipe at isoflux conditions, is analyzed by Large Eddy Simulations (LES), with an extended Smagorinsky model. A thermally-dependent fluid is studied at various Pearson numbers ($0 \leq Pn \leq 5$), for two power law indices ($n=0.75$ and 1), at Reynolds and Prandtl numbers $Re_s=4000$ and $Pr_s=1$. The LES predictions are validated through comparisons with the literature at $Pn=0$. The LES predictions allow a better understanding of the physical mechanisms involved in the non-Newtonian thermally-dependent fluid flows: with increasing Pn, the relative viscosity is reduced close to the wall and enhanced towards the pipe center, reducing the turbulent fluctuations and the turbulent heat transfer in the bulk and, as a consequence, the friction factor and the Nusselt number.

Keywords: LES, non-Newtonian, thermally-dependent power-law fluids, turbulence, pipe flow, heat transfer.

1 Introduction

The turbulent flows of non-Newtonian fluids are encountered in a wide range of engineering applications such as flows through ducts, pumps, turbines and heat exchangers, in the petroleum, chemical and food industries. When the Reynolds number is enough large, large eddy simulation (LES) provides an effective tool to predict the effect of the flow parameters on the turbulent fields. Ohta and Miyashita [1] developed a turbulence model that can reproduce DNS results in non-Newtonian fluid flows. They performed LES with a Smagorinsky model, extended according to their DNS results. They showed that this model can more accurately predict the velocity of turbulent flows, for fluids described by Casson's and power-law models, than the standard Smagorinsky model. Gnambode et al. [2] used this extended Smagorinsky model to predict the turbulent pipe flow of power law fluids, for various flow indices ($0.5<n<1.4$), Reynolds numbers ($4000<Re_s<12000$) and Prandtl numbers ($1<Pr_s<100$).

Heat transfer in a turbulent pipe flow of a non-Newtonian fluid has been very few studied by LES. Some theoretical and experimental [4, 5] works focused on the turbulent heat transfer in pipe flows of non-Newtonian fluids for a constant viscosity case.

However still less study has accounted for the temperature dependent viscosity observed in applications.

The objective of this work is to numerically investigate by LES, with an extended Smagorinsky model, turbulent heat transfer in the fully developed pipe flow of a thermally-dependent power law fluid at n=0.75 and 1, Re_s=4000, and Pr_s=1, for $0 \leq Pn \leq 5$, where the Pearson number, Pn, is the dimensionless number measuring the temperature effect on the consistency of a non-Newtonian fluid. The aim is to gain more insights into such complex fluid flows whose viscosity is a function of both the temperature and the shear rate.

2 Governing equations and numerical procedure

The present study deals with the fully developed turbulent flow and heat transfer of power law fluids in pipes whose wall is heated at a constant heat flux φ_w. The filtered non-Newtonian equations are made dimensionless using the centerline axial velocity of the analytical fully developed laminar profile, $U_{cl}=(3n+1)U_b/(n+1)$, the pipe radius, R, and the reference temperature, $T_{ref}= \varphi_w R/\lambda$, for the velocity, length and temperature scales respectively, where U_b is the bulk velocity and λ the fluid thermal conductivity. The filtered equations (with the continuity eq. $\partial\overline{u_i}/\partial x_i = 0$) write:

$$\frac{\partial\overline{u_j}}{\partial t}+\frac{\partial\overline{u_i}\,\overline{u_j}}{\partial x_i}=-\frac{\partial\overline{P}}{\partial x_j}+\frac{1}{Re_s}\frac{\partial}{\partial x_i}\left[\overline{\gamma}^{n-1}e^{Pn\overline{\Theta}}\left(\frac{\partial\overline{u_i}}{\partial x_j}+\frac{\partial\overline{u_j}}{\partial x_i}\right)\right]+\frac{\partial\overline{\tau_{ij}}}{\partial x_i} \tag{2}$$

$$\frac{\partial\overline{\Theta}}{\partial t}+\frac{\partial}{\partial x_j}\left(\overline{u_j}\,\overline{\Theta}-\bar{\tau}_{\Theta j}\right)-\overline{u_z}\frac{d}{dz}<T_w>=\frac{1}{Re_s Pr_s}\frac{\partial^2\overline{\Theta}}{\partial x_k\partial x_k} \tag{3}$$

where the dimensionless temperature is defined by $\Theta=(<T_w>(z)-T(r, \theta, z, t))/T_{ref}$, T_w is the wall temperature and < > is an average in time and in the periodic directions. The subgrid heat flux tensor is defined by $\tau_{\Theta j}=-\alpha_t(\partial\Theta/\partial x_j)$, with $\alpha_t=\upsilon_t/Pr_t$ the turbulent thermal diffusivity linked to the turbulent viscosity, υ_t, by the turbulent Prandtl number, Pr_t, which is constant for a given flow index n. Preliminary LES carried out with the dynamical Smagorinsky model [2] allowed us estimating Pr_t: for n=1, Pr_t=0.7 and for n=0.75, Pr_t=1.5. The Reynolds and Prandtl numbers of the simulations are defined as $Re_s=\rho U_{cl}^{2-n}R^n/K_0$ and $Pr_s=K_0/\rho\alpha R^{n-1}U_{cl}^{n-1}$. The apparent viscosity η of the fluid is modeled by a power-law: $\eta=K\gamma^{n-1}$, where $K=K_0\exp[Pn(\Theta-\Theta_b)]$, with K_0 the consistency at the bulk temperature T_b, $Pn=bT_{ref}$ the Pearson number and b the parameter of the thermo-dependence. $\gamma=(S_{ij}S_{ij})^{1/2}$ is the shear rate, with the strain rate tensor $S_{ij}=(u_{i,j}+u_{j,i})/2$. The subgrid stress tensor is equal to $\overline{\tau_{ij}}=-2\upsilon_t\overline{S_{ij}}$. In the non- Newtonian Smagorinsky model [1], $\upsilon_t = C_s f_s (f_\eta\, \Delta)^2\overline{S_{ij}}$, where f_s is the van Driest wall damping function and $f_\eta=\eta/\eta_w$ the correction function for the change in viscosity.

An in-house finite difference code is used to solve the above LES model in a 3D cylindrical domain of axial length L_z=20R so that the largest thermal structures are well captured. Periodic B.C. and a uniform grid are used in θ and z directions. No slip

B.C. and a non-uniform grid refined close to the wall are used in the radial direction. The mesh size is $N_\theta \times N_r \times N_z = 65^3$. More details on the non-Newtonian Smagorinsky model, numerical methods, simulation parameters and validations are given in [2].

3 Results and discussion

3.1 Mean velocity and RMS profiles

The mean streamwise velocity profiles, $U^+ = U/U_\tau$, scaled by the friction velocity $U_\tau = (\overline{\tau_w}/\rho)^{1/2}$, are depicted in Fig. 1, for Re_s=4000, Pn=0 and Pr_s=1, as a function of the wall distance $y^+ = \rho U_\tau (r-R)/\eta_w$ with $\eta_w = (K\gamma^{n-1})_w$ the viscosity at the wall. These profiles are in satisfactory agreement with the DNS results by Rudman et al. [3] for n=0.75, and with the well-known universal law for n=1. The profiles of the root mean square (RMS) of the velocity fluctuations also agree well with the DNS data [3], Fig 2. Since, for the shear thinning fluid (n=0.75), the apparent viscosity is smaller close to the wall and larger in the bulk than with a Newtonian fluid (n=1), it is normal that the mean velocity is larger and the RMS smaller in the flow core (for y^+>20) for n=0.75.

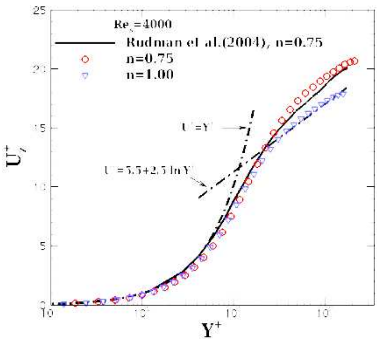

Fig. 1. Effect of n on the axial velocity.

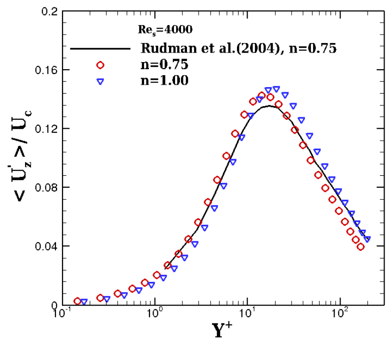

Fig. 2. Effect of n on the RMS of U_z'.

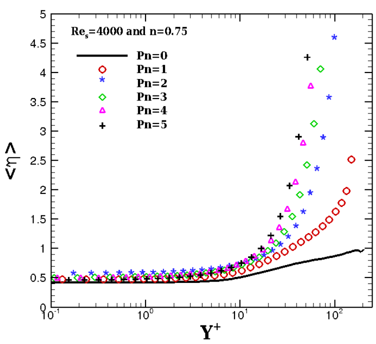

Fig. 3. Effect of Pn on the mean viscosity.

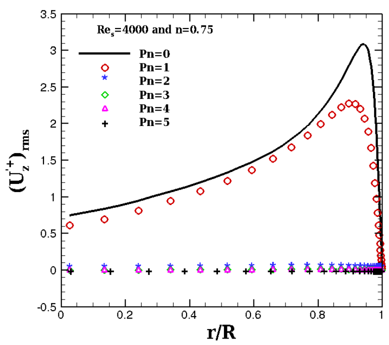

Fig. 4. Effect of Pn on the RMS of U_z'.

For the Newtonian fluid (n=1), the viscosity is constant when Pn=0 because $\eta=K_0$. In accordance with the experiments [4 5], for a given $n\leq1$, the viscosity $\eta=K\gamma^{n-1}$ increases towards the duct center with increasing Pn (see Fig. 3 for n=0.75) because the shear rate γ decreases and the consistency $K=K_0\exp[Pn(\Theta-\Theta_b)]$ increases towards the center, since Pn >0 and the duct wall is heated: $\Theta-\Theta_b = T_b-T$ is negative close to the wall but positive in the core region. The fluid becomes more rigid towards the pipe center and leads to a monotonous decrease of all the velocity fluctuations (normal and parallel to the wall) when Pn increases (see Fig. 4 for the axial velocity fluctuations). Note that all the fluctuations are "killed" towards the pipe center for $Pn\geq3$.

On the other hand, a non-monotonous evolution of the mean axial velocity with increasing Pn is observed in the log-region at n=0.75 (Fig. 5) and n=1 (not shown). This behavior is correlated with the non-monotonous evolution of the mean wall shear stress $\bar{\tau}_w = K\bar{\gamma}_w$ or friction factor $f=2\bar{\tau}_w/\rho\bar{U}^2$ (Fig. 6). On Figs. 5 and 6, f decreases and the flow accelerates when Pn increases from 0 to 2 and f increases and the flow decelerates when Pn increases from 2 to 5. This is due to the competition, in the wall boundary layer, between the decreasing consistency due to heating and the increasing shear rate when Pn increases.

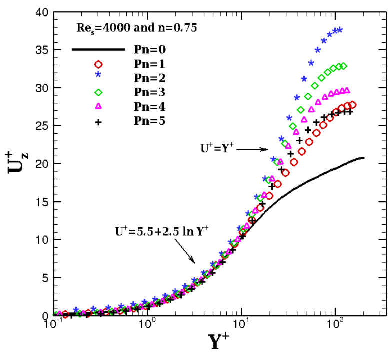

Fig. 5. Effect of Pn on the axial velocity.

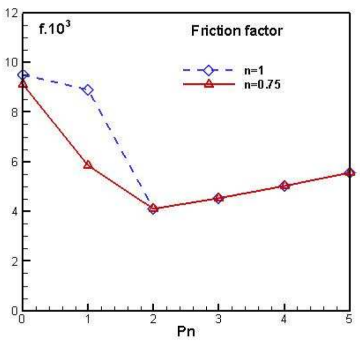

Fig. 6. Effect of Pn on the friction factor.

3.2 Temperature, RMS and turbulent heat fluxes

As already seen above, due to the strong increase of the apparent viscosity in the bulk flow when Pn increases, the velocity and temperature fluctuations decrease from Pn=0 to 2 and become nearly non-existent beyond (Fig. 7). As a consequence, with increasing Pn, the turbulent wall-normal heat flux undergoes a noticeable reduction (Fig. 8): it is strong at Pn=0, twice smaller at Pn=1 and nearly zero beyond due to the simultaneous decrease of the fluctuating radial velocity and temperature. The same behavior is observed for the axial turbulent heat flux $<U'_z\Theta'>$ (not shown here).

To help reading Fig. 9, we remind that $\Theta-\Theta_b = T_b-T$ is negative close to the wall and positive in the core region. Therefore the Θ-increase corresponds to a reduction of the temperature T. Thus Fig. 9 shows that the fluid is the hottest and the most thermally homogenous at Pn=0, thanks to the strong turbulent radial heat flux which controls heat transfer (Fig. 8). When Pn increases from 0 to 2, the temperature T significantly decreases towards the pipe center due to the decrease of the turbulent radial heat flux and the smaller residence time of the fluid particles in the channel because the fluid accelerates (Fig. 5). As a consequence the average Nusselt number, Nu, strongly decreases from Pn=0 to 2 (Fig. 10). When Pn increases from 2 to 5, the flow decelerates and heat transfer is controlled by the mean axial convection because the turbulent radial heat flux is nearly zero: the residence time increases and the fluid is more heated by radial molecular diffusion (Fig. 9). As a consequence, Nu is small and nearly constant (Fig. 10).

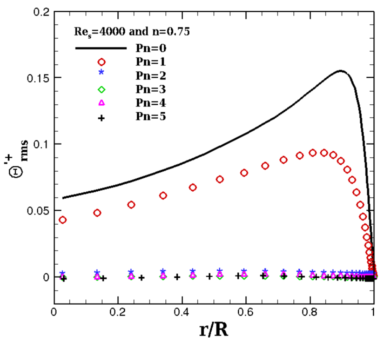

Fig. 7. Effect of Pn on the RMS of Θ'.

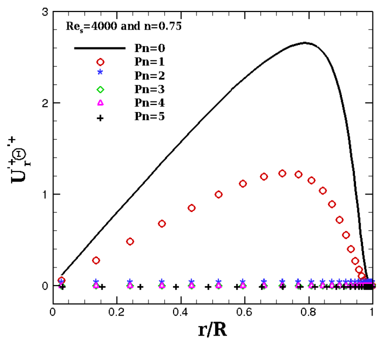

Fig. 8. Effect of Pn on the radial heat flux

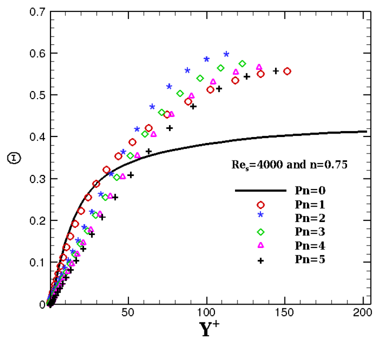

Fig. 9. Effect of Pn on the temperature

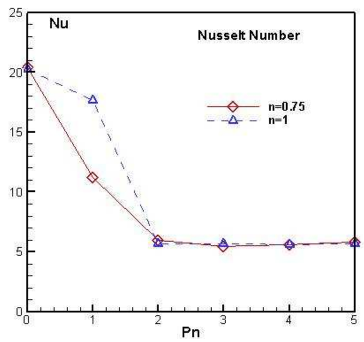

Fig. 10. Effect of Pn on the Nusselt number

3.3 Visualizations

To show the effects of Pn and n on the flow, the resolved axial velocity fluctuations are presented on Fig.8, in the full cylindrical plane (θ, z) at $y^+ \approx 15$. For the Newtonian fluid (n=1), with increasing Pn, the turbulent structures are less random and the streaks are larger indicating a less developed turbulence. For the shear-thinning fluid (n=0,75), the number of streaks is reduced compared with n=1 and much longer streaks appear, particularly at Pn=1, suggesting a reduction of the turbulence and, as a consequence, of the heat transfer. The reduction of the turbulence is more pronounced at n=0.75 than at n=1, due the augmentation of the viscosity towards the pipe center.

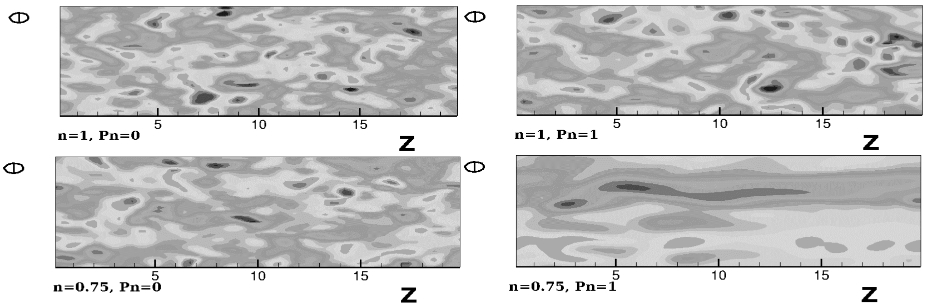

Fig. 11. 2D field of the resolved axial velocity fluctuations in the plane (θ, z) at $y^+ \approx 15$.

4 Conclusions

This study is the first contribution that uses LES, with an extended Smagorinsky model, to simulate the fully developed turbulent flows and heat transfer of thermally dependent power-law fluids in pipes, under isoflux conditions. These LES enable to analyze these complex fluid flows with shear rate and temperature dependent viscosity. The Pearson number effect on the flow and thermal fields is particularly studied. For the non-thermally dependent viscosity case (Pn=0), the axial velocity profiles for the shear-thinning and Newtonian fluids, as well as the friction factors and Nusselt numbers, are in reasonably good agreement with the findings of the literature. With increasing Pn (thermally dependent viscosity case), the mean viscosity is enhanced towards the pipe center. As a consequence, the RMS of the temperature Θ'^+, just like the rms of U_z', is reduced when approaching the pipe center, and the Nusselt number undergoes a significant reduction when Pn increases. From Pn=0 to 2, this leads to a decrease of the turbulent radial heat flux, the acceleration of the flow and the temperature reduction in the log-region.

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