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Optimal Design of Piezoelectric Transformers: A Rational Approach Based on an Analytical Model and a Deterministic Global Optimization

Francois Pigache, Frédéric Messine, and Bertrand Nogarede

Abstract—This paper deals with a deterministic and rational way to design piezoelectric transformers in radial mode. The proposed approach is based on the study of the inverse problem of design and on its reformulation as a mixed constrained global optimization problem. The methodology relies on the association of the analytical models for describing the corresponding optimization problem and on an exact global optimization software, named IBBA and developed by the second author to solve it. Numerical experiments are presented and compared in order to validate the proposed approach.

I. INTRODUCTION

SINCE the first demonstrator of piezoelectric transformer made by Rosen in 1956 [1], the improvement of ceramics properties and various available shapes have extended the applications of these devices. Its low bulk, high efficiency and power density, high degree of insulation, and nonmagnetic properties fulfill the actual needs of the miniaturized devices. Tube photomultipliers supplies, backlight of LCD screens, cold cathode lamps, AC/DC or DC/DC converters, galvanic insulation for sensors, and gate drivers are some examples of the broad range of applications which need specific requirements.

The applications require different conditions of operating frequency, voltage ratio, acceptable temperature rise, etc. All of these properties are strongly dependent on the dimensioning, the shape, the mechanical, and the electrical properties. For example, by considering only a radial mode transformer, the design parameters are the radius, the thickness, and the number of layers of primary and secondary sides. The ceramics available on the market offer many combinations of mechanical and electrical properties which influence the global stiffness, electromechanical coupling, and the dielectric and mechanical losses. Moreover, the performances of piezoelectric transformers are strongly affected by the supply voltage, the operating frequency, and the load. Therefore, a precise dimensioning

of the transformer must be chosen in accordance with the expected conditions.

As a result, compromises must be done by the choice of these different parameters in order to obtain the best performances according to the main criteria. This problem becomes quickly complex if some constraints are not fixed empirically and, in particular, when a multi-objective optimization is expected (minimal primary capacitance with a maximal efficiency, compactness, etc.). Moreover, this optimization problem groups continuous and discrete variables, corresponding to the geometrical parameters and the ceramic type.

This paper deals with a rational methodology to design radial mode transformers, dimensioned for an electronic ballast application. Our first wish is to stay free from arbitrary constraints which are not directly imposed by the schedule of conditions. Then, in addition to the constraints of the application, technical limitations of manufacturers will be taken into account in the design elaboration. The results obtained are discussed and compared with another design method deduced for the same type of applications in order to emphasize the advantage of our rational approach.

Nowadays, the design of an electromechanical actuator is understood and expressed as an inverse problem: From the characteristic values fixed by a schedule of conditions, obtain the structure, the composition, and the dimensions of the actuator.

This inverse problem of design is formulated as a mixed constrained global optimization problem:

$$\left\{ \begin{array}{l} \min_{\substack{x \in \mathbb{R}^{n_r}, z \in \mathbb{N}^{n_e}, \\ \sigma \in \prod_{i=1}^{n_c} K_i, b \in B^{n_b}}} f(x, z, \sigma, b) \\ g_i(x, z, \sigma, b) \leq 0 \forall i \in \{1, \dots, n_g\} \\ h_j(x, z, \sigma, b) = 0 \forall j \in \{1, \dots, n_h\}, \end{array} \right. \quad (1)$$

where f is a real function, K_i represents an enumerated set of categorical variables (the type of piezoceramic in the present study), $B = \{0, 1\}$ the boolean set, and \mathbb{N} and \mathbb{R} the integer and real sets, respectively.

In [2], we solve exactly this problem by associating analytical models of electrical machines with a deterministic global optimization code, named IBBA for interval branch and bound algorithm which has been developed by the second author [3].

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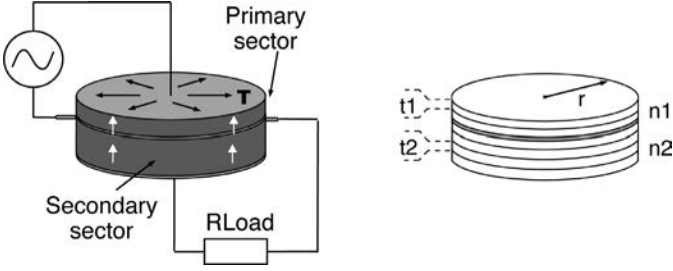


Fig. 1. Architecture and parameters of radial mode transformers.

In this paper, IBBA is used for solving the inverse problem of design of a radial mode transformer. In Section II, the analytical model of a transformer is detailed. IBBA is then recalled in Section III and applied to solve problems with categorical variables. Some numerical tests are presented in Section IV in order to validate our approach. These results are compared with those presented in [4].

II. ANALYTICAL MODEL

A. Radial Mode Transformer

All piezoelectric transformers (PT) are based on a double electromechanical conversion using the direct and inverse effect properties of the piezoceramics. This double electromechanical coupling comes from the mechanical association of two piezoelectric elements which compose the primary and secondary sectors. These lead-zirconate-titanate (PZT) elements are able to work according to three elementary coupling modes: longitudinal, transversal, and shear.

The radial mode transformer is made from two ring-shape ceramics polarized in the thickness direction, and the mechanical stress is generated along the radius. This transformer is classically used for high-power applications (some ten watts) with relatively low voltage gain. The provided power density is approximately three times higher than for a Rosen-type transformer with the same volume [5]. The ratio between the number of primary and secondary layers is an easy way to choose the ideal voltage gain N .

Fig. 1 shows the principle of electromechanical coupling and the different geometrical variables; r the radius, and $t_1 n_1$ and $t_2 n_2$, respectively, the thickness and the number of primary and secondary layers. The optimal design problem relies on the interpretation of the PT by an electrical equivalent circuit.

B. Analytical Model of the Transformer

The characterization and simulation of piezoelectric transformers is currently based on the electrical equivalent model describing the dielectric and mechanical resonance behaviors. Around one vibratory mode, this equivalent scheme can be illustrated by using passive components and an ideal transformer; refer to Fig. 2. The input

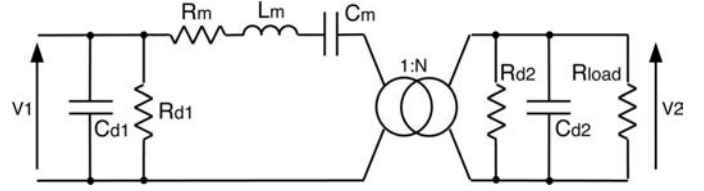


Fig. 2. Electrical equivalent circuit around one vibratory mode.

TABLE I
EQUATIONS OF EQUIVALENT CIRCUIT.

Primary capacitance	$C_{d1} = \frac{r^2 \cdot n_1 \cdot \pi \cdot \epsilon_{33}^T}{t_1} \left[1 - \frac{d_{31}^2}{(1-\sigma)\epsilon_{33}^T \cdot S_{11}^E} \right]$
Secondary capacitance	$C_{d2} = \frac{r^2 \cdot n_2 \cdot \pi \cdot \epsilon_{33}^T}{t_2} \left[1 - \frac{d_{31}^2}{(1-\sigma)\epsilon_{33}^T \cdot S_{11}^E} \right]$
equivalent inductance	$L_m = \frac{\rho \cdot S_{11}^{E^2} \cdot (1-\sigma)^2 \cdot (n_1 t_1 + n_2 t_2)}{16 \cdot \pi \cdot (n_1 d_{31})^2}$
Equivalent capacitance	$C_m = \frac{32 \cdot r^2 \cdot (d_{31} \cdot n_1)^2}{\pi \cdot S_{11}^E \cdot (n_1 t_1 + n_2 t_2) \cdot (1-\sigma)}$
Equivalent impedance	$R_m = \frac{\sqrt{2\rho \cdot S_{11}^{E^3} \cdot (1-\sigma)^3} \cdot (n_1 t_1 + n_2 t_2)}{32 \cdot Q_m \cdot d_{31}^2} \cdot \frac{1}{n_1^2 \cdot r}$
Prim. coupling factor	$\psi_1 = 2\sqrt{2}\pi \cdot r \cdot n_1 \cdot \frac{d_{31}}{S_{11}^E (1-\sigma)}$
Sec. coupling factor	$\psi_2 = 2\sqrt{2}\pi \cdot r \cdot n_2 \cdot \frac{d_{31}}{S_{11}^E (1-\sigma)}$
Ideal voltage gain	$N = \frac{\psi_1}{\psi_2} = \frac{n_1}{n_2}$

voltage $V1$ is a sine wave in the vicinity of the resonance frequency in order to avoid the excitation of spurious vibratory modes.

All of these equivalent parameters can be expressed from the geometrical, electrical, and mechanical properties of the PT, obtained by the derivation of a physics-based equivalent model or the Hamiltonian approach. The study developed by Lin in [5] is relatively complex and will not be detailed in this paper. The main equations are recalled in the Table I.

Considering only one vibratory mode, the relations in Table I are relatively simple and they are based on some main piezoelectric parameters, collected in Table II.

Another assumption is considered about the mechanical quality factor. Indeed, the transformer is composed of two ceramics, electrodes, and sometimes an insulating layer which can decrease the global mechanical quality fac-

TABLE II
PROPERTIES OF PIEZOELECTRIC CERAMICS.

	Definition	Unit
ϵ_{33}^T	Relative Permittivity	$F \cdot m^{-1}$
S_{11}^E	Elastic compliance	$m^2 \cdot N$
d_{31}	Piezoelectric coefficient	$C \cdot m^{-2}$ or $m \cdot V^{-1}$
ρ	Density of materials	$kg \cdot m^{-3}$
σ	Poisson's coefficient	
$\tan(\phi)$	Dielectric losses	
Q_m	Mechanical Quality factor	

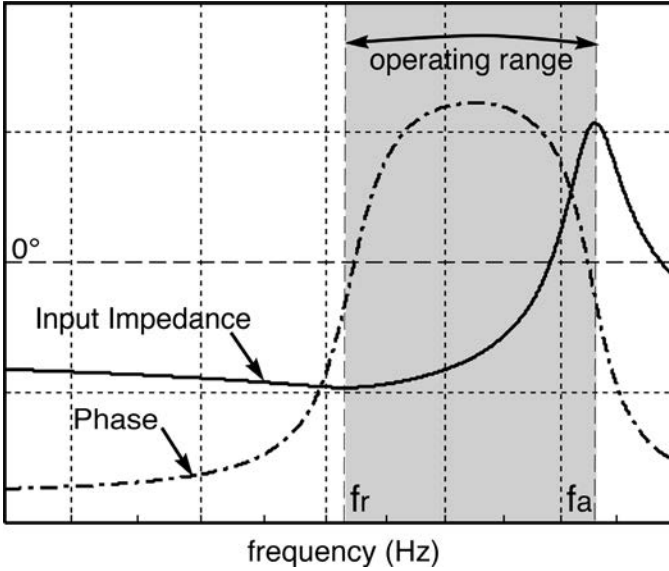


Fig. 3. Input impedance and phase as functions of the operating frequency.

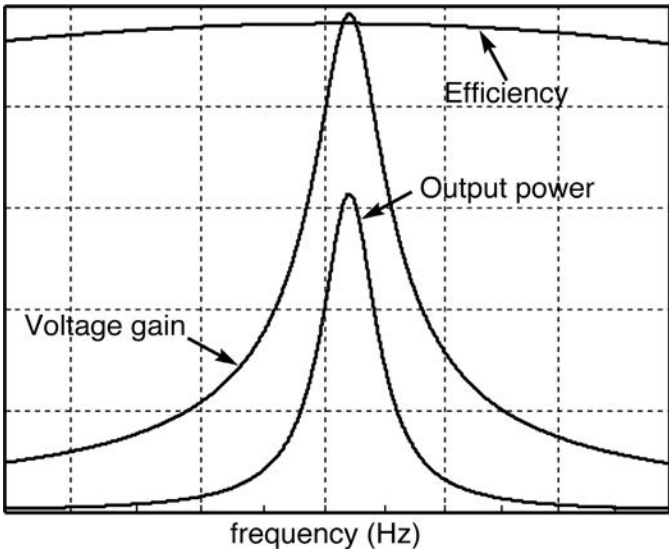


Fig. 4. Typical characteristics as functions of the operating frequency.

tor. In our study, gluing and assembling are considered to be perfect, and the Q_m values are obtained from data of the manufacturers.

The simulations of the equivalent circuit provide typical curves and illustrate the electrical behavior of the PT. First, the input impedance is essential to dimension the supply of the PT. Fig. 3 illustrates the input impedance and the phase of the classical PT. The frequency zone bounded by the resonance and the antiresonance frequencies corresponds to the zone of interest, as confirmed by Fig. 4. These two specific frequencies are defined later.

The resistive load and the operating frequency are important variables which strongly affect the performances such as the efficiency and the voltage gain. It can be noticed that the voltage gain and the output power are

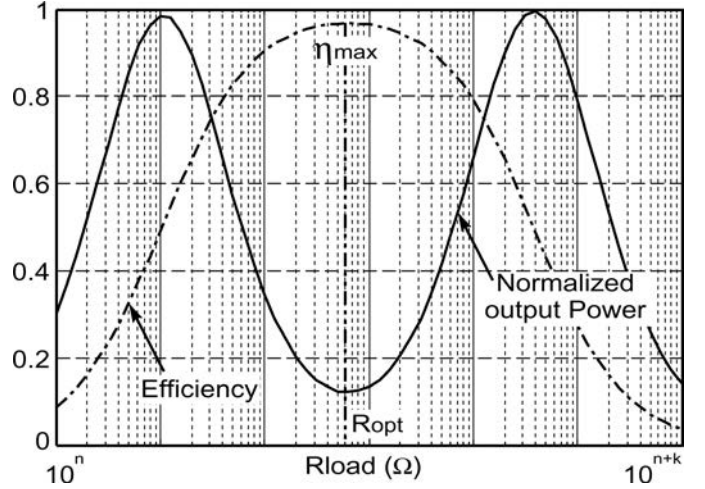


Fig. 5. Typical characteristics as functions of the resistive load.

strongly increased for a specific operating frequency. Consequently, the choice of this operating frequency will be a criterion of optimization. Another essential characteristic is the variation of the output power and the efficiency depending on the resistive load. This load affects the resonance properties of the PT. Indeed, the resonance frequency is minimal when the secondary is short-circuited, and is slightly increased with the resistive load. As a consequence, the schemes on Fig. 5 are given as functions of the resistive load and by following the resonance point.

The maximal efficiency is reached at the point where the output power is minimal. For an optimal functioning of the PT, the dimensioning must make it possible to fit the optimal value with the rated resistive load.

This optimal efficiency point is characterized by the electrical quality factor expressed as follows [4]:

$$q_i \approx R_{load} \cdot C_{d2} \cdot 2\pi \cdot f \approx 1. \quad (2)$$

The optimal efficiency is reached when the secondary capacitance is dimensioned in accordance with the rated load and frequency. In order to obtain the optimal electromechanical conversion, this frequency will be chosen close to the resonance frequency. From this first definition, other functions are deduced. The equivalent serial capacitance:

$$C_{eq} = \frac{C_{d2} \cdot N^2 \cdot C_m (1 + q_i^2)}{q_i^2 \cdot C_m + N^2 \cdot C_{d2} (1 + q_i^2)}. \quad (3)$$

The resonance frequency:

$$f_r = \frac{1}{2\pi \sqrt{L_m \cdot C_{eq}}}. \quad (4)$$

The antiresonance frequency:

$$f_a = \frac{1}{2\pi \sqrt{L_m \left(\frac{C_{d1} \cdot C_{eq}}{C_{d1} + C_{eq}} \right)}}. \quad (5)$$

The equivalent circuit on Fig. 2 is a useful approach for simulations of electrical behavior under some acceptable

assumptions. However, the technical limitations specific to the PZT ceramics are not taken into account in this linear model. These limitations, itemized below, will be included in the algorithm as constraints:

- Dielectric strength: limitation of the applied voltage,
- Coercive field strength: critical voltage of depolarization,
- Delamination: critical maximal stress,
- Temperature rise: depolarization at the Curie point.

If multilayer ceramics are used for the primary and the secondary sectors, a maximal thickness must be considered in order to avoid the delamination of the ceramics. Furthermore, the diameter of the transformer must be greater than the global thickness ($D > 5T$) to support the radial coupling mode. All of these technical limitations will be considered as constraints in the optimization algorithm, and the model is developed keeping in mind these conditions.

The input and output powers (respectively, P_{in} and P_{out}) are calculated for the selected rated load. From these values, the global temperature rise is obtained by the following equation:

$$\Delta\Theta = P_{\text{in}} \frac{1 - \eta/100}{h_{\text{conv}} A}, \quad (6)$$

where η is the efficiency at the rated functioning point, h_{conv} the convection factor of ceramic ($\approx 15^\circ\text{C}\cdot\text{W}^{-1}\cdot\text{m}^{-2}$), and A the total area. This maximal temperature rise is an essential constraint for the dimensioning, to avoid the critical point of depolarization. Moreover, it depends on the input power and consequently on the input voltage (120 V in the present case).

C. Most Important Criteria to be Optimized

From the previous characteristics of electromechanical behaviors, the main criteria are deduced. First of all, the rated resistive load is deduced according to the expected application.

1. Maximal Efficiency: The essential criterion for any application is to obtain the maximal efficiency for the optimal resistive load R_{opt} . This condition means to fix the electrical quality factor near to the unit value, corresponding to (2). To reach this value, the operating frequency and the secondary dielectric capacitance are the fitting variables. After fixing the electrical quality factor, the maximal efficiency must be as great as possible; this optimal point is calculated as below:

$$\begin{aligned} \eta_{\text{max}} = & R_{\text{opt}} R_{d2}^2 N^2 R_{d1} (R_m^2 N^4 X^2 R_{\text{eq}}^2 + (R_{\text{eq}} + N^2 R_m)^2) / \\ & (((1 + R_{\text{eq}}^2 X^2) (R_{\text{opt}} + R_{d2})^2 R_m^2 N^4 + R_{\text{eq}} (2N^2 R_m + R_{\text{eq}}) \\ & (R_{\text{opt}} + R_{d2})^2) (R_{\text{eq}}^2 + (R_{d1} + 2R_m) N^2 R_{\text{eq}} + \\ & R_m N^4 (R_m + R_{d1}) (1 + R_{\text{eq}}^2 X^2))), \end{aligned} \quad (7)$$

$$\text{with } R_{\text{eq}} = \frac{R_{\text{opt}} R_{d2}}{R_{\text{opt}} + R_{d2}} \quad \text{and} \quad X = C_{d2} \cdot 2\pi \cdot f_r.$$

Contrary to the analytical hand-optimization approach, the computational method allows taking into account the dielectric losses by (7).

2. Electromechanical Coupling Factor: This parameter strongly influences the power density of the transformer, according to the following proportionality,

$$\frac{\text{Power}}{\text{Volume}} \propto k_{\text{eff}}^2 \cdot \epsilon \cdot f_r, \quad (8)$$

with ϵ the permittivity and f_r the resonance frequency. Furthermore, it also determines the operating frequency range where the performances are optimal correspondingly to:

$$k_{\text{eff}} = \sqrt{\frac{(f_a^2 - f_r^2)}{f_a^2}}. \quad (9)$$

Hence, to obtain the best efficiency on a widest range of frequency variation, this parameter must be maximized.

3. Primary Dielectric Capacitance: Depending on the converter topology selected to supply the PT, the primary capacitance is an important element of the optimal design. Indeed, this capacitance induces high reactive current which implies the oversizing of the inverter components (increasing of commutation losses). As a consequence, for a magnetic-less topology, the dielectric capacitance must be minimized. In the case of a resonant inverter, an inductance correctly dimensioned with the primary dielectric capacitance is used in order to operate at the resonance or antiresonance frequency of the PT. For this structure, the input electrical quality factor also implies the minimization of the capacitance.

4. Volume: The volume can also be a criterion of optimization in order to obtain a compact solution for onboard devices. However, the dielectric losses and the internal friction of the PT induce a temperature rise which becomes critical near the Curie point. So, the minimization of the global volume must take into account the technical limit of temperature directly linked to the dissipation area.

The expected optimal design of the transformer sometimes needs to consider simultaneously several criteria. In this case, the optimization problem becomes quickly complex and an analytical method is inefficient if strong simplifications are not done.

III. THE EXACT GLOBAL OPTIMIZATION METHOD: IBBA

Different methods have already been used to solve the design problem of piezoelectric transformers, such as an analytical approach [5], [6] or evolutionary strategy-based algorithms [7]. The analytical approach is applicable if some preliminary variables are fixed (fixed operating frequency, type of ceramic, etc.) and the model is also simplified. Consequently, the arbitrary choice of these constant

parameters limits the possible configurations that can satisfy the performances, especially when the consideration of several optimal criteria is required.

In comparison, the algorithmic method detailed below permits keeping a wide set of variables and thus extracting the exact optimal result among a broad range of intermediate solutions that satisfy the constraints. This method is based on a deterministic approach that permits reaching the global solution for continuous problems. It relies on analytical equations, and interval arithmetic and bound and branch techniques. In addition to the optimal dimensioning, this approach permits integration of discrete variables and, in our application, allows selection of the suitable piezoelectric material type among a broad range of ceramics. In this way, the properties announced by manufacturers are collected and tested together, and the best category is kept for the optimal solution. This advantage is important compared to the simple analytical method because this last solution is generally developed for a particular ceramic and, moreover, by neglecting the dielectric losses. As a result, the maximal efficiency is optimized only by reducing the mechanical losses. These losses and the induced temperature rise are particularly nonnegligible when multilayered ceramics are used.

A. Interval Branch and Bound Algorithm and Extensions

The exact global optimization algorithm used for this work was already developed for other applications [2] [8]–[10]. This algorithm is an extension of an interval branch and bound algorithm named IBBA; see [11]–[13]. It is based on interval analysis which permits computation of bounds for a continuous function over a box (i.e., an interval vector). In [2], this method has been extended to deal with mixed variables: real, integer, categorical, or logical. The detailed algorithm IBBA is given in [2], [3], and is briefly recalled here, supplemented with some particular extensions dedicated to this application.

Algorithm IBBA.

1. Set $X :=$ the initial domain in which the global minimum is sought,
 $X \subseteq \mathbb{R}^{n_r} \times \mathbb{N}^{n_e} \times \prod_{i=1}^{n_c} K_i \times B^{n_b}$.
2. Set $\tilde{f} := +\infty$.
3. Set $\mathcal{L} := (+\infty, X)$.
4. Extract from \mathcal{L} the box for which the lowest lower bound has been computed.
5. Bisect the considered box chosen by its midpoint, yielding V_1, V_2 .
6. For $j:=1$ to 2 do
 - (a) Compute $v_j :=$ lower bound of f over V_j .
 - (b) Compute the lower and upper bounds for the interesting constraints over V_j ; deduction steps using the constraints permit to reduce V_j , [3], [14]
 - (c) if $\tilde{f} \geq v_j$ and no constraint is unsatisfied then
 - insert (v_j, V_j) in \mathcal{L} .
 - set $\tilde{f} := \min(\tilde{f}, f(m))$, where m is the midpoint of V_j , if and only if m satisfies all the constraints.
 - if \tilde{f} is changed then remove from \mathcal{L} all (z, Z) where $z > \tilde{f}$.
7. if $\tilde{f} < \min_{(z, Z) \in \mathcal{L}} z + \epsilon_f$ and the largest box in \mathcal{L} is smaller than ϵ , then STOP.
 Else GoTo Step 4.

Because the algorithm stops when the global minimum is sufficiently accurate less than ϵ_f , and also when all of the sub-boxes Z are sufficiently small, all of the global solutions are given by the minimizers belonging to the union of the remaining sub-boxes in \mathcal{L} , and the minimal value is given by the current minimum \tilde{f} . Generally, only \tilde{f} and its corresponding solution are considered.

The algorithm works by the four distinct following phases, which are completely detailed in the following sub-sections:

1. *Bisection Rules:* This phase is important because it determines the efficient way to decompose the initial problem into smaller ones. In our implementation, all of the components of a box are represented by real-interval vectors. Thus, attention must be paid when the components represent integer, boolean, or categorical variables.

The classical principle of bisection (in the continuous case) is to choose a coordinate direction parallel to which Z has an edge of maximal length. Then, Z is bisected normal to this direction [13]. In our applications (the design of electromechanical devices), a lot of variables are nonhomogeneous, coming from different physical characteristics (number of primary and secondary layers, for example). Hence, the accuracy given by the designer, and represented in the algorithm by the variable ϵ , must be a real vector representing the desired precision for the solution at the end of the algorithm; ϵ_k is fixed to 0, if it represents a discrete (integer, boolean, or categorical) component. Therefore, the bisection rule is modified considering continuous or discrete variables ($\epsilon_k = 0$).

Let us denote by $\omega_i^X, \omega_i^Z, \omega_i^\Sigma$, and ω_i^B the given weights for the real variables x_i , the integer variables z_i , the categorical variable σ_i , and the boolean variables b_i , respectively.

First, the following real values are computed:

$$\Omega = \left\{ \omega_i^X \times \frac{X_i}{\epsilon_i}, \omega_i^Z \times (|Z_i| - 1), \omega_i^\Sigma \times (|\Sigma_i| - 1), \omega_i^B \times (|B_i| - 1) \right\}, \quad (10)$$

where the application $|\cdot|$ denotes the cardinal (i.e., the number of elements) of the considered discrete sets.

The k^{th} variable, which will be bisected, corresponds to the largest real value of the set Ω . Then, it is bisected as follows:

1. $Z_1 := Z$ and $Z_2 := Z$
2. if $(\epsilon_k = 0)$ then (for discrete variables)

$$\begin{aligned} \text{(a) } (Z_1)_k &:= \left[z_k^L, \left\lceil \frac{z_k^L + z_k^U}{2} \right\rceil \right]_I \\ \text{(b) } (Z_2)_k &:= \left[\left\lfloor \frac{z_k^L + z_k^U}{2} \right\rfloor + 1, z_k^U \right] \end{aligned}$$

3. else Z_k is divided by its midpoint, this directly produces Z_1 and Z_2 ,

where $Z_k = [z_k^L, z_k^U]$, respectively $(Z_1)_k$ and $(Z_2)_k$, denotes the k^{th} components of Z , respectively of Z_1 and Z_2 . $[x]_I$ represents the integer part of the considered real value x .

Remark 1: It is more efficient to emphasize the bisection for discrete variables because this involves a lot of important modifications of the so-considered optimization problem (1). In the following numerical examples, the weights for the discrete variable are fixed to 100.

2. Computation of the Bounds: The computation of the bounds is the fundamental part of the algorithm, because all of the techniques of exclusion depend on it.

An *inclusion function* is an interval function such that it encloses the range of a considered function over a box Y . For a considered function f , a corresponding *inclusion function* is denoted by F ; hence, one has $[\min_{y \in Y} f(y), \max_{y \in Y} f(y)] \subseteq F(Y)$, and also, that for all $Z \subseteq Y$, $F(Z) \subseteq F(Y)$, [15].

In the case when continuous functions are considered, interval arithmetic can be directly used to construct interval inclusion functions; please refer to [3], [11], [13], [15] for more details on this subject. In our problem (1), mixed function has to be considered, and then, new kinds of inclusion functions must be introduced. For a mixed function coming from problem (1), these inclusion functions were introduced in [2] and [8].

The following paragraphs recall the techniques used in [2] to construct inclusion functions for mixed functions of type (1), by using interval analysis, for example, [15].

Let \mathbb{I} be the set of real compact intervals $[a, b]$, where a, b are real (or floating point) numbers. The arithmetic operations for intervals are defined as follows:

$$\left\{ \begin{array}{l} [a, b] + [c, d] = [a + c, b + d], \\ [a, b] - [c, d] = [a - d, b - c], \\ [a, b] \times [c, d] = [\min\{a \times c, a \times d, b \times c, b \times d\}, \\ \quad \max\{a \times c, a \times d, b \times c, b \times d\}], \\ [a, b] \div [c, d] = [a, b] \times \left[\frac{1}{d}, \frac{1}{c} \right] \text{ if } 0 \notin [c, d]. \end{array} \right. \quad (11)$$

The above definitions (11) show that subtraction and division in \mathbb{I} are not the reverse operations of addition and multiplication. Unfortunately, the interval arithmetic does not conserve all of the properties of the standard one. The division by an interval containing zero is undefined, and then, an extended interval arithmetic has been developed; refer to [13].

A fundamental theorem of interval analysis is that the *natural extension* of an expression of f into intervals, consisting of replacing each occurrence of a variable by its corresponding interval (which encloses it), and then by applying the above rules of interval arithmetic, is an inclusion function; special procedures for bounding trigonometric and transcendental functions allow the extension of this procedure to a great number of analytical functions. The proof of inclusion is given in [13].

The bounds evaluated in such a way (by the natural extension of an expression of f) are not always accurate in the sense where the bounds can become too large, and several other techniques are often used; refer to [3], [13], [15] for a thorough survey and discussion on this subject.

For our considered design problems, the natural extension into interval has been sufficient.

Interval arithmetic is only defined for continuous real functions, and then, inclusion functions must be extended to deal with discrete variables.

For boolean and integer variables, one must just relax the fact that these variables are discrete: the discrete boolean sets $\{0, 1\}$ become the continuous interval compact sets $[0, 1]$, and the discrete integer sets $\{0, \dots, n\}$, $\{1, \dots, n\}$, or more generally $\{z^L, z^L + 1, z^L + 2, \dots, z^U\}$ are relaxed by the following compact intervals: $[0, n]$, $[1, n]$, and $[z^L, z^U]$. Hence, a new inclusion function concerning mixed variables—boolean, integer, and real—can then be constructed.

For categorical variables, intermediate real univariate functions must be introduced, such as $a_i : K_k \rightarrow \mathbb{R}$. In our practical cases, these real functions depend on only one categorical variable, but this could be easily generalized and extended. Therefore, in an expression of a considered function f , one has:

$$f(x, z, \sigma, b) = \dots *_i a_i(\sigma_k) *_j \dots *_k a_j(\sigma_m) *_l \dots *_m a_l(\sigma_k) \dots,$$

where $*_i$ denotes an operation, for example, $+$, $-$, \times , \div ; hence, f produces a real result.

In IBBA, all categorical variables σ_k are denoted by an integer number beginning at 1 to $|K_k|$. Each of these numbers corresponds to a category previously defined.

In this study, a loop is done on each possible value of the categorical variables. Therefore a constrained global optimization problem with only real and integer variables is solved iteratively. Each time the best value for the objective function is kept.

Therefore, bounds are computed using directly the interval arithmetic tools by relaxing the boolean and integer sets as explained above.

Remark 2: There exist four other different approaches for solving this type of problem (1), detailed in [16]. They are not used in this first study of design.

Remark 3: This is the first time that IBBA has been used for solving an electromechanical design problem with a categorical variable which owns a lot of possible distinct values.

3. Exclusion Principle: The techniques of exclusion are based on the fact that it is proved that the global optimum cannot occur in a box. This leads to two main possibilities by considering a sub-box denoted by X, Z, σ, B (where σ is iteratively fixed to a value):

1. The solution already found, denoted by \tilde{f} cannot be improved in this box: $F^L(X, Z, \sigma, B) > \tilde{f}$; i.e., the lower bound of f over the sub-box X, Z, σ, B is greater than a solution already found; no point in the box can improve the current solution \tilde{f} .
2. It can be proved that a constraint could never be satisfied in the sub-box X, Z, σ, B : $G_k^L(X, Z, \sigma, B) > 0$ or $0 \notin H_k(X, Z, \sigma, B)$.

In the present case, the computation of the bounds is exact and sure; no numerical error can produce wrong lower or upper bounds. Thus, the associated global optimization algorithm is exact and the global optimum is then perfectly enclosed with a given accuracy.

4. *Improving of Bounds for η_{\max}* : The expression of η_{\max} (7) contains a lot of occurrences of some identical variables. Hence, interval analysis generally produces in such a case interval results that are too wide and without interest even if the considered box is quite small. An idea that is used here with efficiency is to construct an underestimation and an overestimation function of η_{\max} such that these two new functions own expressions that reduce the number of occurrences of the variables. Therefore the use of interval analysis over these new expressions produces sometimes better bounds than those performed using directly the expression of η_{\max} .

For η_{\max} we construct the two following estimation functions:

$$\eta_{\text{over}} = \frac{100R_{\text{opt}}}{R_m N^2(1 + q_i^2) + R_{\text{opt}}}, \quad (12)$$

with q_i as the electrical quality factor explained in (2). Eq. (12) corresponds to the efficiency without considering the dielectric losses. Therefore, the result of η_{over} will be always over the η_{\max} .

The under estimation is formulated from an unacceptable value of electrical quality factor $q_i = 2$,

$$\eta_{\text{under}} = \frac{100R_{\text{opt}}}{R_m N^2(1 + 2^2) + R_{\text{opt}}}. \quad (13)$$

This reduction of bound values is computed as follows:

$$\begin{array}{l} \text{if } \eta_{\max}^U > \eta_{\text{over}}^U \text{ then } \eta_{\max}^U := \eta_{\text{over}}^U \\ \text{if } \eta_{\max}^L < \eta_{\text{under}}^L \text{ then } \eta_{\max}^L := \eta_{\text{under}}^L \end{array}$$

This principle of reduction of bounds can be applied to all other functions. This is the first time that IBBA is used with these reduction methods, and numerical results are obtained faster than without it. The formulation of the optimization problem and the results are detailed in the next section.

IV. RESULTS OF OPTIMAL DESIGNS

A. Formulation of the Optimization Problem

In order to formulate the design problem, the first step is to declare the different variables which are real, integer, and of category:

- continuous variables:
 - the radius r ,
 - primary and secondary layers thickness t_1 and t_2 ,
 - resistive load R_{load} ,

TABLE III
BOUNDED VALUES OF THE VARIABLES.

Variables	Values
Radius r	$[4.5e^{-3}; 30e^{-3}]$
Layer thickness t_1	$[20e^{-6}; 2e^{-3}]$
Layer thickness t_2	$[20e^{-6}; 2e^{-3}]$
Layer number n_1	$[1; 50]$
Layer number n_2	$[1; 50]$
Resistive load R_{load}	$[500; 500]$
Ceramics type cat	$[1; 13]$

TABLE IV

Functions	Constraints	Functions	Constraints
Maximal height	$h_{\max} < 15 \text{ mm}$	Maximal height	$h_{\max} < \frac{D}{5}$
Resonance frequency	$80k < f_r < 120k$	Coupling factor	$0.38 < k_{\text{eff}} < 1$
Antiresonance frequency	$f_r < f_a$	Maximal heating	$\Delta\Theta < 200^\circ\text{C}$
Ideal voltage gain	$3 < N < 6$	Layers thickness	$t_1 = t_2$

- integer variables:
 - number of primary and secondary layers n_1 and n_2 ,
- variable of category (integer):
 - type of ceramic.

These variables are bounded by minimal and maximal values, defined by the schedule of conditions and the technical limitations. The most important functions are bounded, too, in accordance with the specifications. In the frame of electronic ballast and specifications considered by [4], the values are presented in Table III.

The IBBA will be performed with a selection of 13 types of piezoelectric ceramic. This selection is composed of soft and hard ceramics produced by two manufacturers (Ferroperm Piezoceramics A/S, Kristgaard, Denmark); and APC International, Ltd., Mackeyville, PA.

Added to these bounded values of the variables, a set of constraints are also defined in Table IV in order to obtain an efficient behavior of the transformer.

In comparison with the classical analytical method which is deduced with an imposed resonance frequency, we suggest here a range of convenient frequency f_r in accordance with the specifications of the applications (zero voltage switching conditions). This newly defined bounded function allows the addition of one degree of freedom in the optimal design problem. The electric quality factor q_i is bounded, too, and the values are chosen to satisfy an optimal efficiency for the rated resistive load.

We also fix a maximal height of the transformer to avoid the delamination. Moreover, this maximal height is con-

TABLE V
RESULTS FOR SINGLE-CRITERION DESIGN.

	Unit	Max η	Max k_{eff}	Min C_{d1}	Reference
r	mm	13.07	11.9	11.8	10.5
t_1, t_2	mm	0.615	1.116	0.558	1.5
n_1		3	3	3	4
n_2		1	1	1	1
Ceramic type		PZ24	PZ26	PZ24	ACP-841
f_r	kHz	83.4	81.37	92.42	98.3
N		3	3	3	4
R_{d1}	k Ω	103.6	50.46	104.3	45.4
R_{d2}	k Ω	310.8	151.4	312.8	181.6
C_{d1}	nF	9.2	12.9	8.25	10.2
C_{d2}	nF	3.07	4.3	2.75	2.54
η_{max}	%	98.8	98.7	98.7	96.5
k_{eff}		0.4	0.448	0.4	0.4
q_i		0.804	1.1	0.8	0.786
$\Delta\Theta$	$^{\circ}\text{C}$	163	183	199	900

strained by a ratio with the diameter of the transformer to promote the radial vibratory mode. The temperature rise at the functioning point must be under 200 $^{\circ}\text{C}$ to avoid the Curie point. This critical thermal point is considered equal for each type of ceramic. Finally, the thickness layers of the primary and secondary sectors are equal for the simplification of manufacturing and computations.

B. Single Criterion and Multicriteria Optimizations

This optimization is performed in the frame of electronic ballast application and the results are compared with the design obtained analytically by [4]. The optimal resistive load, bounded values of frequency, and voltage gain are in accordance with the requirements presented in Table IV. The design optimization performed in this paper is about three essential criteria. First, the algorithm method is applied to each single criterion, i.e., the maximal efficiency, the maximal electromechanical coupling factor, and the minimal input capacitance. Then, an optimal design is carried out in order to satisfy the best compromise between these criteria. Table V groups the obtained results. These designs are in accordance with the imposed constraints (optimal load, satisfactory voltage gain, etc.).

Several comments are noteworthy from the results of single criterion optimizations:

- The optimization algorithm has given some slightly more efficient results than those in [4], depending on the chosen criterion.
- Since the secondary capacitance must be in accordance with the rated load (500 Ω), it is noticeable that stacked ceramics are more suitable than multi-layer ceramics for this type of application.
- For each optimal design, the efficiency is excellent and, therefore, it does not represent a sufficient criterion of optimization. Consequently, a real upgrading of transformer concerns the other parameters such

as frequency, volume, dielectric capacitance, coupling factor, or temperature.

- The expected voltage ratio requires few layers. This implies very high dielectric impedances and good efficiency. Consequently, primary and secondary dielectric impedances may be neglected in the present study so as to simplify the expression of efficiency. This is not an appropriate simplification in the case of an architecture with many layers.
- Among the ceramic types considered (13 types), only hard ceramics present satisfying performances (PZ26, PZ24, and ACP-841).
- Among the widest possible combinations among the parameters of Table III, the dimensions are slightly similar for the different optimized designs. This means that the system of equations is strongly constrained. In the present case, resonance frequency, electrical quality factor, and thermal limitations impose strong constraints on the dimensions.
- About the temperature rise, the maximal value is strongly reduced for the optimal solutions compared to the results of [4]. This value is obtained for the rated resistive load at the resonance frequency. This particular operating point will not be reached since the voltage ratio is higher than expected at this point. However, it underlines a destructive functioning point that must be avoided.
- The compactness is almost similar to the reference solution in [4]; however, the temperature rise is less. Furthermore, the reduction of total thickness permits reducing the risk of delamination.

The performances of each design are satisfactory regarding efficiency, coupling factor, and capacitance. A multicriteria optimization is investigated in order to verify if another design gives a better compromise.

The multi-objective optimization is obtained with the formulation of a new function deduced from the single-criterion results. This “weight factor method” permits considering differently the weight of the criteria.

$$\text{multi}(X) = -\frac{\eta_{\text{max}}(X)}{\max(\eta_{\text{max}})} - \frac{k_{\text{eff}}(X)}{\max(k_{\text{eff}})} + \frac{C_{d1}(X)}{\min(C_{d1})}, \quad (14)$$

where $\max(\eta_{\text{max}})$, $\max(k_{\text{eff}})$, and $\min(C_{d1})$ are the three results obtained during the single-criterion optimization (refer to Table V). The minimization of this function (14) gives a compromise between the criteria, depending on the weights attributed to each criterion. Consequently, several optimal results can also be obtained. However, in this study the dimensions obtained are closer regardless of the criterion considered. So, the multi-objective optimization does not bring an important upgrade of performances.

For this reason, the dimensioning obtained for the maximization of coupling factor k_{eff} is selected to illustrate the results of the proposed method compared with the reference solution. Thus, the theoretical performances (according to the electrical equivalent circuit) are shown in the following figures and discussed.

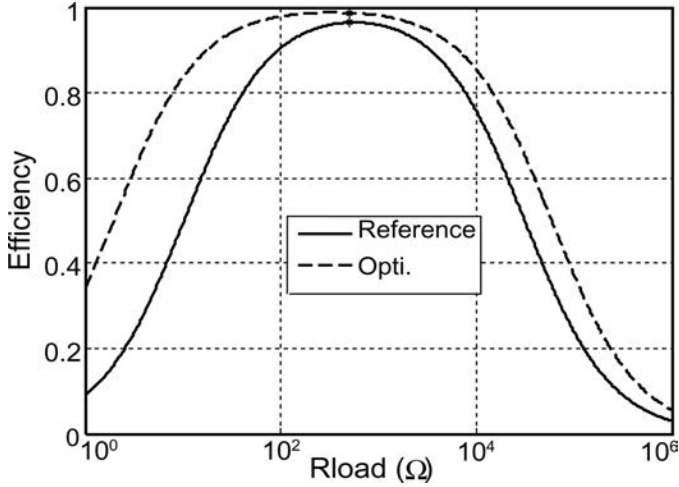


Fig. 6. Efficiency as a function of the resistive load.

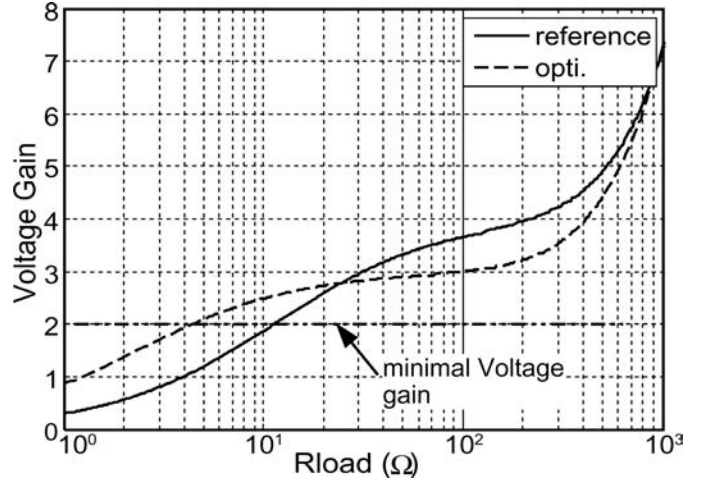


Fig. 8. Voltage gain as a function of the resistive load.

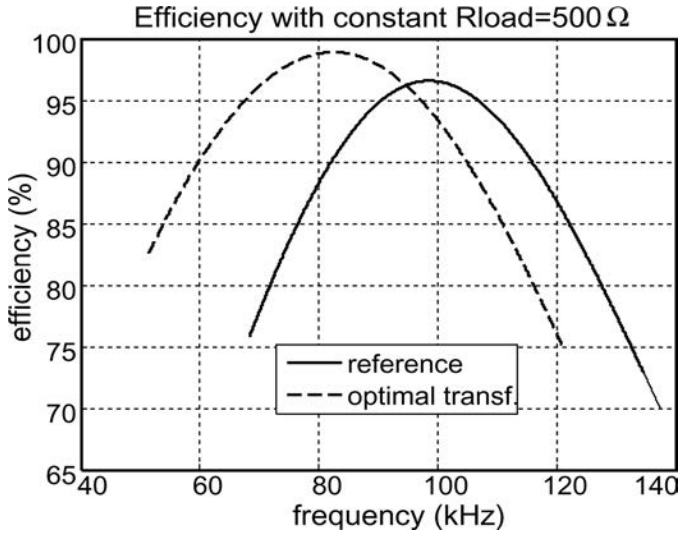


Fig. 7. Efficiency as a function of the operating frequency.

Fig. 6 shows that the optimal solution gives a maximal efficiency slightly better than the reference in [4]. This maximal efficiency is obtained for an optimal R_{load} near the rated value (500Ω), thanks to the electrical quality factor $q_i \approx 1$. The evolution of efficiency at constant resistive load function of the operating frequency is illustrated in Fig. 7. This shows again a greater maximal efficiency, but reached at a lower frequency for the optimal PT. This characteristic still depends on the schedule of conditions and permits limiting the losses in an eventual upstream inverter. Furthermore, the efficiency is high on a widest range of frequency compared with the PT designed in [4].

Verification that the voltage gain for the rated load value is superior to the expected minimal value is shown in Figs. 8 and 9; this voltage gain can be reached by an appropriate operating frequency.

From these results, we can conclude that better control of performances is attained by the proposed algorithmic approach. The association of technical constraints makes

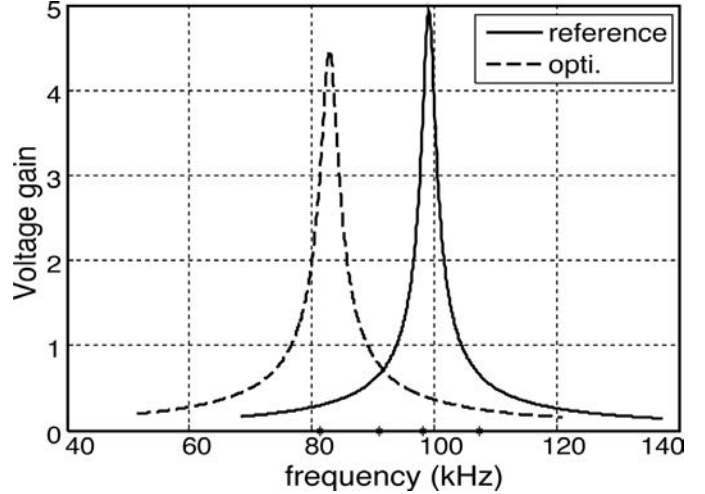


Fig. 9. Voltage gain as a function of the operating frequency.

it possible to determine the best design, avoiding the expensive phase of prototype making. Obviously, the results obtained by this approach depend on the accuracy of the analytical model and on the reliability of the piezoelectric characterization data.

As for all manufactured products obtained by sintering, this conception of ceramics implies an inevitable tolerance of the dimensions. This technical requirement can also be considered in the algorithm design problem: indeed, an epsilon value can be added to each inaccurate variable. Thus, the final result of the algorithm method will be an interval solution for each criterion, depending on the maximal bounded tolerances.

V. CONCLUSION

All piezoelectric transformers require a precise design so as to obtain the optimal performances for specific applications. The considered resonant structures are strongly affected by the operating frequency and the load. There-

fore, this paper deals with the use of IBBA in order to solve the design of PT by the formulation of an inverse problem.

Contrary to a classical analytical optimization which takes into account a single criterion and generally requires a preselected starting point, the present method gives an exact solution (if it exists) for design problems. The IBBA includes a wide set of constraints such as the schedule of conditions, the technical limits, and the realistic properties of the available materials. The variables defined by interval offer more degrees of freedom to satisfy the optimal design without arbitrary constraints.

A typical application of electronic ballast has been chosen in order to illustrate the proposed design method. First, simulations provided from the electrical equivalent circuit have given an overview of the behavior and have also underlined the most important criteria of an optimal functioning. Then, the IBBA has been briefly detailed, including the formulation of the optimal design problem supplemented with some extensions. As a first conclusion, it was noticed that global performances of the chosen test transformer are quite satisfactory compared to the design results of [4]. However, by adding a set of bound parameters in accordance with the conditions, the performances can be significantly improved by a rational choice of both material and dimensions. The present method has shown good accuracy for the chosen step-up transformer application, and it can be adequately applied for specific applications (low-voltage, high frequency, PT matrix, etc.) and other architectures of PT. This study is concentrated on a particular vibratory mode but can be extended to several architectures of transformers for a broad whole of applications. Moreover, a great additional advantage of the proposed design algorithm lies in the possibility of taking into account the technological aspect by defining a precision for each geometrical parameter.

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