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CONSTRUCTION AND JUSTIFICATION OF CENTRAL ANGLE THEOREM IN DYNAMIC GEOMETRY ENVIRONMENT

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Understanding how students construct abstract mathematical knowledge in classroom setting and the use of various instructional tools in facilitating such abstract constructions have been central themes in mathematics education research. This study used Abstraction in Context (AiC) as a theoretical framework to study studying high school students' processes of constructing and justifying of central angle theorem in dynamic geometry environment. Data analysis revealed that the although the central angle theorem was proposed and verified through dragging and measuring tools enabled by a GeoGebra driven simulation, the need to produce a formal proof establishing its validity was not realized by the group, suggesting that the dragging and measuring features of the dynamics geometry environment may pose challenges for teacher and students to construct abstract formal proofs.

Keywords: Geometry and Geometrical and Spatial Thinking, Reasoning and Proof, Technology

Understanding how students construct abstract mathematical knowledge in classroom settings and ways to facilitate the development of abstract generalizations and justifications among learners has been a persistent theme in research in mathematics education for decades (Mitchelmore and White, 2000). Indeed, as it pertains to abstracting and generalizing regarding geometric properties and relationships, scholars have argued, with empirical data, that the presence of dynamic geometry software (DGS) (e.g. GSP, Cabri, GeoGebra) can provide productive means for allowing students to explore, notice relationships, conjecture about potential relationships and construct generalization regarding geometric properties and relationship, with the understanding that such environments also presents challenges when moving students from inductive verification to deductive formal proof (González and Herbst, 2009). However, a majority of work in this area has considered knowledge construction of individuals or small groups of students in presence of DGS. Not many studies have been reported with a focus on exploring the dynamics of generalizing and justifying within a whole group setting, taking into account the impact of peer-teacher interactions on what might be extracted from the DGS and what is assumed collaboratively. The current study aimed to address this gap and is guided by one question: how do students and teacher in a high school geometry classroom construct and justify central angle theorem while working in DGS?

Theoretical Framework

Hershkowitz, Schwarz, Dreyfus (2001) proposed Abstraction in Context (AiC) as a theoretical-methodological framework to investigate the genesis of abstraction. The core of the model consists of three epistemic actions: *recognizing*, *building-with*, and *constructing*. *Recognizing* a familiar structure occurs when a student realizes that a structure is inherent in a given mathematical situation. Recognizing is often, though not always, at the level of empirical thought. *Building-with* consists of combining existing artifacts in order to meet a goal. When operating at the *building-with* level, the student is not enriched with new, more complex structural knowledge. She or he uses available knowledge to produce a viable solution to the problem at hand. *Constructing* consists of assembling knowledge artifacts to produce a new structure, which could be new methods, strategies, or concepts. The three epistemic actions are dynamically nested in such a way that building-with includes recognizing, and constructing includes both recognizing and building-with. This model has been used

to investigate processes of abstraction in a wide range of situations (Dreyfus, 2012). This study will use AiC to examine the process of constructing and verifying central angle theorem in DGS.

Methodology

Data for the study comes from an ongoing program of research which aims to understand how students and teacher in a regular classroom settings interactively work towards constructing and justifying generalizations pertaining to geometric properties and relationships. The participating site is a high school geometry class consisting of 21 students and 1 teacher. All mathematics lessons have been video-recorded daily to capture interactions taking place during instructional activities. All student artifacts have been collected to examine the types of inferences and connections students make individually/collectively based on group interactions.

The data used in this report comes from a particular lesson whose objective was for students to learn central angle theorem. Prior to this lesson students had explored and seemingly learned about (1) all the radii of a circle are congruent, (2) two radii and a chord make an isosceles triangle and the base angles of the isosceles triangle are congruent, (3) inscribed angles subtended by a diameter are right angles. A GeoGebra simulation was created as in Figure 1a for students to explore the relationship between the measure of the central angle and the measure of the circumference angle. In the exploration, the center, point C and the diameter of the circle were locked and point D and E were not fixed. Students were first assigned to work in small groups to explore in the pre-made GeoGebra file and answer the following: (1) As you move D around on the top half of the circumference, what happens to the measure of angle E? (2) As you move E around on the bottom half of the circumference, what happens to the measure of angle O? (3) What do you think is the relationship between angle E and O? And then the teacher initiated a whole-class discussion around their observations.

Findings

When exploring in GeoGebra simulation all the 21 students have *recognized* when moving D around both angle E and angle O become larger or smaller depending on the moving direction and that when moving E around the measures of angle E and angle O stay the same. And when asked the relationship between angle E and O, 18 students had *constructed* the idea that angle E is half of angle O. However, it is unclear students understood why angle E is half of angle O. In the whole group discussion, the teacher asked students to share their observations and tried to help students to see why angle E is half of angle O. The following transcript demonstrates how central angle theorem had been collectively verified in whole group discussion.

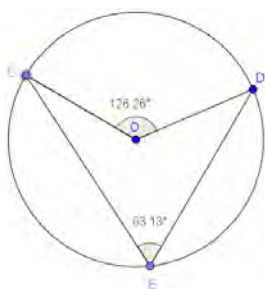


Figure 1a

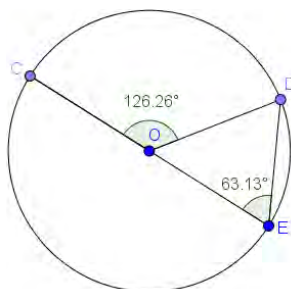


Figure 1b

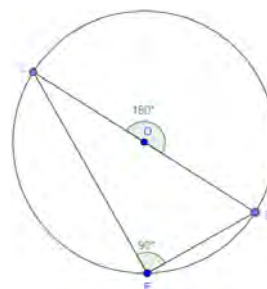


Figure 1c

Teacher: If we move angle D, what happens in general to the two angles that are given to you?
(move D around the circle)

SS: They change.

Teacher: Yes, they are changing. So they are not remaining the same. What about when I move around angle E (move E around the circle)?

SS: They stay the same.

Teacher: They stay the same. Ok, let's put this at...here is a good one, because as just said we can move around E and nothing will change even if I make it overlap like this. That means I can make this as a diameter if I want to. Now it's a diameter [move E to make CE as a diameter as in Figure 1b]. What can I say about what's going on here? Think about last week and raise your hand. [no hand raising in 15 seconds] What congruence marks can I put here? [a few hands raising] Chase, what congruence mark can I put here?

Chase: The middle point and from the middle point to the other side [the teacher then put congruent mark on OE and OD].

Teacher: Anything else, Oliva?

Oliva: OD [the teacher put congruent mark on OD].

Teacher: Ok, these are all congruent. So what's going on here now? Does it help us connect? What do you see Jodi?

Jodi: Isn't like since both them are congruent you can put both angles would be equal?

Teacher: So which angles would be the same?

Jodi: The 63

Teacher: This one here 63.13. Now could I find this angle here [point to angle DOE] suppose I don't know this is 126.26 [point to angle DOC]? How would I do that? Andrew, what do I do?

Andrew: add up 63.13.

Teacher: 63.13 plus 63.13 which is 126.26. Is 126.26 already on there? Hmm, so the sum of these two angles give me this [point to angle DOC]. Are these two angles [point to angle OED and ODE] always going to be congruent no matter where I move that?

SS: Yes.

Teacher: Yes, these two should be congruent. So what do you think the relationship between this angle here E and this [point to angle DOC] we call this a central angle because it comes from the center. What do you think the relationship might be between angle E and the central angle? What do you think, Ethan?

Ethan: It's half of the central angle.

Teacher: So the central angle is twice as much as the circumference angle.

Ethan: Yes

Teacher: So even if I have it looking funky like this [move E up] is that relationship still the same? Is the relationship down here still the same [move E down]?

SS: Yes

Teacher: Yes, that will hold. Here is another way we can prove it. Last week when we looked the diameter as the leg of a triangle [move D to make CD a diameter of the circle as in Figure 1c]. What did we say about this angle here [point to angle E]? What would it always be if this is a diameter? Carol, what do you think?

Carol: 90 degrees.

Teacher: So this is 90 [point to angle E] and this is 180 [point to angle COD]. Does that relationship hold? Is the central angle twice the outside angle?

SS: hmm.

To help students understand why angle E is half of angle O, the teacher created the first special case of central angle theorem and then asked students what congruence marks could be put on the diagram (turn #5). The question triggered students to *recognize* that OC, OE and OD are radii of the

circle (turn # 6 and #8) and therefore triangle DOE is isosceles. *Building-with* these ideas, students concluded angle ODE and angle OED are congruent (turn #10). The teacher then drew students' attention to connect the sum of the two base angles and the central angle (turn # 13 and #15). And students finally articulated that the circumference angle is half of its central angle (turn #18). The teacher then introduced a second special case to connect central angle theorem with inscribed right triangle (turn #23 and #25). Students *recognized* that E is 90 degrees, which is half of 180 degrees. This special case again verified that a central angle is twice as much as its circumference angle.

Discussion

Studies have shown that dragging and measuring features of DGS provides students opportunities to make and empirically verify conjectures and therefore construct new mathematical ideas (González and Herbst, 2009). However, the dragging and the measuring features of this environment also pose challenges for students to produce an abstract formal proof. Even though the teacher introduced two special cases to demonstrate why central angle theorem holds, a more general proof was missing in the whole-class discussion. The first special case has the very potential to be developed into a general proof had the teacher made it clear that why the sum of the two congruent bases angles of the isosceles triangle equals to the central angle. It's unclear in whether the teacher and students used the angles measures or a theorem (i.e. an exterior angle of a triangle is congruent to the sum of the two angles of the triangle that are not supplemental to the given angle) to conclude the sum of the two congruent bases angles of the isosceles triangle equals to the central angle (turn #13, #14, #15). If the conclusion was drawn based on the theorem, a general formal proof can be more easily developed from this special case. However, if the conclusion was drawn based on angle measures of the three angles, a deductive proof might be more difficulty to be obtained.

The data revealed even though central angle theorem was empirically constructed and verified, an abstract mathematical proof of the theorem was not collectively constructed by the teacher and students. In the process of verifying the theorem the teacher and students were mostly *recognizing* previously learned knowledge and partially *building-with* existing mathematical ideas. It seemed that the measuring and dragging functions prevented the teacher and students to construct an abstract formal proof. Further research is needed to explore how to move students to construct abstract proof in dynamic geometry environment.

References

- Dreyfus T (2012) Constructing abstract mathematical knowledge in context. In: *Proceedings of the 12th international congress on mathematical education*, Seoul.
- Dreyfus, T., Hershkowitz, R., & Schwarz, B. (2001). Abstraction in context: The case of peer interaction. *Cognitive Science Quarterly*, 1(3), 307-368.
- González, G., & Herbst, P. G. (2009). Students' conceptions of congruency through the use of dynamic geometry software. *International Journal of Computers for Mathematical Learning*, 14(2), 153-182.
- Mitchelmore, M. C., & White, P. (2000). Development of angle concepts by progressive abstraction and generalization. *Educational studies in mathematics*, 41(3), 209-238.