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Teachers of Mathematics Working and Learning in Collaborative Groups



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Teachers of Mathematics Working and Learning in Collaborative Groups

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COLLABORATIONS BETWEEN TEACHERS AND WITH ACADEMICS IN AN IREM GROUP ON A COLLABORATIVE PROBLEM-SOLVING DEVICE

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The IREMs are a French experience of sustainable collaborative work between teachers and with academics started 50 years ago. We investigate this experience through the study of the collaborative work developed in an IREM group in Montpellier (ResCo), which provides a device for collaborative problem solving between classes, and facilitates collaboration between teachers.

Introduction: context and questions

The IREMs (Instituts de Recherche sur l'Enseignement des Mathématiques) are a well-known structure in the history of mathematics education (see Cortella & Arnoux, 2019). Created in France in the 1960's to facilitate the modern mathematics reform (the IREMs' network turned 50 years in 2019), they remain a specific mathematics-education model. Existing in many universities in France, IREMs are places where mathematics teachers work together and with researchers or university teachers in mathematics, and nowadays also researchers in mathematics education and teacher trainers (all named *academics* in the following), with a key role of collaboration:

Since [their creation], the IREM have contrived to be one of the most active and unavoidable actors in mathematics education in the country, by responding to the three missions entrusted to them:

- research to improve the teaching of mathematics in non-hierarchical groups of university researchers and teachers of the first or second degree school or in higher education;
- Train teachers, especially using the results of network research
- Disseminate the results of research in mathematics, mathematics education or history. (Cortella & Arnoux, 2019)

The IREMs and their network have played a strong role in the development of the didactics of mathematics in France (Brousseau, 2004, Artigue & al, 2019) and they contribute now in the interaction between ground and research, by contributing to the dissemination of research results and foster relationships between research and action (Artigue, 2017).

In the IREM of Montpellier, the ResCo group (for *Résolution Collaborative de problèmes*, Collaborative Problem-Solving) started in the 2000's. In addition to the collaborative work inside the group between teachers and with academics, the ResCo group supports a device of collaborative Problem-Solving between classes and provides every year an original problem that is studied inside this device (ResCo, 2014). The device consists in providing every year a collaborative problem solving session (Collaborative PS session) starting with an initial problem given to the many classes involved (up to 120 classes *i.e.* more than 3000 students some years), about which the classes, associated by groups of three, exchange questions and answers, followed by the sending of a "relaunched" version of the problem, on which the classes search and exchange. The communication between classes, through the teachers, is made on an online forum. This device also includes a teacher training device, on two or three days, oriented towards fostering problem solving in the classroom and preparing teachers to participate in the collaborative PS session.

Currently, the aim of the ResCo group is to foster problem solving and modeling in the classrooms and in teachers' practices (for more details about ResCo see ResCo, 2014, Modeste & Yvain, 2018).

For the ICMI study, we will not detail the collaborative work between classes or students promoted by the device, but we will focus on the collaborative work between the teachers whose classes are involved in the collaborative PS session and between members (teachers and academics) of the ResCo group, in supporting the device and in its others activities. To this end, we will question the history and the dynamic of the group and examine its evolution, and the influence of the involved academics on the group trajectory. Let us consider the following questions:

- What have been the evolution of the ResCo group and its device since its beginning, leading to the current organization? How the academics involved in the collaboration have influenced the trajectory of the group? This will be discussed in the first section (Dynamics and evolution).
- In the ResCo group and device, what are the collaborations at stake? How do they work? This will be discussed in the second section (Nature of the collaborations at stake).
- What are the effects of this collaborative work for the teachers involved in the device? For the members of the ResCo group? For the device itself, and the work of the group? This will be discussed in the conclusion.

Methodology

The results presented are based on the analysis of data of various types: the problems given in the collaborative PS sessions, the contents of the messages exchanged by the teachers through the online forum (including students' productions exchanged by the teachers), the results of a satisfaction questionnaire given every year to the teachers, the notes of the work of the IREM group exchanges during the elaboration of the problem (and the versions of the problem that are discussed), and videos of two teachers implementing the PS session with their students. The analysis of this data have been made in previous work, quoted in the paper. It has mostly been a qualitative analysis, but questions and answers have been analyzed quantitatively, a large part of these analyses have been developed by Yvain-Prébiski (2018).

Dynamics and evolution of the group and influence of the academics

The idea of collaboration between teachers was present from the beginning of the ResCo group. It arose from a project called SFODEM¹ developed in the IREM of Montpellier between 2000 and 2006 (Guin & Trouche, 2005). In this context, in 2001, a "problem-solving" group has been created within the SFODEM (with teachers already involved in this topic) to disseminate problem-solving in teachers' practice based on collaboration between teachers. It was supported and motivated by the development of digital tools (messaging services, chat rooms, forum). The objectives were 1) to foster mathematics problem solving in classrooms, 2) to conceive and disseminate resources, 3) to develop and facilitate a community of practices (Wenger, 1998), and 4) to support teachers with in-service teacher training. We will see that these four aspects are invariant in the history of the group.

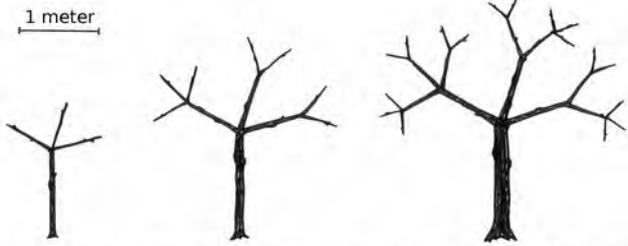
We analyzed the evolution of the group focusing on the objectives of the group, the points of view adopted and the research work associated to the group, the academics involved (as he has, in general, the responsibility of the group, and at least, the responsibility of the quality of the

1 Suivi de Formation à Distance pour les Enseignants de Mathématiques (Monitoring of distance training for mathematics teachers): <http://www.math.univ-montp2.fr/sfodem/GuideSfodem>

mathematical contents), and, as it is central in the topic of the group, the type of problem proposed to the collaborative PS session. We synthesize in table 1 the different periods of the group.

Table 1. Evolution of the group: particularities, and emblematic problems.

Period, context	Objectives / points of view / research	Academics (initials)	Types of problems
2001 to 2002 SFODEM	To develop PS in the practices of teachers, strong anchoring in the idea of exchanges on practices between teachers, use of new digital tools. Emblematic problem: The Art Gallery Problem (see https://en.wikipedia.org/wiki/Art_gallery_problem)	DT, university teacher, mathematician.	Classical problems taken from the mathematical folklore, discrete problems.
2006 to 2009 ResCo group	Context of IBL and PS. Search for realistic problems, concrete situations. Questioning on the link between the realistic problem and the mathematical problem. implementation of a Q-A phase in the session (still existing now). Emblematic problems: - Evolution of a lemming population, - Prevision of loss and gain in a gambling problem (roulette-like).	JD, university teacher, mathematician.	Classical problems from applied mathematics, probabilities.
2009 to 2011 ResCo group	Focus on PS and experimental dimension (heuristic) in PS. Mathematical problems contextualized “out of the mathematical frame”, “intentionally vague” that has to be “mathematized”. Also, solutions of many problems are becoming easily available on internet. Creation of the “relaunched” problem, stabilizing the mathematized problem (still existing now). Emblematic problem: “The Artist. A modern artist wants to create a work of art on a round support, by putting in nails on the rim and stretching strings between the nails. He intends to paint each zone with a different color. How many colors does he need?”	VDG, teacher trainer, didactician, university teacher. Collaborations with another IREM group in Lyon working on IBL and PS.	Mathematical problems identified for their mathematical interest for students’ research and experimental activity (“open problems”, “problems for searching”...) in “a context that is fictional but realistic”.
2012 to 2014 ResCo group	Same context. Formalization of the concept of “Realistic Fictions” (Ray, 2013, and described in Modeste & Yvain, 2018) as “situations that are <i>a priori</i> not mathematical, with a fictional but realistic context, which need a modeling phase to be efficiently handled, and the modeling phase can lead to various mathematical problems according to the choices made”. Emblematic problems: - Anubis Stones: Chomp (https://en.wikipedia.org/wiki/Chomp) in a archeological context. - Leaks at Fukushima (see Gardes & Yvain, 2014; Yvain-Prébiski, 2018): “Under the cooling pound of the reactor 4 of the nuclear power station of Fukushima, TEPCO engineers led by Toshio Nishizawa have founded micro-leaks contaminated by highly radioactive Cesium 137. They decided to send a robot to aspirate the radioactive droplets. Sadly, its guidance device has been damaged by the radioactivity, and it moves by step of 10 cm in the directions north, east, south and west randomly selected. The engineers only know that the robots starts on its charging base and is programmed to make a finite number of moves. Worried, they wonder how many ways the robot has to get to a droplet. Help them!”	EM, university teacher, mathematician.	Problems from the mathematical folklore (combinatorics, games) or classical issues in mathematics (random walks), contextualized according to “realistic fictions”.
2014 to today	Same context with a focus on modeling and making students aware of the necessity of making choices in modeling a situation. Modeling motivated by a goal: predicting, optimizing...	SM, teacher trainer, didactician, university teacher, and	Realistic fictions with two more aspects (Yvain, 2018): the problem is an adaptation of a professional modeling

ResCo group	Grasping of the notions of horizontal and vertical mathematization (Treffers, 1978) adapted by Yvain-Prébiski (2018).	SYP, teacher trainer, didactician.	situation; the values of the didactic variables (Brousseau & Warfield, 2014) are chosen to foster modeling activity.
<p>Emblematic problem: “The tree. Botanists from Botanical Garden have discovered an exotic tree. To study this new species, the botanists have sketched the tree every year since 2013. The botanists want to build a greenhouse to protect it. They believe it will have reached its full size by 2023. To help them, predict how the tree will be in 2023.”</p> <div style="text-align: center;">  <p>Sketches of the tree in November 2013, 2014 and 2015.</p> </div>			

Notes. PS: Problem Solving, IBL: Inquiry-Based Learning, Q-A: Questions-and-Answers.

Since 2014, the focus has been on Modeling and Solving the mathematized problem. The initial problem given to the classes is called “Realistic Fiction”. It is design collaboratively in the ResCo group, taking into account epistemological considerations on epistemology (Yvain-Prébiski & Modeste, in press), the previous experiences of the group, and the expertise of the teachers. This problem lead to a Q-A phase between classes, followed by a “relaunched realistic fiction” that stabilizes a mathematized version of problem with common choices in order to go on working collaboratively between classes. The latter is conceive according to our *a priori* analysis of the produced problem (taking into account the levels of the classes involved) and the exchanges observed during the Q-A phase, in order to remain as close as possible to the students perception of the problem (see figure 1 for a sketch of the ResCo device).

Discussion on the collaborative work

We can notice many aspects and effects of the collaborative work between teachers and with academics. First, we can see invariant global goals in the history of the group: fostering PS in teachers practices, produce and disseminate resources, develop a community of practice, and developing in-service teacher training (the device has always gone along with a teacher-training device). Second, the influence of the academics involved in the group (and its status) strongly impacts the direction of the group (see table 1). Third, beyond that, there is a sort of collaboration over time that can be seen as a dialogue between the different collaborators of the group, because the teachers-members change independently of the academics, but also because the memory of the group is kept up. This continuity and legacy over time permits perpetual improvements over the one-year cycle of the collaborative PS session, foster reflexive perspectives and formalization on the objects involved, and help new directions to develop. Forth, it brings to light a specific model of collaborative work between teachers and with academics in a central core, and with a larger group of teachers involved in a collaborative RP session every year. The next section will examine the nature of the collaborations at stake in the current situation.

Nature of the collaborations at stake

We distinguish three levels of collaboration allowed by the ResCo device: collaboration between teachers and members of the ResCo group, then collaboration between academics and teachers within the ResCo group, and collaboration between teachers. The online forum plays a major role in

these levels of collaboration by allowing: Inter-class exchanges, Exchanges between teachers, whether at the level of practical, organizational, pedagogical and didactic aspects, Exchanges between teachers and the ResCo group, Access for teachers to all the productions of the classes posted online that can serve as resources to teach classes but also as a support for professional reflection (didactic and pedagogical), Having a memory of the activities that took place each year can thus serve as a resource for teacher-training.

In order to characterize the different types of interactions involving teachers, we will rely on the following criteria for each moment of the collaborative PS session: Which actors? At which stages of the ResCo device? About what? For what purpose? What role have the collaboration? The collaborations analyzed in this section are synthesized in figure 1.

Collaboration between the members of the ResCo group and the teachers

Before the collaborative PS session. The members of the ResCo group provide to the teachers various documents explaining the objective of the collaborative session, the organization of the collaborative PS session and its schedule. This happens either during workshops or communication at conferences and professional meetings or during the training device. This is essential to enable teachers to appropriate the organization of the device and to include it in their syllabus. In these documents, there are also examples of student's productions on the previous collaborative session with analysis elements and some elements of *a priori* analysis of the realistic fiction that will be proposed for the new session. Actually this is not yet the final version of the realistic fiction: the ResCo group presents a draft version of the realistic fiction and relies on feedback from teachers during the training device to develop the final version.

During the collaborative PS session. The exchanges between teachers and the ResCo group are most often related to the organization or functioning of the platform. The ResCo group deposits on the platform the statement of the realistic fiction, then the *relaunched realistic fiction*. In order to support teachers in the implementation of the collaborative PS session, the ResCo group sends to teachers a document to support them in the work following the relaunched realistic fiction and documents to enable them to finalize the collaborative session in their classrooms (expert solution, synthesis of students' productions, mathematical knowledge and skills at stake).

After the collaborative PS session. The ResCo group gives teachers the opportunity to share their analysis of the work done, by sending them an online questionnaire. These exchanges enable the group to improve conditions for the device to work better and also, to adjust the content of the documents given before the collaborative PS session.

Collaboration between teachers and with academics within the ResCo group

Before the collaborative PS session. At meetings of the ResCo group, the academics propose to the group a professional modeling problem. From this, all the group members will collaborate to elaborate the future problem of the session. Based on the expertise of the teachers, the group conceive a first statement, whose collaborative *a priori* analysis will identify its potential for learning mathematics. Several meetings are necessary to reach the final version used in the collaborative PS session. Collaborations between academics and teachers of the ResCo group are also very strong in the preparation of the training courses. Teachers and academics assume the role of trainer and co-construct the content, relying heavily on the experiments of the group's teachers.

During the collaborative PS session. Each member of the group analyses the productions submitted to the platform during the question-and-answer phase in order to co-construct the relaunched realistic fiction. This collaboration is one of the most important as the group's teachers themselves involve some of their classes in the session.

After the collaborative PS session. Similarly, for the preparation of the documents to finalize the session, the group relies heavily on feedback from ResCo teachers and also on all the productions uploaded on the platform. The ResCo academic writes an expert solution of the problem and the ResCo teachers synthesize and analyze the various students' productions from the point of view of the mathematics involved. Each document is discussed between the members of the group and validated by each one. This collaboration, which is based on everyone's expertise, is very fruitful and necessary to think out the favorable conditions to implement the collaborative PS session.

Collaboration between the teachers involved in the ResCo device

Before and after the collaborative session. Sometimes, teachers who participated in a previous collaborative PS session interact with their colleagues to introduce them to the ResCo device.

During the collaborative session. Teachers use the forum to deposit their students' work and also to communicate with each other, for example:

- Share any difficulties they may have in implementing the system: "The students have answered your questions succinctly, they have difficulty getting into the response phase, they tell me they can't know." (Excerpt from a message submitted during the question-and-answer phase)
- Sometimes support each other, as illustrated by these exchanges between two colleagues:
P1 to P2: "These are the questions from the 9th graders of middle school xxx. Surprisingly, for my part, many strategies came up when I read the statement. Few questions emerged. I have the feeling that they will arise in the following sessions. Have a good reading and good research."
P2 to P1: "I am in a bit the same situation as you, few questions were asked, they especially wanted to start measuring or looking for relationships between lengths. I think answering your questions can't do any harm to them! Have a good day, and good research."

In addition, via the forum, the system offers teachers the opportunity to consult all of the students' work, to share and exchange on their practices. Similarly, when the ResCo group sends the documents to finalize the collaborative PS session, teachers have at their disposal elements for a *posteriori* analysis of the work carried out, which allows them to reflect on the proposed activity.

Conclusion: Effects and outcomes of the collaborations

The collaborative work between teachers in collaborative PS sessions, and inside the ResCo group between teachers and with academics, has been developing for almost 20 years through the ResCo device. This model of multiple-collaboration has enabled many effects and outcomes.

On teachers practices. A specific relation develops between the IREM group and the teachers. Indeed, the ResCo group is responsible for the choice of the problem and its organization. Teachers can choose to step back by informing students that they are not decision-makers, neither of the choice of the problem nor of the different phases of work proposed. The work of Yvain-Prébiski (2018) and Yvain-Prébiski & Chesnais (in press), shows that this negotiation of the *didactic contract* of the class is mainly manifested in the exchanges with the class, which each teacher manages in a singular way with an objective that seems common: to encourage the *devolution* to the

students of the work requested, based on the collaborative dimension of the system. The support provided by the ResCo group throughout the implementation of the ResCo device thus reduces any hesitation on the part of some teachers to participate in the device. This can lead these teachers to dare to conduct a PS activity in the classroom, and change their practices over time, to develop PS activities in the classroom, anticipate modeling issues, or simply give students more autonomy.

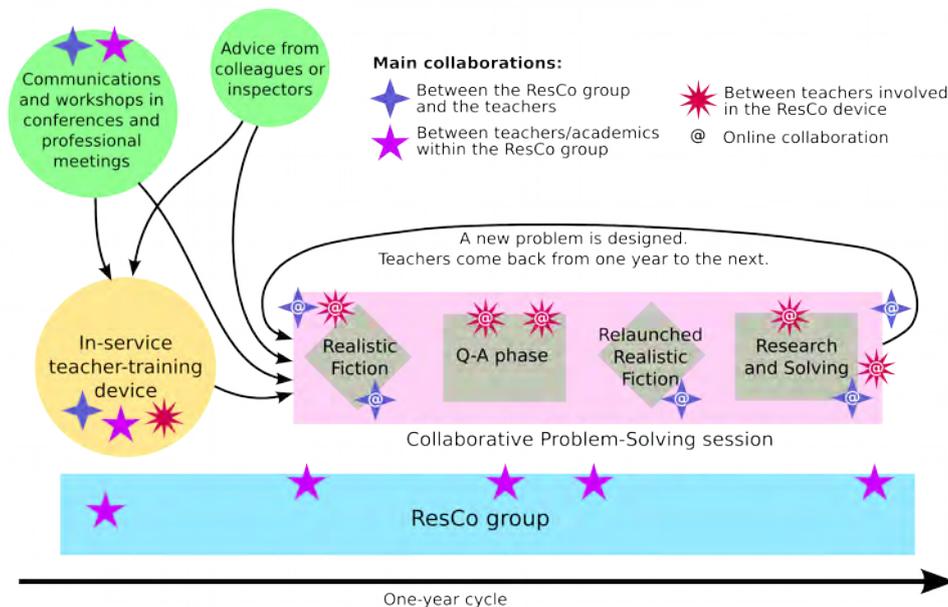


Figure 1: The ResCo device and the various collaborations at stake.

On teachers professional development. Collaborations also impact on the teachers’ trajectory. By co-option, teachers who have participated for several years in the device can join the ResCo group and participate in its activities. This contributes to the training and professional development of these teachers, which can lead them to become teacher-trainers or to be initiated into research. We illustrate these professional developments, with the trajectories of four teachers (figure 2). Teacher T1 participated in the ResCo group, and this led him to complete a master’s degree in didactics of sciences. He later moved and got pedagogical and administrative responsibilities in the national education system. Teacher T2 participated in the ResCo device, then became a member of the ResCo group. This led her to complete a master’s degree in didactics of sciences and to become a teacher trainer. Then T1 prepared and defended a PhD whose field of experimentation was the ResCo device. Teachers T3 and T4 participated in the ResCo device, then became members of the ResCo group. T3 plans to prepare a master’s degree in didactics of sciences.

On the work of the ResCo group. As explained previously, the collaboration in the IREM group is non-hierarchical and based on the diversity of the expertise of the group. The cyclic organization of the collaborative RP session permits to improve sequentially the organization of the session, the support to the teachers, and the work on the design the annual problem, with collaborative analyses of the problem and its implementation before, during and after the session. The central role of the teachers of the group having students involved in the collaborative RP session is fundamental to this end. Finally, the dynamic of the group is also influenced by its past, some transmission of the memory of the group is done with newcomers, and enables continuity and progress over years.

Perspectives. It would be interesting to compare this model of collaboration and this type of device with other experiences over the world in mathematics education and teacher training and

professional development in order to characterize the criteria that makes work such collaborations between teachers, but also to understand how it can be scaled up or replicated.

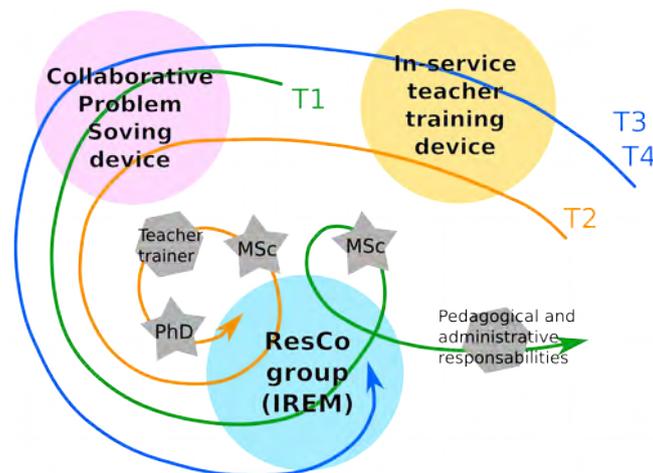


Figure 2: Trajectories and professional development of teachers.

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