



HAL
open science

From language as right to language as resource in mathematics education research

Núria Planas, Mapula Ngoepe

► **To cite this version:**

Núria Planas, Mapula Ngoepe. From language as right to language as resource in mathematics education research. 2020. hal-02569003

HAL Id: hal-02569003

<https://hal.science/hal-02569003>

Preprint submitted on 10 May 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

From language as right to language as resource in mathematics education research

Núria Planas, Universitat Autònoma de Barcelona

Mapula Ngoepe, University of South Africa

Today, the richness of languages, cultures and communities produces a complex heterogeneous picture of what it means to teach, learn and think school mathematics. This picture is one of the reasons for terms like superdiversity entering classroom research in mathematics education (Barwell, 2016) to describe the plethora of intersecting types of diversity in the midst of discourses of uniformity and homogeneity. Despite the evidence of superdiversity in our world, pedagogies based on the belief that monolingualism is achievable and preferable are strongly rooted in history and tradition. In mathematics teaching and learning in particular, multilingualism is still considered exceptional and monolingualism remains the norm. In order to contribute to the body of works that challenge the monolingual norm, in this chapter we address two questions: *What is the role of language in the mathematics classroom? What is the role of the languages of the learners?* Central to our argument is the understanding of the language of the mathematics classroom as conformed by all the languages *in* the classroom and their uses.

Learners and people in general have the right to use their languages, and importantly the right to learn in settings that are linguistically, culturally and pedagogically responsible. When only the language of instruction or only some languages of only some learners are valued as tools for thinking, teaching and learning: “What students are afforded the right to learn mathematics robustly, actively, and with understanding? What students are obligated to learn mathematics in less productive ways?” (Langer-Osuna & McKinney de Royston, 2017, p. 647). Our experiences of research and teaching in mathematical lessons of Catalonia and Gauteng, where participants are able to draw on more than one language as they teach, learn and interact, frame our approach to the issues of power and equity captured in the quote above. In our two contexts, basic language and educational rights similarly exist in principle, with bi/multilingual policies and culturally responsive discourses on paper. Nonetheless, Catalan in Catalonia and English in Gauteng are the languages of teaching and learning privileged in practice, and the forms of knowledge of the mainstream groups are those primarily valued over the course of school mathematics.

Throughout this chapter, we discuss the potential of the languages of the learners alongside the language of the teacher and of mathematics in the production of a language of the mathematics classroom that supports mathematics learning. After this introduction, we focus on findings from research to argue in favor of varied uses of diverse languages in mathematics teaching, learning and thinking. We claim that any instance of language use, regardless of the politics attached to it (Ricento, 2014; Ruíz, 1984), is a potential resource to be realized for learners in their mathematics learning (Planas, 2018). In the examination of the role of language, we take pieces of data to consider the impact of practices of switching languages that incorporate the home languages, and of practices of diversifying modes of communication on the creation and distribution of mathematics learning. The illustration of data collected in Catalonian schools is utilized to reflect on findings from field observations, conversations with

teachers and learners and lessons in Gauteng as well. We conclude with some implications regarding the adoption of the multilingual norm and the use of related multilingual practices in mathematics education.

The languages in the language of the mathematics classroom

Following the publication of *Speaking mathematically: Communication in mathematics classrooms* (Pimm, 1987), the study of the teaching and learning of the language of mathematics became strengthened in the field of mathematics education. Since then, this line of research has gained complexity through the articulation of the languages of the learners, the teacher and the mathematics (see the review of the early domain by Austin and Howson, 1979) to imply the plural language of the mathematics classroom (Planas, Morgan & Schütte, 2018). All these languages are connected, each has a relationship to another, although it is not necessarily obvious how they connect over the course of a lesson. The reference to a triad of inseparable classroom types of language is currently of much importance and helpful in many respects. Attention to this triad has been decisive in the structuring of research on mathematics education and language diversity as visible in the volume edited by Barwell and colleagues (2016). Specifically, the focus on the languages of learners has contributed to unveiling the social and educational disadvantages created for learners who do not speak the language of the teacher at home.

While language is a potential tool of communication and of facilitation of mathematical activity, this role of language is especially complex in the mathematics classrooms where the language of the teacher and of instruction is not the home language of all learners. The broad conceptualization of the language of the mathematics classroom as a pedagogic resource (Planas, 2007, 2011, 2014, 2018; Planas & Civil, 2013; Planas & Setati-Phakeng, 2014), thus, does not imply that mathematics learning is always encouraged in any language use. Language use does not necessarily translate into mathematics learning and, indeed, it may be hindered when practice is primarily oriented to teaching, assessing and developing proficiency in the language of instruction. The realization of language as a resource for the learning of mathematics requires effective practices oriented to teaching, assessing and developing the language of mathematics. Hence we need to understand the practices playing a role in making use of language to enhance mathematics learning, compared to the practices that subsume the learning of mathematics under the learning of the language of instruction. Focusing on the practices responsible for differences in the effects of language on mathematics learning, several chapters in Hunter, Civil, Herbel-Eisenmann, Planas and Wagner (2018) describe practices in multilingual lessons that helped to overcome tensions between the languages present in the discussion and communication of mathematical tasks. Some of the practices of flexibly using the languages of the learners appear combined with reasoning algebraically, building models, telling examples, predicting patterns and drawing representations. Walsh (2011) also provides a variety of classroom practices where participants interact through their diverse languages and, significantly, with the aid of drawings, diagrams and other forms of visual languages introduced in explanations to the whole group. These are works in which language diversity has two main interpretations regarding diversity of verbal languages and diversity of modes of communication and communicational abilities.

The practice of working in groups with learners using their home languages is largely addressed in the discussion of the diversity of spoken languages in the mathematics classroom. A strong relationship is indicated between the use of home languages and the mathematical participation of learners. In low-income Latino communities of the

U.S., Civil (2012) reports the contrast between learners' participation in English and Spanish: "When presenting to the whole class in English, their communication was tentative and stilted ... When presenting in Spanish or talking in their small groups (where students turned automatically to Spanish), it was a completely different story" (pp. 50-51). This is also consistent with what Planas and Setati (2009) found regarding Latin American migrant learners in Barcelona, the main city of Catalonia, who switched to the home language as soon as the conceptual level of mathematical explanations increased. The mathematical ideas brought up in the home language during group work, on the contrary, did not arise as long as this language remained unused by the teacher and the learners in whole class interaction. This happened with bilingual teachers who shared the language of Latin American learners and who occasionally replied in their language, but also with Arabic-speaking learners and teachers who did not share a main language with the majority of learners. Migrant learners who did not own the language of instruction hardly participated in the whole class and their reasoning was visible at the level of group work only, where activity was relatively free of instruction and direct assessment. Interestingly, in small groups in which learners did not share the home language with all their peers, the communication included unusual drawings and mathematically meaningful sketches. In this way, learners found modes of communicating their reasoning other than speaking either the home language or the language of instruction. Those drawings were as much a part of the set of language resources used by the learners as were their spoken languages.

In her work with children of working class immigrant families, Civil (2012) reminds us of the need to look at the politics of the situation. While learners in classrooms from her research share a home language, this is a consequence of policies of segregation with Latino children mostly concentrated in certain neighborhoods and school districts. In spite of the socially ambitious educational policies developed for Gauteng and Catalonia, in our regions the politics of the situation also reveals geographic concentration of language groups. Together with the challenge of meeting the needs of schools with learners who do not own the language of instruction, many other challenges therefore appear in relation to how the minority languages spoken by the majority of learners are to be considered. In historically disadvantaged black African schools, for example, teachers are often fluent in the home languages of their learners but prefer to teach mathematics in English, which is a language that some of them are not proficient in. Setati and Adler (2000) provide findings from mathematics classrooms in these schools with teachers who experimented work in linguistically homogeneous groups. The discussions in group work were in the home languages, simultaneously or not with English, while interaction with the whole group was through speaking and writing on the board in English. Still in South Africa and similarly to what was found in the work with learners in Barcelona, Mparutsa (2011) reports practices of alternating modes of communication in the absence of a common language. In her exploration of access to school mathematics when the teacher does not share a home language with learners, Mparutsa found that learners move between their languages in peer work and between verbal and visual modes of communication to be able to interact with and understand what is mathematically going on in the lesson.

The examples in the next two sections respectively focus on two practices: 1) group work with learners using their home languages, and 2) varied modes of communication in the absence of a common language. We combine pieces of data that reflect stories of participation and realizations of the languages of learners as a resource for mathematics learning. Despite the underlying force of the monolingual norm, the learners in the

examples flexibly use their home languages and communicational abilities. Although differences at many levels can be argued between Catalonia and Gauteng, the effects of these two practices raise significant commonalities. The relevance of the language of instruction in the lessons, the tensions around the views of languages and of speakers, and the ways in which the learners nevertheless draw on their home languages and varied communicational abilities are similar. The deliberate promotion of these practices might be a solution to the disadvantage experienced by learners of minority language groups in settings of school mathematics and, more generally, to the loss of learning opportunities experienced by all learners that see their possibilities of interaction and exchange reduced. We believe that the observations in Catalonia and Gauteng might illuminate the interpretation of other lesson contexts that at first sight might appear as different.

Learners using their home languages

The moves between the languages of the learners and the languages of mathematics and of the teacher operating together within the language of the mathematics classroom provide the context for the possibilities of using the home languages. Language switching, languaging, translanguaging or codemeshing are some of the terms for these moves to mean what learners do with languages, registers, words, pronunciations, grammars, etc. Regardless of the term used to name the practice, we are interested in reflecting on its pedagogic value for mathematics teaching, learning and thinking. The home languages and the language of instruction may not share the same space at the same time as a presence visible to participants. However, an important issue with respect to the use of the home languages is the use of the language of instruction. All these languages are interconnected in the activity of the learner in the classroom, and they all need to be viewed from the perspective of their contribution to mathematics thinking and learning. Even for learners who are in the early process of learning the language of instruction, this language is never absent or small. It develops in the general school practices of watching, listening, reproducing and imitating the oral and written texts of the teacher and the classroom. The same applies to learners whose home language is not the language of instruction and, based on their interactions, apparently draw on this language only.

The pieces of data in this section are published in Planas and Setati (2009). They illustrate how the home languages function alongside the language of instruction at selected times of mathematics teaching, learning and thinking. In many of the lessons observed over these years, learners switched to their home languages when discussing in small groups. The home languages were less common in situations of whole group and of writing, in which the language of instruction mostly prevailed. In our research the flexibility in the use of the languages in the classroom is thus particular to oral interaction in group work. In reflecting back on the data in Excerpts 1 and 2 below, we can think of many other examples in lessons of Catalonia and Gauteng, where group work is one of the methods targeted by the local curricula to promote participation and engagement. One important conclusion from our observations is that learners engage meaningfully with mathematical content when talking in their home languages. Moreover, in the rare cases in which learners write in their home languages or use them in whole class discussions, they provide substantial details of the mathematical reasoning developed in the small group. These results provide arguments for the deliberate promotion of different languages working together so that learners can harness all their languages in their learning.

Learners engaging meaningfully with mathematical content in group work

This first piece of data serves us to argue that learners use their home languages in group work for engagement with conceptual reasoning that is difficult to them. Excerpt 1 shows an instance of group work on the mathematical representation of a tornado in a lesson in Barcelona. The utterances are first given in the languages in which they were made. Italics and non-italics in the English version indicate the moves between Catalan – the language of instruction – and Spanish – the home language of these learners –. Language is diverse at several other levels if we consider the combination of drawings of arrows with gestures to imply the spiral motion. While these two sides of language diversity – spoken languages and modes of communication – are necessary to capture the role of language in mathematics learning, we leave the discussion of the latter for the next section.

Excerpt 1

- Máximo: Hem de decidir les fletxes que dibuixem i ja està. [We need to decide the arrows that we draw and that's all.]
- Eliseo: Primer pensem les fletxes, després les dibuixem i després en parlem. [First we think about the arrows, then we draw them and then we talk about it.]
- Máximo: Esta idea de las flechas no es fácil. Tenemos que imaginar los diferentes movimientos que existen dentro del tornado. [*This idea of the arrows is not easy. We have to imagine the different movements that exist within the tornado.*]
- Eliseo: Una flecha tiene que ser una línea recta para que el tornado baje. Tenemos la t para la traslación. [*An arrow needs to be a straight line for the tornado to go down. We have the t for the translation.*]
- Luna: La pregunta pide representar un tornado, ¿no? [*The question asks to represent a tornado, doesn't it?*]
- Nicolás: Sí, diu que s'ha de representar matemàticament un tornado. [Yes, it says that we need to mathematically represent a tornado.]
- Luna: No és parlar d'un tornado, és representar-lo matemàticament. [It is not to talk about a tornado, it is to mathematically represent it.]
- Eliseo: Nos puede ser útil representar un tornado antes de dibujarlo. [*The drawing of a tornado can be useful before its representation.*]
- Nicolás: Está claro que con una sola flecha no basta, porque un tornado es más que una traslación. [*It is clear that only one arrow is not enough, because a tornado is more than a translation.*]
- Eliseo: Hay que pensar en cómo dibujaríamos una espiral. Dibujaríamos curvas. [*We need to think about the drawing of a spiral. We would draw curves.*]

Máximo is a second-generation Colombian boy, Luna is a girl born in Peru, and Nicolás and Eliseo are two boys born in Colombia. They all testified that they spoke Spanish at home and that they most often spoke Catalan and Spanish at school. During group work, they used Spanish and Catalan in the discussion of the task with the home language dominating the processes of reasoning and, specifically, the processes of making sense of the task, its nature and the procedure to be followed in the resolution. They used what they knew to make sense of what they were asked to do, and the home language is certainly part of what all learners know. They engaged in the discussion of the dynamics of a tornado and how to represent it mathematically. In particular they developed a spoken description of the cylindrical spiral shape, which they refined into the description with gestures of a helical spiral motion. They were then challenged by the representation of the helical case as a rotation composed with a translation in the two dimensions of the plane. They started by proposing a composition of arrows that showed the direction of linear motions, and they soon realized that they had to solve the problem of the different directions needed for the composed rotation, a transformation for which the two dimensions of the plane were not enough. The discussion continued

for some minutes with references to angles, parallelism and perpendicularity in the planar representation of spatial motions. These learners, however, were not engaged the same in the final discussion guided by the teacher in Catalan. We illustrate this point with the next excerpt.

Learners briefly reporting their mathematical discussion in whole group

Regarding participation and reasoning, this second piece of data allows us to argue the contrast between the engagement through the home language in group work and the engagement through the language of instruction in the whole group feedback. In our observations, the active role that the languages of the learners play during the small group discussions tend to decrease when the learning environment is the whole class. Excerpt 2 shows part of the later oral exchange of the group in Excerpt 1 with the teacher, during which only the language of instruction was used. The teacher asked a representative of the group to put up their answers on the board and explain it to the rest of the class. Eliseo hardly shared a brief part of the reasoning. He disregarded the discussion of the cylindrical and helical spirals produced in the group and did not explain how they had addressed the challenge of representing the tornado with the two dimensions of a plane only.

Excerpt 2

- Teacher: Eliseo, per què no dius res? Sé que heu estat treballant en el vostre grup. [Eliseo, why don't you say anything? I know you've been working in your group.]
- Eliseo: Hem dibuixat algunes fletxes. [We've drawn some arrows.]
- Teacher: I quina heu triat finalment? Vols dir-ho en castellà? [And which have you finally chosen? Do you want to say it in Spanish?]
- Eliseo: Sabem que han de ser almenys dues fletxes i una és vertical perquè el tornado va cap avall. [We know that there are at least two arrows and one is vertical because the tornado goes down.]
- Teacher: Una translació vertical? [A vertical translation?]
- Eliseo: Una vertical. [Vertical.]
- Teacher: Els altres possibles moviments? [The other possible motions?]
- Eliseo: El tornado gira; creiem que una fletxa ha de ser la de la rotació. [The tornado turns around; we think that one of the arrows has to be the rotation.]
- Teacher: I què més? Abans parlàveu molt. [And what else? You were talking a lot.]
- Eliseo: Res més. [Nothing else.]

Although the teacher used Catalan, she prompted the learners to choose the language for communication. These prompts of the teacher to use languages other than Catalan did not lead to use the home language in the whole group feedback. Compared to peer work, Luna, Máximo, Nicolás and Eliseo were less engaged in the presentation involving the whole class, for which Eliseo was the representative not without resistance. As a consequence, the other learners missed the opportunity to listen to, for example, the descriptions of the two spiral curves, the helical and the cylindrical, considered in that group. In fact, the term 'spiral' and the variety of meanings for it, or the challenges involved in representing the curve of the tornado on a plane did not appear in the whole class feedback. At the end of the lesson, in a conversation with the teacher, she showed concern about the knowledge of the language of instruction and interpreted that this knowledge determined the participation of learners in the whole group.

- Teacher: Necessiten més confiança amb la llengua. Tot arribarà... i se'n sortiran amb els estudis. No participen més per la llengua. [They need more confidence with the language. All in good time... and they will get on with their education. They do

not participate more because of the language.]

Núria: Què passa amb la llengua? [What happens with the language?]

Teacher: A les posades en comú, els altres parlen català més que ells. El saben parlar una mica. [In whole class discussion, others speak Catalan more than they do. They know how to speak a little.]

This conversation shows most of the issues observed repeatedly with other teachers in Catalonia and Gauteng. The teacher in Barcelona did not question the mathematical activity and performance of her learners. Rather, she claimed that some of them had precarious knowledge of the language of instruction and needed more knowledge of this type for participation as well as for continuity with their studies. In our research various teachers relate the use of the home languages in group work to poor knowledge of the language of instruction. The dismissal of some languages as useless “for getting on with an education” and the elevation of one language over the others for learning reasons are also common findings. The meanings that, either intentionally or not, were communicated by the teacher in the conversation of that day were even more complex. In the case of her learners, it was unclear how much knowledge of the language of instruction was enough and how should that knowledge be gained. Despite there is room for a diversity of opinions as to how much knowledge is enough according to a designated standard form of a language, we can agree that learners need some knowledge of the language of instruction. However, those four learners could speak Catalan well though this language did not play a part in their lives outside the school. Máximo entered the local school system when he was a child and his peers, Luna, Nicolás and Eliseo, learned Catalan in the lessons for “late arrivals” at the school and passed the prescriptive local tests of proficiency in this language. Thus, it may not be precise to argue that the home languages played a remedial role in the group work of these learners. Learners may draw on their languages for different aspects of their lives, and may combine these languages for different purposes in their learning of mathematics. Those of us who speak two or more languages know about the socially differentiated use of languages.

Learners using their communicational abilities

In the study of language diversity in mathematics education, research has tended to consider diversity in relation to spoken and written languages rather than uses of language expressed through other modes of communication. In the previous section, the pedagogic value of using the learners’ spoken home languages has served to question the ubiquity of the monolingual norm. Now we expand the problematization of the monolingual norm with the focus on the interaction between verbal and non-verbal modes in lessons in which learners may not share a home language. We particularly reflect on the pedagogic value, for mathematics teaching, learning and thinking, of using modes beyond orality and writtenness. In any classroom, communication uses a variety of modes. There is no verbal text in gaze and gesture or in the body languages of signs of the deaf people. Cartesian graphs and geometric designs are also examples of non-verbal tools, culturally encoded to convey mathematical meanings. Indeed, the language of mathematics has been characterized as one in which verbal modes are accompanied by visual and symbolic modes, stimulated in recent years by the growth of digital technologies (Morgan & Kynigos, 2014). Much has been said about the visual and the symbolic in mathematics thinking and learning, but little is still known about the construction of learning spaces oriented to develop abilities other than speaking and writing. The creative use of language in interaction has been less well studied, as well as the relationship between the verbal and the visual. The mathematical abilities of learners

are mostly considered in their verbal dimension, and learning is primarily assessed through verbal and written performance.

In what comes we provide a piece of unpublished data from one of the lessons in Planas (2014). We use it to illustrate how original personal images function alongside explanations in one or more verbal languages to support mathematical reasoning in group work. The drawings generated by the learners in the prior examples with the representation of the tornado would also serve. Language diversity in terms of modes of communication that include idiosyncratic configurations of individual learners is not a very frequent result in the lessons observed in Catalonia and Gauteng. In our contexts, verbal and verbal-symbolic texts dominate the language of the mathematics classroom except for the occasions in which the teacher prompts learners to work with diagrams of number arrays, bar graphs, tables with organized data or prototypical drawings of concepts in school mathematics like quadrilaterals with specific orientations. Overall, our example with the production of unusual diagrams in multilingual group work suggests the role of the communicational abilities of learners in mathematics thinking and learning. It is sound to argue that the role of language in mathematics learning continues in the influence and juxtaposition of the verbal languages, the prototypical images and the creative configurations of learners. The effects of idiosyncratic drawings of learners in the mathematical discussion below reveal the strength of the creative use of language. Once more, although this use of language may become particularly pertinent in a learning environment where the learners do not share the spoken language at home, its promotion may be important for all learners in their struggle with the language of mathematics.

Learners supporting mathematical reasoning with idiosyncratic images in group work

Similar to Excerpts 1 and 2, the teacher of the classroom in this example regularly proposed lesson dynamics that made it possible for learners to communicate their thinking with others. Almost at every lesson, she asked the class to develop and listen to different ways of solving the tasks, first in small group work and then in whole group sharing. Not all learners shared in front of the whole class. As noted earlier in the chapter for the group with Eliseo, most learners whose home language was not the language of instruction, hardly volunteered to present the work done. The piece of data below shows three learners in Barcelona involved in the task of converting algebraic expressions into word texts. Roberto, a learner born in Ecuador, is discussing with Joana and Miquel an alternative to “any odd number” for $2x+1$. Roberto is one of the learners that never shared the group work with the whole class, although he was never absent and he meaningfully engaged in the discussions with his home language regardless of the language used by his peers.

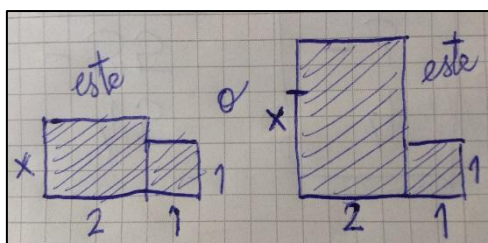


Figure 1. Roberto's drawings

Although this is an instance with learners that spontaneously moved between two spoken languages in the interpretation of technical vocabulary (“regular”) and more generally in the discussion of answers to the task, we focus on how the interacting

modes of communication strengthened the reasoning that was difficult to them. The drawings of Roberto in Figure 1 served to initiate a geometrical interpretation that was later included in the final written performance. This learner first drew a rectangle with two sides of length x and 2 and adjacent to a square with side 1 , and then marked the two areas. During the discussion in Excerpt 3, he drew a second rectangle for a value of x higher than the prior considered. At the end Joana made the drawing with colours and shapes in Figure 2 to represent “The area of either a rectangle or an L form” as one possibility for the conversion of $2x+1$. Importantly, the thinking through drawings introduced the discussion of the numerical set for the values of x , which was not specified in the given task. The more sophisticated was the understanding of x , the more complex was the drawing done.

Excerpt 3

- Joana: Però nombre senar vol dir per tots els nombres senars i el teu dibuix és només per uns casos. No sé si vale. [But odd number mean all odd numbers and your drawing is for some cases only. *I don't know if it works.*]
- Roberto: El cuadrado de costat u tiene que ser siempre así. [The square with side one has to be always like this.]
- Miquel: Però el rectangle no. [But the rectangle has not.]
- Joana: No vale. Ha de ser general. [It doesn't work. It has to be general.]
- Roberto: Pues necesitamos dos dibujos... A lo mejor tres si el rectángulo es perfecto. [So we need two drawings... Maybe three if the rectangle is perfect.]
- Miquel: Perfecte? [Perfect?]
- Roberto: Si es dos y dos, es perfecto. [If it's two and two, it is perfect.]
- Joana: Sí, regular, un quadrat. Però encara no sé això què voldria dir. Com diferents grups de nombres senars? [Yes, regular, a square. But I still don't what that would mean? Like different groups of odd numbers?]
- Roberto: ¿Por qué? Yo hablo de áreas regulares. [Why? I speak about regular areas.]
- Joana: Sí, però quin dibuix tens pel número u? I el número tres? [Yes but what is your drawing for number one? And number three?]
- Roberto: Pues quatre dibuixos. No quiero un dibujo para el número u, lo quiero para equis cero. Lo que cambio es un lado. Yo tengo muchas más equis. [So four drawings. I don't want a drawing for number one, I want it for x zero. What I change is one side. I have many more x 's.]
- Joana: Tinc una idea. Ho podem fer general. Puc agafar-te la llibreta? [I have an idea. You can make it general. Can I take your notebook?]

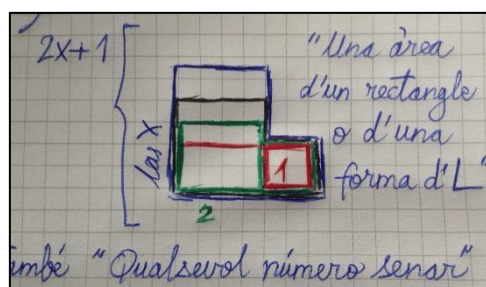


Figure 2. Joana's drawings

This example clearly shows the co-presence of mathematical symbolism, visual display and verbal language in mathematics learning. Moreover, it reminds us that multimodality is not synonymous with the digital world (computers, podcasts, virtual media, and other types of digital literacy). We are aware of the multiple possibilities of the geometry dynamic software and graphic calculators at present, which would easily create representations for $2x+1$ on the screen that learners could quickly modify and

visualize. Roberto and Joana introduced rather rudimentary sketches in their mathematical talk, but these sketches proved to be a significant start in their geometrical visual thinking. Without the aid of the software or the calculator, they also found a dynamic way to manipulate and change the length of one side of the rectangle. Any understanding of how they made meaning of the task in that lesson would be incomplete without attending to the modes and languages at play. Taken alone, none of these modes and languages would be sufficient. The use of only the images or only the languages of the learners might have been relatively limited in service of the reasoning developed. The reproduction of the drawing in Figure 2 in the written report for the teacher indicates that, for the learners in this group, the image created was just as important as the texts with the answers. As with the learners in the prior section, the drawings here reveal mathematical reasoning that some learners may have difficulty writing or speaking about in any of their languages.

In conversations with other learners regarding images created for the discussion of mathematical tasks in group work, we found that some of them related the utility of some of their drawings in moments of difficulty with the understanding of the spoken languages and the mathematics. These visual representations replaced verbal descriptions. One of the learners from Latin America said that she often made drawings that were “understood by them [the learners in the group] to “avoid the mess with the words.” Another learner of the Catalan school system whose home language was Amazigh brought up questions of creativity and normativity in relation to the production of images unintended by the teacher. In a conversation about her low participation in the mathematics classroom, she responded that she was active in the group work with peers. She added, “Sometimes teachers say, hey, draw a picture, and I am good at drawing... But sometimes they do not mean any picture. Then I need to know exactly, and I do my best.” This quote arises the question of the limits of creativity in language use in a given classroom culture.

Towards the newer norm of multilingualism

We started with two questions: *What is the role of language in the mathematics classroom? What is the role of the languages of the learners?* We have argued the potential role of the language of the mathematics classroom as a resource for mathematics teaching, learning and thinking. Specifically for the languages of the learners, we have presented two multilingual practices that can serve to the realization of the language of the classroom as a resource. These practices are grounded on two major approaches to the understanding of language diversity in mathematics education. Regarding the verbal dimension of language diversity, we have discussed the benefits of the flexible use of home languages. Regarding the wider communicational dimension of language diversity, we have reflected on the benefits of the creative use of modes of communication other than speaking and writing mathematics (in the language of instruction). Since some of the fundamental properties of language include its flexible and creative expression and the fact that it is multimodal (Halliday, 1978), these are possible and achievable practices.

A number of reasons, however, indicate that the monolingual norm still prevails in mathematics education. When we refer to the term ‘multilingual norm’ we are indicating an approach to language very different to an arrangement of abstract linguistic symbols according to grammatical rules. The research community and the school context are in the early stages of producing newer norms aligned with flexibility and creativity in language use during mathematics teaching, learning and thinking. It is widespread the idea that learners who are not ‘native’ speakers of the language of

instruction are deficient communicators of school mathematics. Moreover, it is common to find practices of slow and simplified language use, along with curricular remedial arrangements, in lessons with learners whose home language is not the language of instruction. The primary goal is often to make these learners monolingual as they work with academic written texts, rather than to support them in their learning of mathematics. The message sent is that the language of instruction is the appropriate language for the learning of mathematics. In this chapter, we have shown multilingual learners in interaction who are proficient communicators of the language of mathematics through their many languages and abilities. An important point in our arguments is that multilingualism cannot be separated from multimodality. Learners naturally switch between languages and modes in their learning processes. Our research indicates that the learners use their home languages as well as idiosyncratic drawings for engagement with mathematical reasoning that is difficult to them, but also that the presence of these resources is lower in the whole group.

Following our observations, many multilingual practices are spontaneous initiatives of learners and develop with minimal pedagogical intervention from teachers. A quick interpretation might give the impression that these practices do not have to be taught by the mathematics teacher. Far from this, consistent progress towards the multilingual norm strongly requires coordination of mathematics teachers, educators and researchers for the design and implementation of mathematical curricula that explicitly refer to the flexible and creative use of languages and modes of communication. Otherwise, multilingual practices can be confined to interaction in the small group, and implicitly censored in the whole class. This cannot be done in practice without knowledge and awareness of all the languages that pedagogically matter: the languages of the learners, the language of the teacher and the language of mathematics. In our two contexts, there is a gap between the language knowledge and awareness needed and the orientations about language and language diversity in mathematics teacher education. Accordingly, mathematics teachers are not trained to be able to reflectively think about language and language diversity in teaching and learning. The processes of learning to teach mathematics are primarily mapped to the processes of becoming a teacher of mathematical contents and a learner of mathematical practices. As explained in Essien, Chitera and Planas (2016) and in Barwell et al (2016), a similar monolingual bias can be found in other countries where the language of instruction is produced as the privileged tool for teaching, learning and thinking mathematics. We thus have a long way to go in developing teaching knowledge and teaching strategies out of the monolingual models for the progressive deliberate introduction of multilingual practices in service of mathematics learning.

More could be said about the role of language and specifically of the languages of learners in mathematics learning. It is our hope to have motivated views that are socially, linguistically and pedagogically responsible with all learners. There are good reasons why teaching practices should be developed from the strategies learners themselves use.

References

Adler, J. (2000). Widening the lens – changing the focus: Researching and describing language practices in multilingual classrooms in South Africa. In H. Fujita, Y. Hashimoto, & T. Ikeda (Eds.), *Abstracts of Plenary Lectures and Regular Lectures, ICME9* (pp. 20-21). Makuhari, Japan: ICMI.

- Austin, J. L., & Howson, A. G. (1979). Language and mathematical education. *Educational Studies in Mathematics*, 10(2), 161-197.
- Barwell, R. (2016). Mathematics education, language and superdiversity. In A. Halai & P. Clarkson (Eds.), *Teaching and learning mathematics in multilingual classrooms* (pp. 25-39). Rotterdam: Sense Publishers.
- Barwell, R., Clarkson, P., Halai, A., Kazima, M., Moschkovich, J., Phakeng, M., Planas, N., Valero, P., & Villavicencio, M. (Eds.) (2016). *Mathematics education and language diversity. The 21st ICMI Study*. New York: Springer.
- Chronaki, A., & Planas, N. (2018). Language diversity in mathematics education research: From language as representation to the politics of representation. *ZDM*, 50(6), 1101-1111.
- Civil, M. (2012). Opportunities to learn in mathematics education: Insights from research with “non-dominant” communities. In T. Y. Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 43-59). Taipei: PME.
- Essien, A., Chitera, N., & Planas, N. (2006). Language diversity in mathematics teacher education: Challenges across three countries. In R. Barwell et al. (Eds.) (2016). *Mathematics education and language diversity. The 21st ICMI Study* (pp. 103-119). New York: Springer.
- Halliday, M. A. K. (Ed.) (1978). *The social interpretation of language and meaning*. London, UK: Edward Arnold.
- Hunter, R., Civil, M., Herbel-Eisenmann, B., Planas, N., & Wagner, D. (Eds.) (2018). *Mathematical discourse that breaks barriers and creates space for marginalised learners*. Rotterdam: Sense Publishers.
- Langer-Osuna, J. M., & McKinney de Royston, M. (2017). Understanding relations of power in the mathematics classroom: Explorations in positioning theory. In A. Chronaki (Ed.), *Proceedings of the 9th International Mathematics Education and Society Conference* (vol. 2, pp. 645-653). Volos, Greece: University of Thessaly.
- Morgan, C., & Kynigos, C. (2014). Digital artefacts as representations: Forging connections between a constructionist and a social semiotic perspective. *Educational Studies in Mathematics*, 85, 357-379.
- Mparutsa, S. T. (2011). *Towards the development of instructional strategies for teaching algebra in multilingual classrooms in South Africa*. PhD Manuscript. Johannesburg: The University of the Witwatersrand.
- Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. London: Routledge Kegan & Paul.
- Planas, N. (2007). The discursive construction of learning in a multiethnic school: Perspectives from non-immigrant students. *Intercultural Education*, 18(1), 1-14.
- Planas, N. (2011). Language identities in students’ writings about group work in their mathematics classroom. *Language and Education*, 25(2), 129-146.
- Planas, N. (2014). One speaker, two languages: learning opportunities in the mathematics classroom. *Educational Studies in Mathematics*, 87(1), 51-66.
- Planas, N. (2018). Language as resource: A key notion for the understanding of the complexity of mathematics learning. *Educational Studies in Mathematics*, 98(3), 215-229.

- Planas, N., & Civil, M. (2013). Language-as-resource and language-as-political: Tensions in the bilingual mathematics classroom. *Mathematics Education Research Journal*, 25(3), 361-378.
- Planas, N., Morgan, C., & Schütte, M. (2018). Mathematics education and language. Lessons from two decades of research. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger & K. Ruthven (Eds.), *Developing research in mathematics education. Twenty years of communication, cooperation and collaboration in Europe* (pp. 196-210). London: Routledge.
- Planas, N., & Setati, M. (2009). Bilingual students using their languages in their learning of mathematics. *Mathematics Education Research Journal*, 21(3), 36-59.
- Planas, N., & Setati-Phakeng, M. (2014). On the process of gaining language as a resource in mathematics education. *ZDM*, 46(6), 883-893.
- Ricento, T. (2014). Thinking about language: What political theorists need to know about language in the real world. *Language Policy*, 13(4), 351-369.
- Ruíz, R. (1984). Orientations in language planning. *National Association for Bilingual Education Journal*, 8, 15-34.
- Setati, M., & Adler, J. (2000). Between languages and discourses: Language practices in primary multilingual mathematics classrooms in South Africa. *Educational Studies in Mathematics*, 43(3), 243-269.
- Walsh, M. (2011). *Multimodal literacy: Researching classroom practice*. Sydney: Primary Teachers Association of Australia.