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Can adaptive grid refinement produce grid-independent solutions for incompressible flows?

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Abstract

This paper studies if adaptive grid refinement combined with finite-volume simulation of the incompressible RANS equations can be used to obtain grid-independent solutions of realistic flow problems. It is shown that grid adaptation based on metric tensors can generate series of meshes for grid convergence studies in a straightforward way. For a two-dimensional airfoil and the flow around a tanker ship, the grid convergence of the observed forces is sufficiently smooth for numerical uncertainty estimation. Grid refinement captures the details of the local flow in the wake, which is shown to be grid converged on reasonably-sized meshes. Thus, grid convergence studies using automatic refinement are suitable for high-Reynolds incompressible flows.

Keywords: grid adaptation, grid convergence, uncertainty estimation, hydrodynamic flows

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1 1. Introduction

2 One of the trends in computational fluid dynamics today is the high-
3 fidelity simulation of more and more complex flows. For example, simulations
4 for realistic geometries such as ships with their propellers and appendages
5 have become possible. Physical flow features such as flow separation, vortex
6 shedding and vortex breakup are simulated in unprecedented detail. Finally,
7 multiphysics computations like fluid-structure interaction or the modelling
8 of cavitation become common.

9 The results of such simulations depend on the physical models being
10 used, such as the turbulence model in the Reynolds-averaged Navier-Stokes
11 (RANS) equations. Often, such models are applied in situations which are far
12 more complex than the ones for which they were developed and which may
13 be outside their range of validity. Research of physical modelling, specifically
14 for today's realistic simulations, is therefore of prime importance.

15 To accurately assess the precision of a physical model, we need to know
16 a numerical solution in which the numerical errors are small with respect to
17 the modelling errors: a solution that is close to grid convergence. In simple
18 cases, it is possible for an experienced user to generate meshes which provide
19 sufficiently small numerical errors. However, for more and more complex
20 flows, the grid resolution needed to resolve the flow phenomena, as well as
21 the precise position of these features, is uncertain. Therefore, it is impossible
22 to know beforehand what mesh size is needed where, in order to obtain grid
23 convergence.

24 The key to the study of physical flow modelling in complex cases may
25 be the adaptive refinement of the grid. Mesh adaptation is the technique

26 of modifying locally the mesh density during a computation, in order to
27 optimise the mesh for the flow being simulated. Such techniques are well
28 established, see for example [1]. For the modelling of complex flows, adap-
29 tation techniques can detect where relevant flow features occur, so fine cells
30 can be placed only around these features. This allows the creation of locally
31 very fine meshes if these are needed to get grid convergence for flow details,
32 while the total number of cells is kept reasonable.

33 In this paper, we see if today's grid refinement techniques can assure that
34 the solution of an incompressible flow problem is grid-independent. This
35 means not only obtaining a solution, but also proving that its numerical
36 errors are small enough. For the paper, steady flows are considered and
37 iterative convergence errors are minimised by setting strict tolerances for
38 convergence. Thus, we concentrate on errors due to the spatial discretisation,
39 i.e. interpolation errors and approximate integration.

40 In the absence of single-grid error estimation procedures like [2], estimat-
41 ing spatial discretisation errors requires simulations on several coarse to fine
42 grids. Therefore, we wish to know if grid refinement studies can be performed
43 with adaptive refinement. Specifically, we investigate the following: (a) Can
44 grid adaptation create geometrically similar grids, where the local ratio of
45 cell sizes between two grids is constant and the cell shapes and orientations
46 are the same? (b) Are the results suitable for use with established uncer-
47 tainty estimation methods? (c) Can the global and local flow be computed so
48 accurately that the numerical errors become much smaller than the physical
49 modelling errors?

50 Simulations and grid refinement are performed with the ISIS-CFD un-
51 structured finite-volume incompressible RANS solver developed by the au-
52 thors from ECN-CNRS. This code is used for the realistic simulation of
53 industrial-type flows, it is available as the flow solver of the FINETM/Marine
54 computing suite. Its adaptation method is based on the refinement of un-
55 structured hexahedral grids by subdivision of the cells [3, 4]. Anisotropic
56 refinement, where the cells are adapted to the flow both in size and in aspect
57 ratio, is handled with the metric tensor approach introduced by George and
58 Borouchaki [5], modified for the refinement of hexahedrals.

59 The tests in this paper are therefore based on the type of computations
60 usually performed with ISIS-CFD: incompressible RANS flows, unstructured
61 hexahedral grids, refinement by subdivision, and refinement criteria based
62 on second spatial derivatives. However, the proposed method of uncertainty
63 evaluation is not limited to this type of simulations. To complete the tests,
64 we evaluate if the conclusions reached are also valid for other grid adaptation
65 methods.

66 The flow solver is presented briefly in section 2, the refinement method
67 and its use of metric tensors as refinement criteria are discussed in section
68 3. Section 4 investigates how such a refinement method can be used to
69 produce geometrically similar series of meshes for convergence studies. A
70 two-dimensional airfoil is simulated in section 5, to test whether these se-
71 ries can produce asymptotic convergence of the computed forces and useful
72 estimations of the numerical uncertainty. Furthermore, we search the grid
73 density needed to obtain grid independence for the solution in the wake. Fi-
74 nally, the flow around the KVLCC2 tanker is computed to see how close one

75 can get to grid convergence for a realistic ship flow test case (section 6). The
76 conclusion (section 7) discusses the generality of the results and evaluates
77 the limitations and perspectives of the proposed grid convergence method.

78 **2. The ISIS-CFD flow solver**

79 ISIS-CFD is an incompressible unsteady multifluid Navier-Stokes solver
80 [6, 7, 8]. The solver is based on the finite volume method to build the spatial
81 discretisation of the transport equations. Pressure-velocity coupling is ob-
82 tained through a Rhie & Chow SIMPLE-type method [9]. The discretisation
83 is face-based, so cells with an arbitrary number of arbitrarily-shaped faces
84 are accepted; most computations are performed on unstructured hexahedral
85 meshes. The code is fully parallel using the MPI (Message Passing Interface)
86 protocol.

87 Turbulence is principally modelled with the Reynolds-averaged Navier-
88 Stokes (RANS) equations and advanced turbulence closures, such as the
89 anisotropic EASM model [6]. Furthermore, several Detached-Eddy Simu-
90 lation (DES) models are available. If a free surface is present, it is captured
91 with a mixture-model approach [7]. Finally, techniques such as mesh de-
92 formation allow the 6 DOF resolution of body motion, rotation of bodies is
93 achieved through sliding interfaces, and coupling with other fluid or structure
94 solvers is possible. For brevity, these options are not further described here.

95 **3. Grid refinement method**

96 An adaptive grid refinement technique is included in the solver ISIS-CFD
97 [3, 4]. The method performs isotropic and anisotropic refinement of unstruc-

98 tured hexahedral meshes. Adapted meshes are created by dividing the cells
99 of a coarse original grid into finer cells; this division can be repeated sev-
100 eral times until the desired cell sizes are obtained. Anisotropy is introduced
101 by splitting cells in one direction only. The grid refinement is performed in
102 parallel and includes an automatic dynamic load balancing in order to redis-
103 tribute the refined grid over the processors when some partitions have been
104 refined more than the others. The mesh is adapted regularly during the com-
105 putation in order to follow the convergence of the flow; earlier refinements
106 can be undone to account for the changes in the flow as it converges. Recent
107 developments of the technique include the treatment of this derefinement in
108 an anisotropic way [10] and the flux-component Hessian criterion described
109 below. The remainder of this section gives an overview of the refinement cri-
110 teria which are used, followed by a description of two measures which ensure
111 the quality of the refined grid.

112 *3.1. Metric-based refinement*

113 To obtain anisotropic grid refinement, we use metric tensors as refinement
114 criteria. This technique was introduced for the generation of anisotropic
115 tetrahedral cells [5], it has later been used successfully for the adaptive re-
116 finement of such meshes [11, 12]. The technique also provides a practical and
117 flexible framework for the refinement of hexahedral meshes.

118 In our procedure, the refinement of the cells is decided as follows. First,
119 the 3×3 criterion tensors \mathcal{C}_i in each cell i are computed (in some way) from
120 the flow solution. In a hexahedral cell, let the cell sizes $\mathbf{d}_{i,j}$ ($j = 1, 2, 3$) be
121 the vectors between the opposing face centres in the three cell directions.
122 The goal of the grid refinement is then to create a grid which is uniform

123 under the transformation \mathcal{C} , which implies that:

$$\|\mathcal{C}_i \mathbf{d}_{i,j}\| = T_r \quad \forall i, j, \quad (1)$$

124 where T_r is a constant. In the refinement procedure, this is obtained in the
 125 following way. Each time the procedure is called, the criterion \mathcal{C}_i and the cell
 126 sizes $\mathbf{d}_{i,j}$ are computed on the current grid. A cell i is refined in the direction
 127 j when $\|\mathcal{C}_i \mathbf{d}_{i,j}\|$ exceeds the constant T_r , while a previously refined group of
 128 cells can be derefined in the direction j if $\|\mathcal{C}_i \mathbf{d}_{i,j}\|$ is lower than T_r/d for all
 129 cells in the group. The constant d is chosen slightly larger than 2, to prevent
 130 cells being alternately derefined and re-refined. Equation(1) implies that the
 131 tensors \mathcal{C} are direct specifications of the desired cell sizes: in a converged
 132 refined grid, the cell sizes are inversely proportional to the magnitude of the
 133 \mathcal{C} .

134 3.2. Refinement criteria

135 The refinement criteria are based on the Hessian matrix of second spatial
 136 derivatives. Hessian matrices can be interpreted as rough error indicators,
 137 since they are linked to interpolation errors for linear interpolation [11]. Thus,
 138 they provide some measure of the truncation error for a second-order finite-
 139 volume discretisation.

140 Initially, we based the criterion on the Hessian of the pressure [4], using
 141 the rationale that the pressure is important for the computation of forces
 142 on bodies and also a good indicator of waves and vortices. However, the
 143 pressure criterion is unable to track wakes. To adapt the mesh to pressure-
 144 based flows but also to boundary layers, wakes, and shear layers, a new
 145 criterion is introduced based on the Hessians of both the pressure and the

146 velocity. To give equal importance to the different Hessian matrices, they
 147 are weighted in the way in which they appear in the flux.

148 The Hessian criterion based on the pressure p (PH criterion) is computed
 149 as:

$$\mathbf{C}_{PH} = (\|\mathcal{H}(p)\|)^\alpha, \quad (2)$$

150 where \mathcal{H} is the Hessian operator and the absolute value of a matrix $\|\cdot\|$
 151 corresponds to a matrix having the same eigenvectors as the original one and
 152 the absolute values of its eigenvalues. In the same way, the power α of a
 153 matrix is obtained by taking its eigenvalues to the power α while keeping
 154 the eigenvectors. For second-order reconstructions, the interpolation errors
 155 are proportional to the Hessian, times the cell size squared. Thus, to get an
 156 approximate equidistribution of the interpolation errors, we choose $\alpha = \frac{1}{2}$
 157 [3].

158 The new flux-component Hessian (FCH) criterion is computed from Hes-
 159 sians of the pressure and velocity components. To derive a general refine-
 160 ment criterion, we want to get as close as possible to an indication of the
 161 truncation error. For finite-volume discretisations, this error comes mainly
 162 from interpolation errors in the fluxes, so it contains all the different state
 163 variables (pressure, velocity, turbulent viscosity). However, from a compu-
 164 tational point of view it is desirable to have a simple refinement criterion.
 165 Therefore, as a first step, only the convective and pressure part of the flux is
 166 used and the turbulence modelling is ignored. Furthermore, we do not take
 167 into account all the different products of velocity terms which appear in the
 168 fluxes but assign a common weight ρV to all the velocity Hessians, where ρ

169 is the density and $V = \sqrt{u^2 + v^2 + w^2}$. Thus, the criterion is chosen as:

$$\mathbf{C}_{FCH} = \left(\max(\|\mathcal{H}(p)\|, \rho V \|\mathcal{H}(u)\|, \rho V \|\mathcal{H}(v)\|, \rho V \|\mathcal{H}(w)\|) \right)^\alpha. \quad (3)$$

170 The maximum of two tensors is computed using the approximative procedure
 171 defined by [3]. Further testing has to determine if this criterion is adequate,
 172 or if diffusive and turbulence terms must also be added.

173 3.3. Boundary layer protection and minimum cell size

174 Two measures are available to improve the regularity of the adaptively
 175 refined meshes. The first (figure 1) is a protection of the boundary layer
 176 grid, which on the original grid consists of several layers of wall-aligned cells.
 177 For these layers, the refinement method contains an option to forbid any
 178 refinement in the wall normal direction (which would locally increase the
 179 number of layers). Furthermore, in all cases the refinement parallel to the
 180 wall is made the same in each column of cells from the wall to the outer layer:
 181 if one of the cells in a column needs to be refined, all the cells are refined.
 182 Thus, the column / layer structure of the boundary layer grid is preserved.

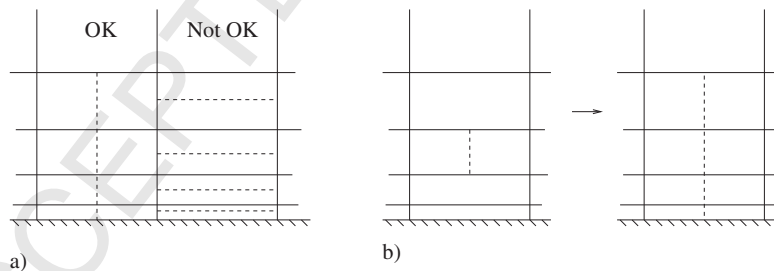


Figure 1: Boundary layer protection: preventing normal refinement (a) and copying parallel refinement in a column (b).

183 A second measure is to impose a minimum cell size: cells smaller than this
184 size are no longer refined. This option prevents spurious refinement if locally
185 large errors appear in the computation of the refinement criterion, which may
186 appear for example in the high aspect-ratio cells of the near-wall boundary
187 layer grid. Also, it prevents infinite refinement around flow singularities.

188 4. Using grid refinement for convergence studies

189 Metric-based grid adaptation methods like the one described in section
190 3 provide a simple and elegant way to create series of meshes for grid con-
191 vergence studies. This section describes such grids and the way they can
192 be created adaptively, plus it highlights certain theoretical limitations of the
193 method proposed. The discussion is completed in section 5, which tests the
194 effectiveness of the approach for obtaining grid-converged solutions in prac-
195 tice.

196 4.1. Geometrically similar grids

197 To allow extrapolation towards the grid-independent solution, grid con-
198 vergence studies require a series of computations on different meshes for
199 which the local truncation errors have the same spatial distribution and
200 vary proportionally to a single global mesh size parameter (see for exam-
201 ple [13, 14]). Since truncation errors in general depend on the shapes and
202 orientations of the cells as well as on their size, this puts two requirements
203 on the meshes:

- 204 1. The ratio of the cell sizes between two meshes must be constant through-
205 out the mesh,

206 2. In any given position, the meshes need to have cells of similar shape
207 and orientation.

208 For a user, creating such meshes is much easier with structured than with
209 unstructured grid generators. In the former, the cell orientation is given by
210 the shape of the grid block(s), the cell size can be varied freely by changing
211 the number of cells in each direction. For the latter, the cell orientations
212 are often not explicitly imposed. Also, it may require great care to obtain
213 fine cells in exactly the same zones for all meshes, since the mesh generation
214 algorithm does not ensure this naturally. In practice, this limits the use of
215 refinement studies with unstructured grids.

216 *4.2. Series of meshes with grid adaptation*

217 Metric-based anisotropic grid refinement methods such as the one out-
218 lined in section 3 can create series of geometrically similar unstructured
219 meshes naturally. Thanks to the use of metric tensors, the threshold T_r
220 globally specifies the fineness of the grid: if the refinement criterion remains
221 constant as the mesh is refined, the mesh size everywhere is proportional
222 to T_r (equation 1). This is the case for any refinement criterion, local or
223 integral-based, which is computed from the flow without explicitly taking
224 into account the mesh. The Hessian criteria are an example. Furthermore,
225 if the refined grids are created by cell division, the refined cells conserve the
226 shape and orientation of the cells in the original grid, so two meshes refined
227 from the same original grid have similar cell shapes. Thus, the two require-
228 ments of section 4.1 are satisfied. This means that series of geometrically
229 similar meshes can be created by starting from the same original grid and
230 simply varying T_r .

231 A limitation of this approach is that the refinement procedure cuts cells
232 in half, so the cell sizes in the original grid can only be reduced by a power of
233 two. Thus, if T_r is reduced by anything other than a power of two between
234 subsequent meshes, the cell sizes are divided by the nearest power of two and
235 the zones with cells of the same size become larger or smaller (see figure 3
236 for an example). While such meshes still resemble each other closely, they
237 are not formally geometrically equivalent.

238 Furthermore, the mesh size is not proportional to T_r in those regions
239 where the original grid is not refined at all. To obtain grids that are similar
240 in these zones, one could make a different original grid for each threshold.
241 However, if such grids are unstructured, they are rarely exactly similar close
242 to the walls, where the grid is deformed to follow the walls. Thus, to improve
243 the similarity in the unrefined parts of the mesh which are by definition the
244 least interesting for the flow, one would reduce the similarity in the most
245 critical parts of the mesh, the near-wall regions. Therefore, we base our
246 convergence studies on the same original mesh for all grids.

247 *4.3. Effects of protective measures*

248 Protective measures also break the proportionality of the mesh size to T_r .
249 Suppressing boundary layer refinement in the wall-normal direction results
250 in the same cell sizes normal to the wall for all meshes. This is often the
251 desired behaviour. For example in refinement studies with law-of-the-wall
252 boundary conditions it is common practice to keep the boundary grids the
253 same between meshes, since the gradient in the first layer of cells on the
254 boundary (and therefore the solution) depends on the thickness of this layer.
255 However, one must be aware of introducing a component to the numerical

256 error which does not disappear on grid refinement.

257 Imposing a minimum cell size has the same effect. Once the finest parts
258 of the mesh have reached this size, they are no longer refined when T_r is
259 reduced, so the truncation errors remain constant in these parts. A solution
260 for this problem is to vary the minimum cell size on the different meshes,
261 proportional to T_r . However, the minimum cell size only exists to counteract
262 occasional large errors in the refinement criterion, it should have the least
263 possible effect on the meshes. Therefore, we keep the minimum mesh size
264 constant and we choose it as small as possible.

265 **5. Two-dimensional case: Nakayama B airfoil**

266 The objective of this section is to test the proposed method for grid-
267 independence studies on a two-dimensional test case, for which very fine
268 meshes can be produced if needed. Three different series of meshes with low-
269 Reynolds and wall-law boundary conditions are created (section 5.1). The
270 adaptively refined meshes are analysed in section 5.2, the convergence of the
271 global forces and the related uncertainty estimation is studied in section 5.3.
272 Section 5.4 considers the grid-independence of two local flow features, the
273 pressure on the foil and the wake. The final section 5.5 gives estimations of
274 the equivalent three-dimensional grid sizes for the grids obtained here.

275 *5.1. Test case and computations*

276 The test case is the Nakayama B airfoil [15]. This case is of particular
277 interest since detailed measurements of the velocity and the turbulence in-
278 tensity have been performed both in the boundary layers and in the wake,
279 up to 2 chord lengths behind the trailing edge. The airfoil is a supercritical

280 profile with chord $c = 61$ cm, placed at $\alpha = 4^\circ$ in the centre of a 137 cm high
 281 test section. The flow is incompressible and $Re = 1.2 \cdot 10^6$. Simulations are
 282 performed with the $k - \omega$ SST turbulence model and with the AVLSMART
 283 scheme [16] for all convective terms.

284 Three series of computations have been performed. The first two series
 285 use a low-Reynolds (no-slip) boundary condition on the airfoil and serve
 286 to compare the pressure (LR PH) and flux-component (LR FCH) refinement
 287 criterion. The original mesh for these computations has 5755 cells and $y^+ \approx 1$
 288 on the walls. A minimum cell size is imposed but this limit is kept low
 289 ($0.0002c$). The last series uses the FCH criterion with a wall-law boundary
 290 condition, for which the original grid has 2623 cells and $y^+ \approx 30$. For this
 291 series (WL FCH), the minimum cell size is $0.001c$, the size of the first layer
 292 of boundary cells. As seen in section 4.3, these cells cannot be refined.

293 All series consist of ten meshes. For each mesh, the refinement threshold
 294 T_r is divided by $\sqrt{2}$ with respect to the previous mesh (thus, the cell sizes
 295 are halved every two meshes). The T_r go from 0.5000 to 0.0221.

296 5.2. Refined meshes

297 This section studies the series of adapted meshes to see if they have the
 298 characteristics described in sections 3 and 4. The difference between the
 299 three series is illustrated by figure 2. For the pressure criterion (figure 2a),
 300 the refinement is concentrated around the leading edge where the variations
 301 in the pressure are the largest. The same refinement is created by the flux-
 302 component criterion (figure 2b), since this criterion includes the pressure
 303 Hessian. However, the boundary layer and the wake are also refined, which
 304 is not the case for the pressure criterion. Contrary to the accepted practice

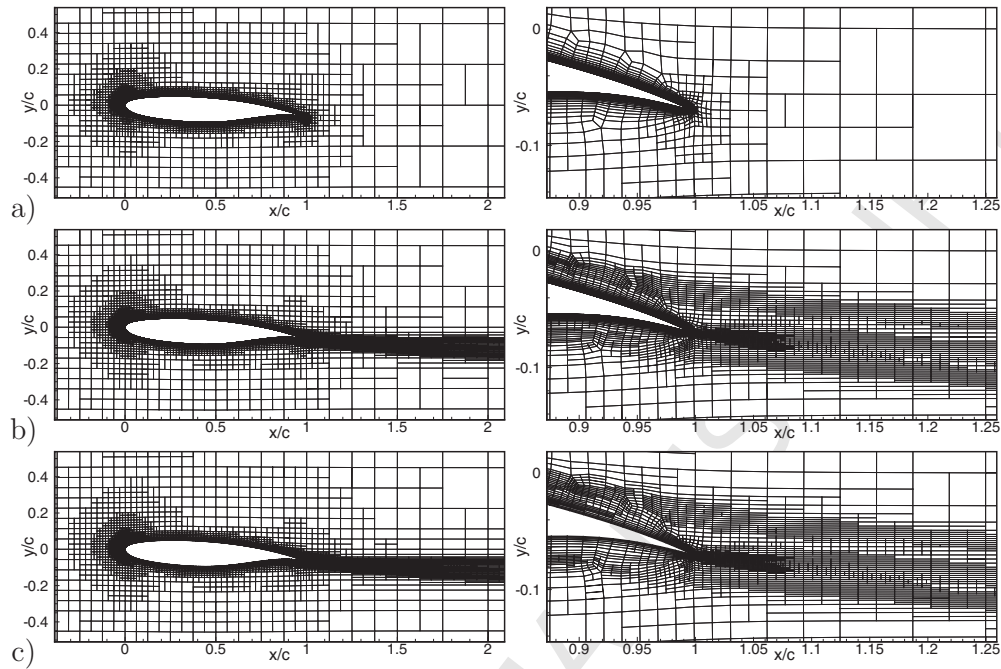


Figure 2: Refined meshes for the Nakayama airfoil with $T_r = 0.125$. Low-Reynolds conditions with the PH criterion (a) and FCH criterion (b), wall law with FCH (c). The right figures show a close-up around the trailing edge.

305 for boundary layer grids, fine cells are concentrated near the outside of the
 306 boundary layer. Finally, the wall-law FCH grids (figure 2c) are generated
 307 with the same criterion, the difference comes from the larger minimum cell
 308 size and the coarser near-wall grid in the original mesh. The figures show
 309 that even in the refined meshes, the difference with the LR FCH meshes
 310 remains limited to the near-wall region.

311 Are the meshes in one series geometrically similar? Figure 3 shows a
 312 detail of three successive meshes in the LR PH series. The threshold for the
 313 third mesh is twice smaller than for the first one and these two meshes indeed

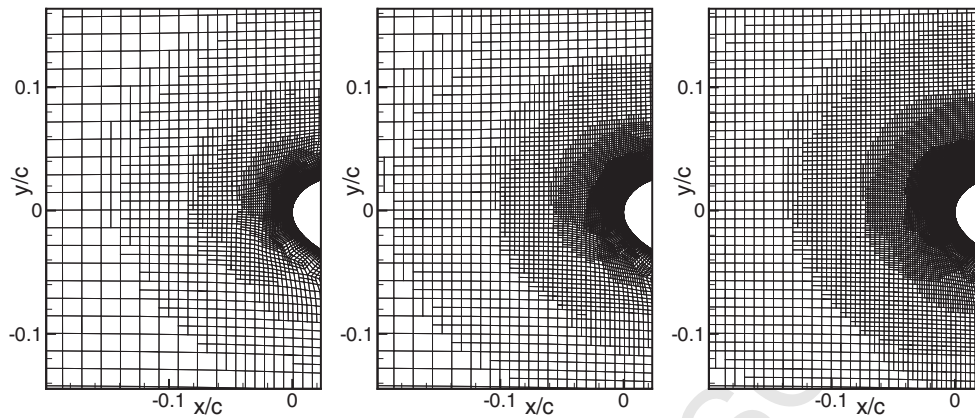


Figure 3: A detail of the LR PH meshes, for progressively finer thresholds. Left to right: $T_r = 0.0625$, $T_r = 0.0442$, and $T_r = 0.0313$.

314 have refinement in the same places (see section 4.2). For example, regions of
 315 cells with the same size start at $x/c = -0.14$ and $x/c = -0.07$ in both grids.
 316 The anisotropic refinement is also the same, as shown by the similar regions
 317 of vertically oriented cells above and below the regions of fine cells. Thus,
 318 the first and third grids are geometrically similar.

319 For the middle grid, the threshold has a ratio of $\sqrt{2}$ with respect to the
 320 other ones. Thus, the regions of fine cells are not the same, they start at
 321 $x/c = -0.1$ and $x/c = -0.05$ for example. Furthermore, the cells have
 322 the same size in the second mesh that they have in either the first or in
 323 the third mesh, so when going from a ‘coarser’ to a ‘finer’ mesh, in many
 324 places the cell sizes do not change! Locally, the second mesh is therefore not
 325 geometrically similar to the first and third ones. However, globally the three
 326 meshes resemble each other. The cell orientations are the same, the shape of
 327 the fine-cell regions is the same even down to the distribution of anisotropic
 328 refinement. The three meshes still form a series.

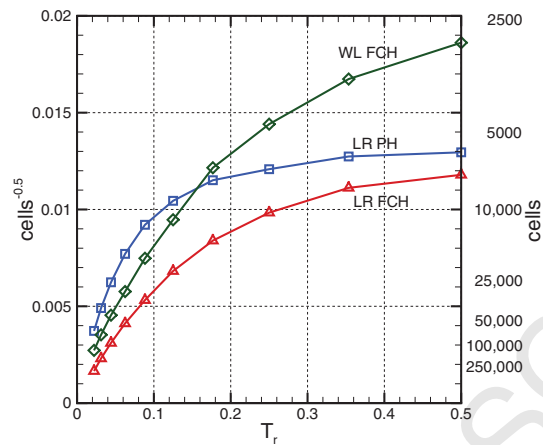


Figure 4: Number of cells in the three series of grids. The vertical axis shows the number of cells to the power -0.5 , the corresponding number of cells is indicated to the right.

329 Finally, with the cell sizes proportional to the threshold (equation (1)),
 330 the number of cells should scale with T_r^{-2} . Figure 4 shows that for the
 331 Nakayama case, this is only asymptotically true as T_r goes to zero. On the
 332 coarser grids, in most of the domain the original grid is not refined. In
 333 these regions the number of cells does not change between grids; geometrical
 334 similarity is therefore not satisfied. However, even the coarsest grid is refined
 335 in those regions which are the most important for the simulation of the flow.
 336 Thus, in these regions the grids are geometrically similar. If this is enough
 337 for convergence studies, will be seen below.

338 5.3. Grid convergence of forces

339 For all computations, the convergence of the force coefficients $C_d =$
 340 $F_x / (\frac{1}{2}\rho V^2 cb)$ and $C_l = F_y / (\frac{1}{2}\rho V^2 cb)$ is studied, where the forces are com-
 341 puted on a wing segment of unit span $b = 1$. Uncertainty estimations are

342 performed following Eça and Hoekstra [14]. A power law:

$$C(T_r) = C_0 + \alpha(T_r)^p, \quad (4)$$

343 with an unknown order p is least-squares fitted through the force coefficients
 344 obtained on the five grids considered. This fit is used to compute the es-
 345 timated error $\epsilon = \alpha(T_{r1})^p$ which is the difference between the value on the
 346 finest of these grids and the extrapolation to $T_r = 0$. Furthermore, the corre-
 347 lation error σ is computed as the standard deviation of the difference between
 348 the actual values on all grids and the power-law values obtained for the T_r of
 349 these grids. Then the uncertainty on the finest grid is estimated as $1.25\epsilon + \sigma$.
 350 The safety factor 1.25 due to Roache [17] is meant to ensure that the uncer-
 351 tainty interval contains the actual error with a 95% probability. When no
 352 satisfactory fit is found, new fits are computed with fixed-power laws and the
 353 factor of safety is increased. For details, the reader is referred to the original
 354 paper [14].

355 Other uncertainty estimation techniques such as those by Stern et al. [18]
 356 or Celik et al. [19] would probably also work with our data. However, the
 357 advantage of the Eça and Hoekstra approach for the current work is, that the
 358 standard deviation σ provides a measure of the quality of the fit. Thus, σ can
 359 be used to see how well our series of meshes produce asymptotic convergence.

360 5.3.1. *Estimated uncertainties*

361 The dependence of C_d and C_l on T_r is shown in figure 5; starting from the
 362 third grid, estimated uncertainties are included. To resemble the usual type
 363 of convergence study where a large number of grids is not always available,
 364 the uncertainties on each grid are computed from not more than five grids:

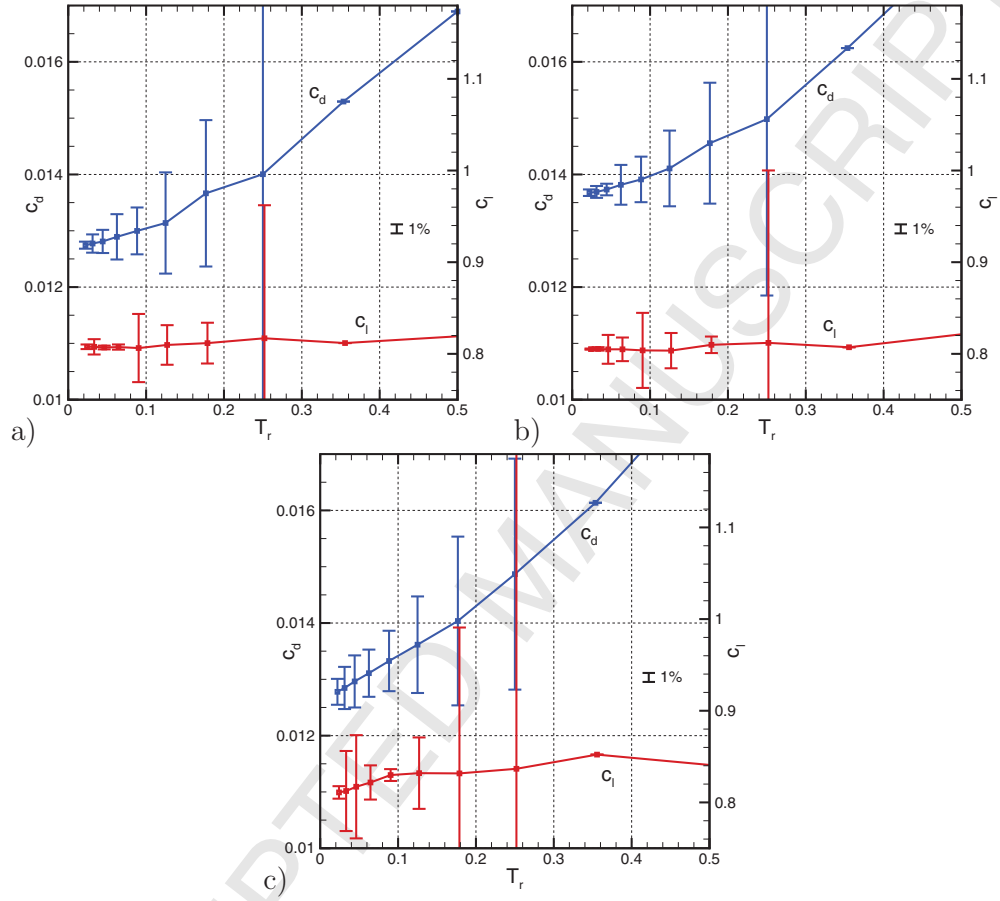


Figure 5: Convergence of drag coefficient C_d and lift coefficient C_l for the Nakayama airfoil, with respect to the threshold T_r . FCH with low-Reynolds grid (a) and wall-law grid (b), PH with low-Reynolds grid (c). The two vertical axes are scaled such that error bars of the same length for C_d and for C_l indicate the same relative uncertainty.

365 the grid itself and the previous four, if these exist.

366 The low-Reynolds grids with the FCH criterion provide the most com-
367 plete simulation of the flow, since the wake and the boundary layer are fully
368 resolved. Figure 5a shows a convergence for C_d that appears asymptotic
369 with some superimposed high-frequency noise. The uncertainty estimation
370 works well here: all the uncertainty intervals contain the actual converged C_d
371 and those computed with five or four grids ($T_r \leq 0.1768$) are not excessively
372 large. For C_l , even on the coarsest grids, the computed values are close to the
373 solution on the finest grid. Unfortunately, this means that the convergence
374 is dominated by noise so the estimated uncertainties are high; the estimation
375 cannot show how good the coarse-grid solutions really are. Realistic error
376 bars are obtained for $T_r \leq 0.0625$. Similar results are obtained for the WL
377 FCH series (figure 5b), so the use of the wall law does not perturb the grid
378 convergence.

379 However, the PH series shows a different convergence behaviour (figure
380 5c). First, C_l varies much more over the grids. And second, the shape of
381 the convergence curve for both forces does not resemble a power law, which
382 means that the uncertainty intervals on coarser grids do not contain the
383 true converged values. So even though the lift is predominantly a pressure
384 force, the pressure-based refinement criterion is not the optimal choice for
385 computing it. On the contrary, a fine resolution of the boundary layers and
386 the near wake is required.

387 Finally, the convergence behaviour on coarse grids is not much worse than
388 on fine grids, even though these coarse grids are mostly equal to the original
389 grid (see section 5.2): the convergence for C_d is smooth, while the values for C_l

390 rapidly approach the converged one. Thus, geometrically similar refinement
 391 of the most crucial regions is enough to get acceptable convergence.

392 5.3.2. Asymptotic convergence and noise

393 To see if the series of adaptively refined meshes produce asymptotically
 394 convergent solutions, we perform different power-law fits for the LR FCH
 395 series. For the least-squares method, the bias towards the finest grids sug-
 396 gested by Eça and Hoekstra is used. The coefficients p and C_0 for each fit,
 397 as well as the extrapolated error ϵ and correlation error σ (see equation (4)
 398 and its discussion) are gathered in table 1.

Table 1: Fits of the power law (4) to the forces on the Nakayama wing, using the results on different sets of grids. A symbol – indicates that no fit could be found. Percentages are expressed w.r.t. $D = 1.2714 \cdot 10^{-2}$ (C_d) and $D = 0.8070$ (C_l).

Finest T_r	grds.	$C_d \cdot 10^2$	p	$C_0 \cdot 10^2$	$\epsilon \% D$	$\sigma \% D$	C_l	p	C_0	$\epsilon \% D$	$\sigma \% D$
<i>All grids</i>											
0.0884	6	1.3326	1.52	1.2659	2.38%	0.92%	0.8063	–			
0.0442	8	1.2963	1.56	1.2714	0.76%	0.57%	0.8069	0.68	0.8036	0.36%	0.23%
0.0221	10	1.2778	1.56	1.2714	0.26%	0.37%	0.8077	1.22	0.8070	0.03%	0.18%
<i>5 grids</i>											
0.0221	5	”	1.36	1.2698	0.36%	0.04%	”	1.00	0.8082	0.05%	0.05%
<i>Every 2nd</i>											
0.0221	5	”	1.49	1.2697	0.33%	0.19%	”	1.48	0.8072	0.01%	0.19%

399 To eliminate noise in the convergence curves, Eça and Hoekstra recom-
 400 mend to use computations on as many grids as possible. Therefore, for the

401 first trial fits, the data on a grid and all the preceding ones is used. The re-
 402 sulting fit for C_d converges rapidly and is fully established with $T_r = 0.0442$
 403 as the finest grid; adding the finest two grids does not change the fit, even
 404 though these points receive the largest weight in the fit. Furthermore a fit
 405 through the finest 5 grids, while not identical to the first fits, is good as
 406 shown by its low value of σ . This indicates that C_d actually behaves as a
 407 power law, perturbed only by small-scale noise.

408 This is different for C_l : the power-law fit has no solution for $T_r = 0.0884$,
 409 the coefficients vary for each added point, and C_{l0} is always lower than C_l ,
 410 even though C_l on the finest grids increases with further refinement. The fit
 411 on the finest five grids looks more reasonable, however. It is plausible that
 412 the data for C_l contains a higher-order error term as well as high-frequency
 413 noise, so that fine grids are needed to reach the asymptotic range.

414 The noise in the data may be created because subsequent grids are not
 415 geometrically similar (section 5.2); indeed, figure 5 shows that most of the
 416 perturbations are positive for one point, negative for the next, etc. To elim-
 417 inate this effect, a last fit was made using only every second grid, so each
 418 grid is twice finer than the previous one. For C_d , this fit is better than the
 419 one using all meshes since its σ is lower, but the improvement is small. For
 420 both forces, the fits are comparable. Thus, in practice it is possible to use
 421 the intermediate, not strictly similar meshes for data fitting.

422 5.3.3. Accuracy

423 What grid size is needed to accurately evaluate modelling errors? Fig-
 424 ure 5 shows that a typical modelling error, switching from low-Reynolds to
 425 wall-law boundary conditions, creates a change in C_d of about 7%. For a

426 reliable comparison of the LR FCH and WL FCH computations, the numer-
 427 ical uncertainties for the two computations should be small compared to this
 428 modelling error. For example, an uncertainty of the order of 1% gives a 1.4%
 429 combined uncertainty for their difference; five times less than the modelling
 430 error. The results in figure 5 show that 1% estimated uncertainty for both
 431 C_l and C_d is reached around $T_r = 0.0313$, i.e. on very fine meshes.

432 However, 1% uncertainty is a very strict requirement which may not be
 433 necessary in practice for all computations. Comparing the computed values
 434 on each grid with the extrapolated value for the finest grid, we see that 1%
 435 error for C_d is reached around $T_r = 0.0442$ and for C_l around $T_r = 0.25$!
 436 And for C_d , using Richardson extrapolation, 1% error in C_{d0} is obtained
 437 from $T_r = 0.125$. Thus, modelling errors can be evaluated with results on
 438 coarser grids than the uncertainty estimates indicate, if one accepts these
 439 results without formal proof of their accuracy.

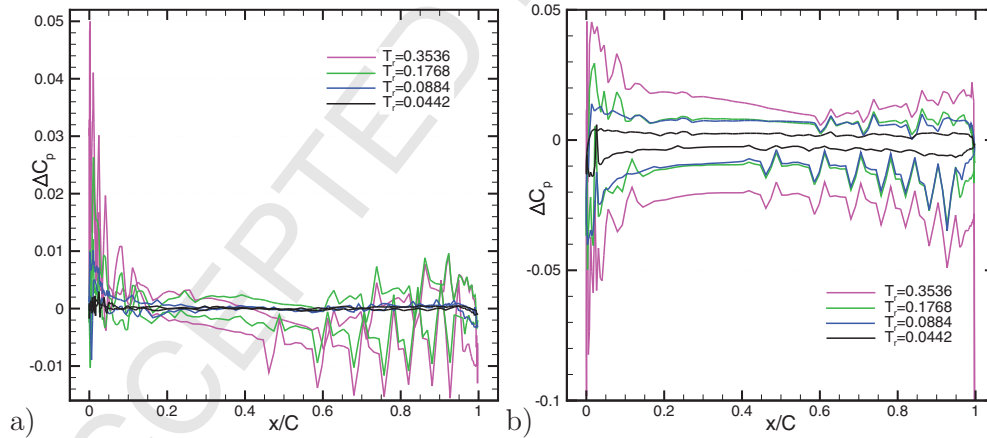


Figure 6: The difference in pressure coefficient C_p with respect to the finest grid $T_r = 0.0221$, for four coarser grids. LR FCH (a) and LR PH (b). Note that the vertical scale of the two figures is different.

440 5.4. *Grid-independence of the local flow*

441 Much of the interest of grid adaptation is in the precise computation
442 of local flow details. This section studies the grid convergence of two such
443 features: the pressure on the airfoil and the flow in the wake. For the distri-
444 bution of $C_p = p/(\frac{1}{2}\rho V^2)$ on the profile, figure 6 shows the difference in the
445 solution between four coarser grids and the finest one. For the LR FCH series
446 (figure 6a), although the pressure signal is noisy, the distance between solu-
447 tions decreases as the grids become finer. The main differences are found in
448 the stagnation point and the suction peak on the leading edge, so the regions
449 of fine cells created at the leading edge (see figures 2 and 3) are justified.
450 From $T_r = 0.0884$ on, the pressure difference with the finest grid is less than
451 1% of the stagnation pressure everywhere.

452 The result for the LR PH series is different (figure 6b). The distance
453 between solutions decreases for finer grids, but the difference with the finest
454 grid is almost uniform over the entire profile and the differences between
455 grids are larger. This confirms what was found in section 5.3.1: even for
456 computing the pressure, the accurate resolution of the boundary layer flow
457 is a necessity.

458 The second feature studied is the near and far wake. Figure 7 gives the
459 velocity and one component of the turbulent fluctuation for the two FCH
460 series (with the PH criterion, the wake is completely diffused). Judging
461 visually from the distance between the curves, on the $T_r = 0.0442$ grid the
462 solution is converged, the $T_r = 0.0884$ grid is close, and the $T_r = 0.1768$ grid
463 already gives the right tendencies.

464 The results are not the same for low-Reynolds and wall-law boundary

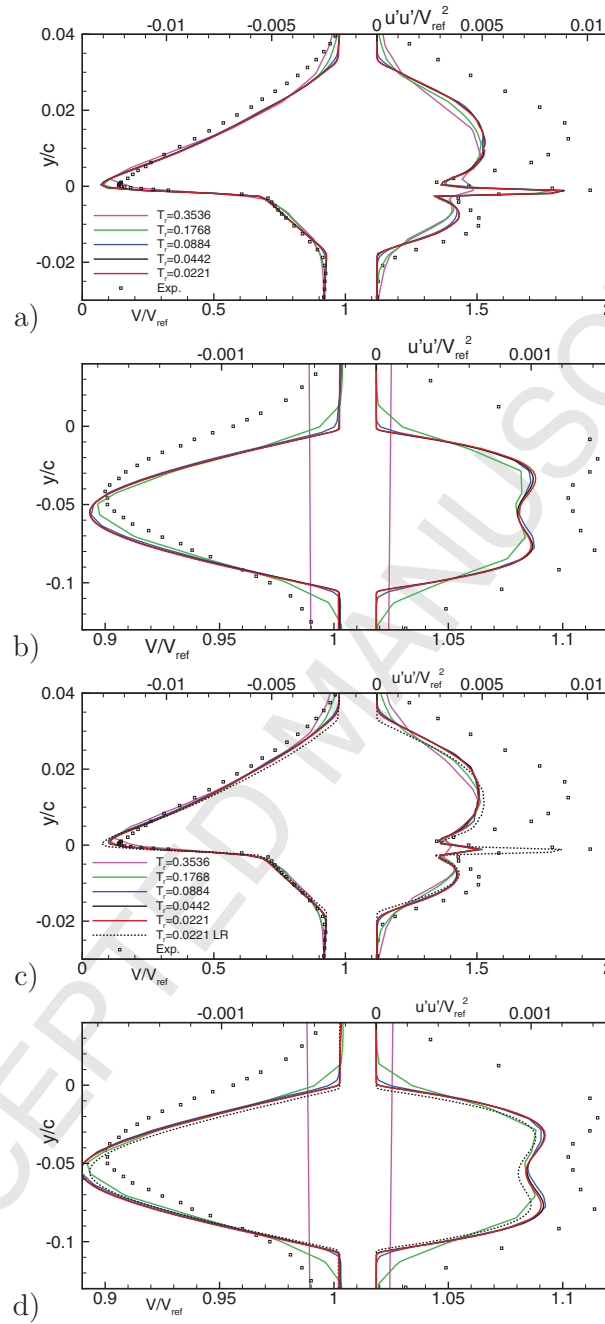


Figure 7: Velocity V (left lines) and fluctuation correlation $\overline{u'u'}$ (right lines) in the wake of the Nakayama airfoil, at $0.01c$ (a, c) and $2.0c$ (b, d) behind the trailing edge. LR FCH (a, b) and WL FCH (c, d).

465 conditions. The most notable effect of the wall law is the attenuation of the
466 turbulence peak at the trailing edge in the near wake, which corresponds to
467 the position where the grids differ most (figures 2b and c). However, even in
468 the far wake a reduction in the velocity defect and the turbulence intensity
469 can be seen due to the wall law.

470 Compared with the experiments [15], the shape of the turbulence profiles
471 is well captured in both cases but the intensity is too low, so the wake and
472 the boundary layer are too thin. Also, unlike the experiments, the edges
473 of the wake are sharp in the numerical solutions. This is only observed
474 because of the extra-fine grid around the edge of the wake (figure 2b); on
475 coarser refined grids and therefore on standard boundary layer grids also,
476 the numerical solution is closer to the experiments. This modelling error is
477 therefore only visible thanks to the adaptive refinement.

478 Thus, from the $T_r = 0.0884$ grids on, the differences between the solutions
479 in one series are small compared with the difference between low-Reynolds
480 and wall-law solutions. Both these differences are small with respect to the
481 distance between the simulations and the experiments. Thus, the $T_r = 0.0884$
482 grid and all finer grids indicate modelling errors in the wake reliably.

483 5.5. Equivalent three-dimensional grid sizes

484 The preceding sections have established that grid convergence for the
485 Nakayama wing can be obtained using adaptive refinement. 1% proven un-
486 certainty in the forces is obtained for $T_r = 0.0313$, while the far wake is
487 converged for $T_r = 0.0884$. A reasonable approximation of both is already
488 obtained when $T_r = 0.1768$. To generalise these conclusions, the thresh-
489 olds should be translated to numbers of cells, or even better, to an order of

490 magnitude for the number of cells in an equivalent three-dimensional grid.

491 To estimate the number of cells in a 3D case which resembles the Na-
 492 kayama airfoil, we suppose that the number of cells in the third direction
 493 (the spanwise direction for a wing) is of the same order as the number of
 494 cells in the flow direction. For want of something better, this number is
 495 approximated by the number of cells on the surface of the airfoil. Multiplying
 496 the number of surface cells by the total number of 2D cells therefore gives a
 497 rough indication of an equivalent 3D grid size. These numbers are given in
 498 table 2; they indicate that the grids for $T_r = 0.0884$ correspond to habitually
 499 used grid sizes, while those for $T_r = 0.0221$ are much finer than what is used
 500 in practice.

Table 2: Nakayama airfoil: estimated number of cells in three-dimensional grids.

T_r	0.3536	0.1768	0.0884	0.0442	0.0221
Low-Re FCH	2.1M	5.6M	26M	164M	983M
Low-Re PH	1.1M	1.6M	3.4M	12M	60M
WL FCH	0.7M	1.8M	8.1M	38M	188M

501 For low-Reynolds FCH computations, getting less than 1% uncertainty
 502 for the forces with our procedure requires more than 100M cells. However,
 503 these grids resolve the wake with very fine cells, which may not be necessary
 504 for obtaining only the forces; also, 1% uncertainty is often too strict (section
 505 5.3.3). For reliable LR FCH predictions of the near and far wake, 20M to
 506 50M cells are needed, so such simulations can be easily envisaged.

507 With the wall law and FCH, less than 1% uncertainty requires around
 508 50M cells while the far wake can be captured accurately with about 10M

509 cells; both objectives are therefore possible. The LR PH computations are
510 efficient, but to obtain 1% uncertainty in the forces they need even finer
511 thresholds than those tested here. Furthermore, the wake is not resolved.
512 Therefore, these computations may be of little interest despite their apparent
513 low number of cells. Note that these are indications only, actual numbers
514 of cells will depend on the real 3D geometry and the flow characteristics, as
515 well as the quality of the original grid and the exact choice of the refinement
516 strategy.

517 **6. The step to 3D: KVLCC2 tanker**

518 A representative three-dimensional case is the flow around the KVLCC2
519 tanker ship. We wish to see if this three-dimensional case confirms the find-
520 ings for the Nakayama airfoil. The KVLCC2 hull has a complex aftbody
521 flow with boundary layer separation and different vortical structures; figure
522 8 gives a global view of the flow field, with the thickening and separation of
523 the boundary layer.

524 The flow is computed for a model of the ship with length $L = 5.571m$,
525 which gives $Re = 4.6 \cdot 10^6$. Since the flow is symmetric, only half the hull
526 is simulated. Free-surface effects are ignored: the water surface is treated as
527 a symmetry plane, like the vertical centreplane. Windtunnel measurements
528 are available from Postech [20], Larsson et al. [21] give a complete description
529 of the test case.

530 The flow is simulated here with a wall-law approach and the anisotropic
531 EASM turbulence model including rotation correction, since this model rep-
532 resents flows dominated by longitudinal vorticity better than the standard

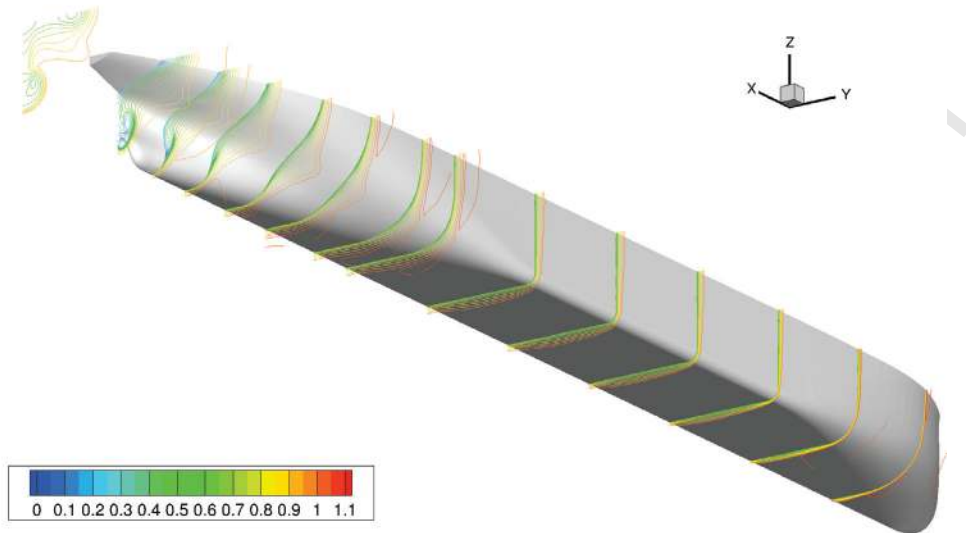


Figure 8: KVLCC2: the (half) hull and the axial velocity field.

533 $k-\omega$ SST model [6]. Convective terms are discretised with a blended central
 534 / upwind scheme. FCH is used as refinement criterion with the minimum
 535 cell size equivalent to the first-layer thickness in the boundary layer grid (see
 536 sections 4.3 and 5.1).

Table 3: KVLCC2: thresholds and mesh sizes.

T_r	3.0	2.0	1.5	1.0	0.75
nb. cells	247k	613k	1.34M	5.02M	10.18M

537 6.1. Refined meshes

538 For the convergence study five different thresholds are used (table 3),
 539 with a factor 4 between the cell sizes of the coarsest and finest mesh. This

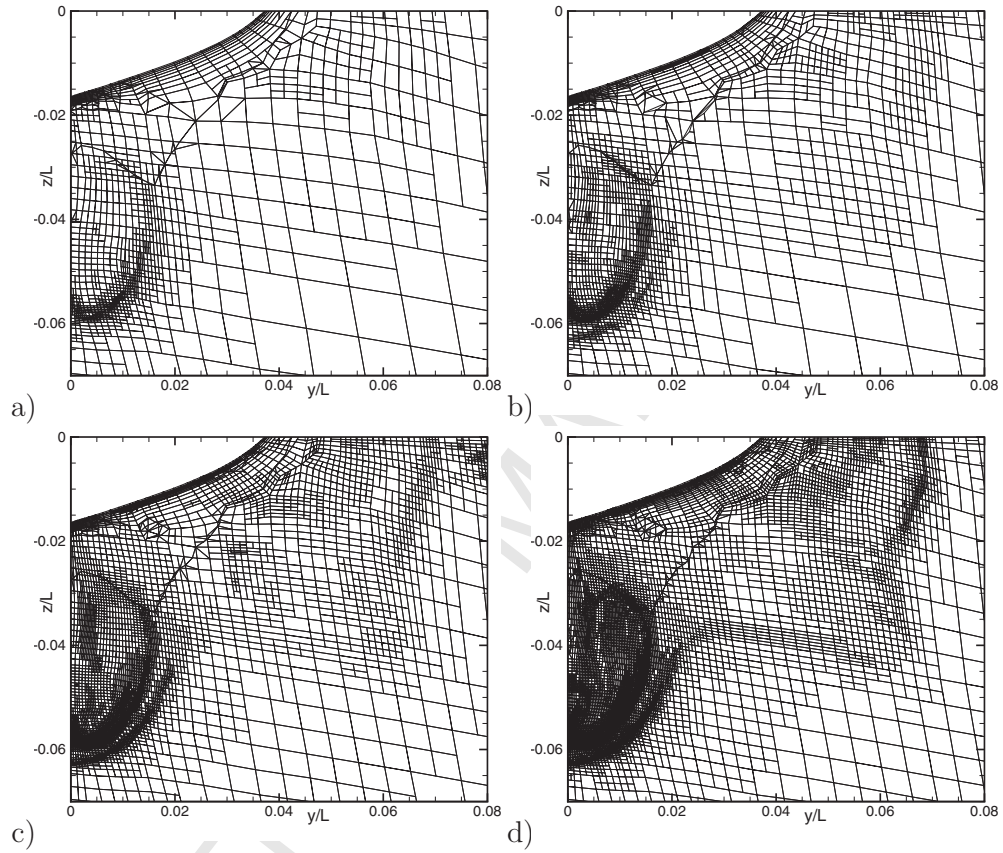


Figure 9: Meshes in the propeller plane $x/L = 0.9825$ of the KVLCC2, $T_r = 2.0$ (a), $T_r = 1.5$ (b), $T_r = 1.0$ (c), and $T_r = 0.75$ (d).

540 represents a more realistic study than the 10-grid Nakayama case, whose
541 coarsest grids would be insufficient to capture the details of the more com-
542 plex KVLCC2 flow, while the finest grids would be too expensive in three
543 dimensions.

544 The refined meshes in the propeller plane (the aftmost cut plane on the
545 hull in figure 8) are shown in figure 9 for the four finest thresholds. These
546 meshes are refined around the principal features of the flow, such as the
547 main aft-body vortex whose centre is near (0.01, -0.04). This vortex creates
548 an open-type separation on the hull, with an associated ear-shaped shear
549 layer that is captured with fine cells. A second shear layer starting at (0.0, -
550 0.065) separates the boundary layer flow around the vortex from the flow
551 outside. Finally, like for the 2D wing, the outer edge of the boundary layer
552 attracts grid refinement everywhere.

553 The pairs of grids in figures 9a and c, as well as b and d, have thresholds
554 T_r which differ by a factor two, so the pairs should be geometrically similar.
555 This is mostly the case; the centre of the vortex with its vertically elongated
556 cells is an example. However, there is a perturbing effect here that was not
557 encountered before: as the mesh is refined, the flow field changes so the re-
558 finement criterion is modified, which leads to non-similar meshes. This effect
559 is stronger than for the 2D case, because the KVLCC2 flow has more shear
560 layers and discontinuities in the velocity gradients, which become sharper on
561 fine meshes. Thus, especially on the outer edge of the boundary the grid
562 size decreases more than linearly with T_r . This is not necessarily a problem,
563 since the sharp resolution of these features improves the quality of the local
564 flow on the fine meshes, while globally seen the four grids form a reasonable

565 series.

566 6.2. Convergence of the flow

567 The convergence of the resistance force $C_T = F_x / (\frac{1}{2}\rho V^2 S)$, with $S =$
568 $0.2682L^2$, is monotone (figure 10a). Numerical uncertainties were again es-
569 timated with the method of Eça & Hoekstra (section 5.3). The error bars
570 computed with 3, 4 and 5 grids overlap, which is an indication that the
571 procedure works well. On the finest grid, the uncertainty is 1.69% with an
572 order $p = 1.7$. However, more noise is present than for the Nakayama case,
573 which may be due to the irregularities in the mesh noted above. Due to the
574 complexity of the case, it is also possible that a higher-order error is present.

575 The convergence of the pressure force coefficient C_{Tp} and the viscous
576 force coefficient C_{Tv} are given in figures 10b and 10c respectively. While
577 the viscous force is about four times larger than the pressure force, their
578 variation across the grids and the computed uncertainties are of the same
579 magnitude. Both figures show that the forces are not in the asymptotic range,
580 but the estimated uncertainties are reasonable, especially for the viscous
581 forces where the uncertainty intervals overlap. The pressure force has some
582 more irregularity. The estimated uncertainties are 6.47% with order $p = 1.51$
583 for C_{Tp} and 1.11% with $p = 2.26$ for C_{Tv} .

584 The grid convergence of the local flow is good (figure 11). Even though
585 the overlapping isolines of the Nakayama case are not obtained, the velocity
586 and turbulence isolines for the finest grids are very close throughout the
587 aft-body flow and also in the near wake. Even the 0.9 axial velocity isoline
588 which represents the outer boundary layer, often underresolved in KVLCC2
589 simulations, is close to convergence thanks to the adaptive refinement. The

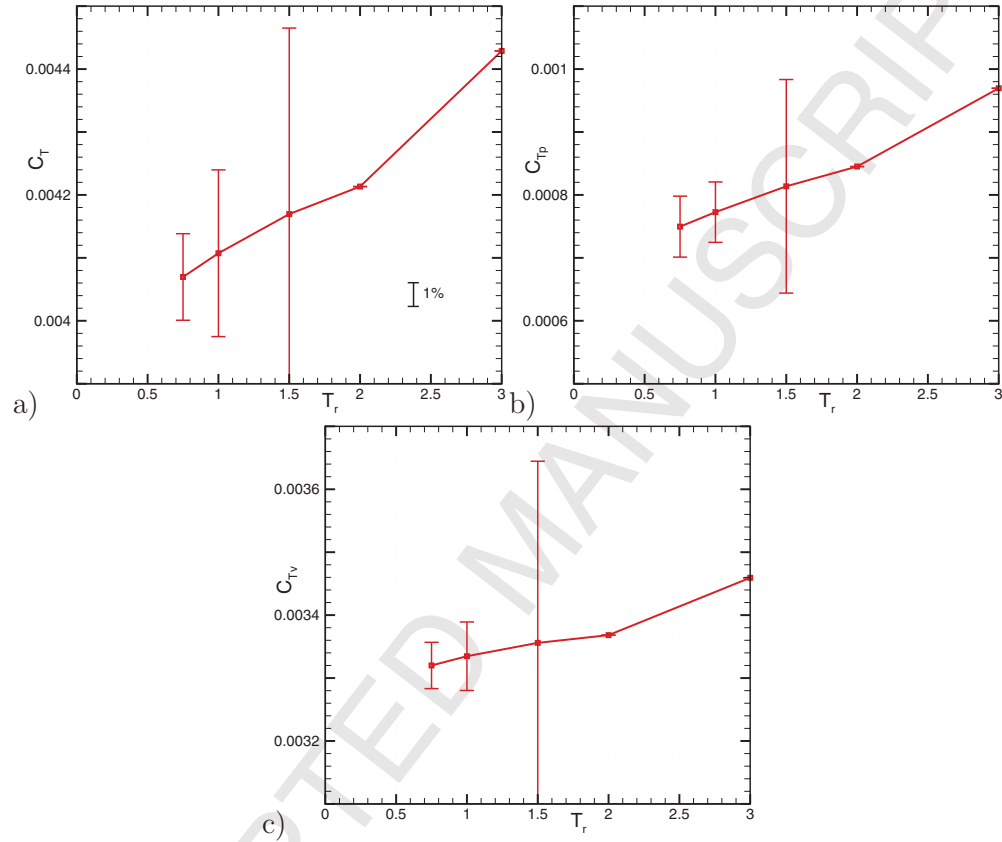


Figure 10: Grid convergence for the KVLCC2: resistance coefficient C_T (a), pressure and viscous resistance coefficients C_{Tp} (b) and C_{Tv} (c), with uncertainty computed as in section 5.3, using all the coarser grids available. The scale of the axes is the same for the three graphs.

590 turbulence kinetic energy in the shear layer is the same on the two finest
591 grids. And finally, the differences between the two finest meshes are small
592 compared to their difference with the wind-tunnel measurements from [20].
593 Thus, the solution on the finest mesh is sufficiently precise to assess modelling
594 errors accurately.

595 These results agree with the findings of the Nakayama case (section 5.5).
596 Convergence of the local flow in the wake is indeed obtained at 10M cells.
597 Furthermore, we expect a 1% uncertainty in the forces around $T_r = 0.5$
598 judging from figure 10. Such a grid would have around 40M cells (eight
599 times the number of cells in the $T_r = 1.0$ grid), which also corresponds to the
600 Nakayama estimate. This confirms the soundness of the 2D-based orders of
601 magnitude for the number of cells, although the KVLCC2 is not a wing so
602 the nearly exact agreement is due to coincidence.

603 7. Conclusion

604 7.1. Evaluation of the results

605 To use adaptive grid refinement for computing grid-independent solutions,
606 the introduction identified three requirements: grid adaptation should create
607 geometrically similar grids, these have to work with established uncertainty
608 estimation methods, and the resulting global and local computed quantities
609 must indeed be accurate. Can these requirements be met?

610 In principle, the adaptive refinement of a coarse, unstructured hexahedral
611 grid using a procedure based on metric tensors is ideal for grid convergence
612 studies. It ensures that cells in all meshes have the same shapes and orienta-
613 tions, while the cell sizes in a mesh are proportional to the global refinement

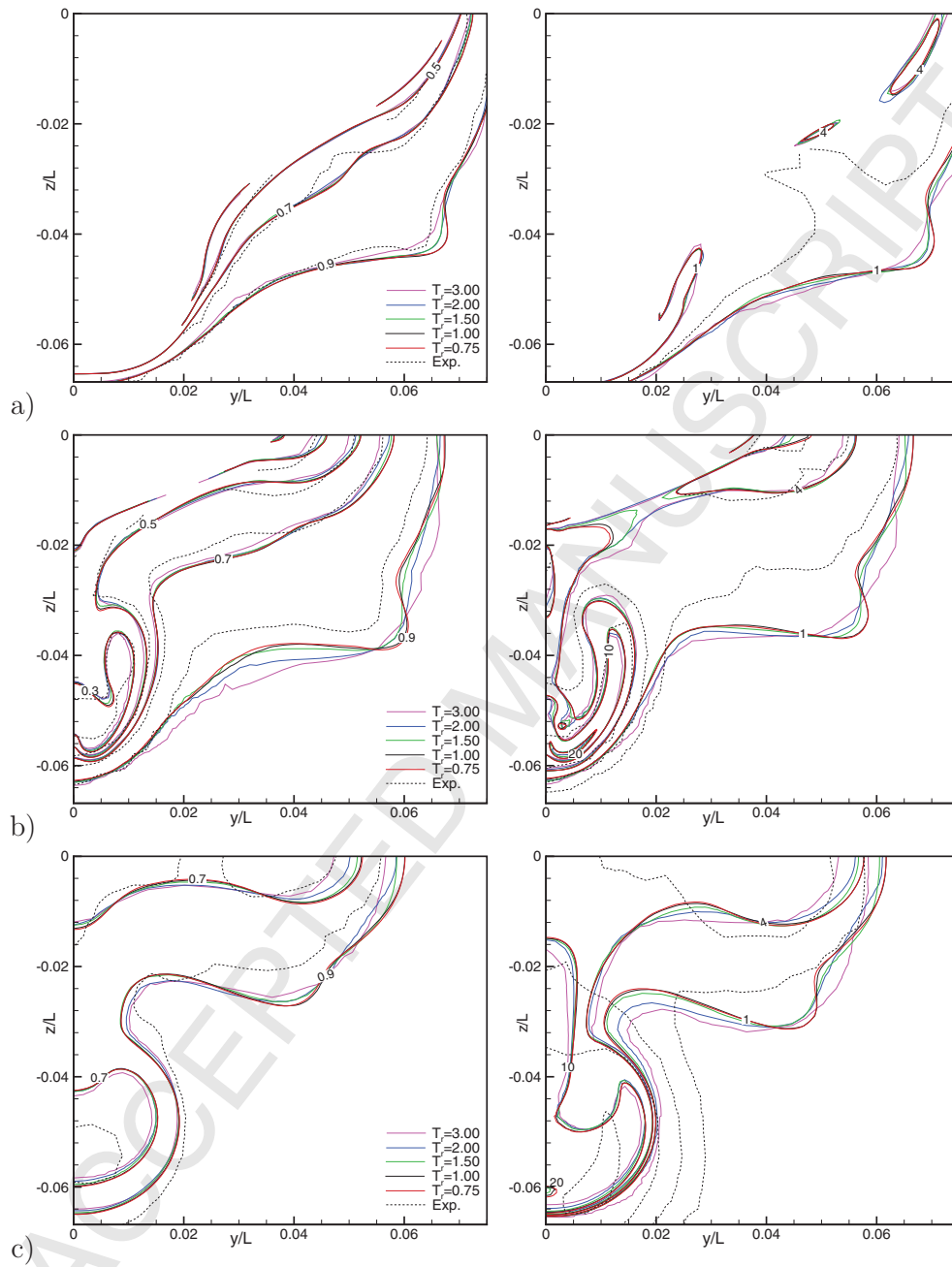


Figure 11: Grid convergence for the KVLCC2 in cut planes at $x/L = 0.9$ (a), in the propeller plane $x/L = 0.9825$ (b), and behind the ship at $x/L = 1.1$ (c): axial velocity (left) and turbulence kinetic energy (right).³⁵

614 threshold T_r . The refined meshes should therefore be similar.

615 However, the possible grid sizes are limited by the refinement method,
616 since refined cells are created by dividing existing cells in half. Thus, if the
617 threshold is reduced by a factor less than 2, the cell sizes are not reduced
618 by this factor; instead, the same cell sizes are kept and the zones of fine
619 cells are enlarged. Furthermore, geometric similarity is perturbed when the
620 flow contains discontinuities and sharp features. These are better resolved
621 on finer grids so the refinement criterion is increased locally, which leads to
622 more refinement than expected on the finest grids. Finally, on coarse meshes
623 the original grid is conserved over much of the domain, while on fine meshes,
624 the refinement may be limited by an imposed minimum cell size.

625 Despite these limitations, series of grids are produced which are globally
626 similar. The consistent orientations of the cells, as well as the equal position
627 of the boundary layer mesh in each grid, are useful to ensure smooth con-
628 vergence. Finally, for cases where the flow is well resolved and the threshold
629 is varied by a factor 2, local geometric similarity has been confirmed by our
630 tests.

631 A least-squares based uncertainty estimation technique was applied to the
632 computed forces, with success. Oscillations in the results are observed when
633 the thresholds between meshes do not vary by a factor 2. However, the least-
634 squares approach removes these oscillations if enough meshes are used. Using
635 only meshes which vary by a factor 2 improves the fits, but the improvement
636 is small. Thus, series of adaptively refined meshes that are not locally similar
637 can be used for uncertainty estimation.

638 The original grid which is not refined everywhere for coarse meshes, does
639 not deteriorate the convergence. The mesh is refined there where this is
640 crucial for the flow, which is enough to ensure grid convergence of the forces.
641 The lift of an airfoil converges quickly with grid adaptation, because the
642 leading-edge pressure peak and the boundary-layer displacement are well
643 resolved on adapted meshes. Finally, the KVLCC2 case shows that the
644 forces are indeed perturbed when the refinement criterion is resolved better
645 on fine meshes. However, this probably means that the flow is not yet in the
646 asymptotic range, so perturbations in the convergence can be expected even
647 if the grids were perfectly similar.

648 The convergence of local flow features, studied by visual inspection of
649 isoline plots, is good. For the two-dimensional airfoil, the computed wake on
650 the finest grids is identical, even far behind the trailing edge. In the more
651 complicated flow around the KVLCC2, the velocity and the turbulence are
652 nearly identical on the finest grids; the differences are small with respect to
653 the distance between computations and experiments.

654 *7.2. Limitations and perspectives*

655 This final section discusses the practical usefulness of grid convergence
656 studies using adaptive refinement. First of all, how large is their field of
657 application? The tests performed here use the ISIS-CFD flow solver and
658 grid adaptation, so they are limited to high-Reynolds incompressible RANS
659 solutions, hexahedral grids, and Hessian-based refinement criteria. How-
660 ever, the idea that series of grids with geometrically similar cell sizes can
661 be obtained by varying a refinement threshold is valid for all metric-based
662 adaptation techniques which use solution-based refinement criteria (i.e. crite-

663 ria with weak mesh dependence). This approach is not even limited to fluid
664 dynamics, it could be used for the simulation of structures etc.

665 Obtaining identical cell orientations between grids requires adaptive grid
666 refinement by subdivision of an original grid, which is typically associated
667 with mesh adaptation for hexahedral cells. However, as shown above, exact
668 local similarity may not be required for convergence studies. Therefore, the
669 proposed technique could also work for tetrahedral mesh generation, espe-
670 cially for anisotropic meshes where the element orientation between meshes
671 is similar, since it is guided by the anisotropy.

672 Thus, any combination of a flow solver, an adaptive mesher and a metric-
673 based refinement criterion can potentially be used to perform convergence
674 studies. The exact procedure needs to be determined for each method.

675 For incompressible flows, we have shown that the technique can produce con-
676 verged local-flow solutions: the numerical accuracy of the computed wake
677 flows is sufficient to assess modelling errors due to turbulence models, wall
678 laws, etc., on meshes with acceptable numbers of cells. Therefore, the tech-
679 nique can be used for research in turbulence modelling of complex flows.

680 For the computation of forces, useful uncertainty estimations are pro-
681 duced. However, some perturbations are generated because the meshes are
682 not perfectly similar. Low uncertainties for wall-law meshes can be obtained
683 with reasonable numbers of cells, while the same is costly for wall-resolved
684 boundary layers. This is because the grid refinement method resolves all
685 the details of the wake, which may not be necessary if the only objective
686 is to compute forces. Thus, convergence studies on structured, non-adapted

687 grids are to be preferred for force computations, if these grids can actually
688 be generated. Otherwise, the proposed procedure is a valid option.

689 Using adaptive meshing for grid convergence studies has a final advantage:
690 it is simple to perform. Once a first computation is set up, simulations on
691 different meshes are obtained by changing only the refinement threshold.
692 Furthermore, a grid convergence study is not expensive, since one is inter-
693 ested in the results on the finest grid. Therefore, with respect to a one-grid
694 computation only coarser grids are added. In three dimensions, the com-
695 putations on all coarser grids together need little more than half the time
696 for the finest grid. Finally, unstructured grids can be used without added
697 difficulty.

698 It is perhaps this aspect, as well as the ability to resolve the details of
699 complex flows, which makes the idea of fundamental importance. Adaptive
700 refinement can make grid convergence studies so straightforward that they
701 become accessible for everyday CFD simulation.

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