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1 Thermal evolution of a metal drop falling in a less
2 dense, more viscous fluid

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8 **Abstract**

9 The initial state of terrestrial planets was partly determined, during accretion,
10 by the fall of metal drops in a liquid magma ocean. Here, we perform systematic
11 numerical simulations in 2D cylindrical axisymmetric geometry of these falling
12 dynamics and associated heat exchanges at the scale of one single drop, for var-
13 ious initial sizes and ambient viscosities. We explore Reynolds number in the
14 range $[0.05 - 48]$, viscosity ratios in the range $[50 - 4000]$, Weber number in the
15 range $[0.04 - 5]$ and Peclet number in the range $[70 - 850]$. We show that heat
16 exchanges between the two phases occurs predominantly at the front section
17 of the drop. Our systematic, parametric study exhibits shows that the ther-
18 mal boundary layer thickness, the depth and time for equilibration, the Nusselt
19 number, and the magma ocean volume affected by thermal echanges, all scale
20 as power laws of the Peclet number. Because of drop distortions, these scaling
21 laws deviate from the classical balances considering only heat diffusion through
22 a laminar thermal boundary layer. Finally, when considering a temperature-
23 dependent viscosity of the ambient fluid, we show that a low viscosity layer
24 surrounds the drop, which influences the thermal evolution of non-deformable,
25 low Reynolds number drops only, and decreases the breakup distance for some
26 limited breakup modes.

28 *Keywords:* Core formation, drop dynamics, thermal exchanges, two-phase
29 flow, numerical modeling.

30 **1. Introduction**

31 Core formation of terrestrial planets is a complex process contemporaneous
32 with planetary accretion (1; 2). Its fluid dynamics and thermodynamics have
33 been addressed in numerous studies (e.g. 3; 4; 5; 6; 7; 8). During the last stages
34 of e.g. Earth accretion, giant impacts likely occurred between the proto-Earth
35 and up to Mars-sized differentiated bodies (9). The kinetic energy released dur-
36 ing such collisions (10; 11), the radioactive heating caused by the disintegration
37 of short-lived radio-elements (12), and the heat dissipation resulting from the
38 conversion of potential energy during core formation and core/mantle separa-
39 tion (13), melt part or all of the Earth mantle (10). Following each impact, the
40 iron core of the impactor thus spreaded and sank into a deep magma ocean.
41 There, the metal further fragmented into drops and diapirs of different sizes,
42 ranging from millimeter drops up to maybe kilometers diapirs, before assem-
43 bling with the Earth proto-core ((5; 7; 14; 15)). Thermochemical exchanges
44 occurred between the fragmented metal drops and the liquid magma ocean dur-
45 ing their sinking, determining the initial thermal and chemical state of the planet
46 ((16; 17; 18; 5)). Past studies have provided many scenarios to characterize and
47 quantify the thermochemical exchanges. (16) and (19) modelled the diffusive
48 equilibration through a laminar thermal boundary layer of respectively, a cloud
49 of uniform drops and a large diapir of iron. (18) further evaluated the influence
50 of drop deformations in (16) scenario. (20) solved the fully coupled dynamical
51 and thermal/chemical equations, but for a fixed spherical geometry only. (5)
52 used experiments where a large volume of immiscible fluid falls into a less dense
53 ambient to show that the smallest scale of turbulence – rather than diffusion

54 through a laminar boundary layer – leads to rapid thermo-chemical equilibra-
55 tion, even before fragmentation. (8) confirmed this conclusion in their analog
56 model, measuring the global cooling of a large volume of hot Galinstan after its
57 fall through a deep tank of viscous oil. Yet a systematic temporal description of
58 heat exchanges at the scale of one falling, freely evolving drop, is still missing.

59 Importantly, the magma ocean viscosity highly depends on its evolving tem-
60 perature and pressure (21). Therefore, the viscosity ratio between the magma
61 ocean and iron drops can vary by several orders of magnitude as a function of
62 depth, of time after impact, etc. Following and extending an abundant literature
63 in different contexts (e.g. 22; 23; 24; 25; 26), analog experiments by (7; 8) and
64 numerical simulations at the scale of one metal drop by (27) showed that the vis-
65 cosity contrast indeed plays an important role in iron drops shape, velocity and
66 fragmentation. (27) predicted that thermo-chemical exchanges should increase
67 with drop deformation and oscillations; but they did not explicitly solve for
68 the fully coupled dynamical and thermal equations. This is the purpose of the
69 present paper. Open questions include: How and where do heat exchanges oc-
70 cur? Do the drop deformation/oscillations indeed favor heat exchanges? What
71 are the characteristics time and depth needed to reach equilibration between the
72 two phases? And, what is the influence of a temperature dependent viscosity of
73 the magma?

74 The paper is organised as follows. Section 2 introduces the physical and
75 numerical models, with the governing equations, the non-dimensional parame-
76 ters, and the numerical method. Section 3 presents in detail a reference case,
77 describing its mechanical and thermal behavior, average temperature evolu-
78 tion, heat transfer at the drop interface, and the magma ocean volume heated
79 during the drop sinking. In Section 4, we present the main numerical results
80 from our systematic parametric study, and derive generic scaling laws for the

81 above detailed parameters. Section 5 then focuses on changes induced by a
 82 temperature-dependent viscosity in the magma ocean. Conclusions and future
 83 works are outlined in final section 6.

84 **2. Physical and numerical models**

85 *2.1. Governing equations*

86 We consider an initially spherical, liquid metal drop of radius R , falling
 87 in an initially motionless, less dense and more viscous surrounding fluid (i.e.
 88 a magma ocean) under the action of gravity. The initial temperature of the
 89 liquid drop and of the magma ocean strongly depends on the growth history
 90 of the protoplanet before the impact and on its initial heating caused by short
 91 lived elements (13). Here, we consider that the liquid metal drop is hotter
 92 than the magma ocean, with uniform initial temperatures in both phases. Both
 93 phases behave as Newtonian, incompressible, and immiscible fluids with uniform
 94 surface tension, and constant density and viscosity within each fluid at first.
 95 Later on in section 5, we also consider a temperature dependent magma ocean
 96 viscosity. The dynamical and thermal evolution of the falling drop and ambient
 97 liquid is governed by the Navier-Stokes and heat transfer equations, describing

- 98 • The mass conservation:

$$\nabla \cdot \mathbf{u} = 0, \quad (1)$$

99 with \mathbf{u} the fluid velocity vector ($\text{m}\cdot\text{s}^{-1}$).

- 100 • The momentum conservation:

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = \nabla \cdot [-P\mathbf{I} + \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)] + \rho \mathbf{g} + \mathbf{F}_{st} \quad (2)$$

101 with ρ the fluid density ($\text{kg}\cdot\text{m}^{-3}$), μ the fluid dynamic viscosity (Pa.s), t
 102 the time (s), P the fluid pressure (Pa), \mathbf{g} the gravitational acceleration
 103 ($\text{m}\cdot\text{s}^{-2}$), \mathbf{F}_{st} the surface tension force ($\text{N}\cdot\text{m}^{-3}$) and \mathbf{I} the identity matrix.

104 • The heat conservation:

$$\rho C_p \left[\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right] + \nabla \cdot [-k \nabla T] = 0 \quad (3)$$

105 with C_p the heat capacity at constant pressure ($\text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$), T the fluid
 106 temperature (K), k the fluid thermal conductivity ($\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$). No heat
 107 source is considered in our model. We ignore the effect of viscous dissi-
 108 pation in this work as we study droplets on small scales of millimetres to
 109 centimetres. The absence of viscous dissipation for these studied drops
 110 does not affect the thermal evolution in the system.

111 To monitor the interface between the falling drop and the magma ocean,
 112 we use the Level Set method, an Eulerian and implicit method frequently used
 113 in multiphase flow problems (e.g. 28). The Level Set function ϕ equals to 1 in
 114 the metal drop and 0 in the ambient liquid, and rapidly changes through the
 115 interface, whose position is determined by the isocontour $\phi = 0.5$. The transport
 116 and reinitialization of the Level Set function ϕ are governed by:

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = \gamma \nabla \cdot \left[\epsilon \nabla \phi - \phi(1 - \phi) \frac{\nabla \phi}{|\nabla \phi|} \right] \quad (4)$$

117 with γ (m/s) and ϵ (m) the reinitialization parameters. γ determines the reini-
 118 tialization amount: a suitable value for γ is the maximum velocity magnitude
 119 experienced in the model. ϵ determines the layer thickness around the interface,
 120 and is equal to half the size of the characteristic mesh in the region explored by
 121 the interface. The density and dynamical viscosity are evaluated using the level
 122 set function:

$$\rho = \rho_m + (\rho_d - \rho_m)\phi \quad (5)$$

123

$$\mu = \mu_m + (\mu_d - \mu_m)\phi \quad (6)$$

124 where subscripts “m” and “d” stand for the magma ocean and the liquid metal
 125 drop, respectively. The surface tension force is determined by :

$$\mathbf{F}_{st} = \nabla \cdot \mathbf{T} = \nabla \cdot (\sigma[\mathbf{I} + (-\mathbf{n}\mathbf{n}^T)]\delta) \quad (7)$$

126 with σ (N/m) the surface tension coefficient, \mathbf{I} the identity matrix, \mathbf{n} the in-
 127 terface normal unit vector, and δ the Dirac delta function, nonzero only at the
 128 fluid interface. The interface normal unit vector is calculated as

$$\mathbf{n} = \frac{\nabla\phi}{|\nabla\phi|}. \quad (8)$$

129 The level set parameter ϕ is also used to approximate the delta function by a
 130 smooth function (29) defined by

$$\delta = 6 |\phi(1 - \phi)| |\nabla\phi|. \quad (9)$$

131 Note that in this work we do not calculate the chemical exchanges between
 132 the two phases. Instead, the main difference between the heat and mass transfer
 133 equations, if the thermal and chemical sources are neglected, is the partition
 134 coefficient between the two phases (30). When the partition coefficient is equal
 135 to 1, we can use the results of this study in geochemical models of planet building
 136 (31).

137 2.2. Physical and non-dimensional parameters

138 The main parameters that characterize the dynamical and thermal evolution
 139 of a falling drop in a more viscous medium are the viscosity, density, thermal

140 conductivity, heat capacity and initial temperature of the two fluids, the initial
 141 drop size, the gravity, and the surface tension between the two phases. In the
 142 geophysical problem of interest (i.e. core formation), the magma ocean viscosity
 143 and the metal drop initial radius vary over a wide range of values, while the other
 144 parameters are roughly constant (even if rigorously, the thermal conductivity
 145 and heat capacity of a magma ocean moderately depend on its composition
 146 (e.g. 32; 33), and the thermal conductivity and heat capacity of metal drops
 147 moderately depend on temperature and pressure (e.g. 34; 35)). Hence in this
 148 study, we vary these two parameters R and μ_m , in the accessible, relevant ranges
 149 4–25 mm and 0.25–20 Pa.s respectively, while we keep all the other parameters
 150 fixed at their representative geophysical values (see Table 1).

Table 1: Symbol definitions and values of the physical and non-dimensional parameters used in this study.

	Symbol	Value or range
Magma ocean density	ρ_m	3500 kg/m ³
Metal drop density	ρ_d	7500 kg/m ³
Metal drop viscosity	μ_d	0.005 Pa.s
Magma ocean viscosity	μ_m	0.25 - 20 Pa.s
Initial drop radius	R	4 - 25 mm
Surface tension coefficient	σ	1 N/m
Magma ocean heat capacity	Cp_m	667 J.kg ⁻¹ .K ⁻¹
Metal heat capacity	Cp_d	800 J.kg ⁻¹ .K ⁻¹
Magma ocean conductivity	k_m	10 W.m ⁻¹ .K ⁻¹
Metal conductivity	k_d	100 W.m ⁻¹ .K ⁻¹
Viscosity ratio	R_μ	50 - 4000
Density ratio	R_ρ	2.14
Reynolds number	Re	0.05 - 48
Weber number	We	0.04 - 5
Peclet number	Pe	70 - 850
Nusselt number	Nu	1 - 6

151 In our simulations, the drop falls from rest, accelerates until reaching a con-
 152 stant terminal velocity, possibly with small oscillations around it, and thermally

153 exchanges heat with the ambient liquid. We pursue continue our simulations
 154 until the drop reaches a stable dynamical regime and its temperature contrast
 155 with the ambient reaches less than 20% of its initial value. During the fall, we
 156 monitor the average terminal velocity V and the average temperature T of the
 157 drop (minus the initial ambient temperature). The dynamical and thermal evo-
 158 lution of each drop is then characterised by the following output dimensionless
 159 numbers:

- 160 • the Reynolds number ($Re = \frac{\rho_m V R}{\mu_m}$) is the ratio of inertial to viscous
 161 forces. Three different regimes are possible: the Stokes regime corresponds
 162 to $Re < 1$ where the viscous effects dominate; the Intermediate regime
 163 corresponds to $Re = 1 - 500$ where both viscous and inertial forces are
 164 important; and the Newtonian regime corresponds to $Re > 500$ where the
 165 inertial forces are dominant. Here, the Reynolds number ranges from 0.05
 166 to 48, hence our drops are in the Stokes to Intermediate regime.
- 167 • the Weber number ($We = \frac{\rho_m V^2 R}{\sigma}$) compares the inertial and surface ten-
 168 sion forces. It governs the deformation, breakup and terminal shape of
 169 a drop (see e.g. 36; 27). When $We < O(1)$, the drop remains spherical
 170 without any change of its morphology, while increasing Weber number
 171 leads to stronger and stronger deformation, then to fragmentation above
 172 a threshold which increases with the viscosity ratio, starting from ~ 3 for
 173 viscosity ratio ≤ 1 (see e.g. 37; 27). Here, the Weber number ranges from
 174 0.04 to 5, considering stable, potentially deformable drops only.
- 175 • the Peclet number ($Pe = \frac{\rho_m C_p V R}{k_m}$) compares the rate of heat advection
 176 to diffusion at the drop scale. Here, the Peclet number ranges from 70 to
 177 850, so heat transfers are strongly affected by advection.
- 178 • the Nusselt number ($Nu = \frac{R \bar{\nabla} T \cdot \mathbf{n}}{(T_{int} - T_m)}$) compares the measured, averaged

179 heat transfer at the drop interface to a purely conductive case, with $\overline{T_{int}}$
180 the mean temperature at the interface and T_m the magma ocean temper-
181 ature far from the drop. Here, the Nusselt number ranges from 1 to 6,
182 hence confirming the important role of advection in heat transfers.

183 All relevant parameter values are given in Table 1.

184 *2.3. Numerical model*

185 We solve Eqs. (1 - 4) using axisymmetric simulations with the COMSOL
186 Multiphysics software, based on the finite element method. The details of our
187 17 runs for this study are listed in Table 2. Each run represents 2 to 4 weeks
188 computation time on a bi-processor, eight-core, 3.2 – 3.6 GHz workstation. The
189 axisymmetric geometry assumption is validated by (38) for a Weber number
190 up to 120. For the dynamics, we use open conditions at the top and bottom
191 boundaries and no-slip conditions at the lateral boundary. For the temperature,
192 we consider no flux conditions at all boundaries. The computational domain
193 must be large enough to allow for convergence without any wall effects. Here,
194 we chose an axisymmetric cylinder of size $(r \times z) = (12R \times 200R)$, which is
195 sufficiently large to reach a statistically steady motion (see our previous study
196 (27)) and to follow equilibration up to a 80% decrease of the initial temperature
197 anomaly.

198 To capture precisely the dynamical and thermal evolution of the drop, a fine
199 mesh is required. For that, we use an adaptive mesh with a high resolution
200 in the drop vicinity. As shown in figure 1, we divide our domain into several
201 regions where the cell sizes vary between $h = 0.015R$ to $h = 1.5R$. As detailed
202 in (27), the simulation is programmed to stop when the drop reaches the bottom
203 of the finest mesh region: the whole mesh pattern is then translated and the
204 simulation is restarted on this new grid (see Figure 1 Right).

Table 2: Dimensional and non-dimensional parameters for all performed simulations used in this study. Pe_h is the grid Peclet number (see section 2.3).

<i>Simulation</i>	$R(mm)$	$\mu_m(Pa.s)$	R_μ	Re	We	Pe	Pe_h	$\Delta T(K)$
#1	8	20	4000	0.05	0.04	70	0.43	100
#2	8	10	2000	0.2	0.15	137.26	0.84	100
#3	8	5	1000	0.76	0.52	254	1.55	100
#3a (T dependent μ_m)	8	5	1000	0.81	0.59	271	1.65	100
#3b (T dependent μ_m)	8	5	1000	0.98	0.85	327	2	1000
#4	8	1	200	9.5	3.23	635	3.89	100
#5	8	0.5	100	21.8	4.26	728.36	4.46	100
#5a (T dependent μ_m)	8	0.5	100	22.4	4.48	747.6	4.57	100
#5b (T dependent μ_m)	8	0.5	100	23.52	4.94	785	4.8	1000
#6	8	0.25	50	47.6	5	793.73	4.86	100
#6a (T dependent μ_m)	8	0.25	50	47.6	5	793.73	4.86	100
#6b (T dependent μ_m)	8	0.25	50	47.6	5	793.73	4.86	1000
#7	4	1	200	2.2	0.34	145.67	0.89	100
#8	6	1	200	5.67	1.53	378.19	2.32	100
#9	10	1	200	12.7	4.61	847.42	5.2	100
#10	25	1	200	45	23.2	3008	18.45	100
#10a (T dependent μ_m)	25	1	200	41.13	19.5	2745	16.8	1000

205 Mesh convergence has been checked. In (27), we showed that a mesh size
206 $h = 0.025R$ (or smaller) allows to capture the falling drop dynamics. Here,
207 we further performed two tests for the resolution of thermal transfers. In the
208 first one, we compare the numerical and analytical heat transfers by thermal
209 diffusion from a motionless spherical drop. The analytical solution for the radial
210 temperature profile from the drop center to a given distance (r) is given by (39)

$$T = 0.5T_0 \left[\operatorname{erf} \frac{R+r}{2\sqrt{Dt}} + \operatorname{erf} \frac{R-r}{2\sqrt{Dt}} \right] - \frac{T_0}{r} \sqrt{\frac{Dt}{\pi}} \left[e^{-\frac{(R-r)^2}{4Dt}} - e^{-\frac{(R+r)^2}{4Dt}} \right] \quad (10)$$

211 with T the temperature anomaly (T_0 its initial value) and D the thermal diffu-
212 sivity ($m^2.s^{-1}$). Figure 2 (Left) shows the excellent agreement of our numerical
213 results.

214 For the second test, we calculated, for different minimum grid sizes $h =$
 215 $0.05R, 0.035R, 0.025R, 0.015R, 0.01R$, the normalized average drop temperature
 216 as a function of normalized time for our reference case #4 in Table 2 (see
 217 details in next section). Figure 2 (Right) shows a reasonable convergence of
 218 the numerical results from $h = 0.015R$, with a relative maximum error $\simeq 3.5\%$,
 219 while the drop mass during the course of this simulation does not change by
 220 more than 0.4% from its initial mass. Therefore, we confirm that our mesh
 221 $h = 0.015R$ captures correctly the thermal evolution of the metal drop.

222 Finally, the grid Peclet number $Pe_h = \frac{Vh}{D_{int}}$, with $D_{int} = \frac{D_d + D_m}{2}$ the mean
 223 thermal diffusivity, is an appropriate parameter in the convection-diffusion equa-
 224 tion to determine whether the heat transfer in the system corresponds to numer-
 225 ical artifacts or not. (40) found very accurate solutions for grid Peclet number
 226 up to 10. As shown in Table 2, our runs in this study remain below this crite-
 227 rion for stability, except for run #10 which we hence do not consider for heat
 228 transfer studies in the following sections.

229 3. Reference case

230 In this section, we present our reference case, from which, in the next section,
 231 we then change the drop size keeping the viscosity ratio constant, and the magma
 232 ocean viscosity keeping the drop size constant. This reference case corresponds
 233 to simulation #4 in Table 2: the drop initial radius is 8 mm, the magma ocean
 234 viscosity is 1 Pa.s, the metal viscosity is 0.005 Pa.s (viscosity contrast 200), and
 235 the initial temperature difference between the metal drop and the magma ocean
 236 is 100 K.

237 The drop motion from rest and its thermal evolution are shown in Figure 3.
 238 The spherical drop accelerates due to gravity and rapidly deforms into a spher-
 239 ical cap. Then, surface tension equilibrates inertia at the drop interface and

240 prevents any further change in morphology. Higher temperatures are concen-
 241 trated at the front of the drop, while the thermal wake behind the drop expands
 242 as a function of time. To characterize this dynamics, we define and compute
 243 the following quantities.

244 3.1. Average ~~falling~~ drop velocity

245 During its fall, the drop mean velocity varies with time depending on the
 246 drop morphology. In our simulations, we compute it as:

$$V(t) = \frac{\int_v U(r, z, t)[\phi(r, z, t) \geq 0.5]dv}{\int_v [\phi(r, z, t) \geq 0.5]dv} \quad (11)$$

247 with $U(r, z, t)$ the local velocity magnitude (m/s), $dv = 2\pi r dr dz$ accounting for
 248 axisymmetric cylindrical geometry, and $[\phi(r, z, t) \geq 0.5]$ the boolean operator
 249 allowing to only capture the iron drop volume. Note that we use in this study
 250 the mean magnitude velocity of drop which is very close to the average vertical
 251 velocity (change of less than 3%). We normalise $V(t)$ by the free fall Newton
 252 velocity $U_N = \sqrt{\frac{\Delta\rho g R}{\rho_m}}$ and time by the diffusion time $t_{dif} = \frac{R^2 \rho_m C_{p_m}}{k_m}$. Figure
 253 4 shows the result for our reference case. The drop rapidly accelerates from rest
 254 up to $t^* = 0.02$, then several small oscillations occur before reaching its asymp-
 255 totic terminal fall velocity, which we use to compute the output dimensionless
 256 numbers.

257 3.2. Average temperature evolution of the metal drop

258 As shown in Figure 3, the liquid drop ~~thermally~~ exchanges heat with the
 259 liquid magma ocean and loses its heat as a function of time and depth. We
 260 determine the mean average temperature anomaly of the drop compared to the
 261 magma ocean as:

$$T(t) = \frac{\int_v T(r, z, t)[\phi \geq 0.5]dv}{\int_v [\phi \geq 0.5]dv} \quad (12)$$

262 We normalize $T(t)$ by its initial value, giving $T^*(t)$. Its evolution for the refer-
 263 ence case as a function of time or as a function of depth of its center of mass
 264 (normalized by the initial radius R) is shown in Figure 5. After a rapid adjust-
 265 ment of the initial temperature jump at the interface, T^* decreases exponentially
 266 towards equilibrium. We determine the best exponential fits according to

$$T^* = e^{(-t/t_c)} \quad \text{and} \quad T^* = C_z e^{(-z/l_c)} \quad (13)$$

267 where C_z is a constant (see Figure 5 in red color). In geophysical science, the
 268 characteristic time (t_c) and length (l_c) for equilibration are very important pa-
 269 rameters (8), used to set the degree of equilibration between iron and silicate of
 270 magma ocean in planet building models (31). Note that chemical equilibrium
 271 is more difficult to reach than thermal equilibrium because of its low diffusiv-
 272 ity and is at least as important to planet formation as thermal equilibration,
 273 especially for small metal volumes. Therefore, in the case of a partition coeffi-
 274 cient between the two phases equal to 1, the results of this work can be used in
 275 geochemical models of planet building.

276 *3.3. Volume of heated magma during the drop sinking*

277 The magma ocean temperature increases during the fall of the drop, espe-
 278 cially in its wake. It is important in geophysics to quantify how much of the
 279 magma is thermally affected by the formation of the iron core: this would for
 280 instance affect the initial structure and heat budget of the Earth’s mantle. In
 281 our simulations, we calculate the volume of magma ocean affected by thermal
 282 exchange as:

$$V_o = \int_v [T > (T_m + T_C)][\phi \leq 0.5] dv \quad (14)$$

283 with T_C a chosen temperature anomaly (K) and $[\phi \leq 0.5]$ the boolean operator
 284 allowing to only capture the magma ocean volume. An example of the captured

285 volume of heated magma ocean is shown in Figure 6 (Left). We performed
 286 tests with several T_C corresponding to 0.1, 1 and 10 K. Results of the volume
 287 normalized by the initial drop volume as a function of the drop depth are shown
 288 in Figure 6 (Right). The three curves corresponding to different equilibrium
 289 degrees are parallel, until a depth equals to 50R where $T_C = 10$ K is strongly
 290 affected by thermal diffusion in the magma ocean. The same behavior would of
 291 course take place at longer time/depth for the other T_C . In the following, for
 292 our parametric study, we compare heated volumes considering $T_C = 1$ K and at
 293 three different depths: 10, 20 and 30 R.

294 *3.4. Heat transfer at the drop interface*

295 To evaluate the Nusselt number, we compute the mean temperature anomaly
 296 and the mean temperature gradient at the interface, using respectively

$$\overline{T_{int}} = \frac{\int_v T\phi(1-\phi)dv}{\int_v \phi(1-\phi)dv} \quad (15)$$

$$\overline{\nabla T_{int}} = \frac{\int_v \nabla T\phi(1-\phi)dv}{\int_v \phi(1-\phi)dv} \quad (16)$$

297 where $\phi(1-\phi)$ allows capturing only the interface region between the two phases.
 298 Figure 7 shows the temperature gradient in and around the drop at a given time
 299 for our reference simulation. Most heat transfer between the metal drop and the
 300 magma ocean occurs at the drop front side, while the back half only accounts for
 301 about 15% of the total. The existence of a hot thermal wake and the external
 302 recirculation behind the drop that encapsulates and entrains magma with the
 303 sinking drop are the main reasons limiting back thermal exchanges (41; 27).

304 A close view of the temperature field and of the thermal boundary layer
 305 is shown in Figure 8. We define the thermal boundary layer thickness as the
 306 distance from the drop interface to a point where the temperature anomaly
 307 reaches 1% of its interfacial value. The boundary layer is very thin at the drop
 308

309 front section, where most thermal exchanges are carried out. It significantly
 310 increases behind the drop. An example of thickness measurement is shown in
 311 Figure 8 (Right) at the thinnest position, i.e. at the front of the drop. In order
 312 to give a global estimate around the drop, we also define the average boundary
 313 layer thickness as

$$\delta_{T_{av}} = \frac{\overline{T_{int}} - T_m}{\nabla T_{int}}. \quad (17)$$

314 4. Parametric study and scaling analysis

315 We now present our systematic exploration of the parameter space, changing
 316 the magma ocean viscosity or the drop initial radius as shown in Table 2, from
 317 our reference simulation #4. We then analyse our results in terms of scaling
 318 laws. In particular we determine the influence of the Peclet number on the
 319 dimensionless parameters defined in the previous section: the time and length
 320 of equilibration, the normalized thermal boundary layer thickness, the Nusselt
 321 number, and the dimensionless magma ocean volume affected by thermal ex-
 322 changes.

323 4.1. Time and length of equilibration

324 As expected from previous studies of drop dynamics, but largely neglected
 325 in geophysical applications (see e.g. discussions in 7; 27), the characteristic
 326 time and length of equilibrium depend on the viscosity contrast between the
 327 metal drop and the magma ocean. As presented in Figure 9, increasing the
 328 viscosity ratio increases the required time for thermal equilibration (Figure 9
 329 Left), mostly because a larger ambient viscosity limits advective heat exchanges.
 330 We can observe in this Figure 9 (Left) two different power laws depending on
 331 the dynamic regime of the drop. In the first regime (non-deformable drop,
 332 $R_\mu > 1000$), the equilibration characteristic time is highly dependent on the
 333 viscosity ratio, whereas in the second regime (deformable drop, $R_\mu < 200$),

334 the equilibration characteristic time depends slightly on the viscosity ratio and
 335 highly on the deformation of the drop. The evolution of the equilibration length
 336 is non-monotonic (Figure 9 Right), because the magma ocean viscosity also
 337 influences the falling velocity. Globally, with a more viscous ambient fluid, the
 338 thermal equilibrium between the two phases occurs less deeply in the magma
 339 ocean, as we predicted in our previous work (27).

340 The drop size at a given viscosity ratio also influences the thermo-chemical
 341 equilibration (see 20, for a fixed spherical shape). Increasing the initial radius of
 342 the liquid metal drop decreases the surface of exchange over volume initial ratio,
 343 and increases the drop falling velocity: we thus expect an increase of both the
 344 characteristic time and length, as confirmed in Figure 10. In (27), we predicted
 345 that increasing the drop initial radius also leads to surface extension due to
 346 drop distortion, hence to faster equilibration compared to a purely spherical
 347 drop: this effect is however limited, because drop deformation mostly occurs at
 348 the back of the drop, while heat exchanges takes place mostly at the front.

349 Those two series of results can be rationalized by considering dimensionless
 350 properties as a function of the Peclet number. Here, we normalize the equilib-
 351 rium time by the thermal diffusion time and the equilibrium length by the initial
 352 drop radius. Results are shown in Figure 11. The thermal equilibrium time for
 353 a high Peclet, spherical drop theoretically scales as $Pe^{-0.5}$ (see e.g. 30). Here
 354 we find

$$t_c^* = 2.45Pe^{-0.59 \pm 0.01}. \quad (18)$$

355 This acceleration of the equilibration time compared to the theoretical model
 356 may be a signature of the drop deformation at large Peclet. Correspondingly,
 357 the length needed to reach the equilibration increases monotonically with Peclet
 358 number following

$$l_c^* = 2.37Pe^{0.41 \pm 0.01}, \quad (19)$$

359 in agreement with a quasi-constant falling velocity, i.e. $l_c = V \times t_c$. In conclu-
 360 sion, when the flow advection measured by the Peclet number increases, thermal
 361 equilibration occurs faster but deeper in the magma ocean, which is of impor-
 362 tance for Earth’s building models (31).

363 *4.2. Heat exchanges*

364 Simply equilibrating large-scale heat advection along the drop surface by
 365 with heat diffusion through the thermal boundary layer perpendicular to it, the
 366 theoretical size of the thermal boundary layer normalised by the drop radius
 367 classically scales as $Pe^{-0.5}$: this was verified numerically by (20) for a rigid
 368 falling sphere. As shown in Figure 12 (Left), we also recover this scaling at the
 369 drop front, where most heat exchange occurs: the best fit gives

$$\delta_T^* = 2.53Pe^{-0.5}. \quad (20)$$

370 With the deformable drops considered here however, the boundary layer thick-
 371 ness strongly varies around the metal drop: we thus compute a characteristic
 372 average value $\delta_{T_{av}}$ from Eq. 17, normalised by the initial radius. Results are
 373 shown in Figure 12 (Right), with the best fit scaling law

$$\delta_{T_{av}}^* = 10.7Pe^{-0.6 \pm 0.01}. \quad (21)$$

374 Over the explored range, the average thickness is at least three times larger
 375 than the front one. The larger-than-expected 0.6 exponent might again be a
 376 signature of drop distortion at large Peclet number.

377 Then by definition, the Nusselt number averaged over the drop surface should
 378 scale as $1/\delta_{T_{av}}^*$. This is indeed recovered, as shown in Figure 13 with the best
 379 fit scaling law

$$Nu = 0.08Pe^{0.63 \pm 0.02}. \quad (22)$$

380 Note that Nu varies over time in many cases in this study: we consider here its
 381 time-mean averaged value.

382 Comparing this result with the one obtained in the previous section, we also
 383 recover that the typical dimensionless time for equilibration t_c^* scales like $1/Nu$,
 384 as expected. This proves the self-consistency of our measurements.

385 4.3. Volume of heated magma ocean

386 In the wake of the drop, the magma ocean is thermally affected by its passage.
 387 Because of heat diffusion within the magma ocean, the affected volume widens
 388 with time. But on the short times, at a given depth of the drop z , it can
 389 be estimated by simply balancing the heat that has passed through the drop
 390 interface with the heat accumulated in this volume of magma ocean, whose
 391 temperature has increased by a given amount larger or equal to T_C . We then
 392 predict

$$V_o^* = \frac{\text{Volume of heated magma}}{\text{Initial drop volume}} \sim 3 \frac{Nu}{Pe} \frac{z}{R} \frac{\Delta T_0}{T_C}. \quad (23)$$

393 with ΔT_0 the initial temperature difference between the two phases. Results
 394 in Figure 14 show a good agreement with a small relative error for various
 395 depths $z = 10, 20, 30R$ and temperature contrast $T_C = 1K$, using the previously
 396 determined scaling for Nu . This indicates the self-coherence of our numerical
 397 results from a fluid dynamics point of view through the energy conservation.
 398 ~~With this scaling law, we conclude that the mixing in the magma ocean induced~~
 399 ~~by the passage of each drop is very small.~~ Note that here the falling drop
 400 encapsulates in its wake a small volume of silicate while the reminder of the
 401 ambient fluid remains largely motionless or laminar. However, the thermal
 402 contamination in the magma ocean induced by the passage of each drop is very
 403 small. This volume will always eventually decrease due to thermal diffusion and
 404 will always go to zero in a long term due to the absence of viscous dissipation in

405 this study. In the context of planetary core formation, the viscous dissipation
 406 effect is very important for the release of gravitational potential energy due
 407 only to the fall of the drop. Thus, long after the drop has equilibrated with
 408 the magma ocean, it still produces a thermal wake as its gravitational potential
 409 energy. Thus, in the segregation of the drops, the phenomenon of diffusion of
 410 the affected volume of the magma ocean does not always tend towards zero.
 411 In addition, the analytical law proposed by the energy balance between the
 412 drop and the magma ocean in Eq. 23, is of interest for planet building models
 413 by passing from the one drop scenario to a cloud of drops of different sizes
 414 resulting from an impactor core with a radius of 10-100 km. Thus, the volume
 415 of the total amount of heat exchanged between this cloud and the magma ocean,
 416 plays an important role on the global heat budget. Our conclusion should be re-
 417 evaluated in the presence of global magma motions, coming e.g. from turbulent
 418 convection, depending on the typical excited time- and length-scales. But this
 419 is beyond the scope of the present paper.

420 **5. Influence of a temperature dependent viscosity**

421 The magma ocean viscosity increases from the surface to the base of the
 422 magma ocean because of the combined effects of temperature and pressure (21).
 423 Those large-scale variations are irrelevant in the context of our local study;
 424 nevertheless, during the drop fall over the typical length of $\sim 200R$ considered
 425 here, the temperature of the liquid magma ocean increases because of heat
 426 exchanges with the drop, therefore its viscosity decreases at a given pressure
 427 (i.e. depth) following the equation of (21) for anhydrous liquid

$$\mu_T = 0.00033e^{(6400/(T-1000))} Pa.s. \quad (24)$$

428 Here, we investigate the influence of such a temperature dependent viscosity

429 on the dynamics, stability and thermal evolution of a drop. For a given initial
430 viscosity ratio, the initial magma ocean temperature is computed using (24),
431 and we then consider an initial temperature difference with the hot metal drop
432 of $\Delta T = 100$ K or $\Delta T = 1000$ K. We focus on 3 reference cases, only changing
433 the viscosity ratio (see Table 2). Those cases are first presented in the absence
434 of temperature dependent viscosity:

- 435 • The non-deformable drop case (simulation #3 in Table 2, $Re=0.76$ and
436 $Pe=254$) is presented in Figure 15. Drop's shape remains mostly constant.
437 The temperature is close to uniform inside the drop because of internal
438 recirculation, and progressively decreases in the wake. Most heat transfer
439 takes place in the front.
- 440 • The weakly deformable drop case (simulation #5 in Table 2, $Re=21.8$ and
441 $Pe=728.36$) is presented in Figure 16. The drop quickly deforms towards
442 a spherical cap, associated with a strong release of heat in its wake. The
443 system then reaches a quasi-steady state, with the most surprising feature
444 being a more rapid temperature decrease in the drop than in the wake,
445 leading to a positive heat transfer from the silicate to the iron in the drop's
446 back. Nevertheless, most heat transfer still takes place at the front.
- 447 • Finally, the strongly deformable drop case (simulation #6 in Table 2,
448 $Re=47.6$ and $Pe=793.73$) is presented in Figure 17. Here, the drop shape
449 keeps oscillating over the whole depth because of competing surface ten-
450 sion and inertial forces. This induces oscillatory temperature changes in
451 the wake, associated to periodic thermal plumes emitted from the drop
452 sides, and strong inhomogeneous fluxes within the magma ocean. Nev-
453 ertheless heat transfers between iron and silicate are still largely focused
454 at the drop's front, explaining while this case is not associated with any
455 specific signature in the previous section.

456 We now re-run those three cases using the previously defined temperature-
457 dependent viscosity.

458 *5.1. Drop dynamics*

459 Figure 18 shows the normalized average velocity of each drop as a function
460 of the normalized time for constant magma ocean viscosity in black lines and
461 for temperature dependent viscosity with $\Delta T = 100$ K ($\Delta T = 1000$ K) in red
462 (blue) dashed lines. The normalized average velocity for the non-deformable
463 drop (Figure 18 Left) with $\Delta T = 100$ K, increases moderately by about 4.8%
464 compared to the constant viscosity case, keeping the spherical shape without
465 change in the morphology. For $\Delta T = 1000$ K, the drop deforms from the
466 back side and its velocity strongly increases by about 26%; concomitantly, the
467 average viscosity around the drop decreases by 93%, explaining this significant
468 dynamical change. On the contrary, for the weakly deformable drop (Figure
469 18 Middle) and the strongly deformable drop (Figure 18 Right), no significant
470 dynamical change is observed, despite a large viscosity decrease in the associated
471 thermal boundary layer (by 76% and 92%, respectively, for $\Delta T = 1000$ K; see
472 also Figure 19). This indicates that the drop dynamics is already mostly inviscid
473 in the absence of temperature-dependent viscosity, as can be guessed from the
474 corresponding values of the Reynolds number (see caption of Figures 16 and
475 17).

476 *5.2. Drop fragmentation*

477 In the most extreme previous case (the strongly deformable drop with $\Delta T =$
478 1000 K), Figure 19 indicates that the hot silicate layer surrounding the drop has
479 a viscosity value of the order 0.05 Pa.s: assuming such a viscosity uniformly in
480 the ambient magma ocean, our previous study (27) predicts a rapid fragmenta-
481 tion, which is not observed here. Actually the viscosity contrast at the interface

482 does not influence the drop stability, because the thermal boundary layer where
 483 the low viscosity is localised is very thin in front of the drop, compared to any
 484 relevant dynamical length scale; besides, the wake has no influence on the drop.
 485 Temperature dependent viscosity only affects the drop’s dynamics and fragmen-
 486 tation when flows and temperature change on comparable scales, in upfront or
 487 sides locations.

488 To further prove this, we performed a simulation with a drop radius of 25
 489 mm and a viscosity ratio of 200 for constant magma ocean viscosity first, then
 490 for $\Delta T = 1000$ K dependent magma ocean viscosity (simulation #10 in Ta-
 491 ble 2). Figure 20 (Left) shows the Jellyfish fragmentation mode when constant
 492 magma ocean viscosity is considered (more details can be found in 27) . The
 493 normalized time ($t_{bk}^* = \frac{t_{bk}V}{2R} \sqrt{\frac{\rho_d}{\rho_m}}$) and distance ($d_{bk}^* = \frac{d_{bk}}{R}$) of breakup equal
 494 to 3.8 and 10.7 respectively. For temperature-dependent viscosity (Figure 20
 495 middle), another Jellyfish fragmentation mode is observed, with time and dis-
 496 tance of breakup equal to 2.3 and 5.6 respectively. Actually, here, the extended
 497 jellyfish membranes form filaments, where thermal effects are very important.
 498 These membranes sink into a less viscous medium whose thickness is compara-
 499 ble to the membranes thickness (see Figure 20 Right). ~~So, the inertia forces in~~
 500 ~~this zone are stronger than the surface tension, and the fragmentation occurs.~~
 501 Thus, the filaments move freely in this zone with low viscous constraint and
 502 therefore separate from the drop volume leading to the fragmentation.

503 Note finally that the local dimensionless numbers, taken at the mean tem-
 504 perature (and associated viscosity) of the thermal boundary layer, become
 505 $Re = 1000$, $We = 19$ and $R_\mu = 10$. From Figures 4, 12 and 13 of our pre-
 506 vious study (27), we find consistent results for the fragmentation mode of the
 507 drop, as well as for its time and distance of breakup, accounting for these di-
 508 mensionless numbers (To compare with our previous study, Re and We should

509 be multiplied in this study by a factor of 2 because in (27), Re and We are
510 calculated by the drop diameter).

511 Hence, temperature dependent magma ocean viscosity influences the drop
512 fragmentation in the filament forming regimes only. In such regimes, the results
513 in (27) as a function of Re , We and R_μ already allow predicting fragmentation
514 mode, breaking time and distance by considering local values of the dimension-
515 less numbers.

516 5.3. Thermal evolution

517 In Figure 21, we compare the normalized mean iron temperature for each
518 case as a function of normalized time. For a non-deformable drop (Figure
519 21 Left), the average temperature for $\Delta T = 100$ K (red dashed lines) and
520 $\Delta T = 1000$ K (blue dashed lines) temperature-dependent viscosity decreases
521 by about 6.1% and 20.8% with respect to the constant viscosity case (black
522 line). As a consequence, it significantly changes the characteristic time and
523 length of equilibration (see Table 3). For a weakly deformable drop (Figure
524 21 Middle), the average temperature evolution with $\Delta T = 1000$ K changes
525 only by about 13.6%; no difference is measured for a strongly deformable drop
526 (Figure 21 Right). As for the fall velocity, temperature dependent viscosity
527 influences significantly the thermal evolution of non-deformable drops only (i.e.
528 the drops in the Stokes regime). Note finally that in the intermediate regime, the
529 characteristics time and length of equilibration decrease slightly with a temper-
530 ature dependent viscosity compared to constant magma ocean viscosity case,
531 whereas in the Stokes regime, the equilibrium characteristic length increases
532 with a temperature dependent viscosity, in contrast to the characteristic time
533 of equilibration (see Table 3).

534 As for the volume of heated magma ocean shown in Figure 22, no significant
535 change is observed in the heated volume for all cases with $\Delta T = 100$ K (red

536 dashed lines). The volume of heated magma ocean increases by about 1.5 to
537 2 times compared to the constant viscosity case (black line) for $\Delta T = 1000$ K
538 (blue dashed lines) due to the initial larger temperature difference; but the
539 curves remain parallel, hence exhibiting no specific dynamical signature.

Table 3: The change in characteristics time and length for cases of temperature-dependent viscosity.

<i>Simulation</i>	ΔT	t_c^*	l_c^*
#3 (constant magma ocean viscosity)	100	0.1	24.25
#3a (T dependent μ_m)	100	0.0975	25.6
#3b (T dependent μ_m)	1000	0.082	27
#5 (constant magma ocean viscosity)	100	0.05	35.14
#5a (T dependent μ_m)	100	0.0496	33.33
#5b (T dependent μ_m)	1000	0.045	31.25
#6 (constant magma ocean viscosity)	100	0.045	32
#6a (T dependent μ_m)	100	0.044	29.4
#6b (T dependent μ_m)	1000	0.042	27.7

540 Finally, we calculate the heat transfer between the two phases "Nusselt num-
541 ber" of all drops in this section for a temperature dependent viscosity. Then, we
542 compare these results with the correlations proposed in the experimental and nu-
543 merical studies of the penetration of a hot diapir through a highly temperature-
544 dependent viscous medium (42; 43). These correlations of the Nusselt number
545 are done for non-deformable spherical drops of low Reynolds number and are
546 given as follows:

$$Nu = 1 + C_1 Pe^{1/3} \quad (25)$$

547 where $C_1 = 0.45 \pm 0.09$.

$$Nu = 0.795 + 0.459 [Pe \left(\frac{\Delta T}{\ln 10} \right)^3 \frac{\overline{\mu_{int}}}{\mu_m}]^{-1/5}]^{1/2} \quad (26)$$

548 The Eq. 25 has been proposed experimentally by (42) and the Eq. 26 has been
549 proposed numerically by (43). The comparison of the Nusselt numbers between

our numerical results and the Eqs. 22, 25 and 26 is shown in Table 4.

Table 4: The comparison of the Nusselt numbers for cases of temperature-dependent viscosity between our numerical results and the Eqs. 22, 25 and 26.

<i>Simulation</i>	<i>Nu: this study</i>	<i>Nu: Eq. 22</i>	<i>Nu: Eq. 25</i>	<i>Nu: Eq. 26</i>
#3a	2.65	2.72	3.91 ± 0.58	3.36
#3b	2.95	3.07	4.10 ± 0.62	2.82
#5a	5.15	5.17	5.08 ± 0.82	4.93
#5b	5.21	5.33	5.15 ± 0.83	3.65
#6a	5.28	5.36	5.16 ± 0.83	5.06
#6b	5.35	5.36	5.16 ± 0.83	4.58

550

551 We observe that our numerical results are consistent with Eq. 26 for the
 552 non-deformable drops and with Eq. 25 for the deformable drops. This proves
 553 the validity of our calculations in this section. We further note that our Nus-
 554 selt number scaling law in section 4.2 (Eq. 22) is valid for this section of a
 555 temperature-dependent viscosity.

556 6. Conclusions and future work

557 We have carried out series of numerical simulations to characterize the ther-
 558 mal exchanges between a falling drop and a viscous ambient fluid, exploring a
 559 new parameter range relevant for the geophysical application of a hot liquid iron
 560 drop falling in a magma ocean. We have shown that because of drop distortions,
 561 thermal equilibration properties slightly change from the theoretical predictions
 562 based on diffusive heat exchanges through a laminar thermal boundary layer.
 563 We have also tested that accounting for a temperature-dependent viscosity in the
 564 magma ocean barely influences the obtained results, except for limited cases like
 565 non-deformable, low Reynolds number drops or Jellyfish fragmentation mode.
 566 Our most relevant results for geophysical application are the scaling laws for the
 567 normalized length of equilibration and for the Nusselt number, which both in-
 568 crease monotonically with the Peclet number as $Pe^{0.41}$ and $Pe^{0.63}$, respectively.

569 Future work should now consider (i) the presence of convection in the ambi-
570 ent magma ocean, which could affect our conclusions providing the associated
571 velocity at the drop scale is at least of similar order as the falling drop velocity,
572 hence requiring extremely turbulent regimes; and (ii) chemical exchanges be-
573 tween iron and silicates, which will determine the initial chemical state of the
574 considered planet (see e.g. 44). Chemical and thermal constitutive equations
575 being similar, similar equilibration scaling laws and dynamics are nevertheless
576 expected.

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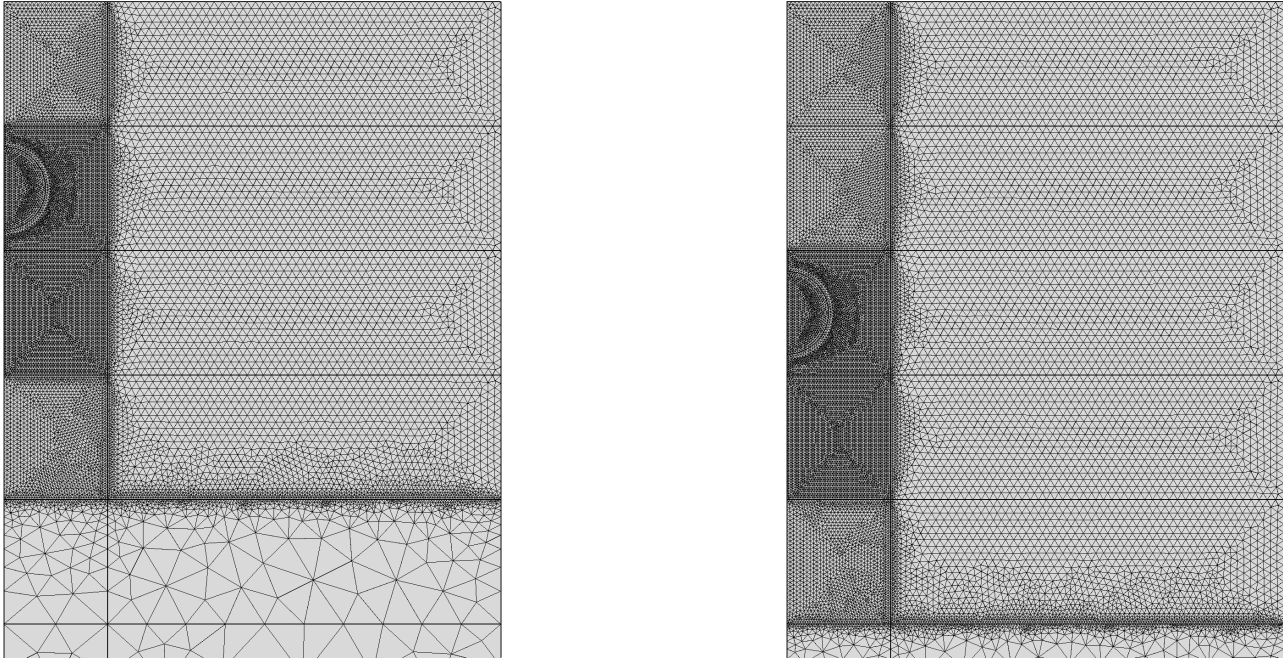


Figure 1: A zoom illustrating our adaptive mesh (left) and the method for mesh evolution over time when the drop reaches the bottom of the finest mesh region (right).

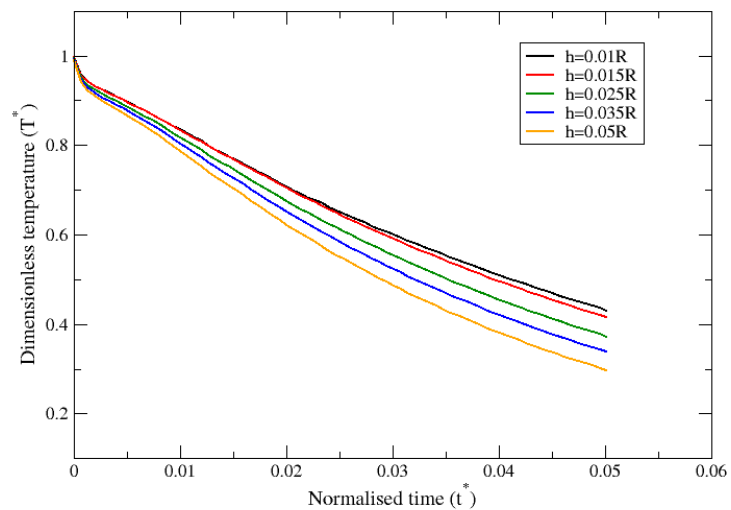
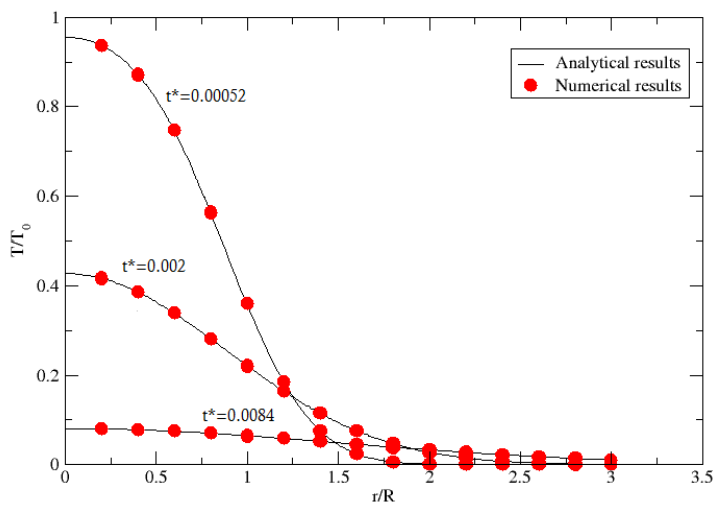


Figure 2: Left: comparison of our numerical results with the analytical results of (39) for the radial profile of the normalised temperature anomaly for a non-deformable and immobile drop. Right: comparison of the time evolution of the normalised mean temperature anomaly determined with different mesh sizes for a deformable and mobile drop (simulation #4 in Table 2).

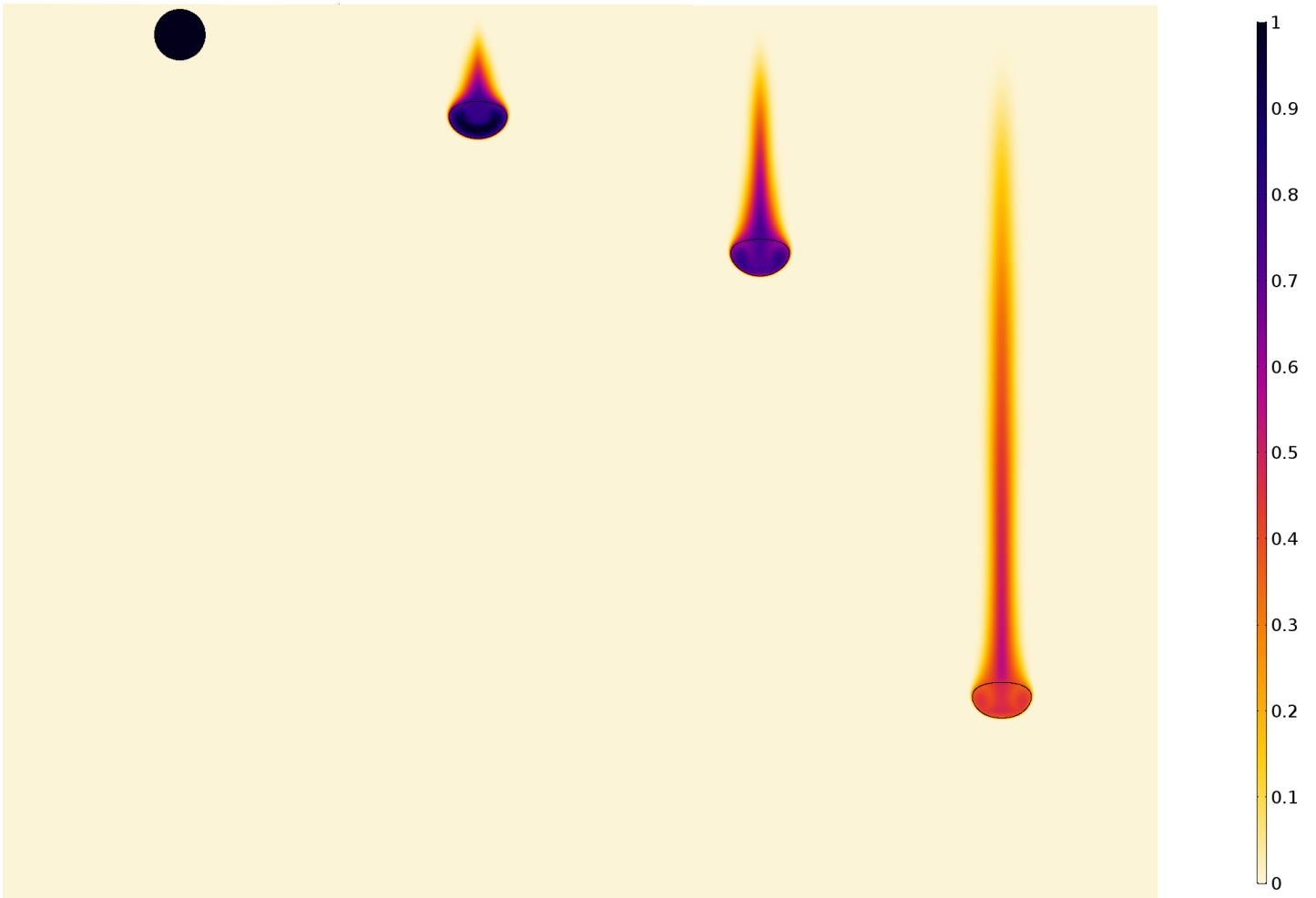


Figure 3: Dynamical and thermal evolutions of the metal drop as a function of time for simulation #4 in Table 2. The color presents the temperature anomaly normalised by its initial value T^* . The black solid line separates the metallic material from the magma ocean. From left to right, the time normalised by the thermal diffusion time is $t^* = 0, 0.01, 0.02$ and 0.05 .

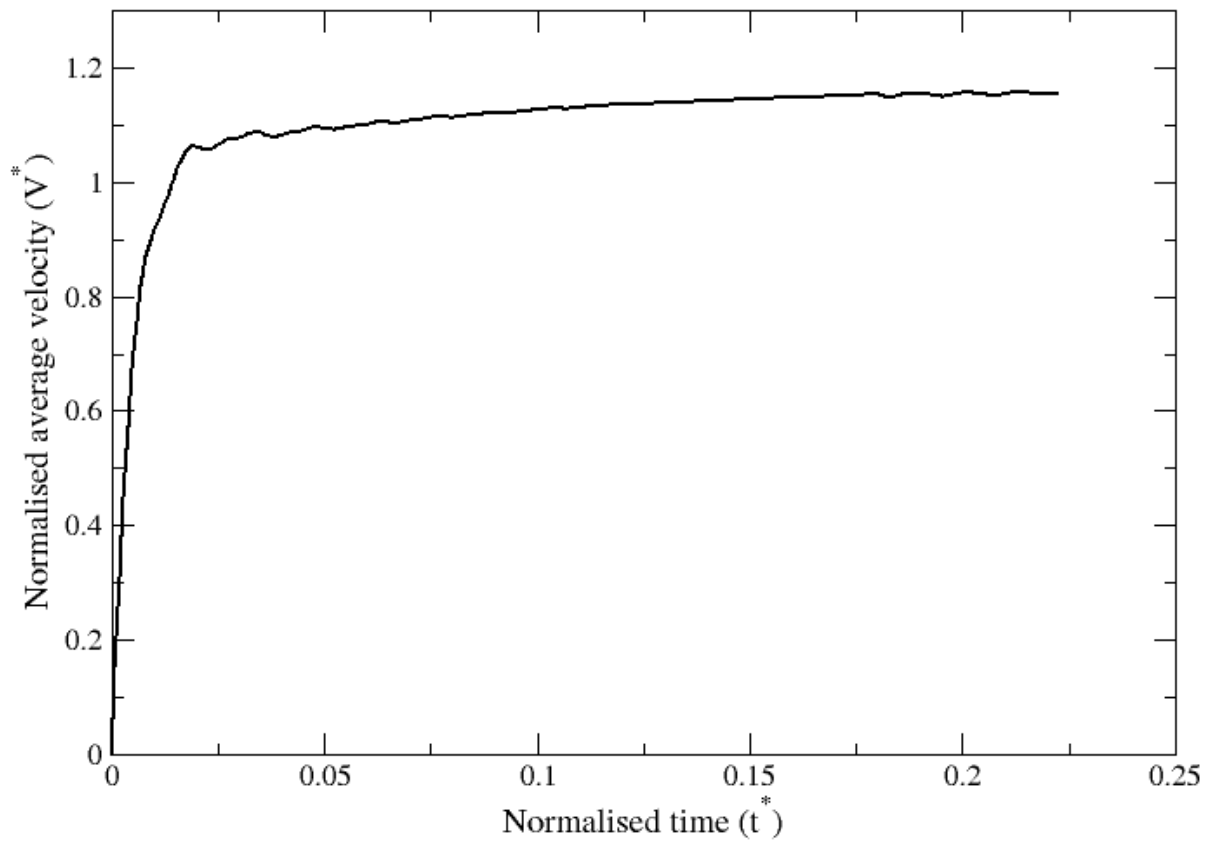


Figure 4: Normalized average velocity of the metal drop as a function of normalized time for simulation #4 in Table 2.

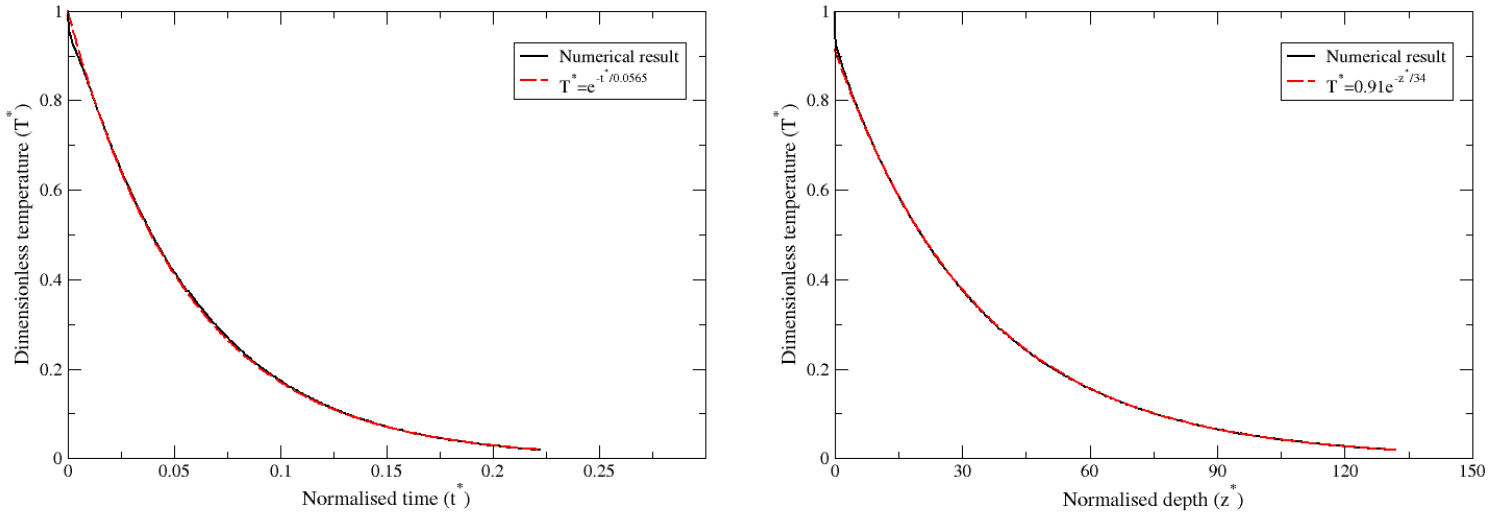


Figure 5: Normalized average temperature anomaly of the metal drop as a function of normalized time (left) and depth (right) for simulation #4 in Table 2. The black lines represent our numerical results and the red dashed lines represent the exponential fits from Eq. 13. The correlation coefficient for the two fits is 0.9999.

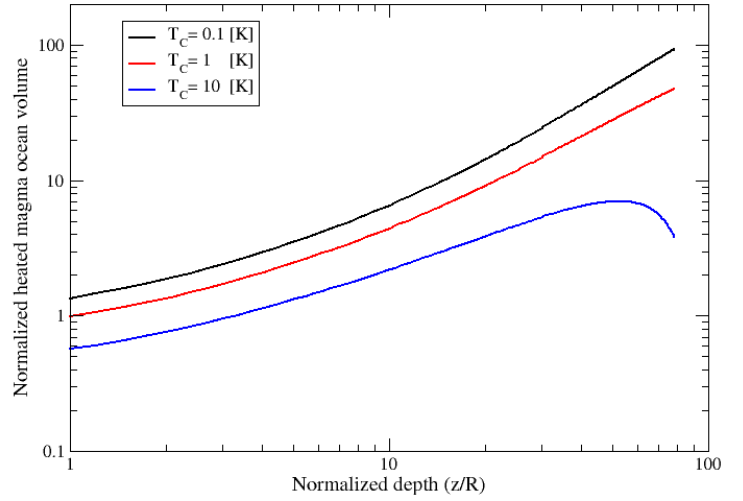


Figure 6: Left: volume of heated magma ocean ($T_C = 1$ K) once the drop reaches a time $t^* = 0.067$ and a depth $z = 37R$ in our simulation #4 in Table 2. Right: normalized heated magma volume as a function of normalized depth for different values of T_C .

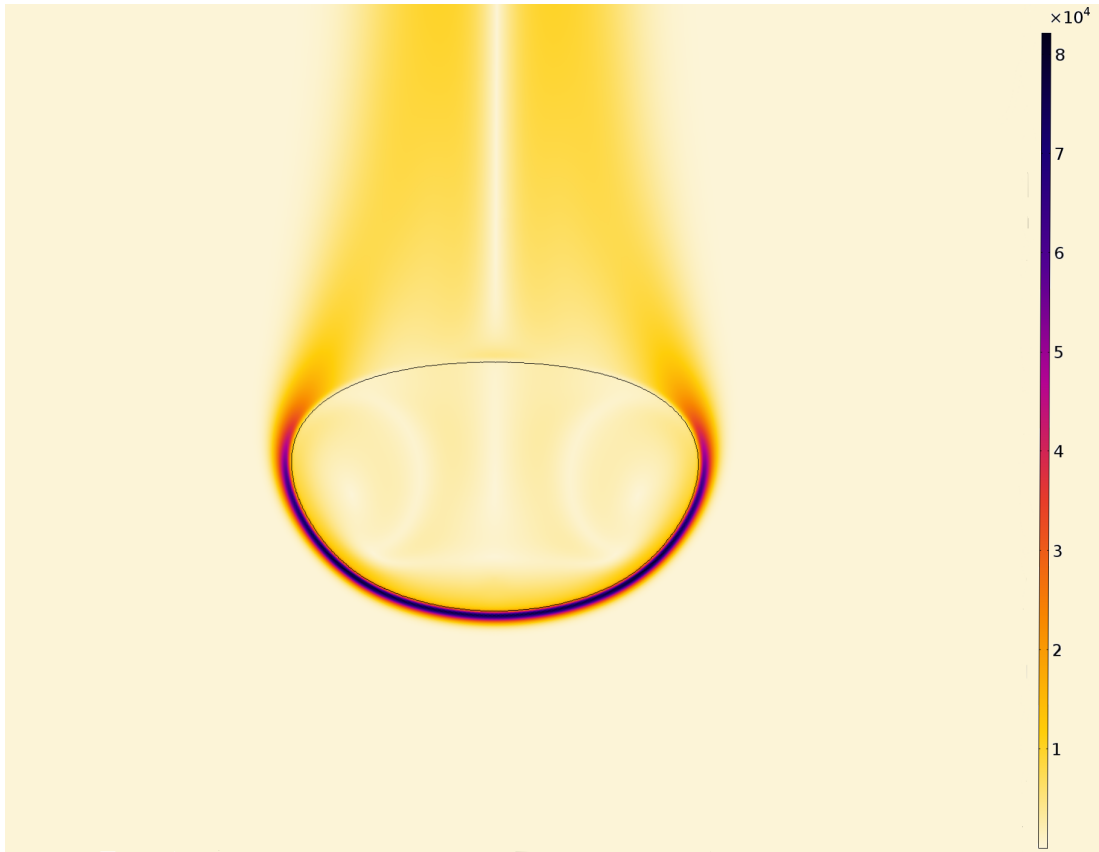


Figure 7: Temperature gradient (K/m) in and around the drop for our reference simulation #4 in Table 2 at time $t^* = 0.055$.

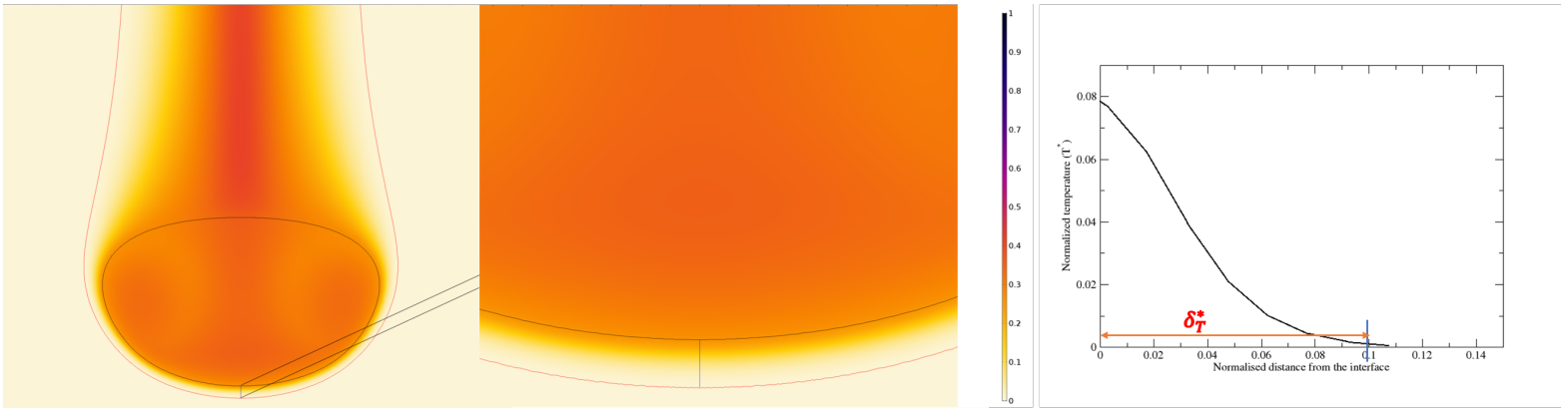


Figure 8: Left: temperature anomaly (colorscale) and contour of the thermal boundary layer (red line) around the drop (black line) for our reference simulation #4 in Table 2 at time $t^* = 0.067$. Middle: zoom at the drop front. Right: temperature profile of the thermal boundary layer in front of the drop and determination of the local boundary layer thickness.

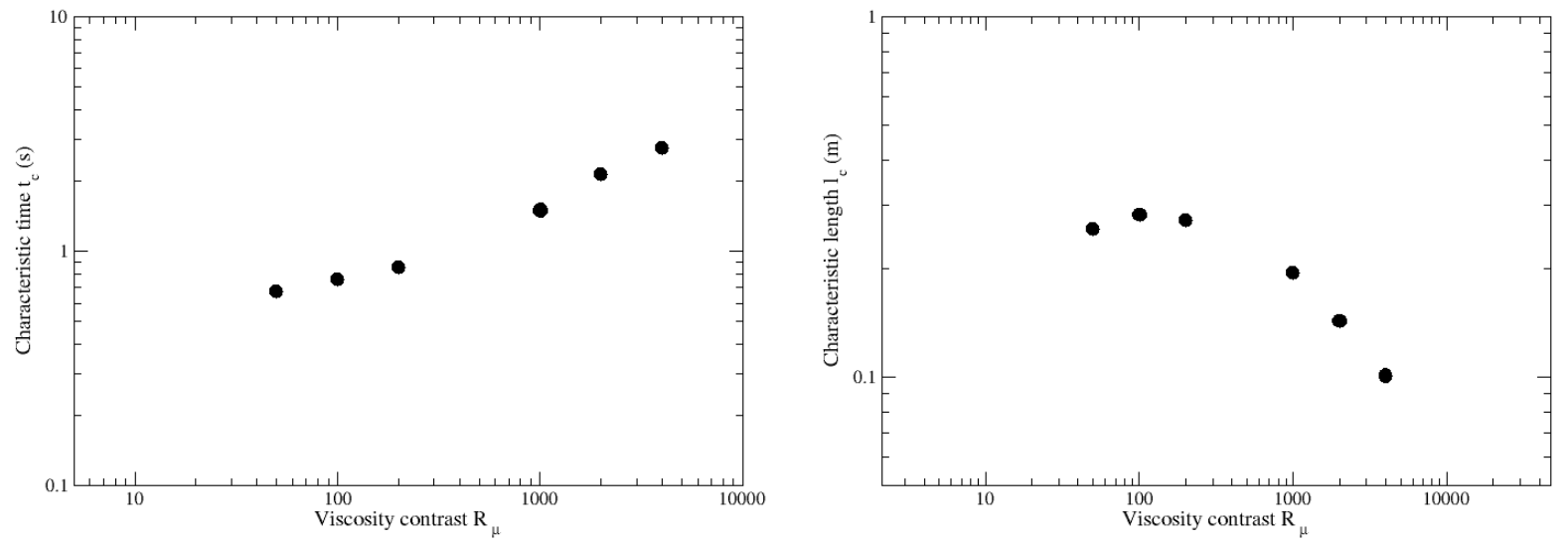


Figure 9: Characteristic time (Left) and length (Right) of equilibration as a function of the viscosity contrast (see Eq. (13)).

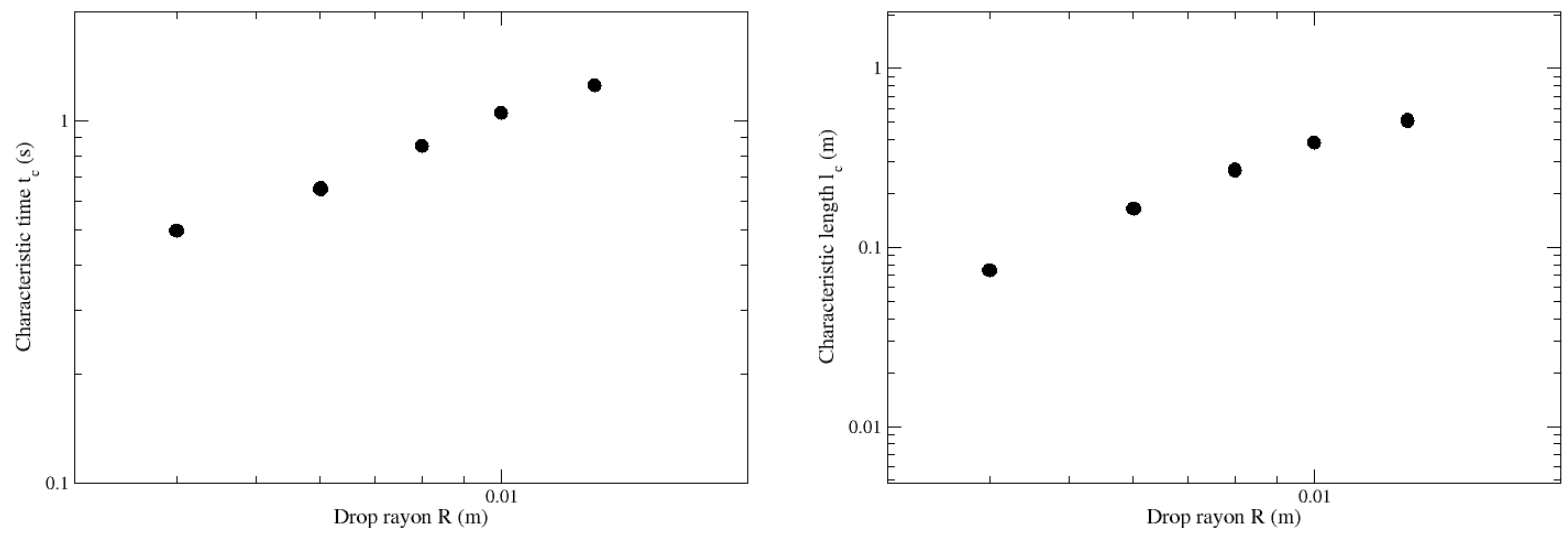


Figure 10: Characteristic time (Left) and length (Right) of equilibration as a function of the drop initial radius (see Eq. (13)).

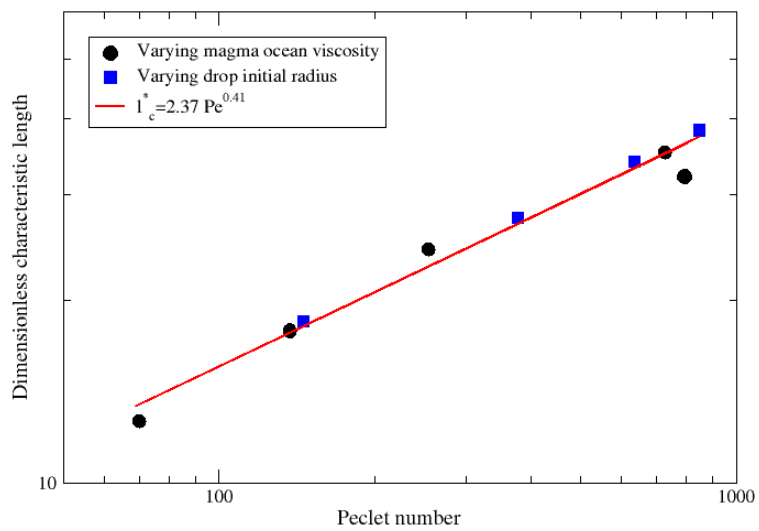
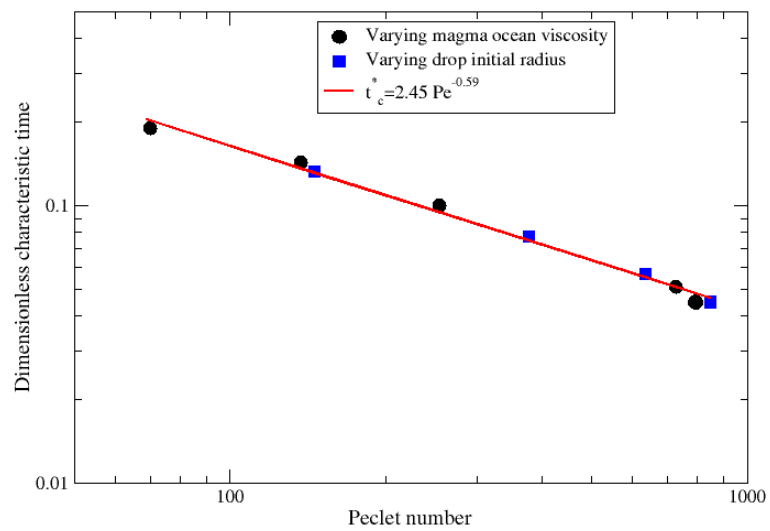


Figure 11: Dimensionless equilibrium time (Left) and length (Right) as a function of the Peclet number.

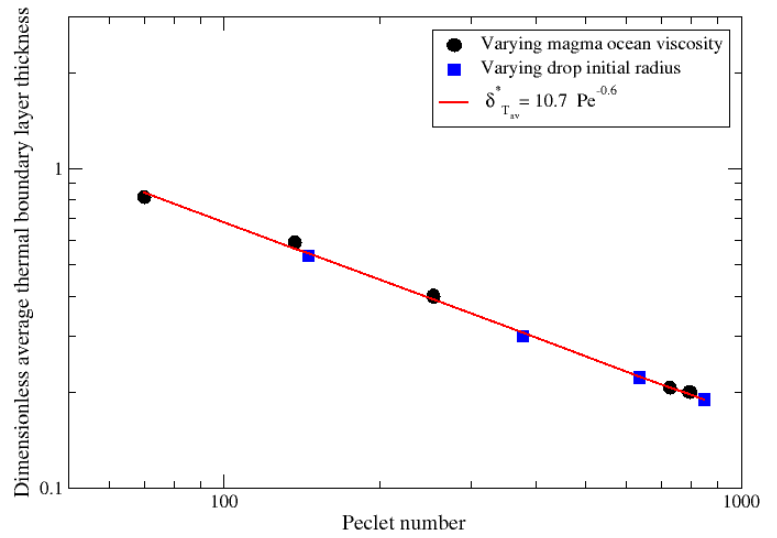
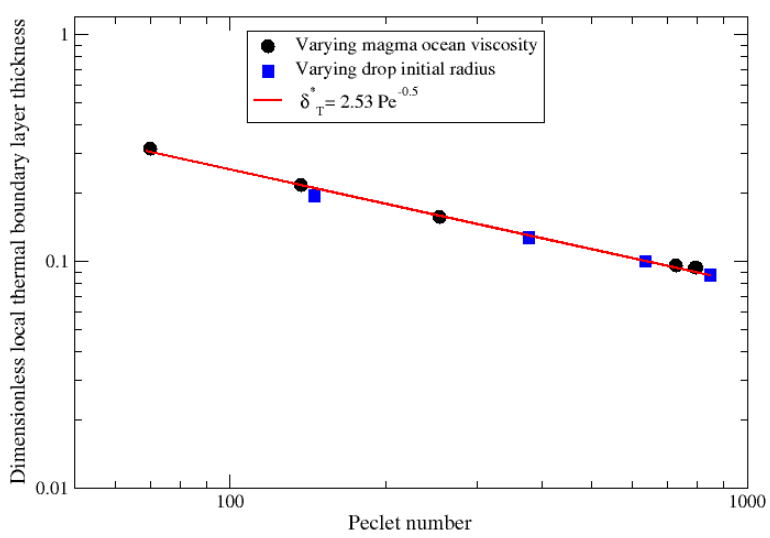


Figure 12: Dimensionless thickness of the thermal boundary layer at the drop front as a function of Peclet number (Left). Dimensionless average thickness of the thermal boundary layer around the drop as a function of Peclet number (Right).

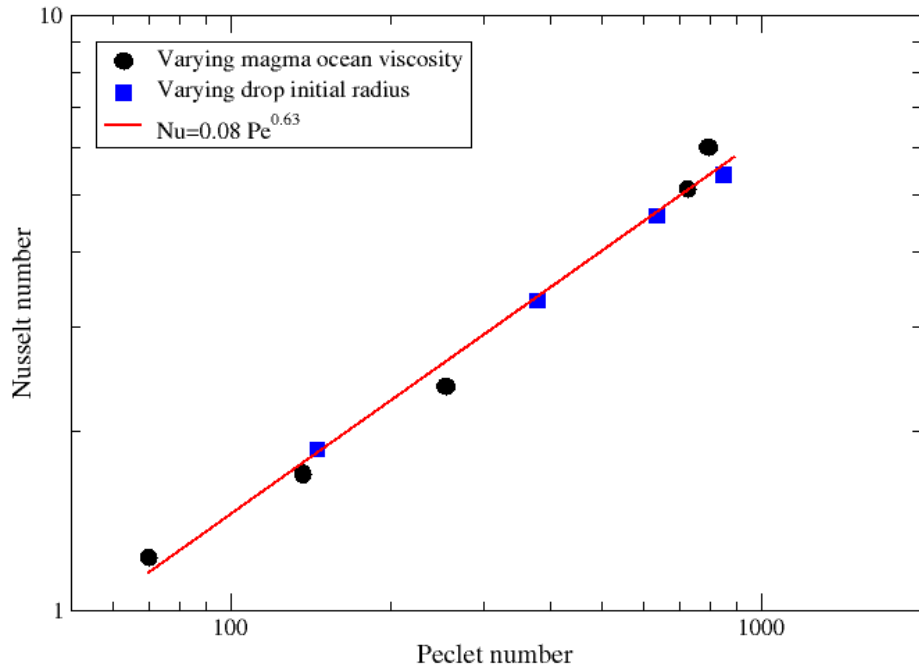


Figure 13: Nusselt number as a function of Peclet number

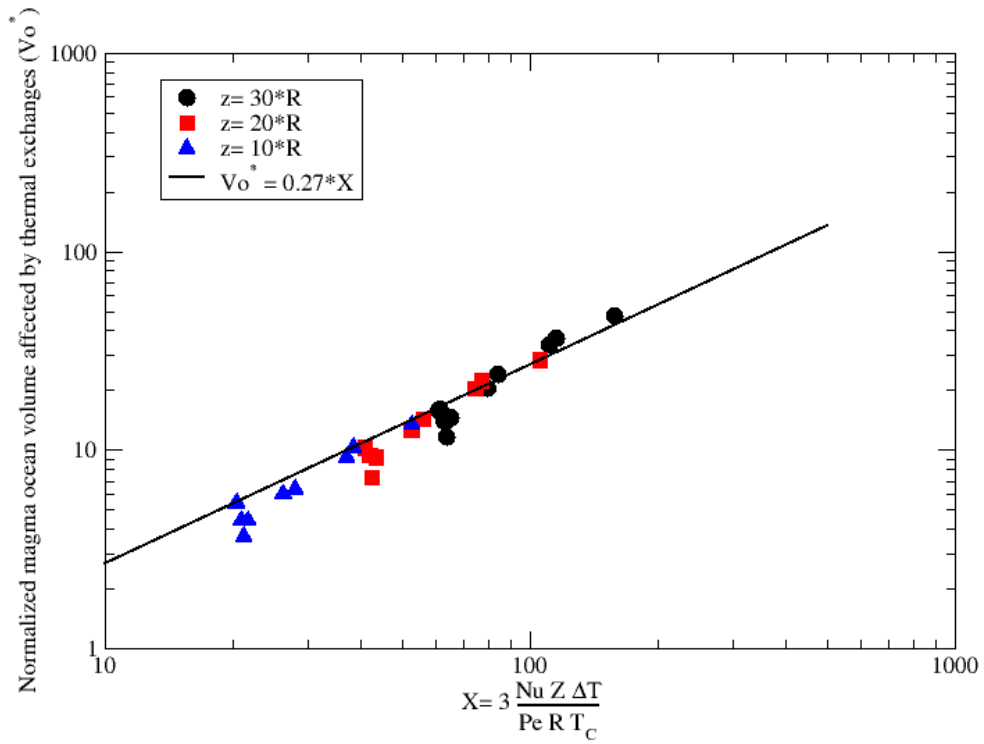


Figure 14: Dimensionless heated magma ocean volume as a function of X , and comparison with our scaling law (23). Our numerical results are represented with different symbols for different given depths.

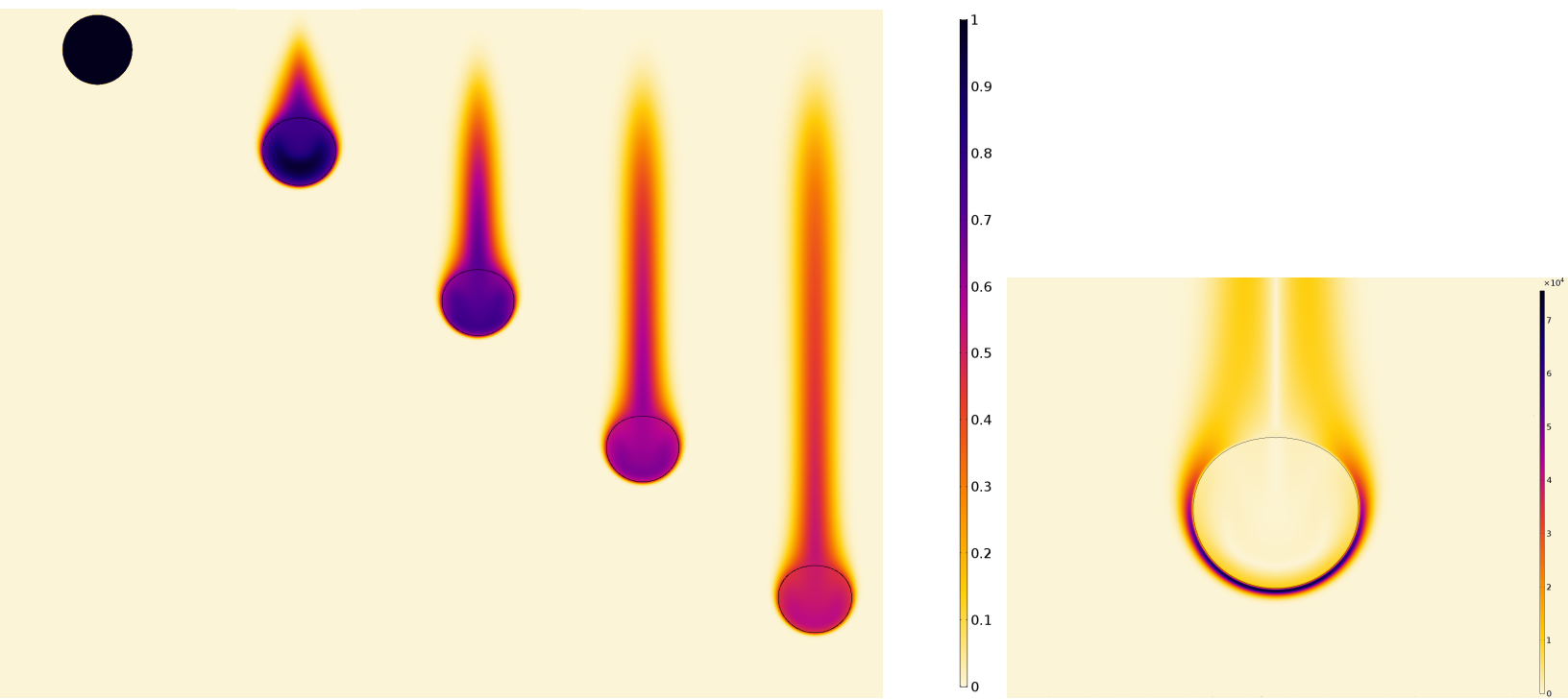


Figure 15: Thermal evolution of the non-deformable metal drop as a function of normalized time (Left). From left to right, $t^*=0, 0.0167, 0.033, 0.05$ and 0.067 . The right figure shows the temperature gradient (K/m) in and around the spherical drop case. In this model, $Re = 0.76$, $Pe = 254$ and $R_\mu=1000$ (simulation #3 in Table 2)

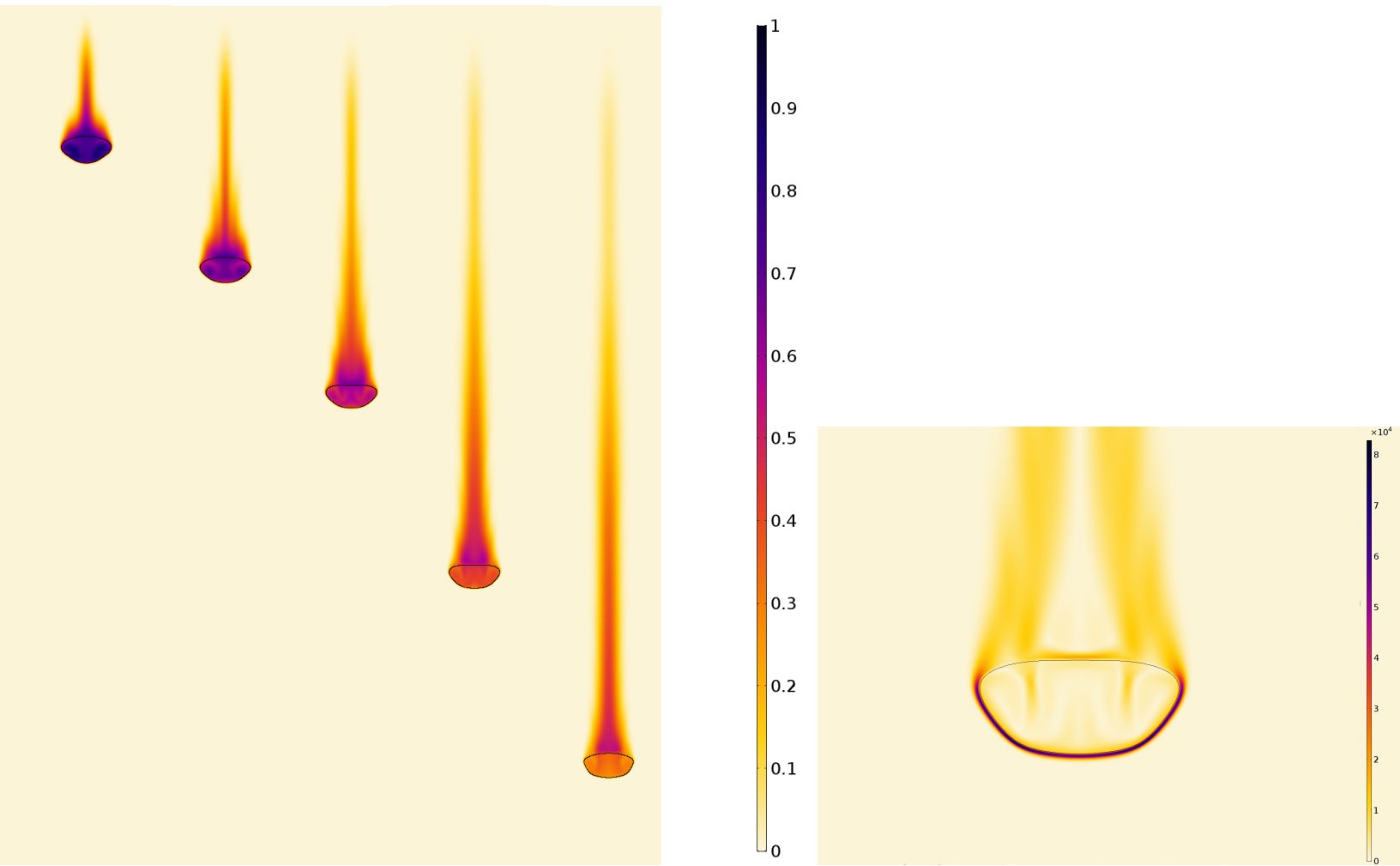


Figure 16: Thermal evolution of the weakly deformable metal drop as a function of normalized time (Left). From left to right, $t^*=0.015, 0.024, 0.033, 0.047$ and 0.06 . The right figure shows the temperature gradient (K/m) in and around the deformable drop. In this model, $Re = 21.8$, $Pe = 728.36$ and $R_\mu=100$ (simulation #5 in Table 2)

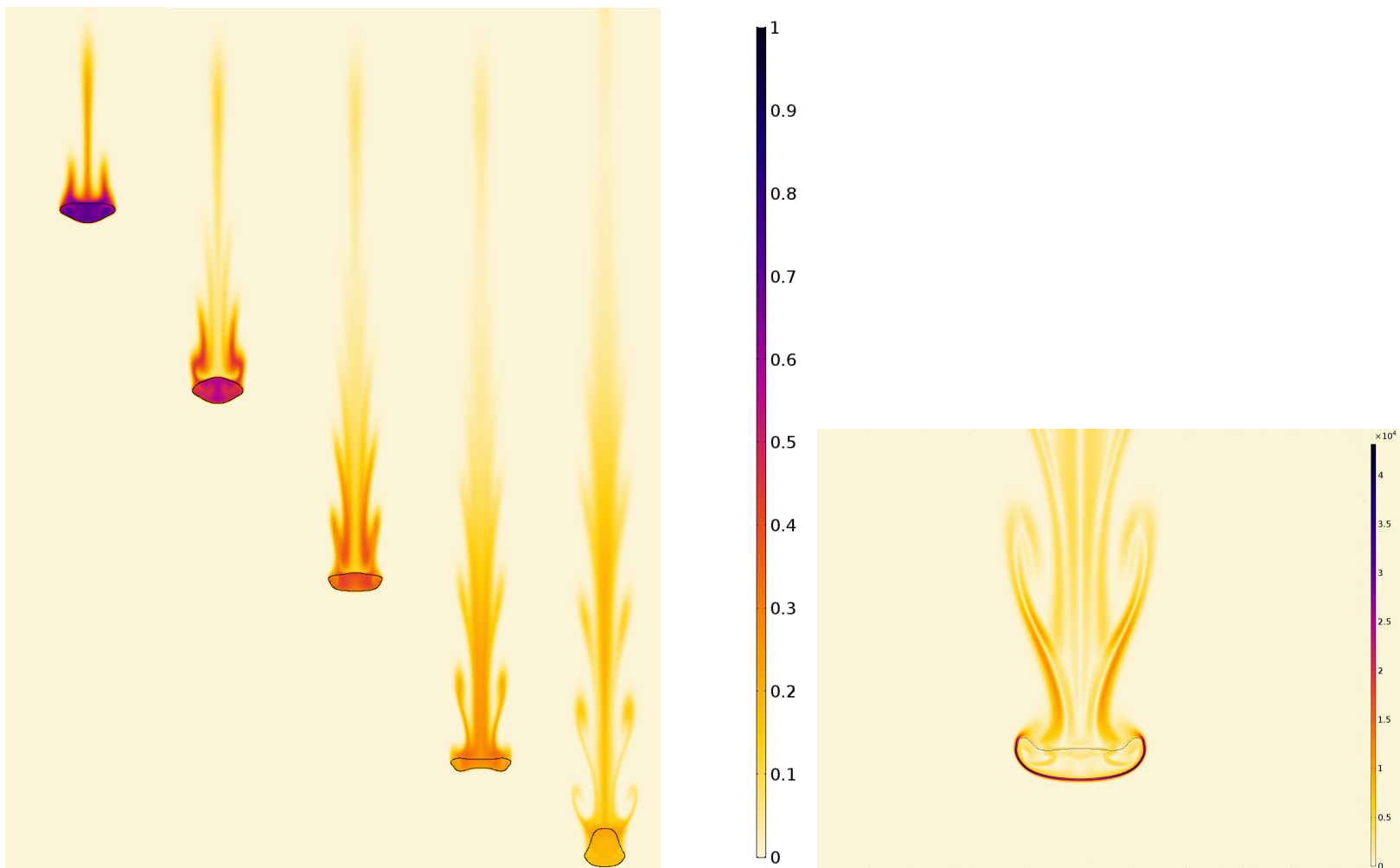


Figure 17: Thermal evolution of the strongly deformable metal drop as a function of normalized time (Left). From left to right, $t^*=0.02, 0.033, 0.047, 0.06$ and 0.077 . The right figure shows the temperature gradient (K/m) in and around the deformable drop case. In this model, $Re = 47.6$, $Pe = 793.73$ and $R_\mu=50$ (simulation #6 in Table 2)

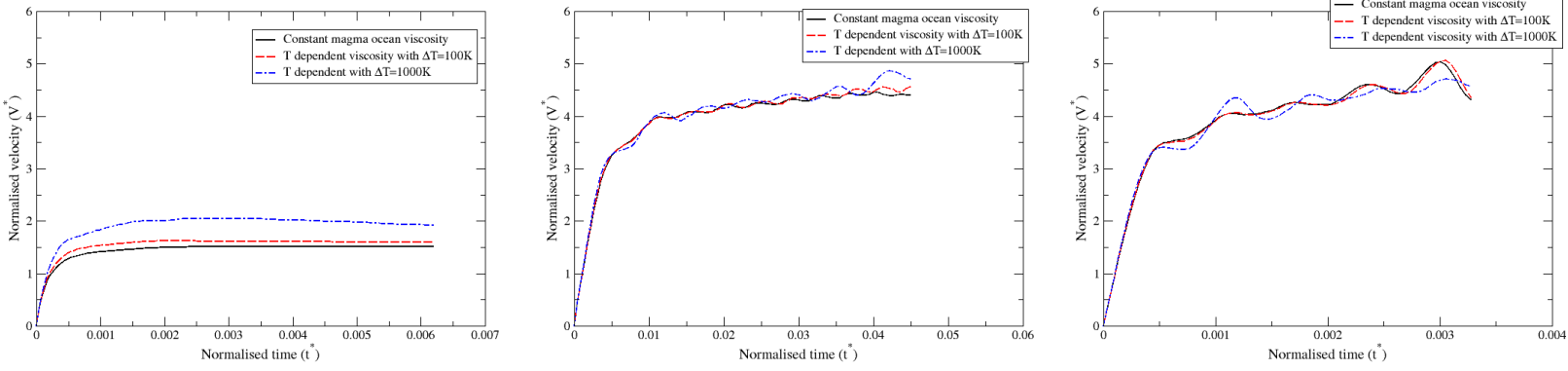


Figure 18: Comparison of the normalized average velocity field as a function of normalized time to show the influence of a temperature dependent viscosity for: a non-deformable spherical drop (Left), a weakly deformable drop (Middle) and a strongly deformable drop (Right). The black lines present the constant magma ocean viscosity. The red (blue) dashed lines present the temperature dependent viscosity with $\Delta T = 100\text{ K}$ ($\Delta T = 1000\text{ K}$).

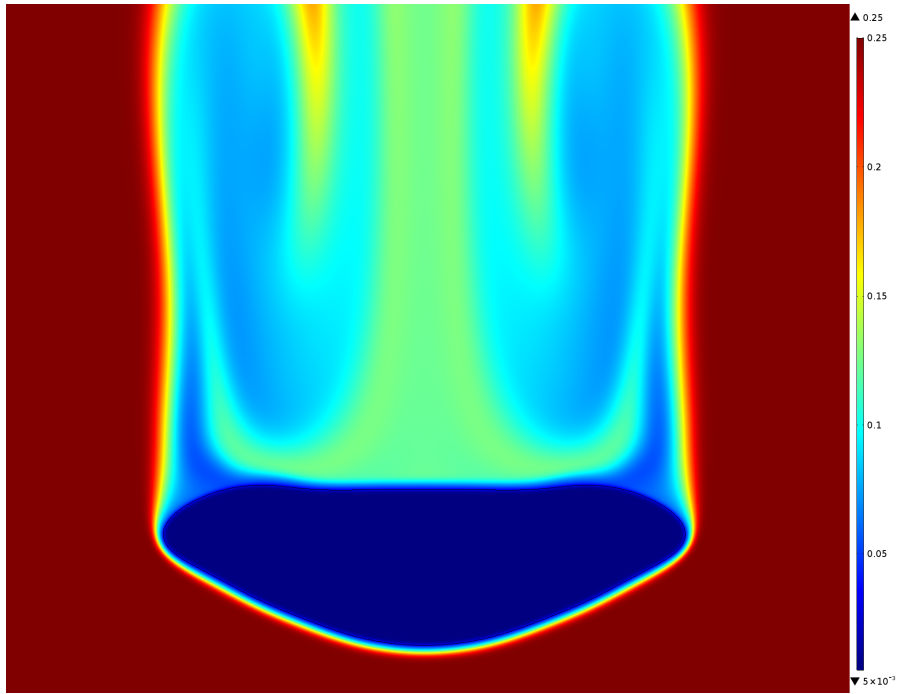


Figure 19: View of the viscosity field (in Pa.s) around the strongly deformable drop when a temperature dependent viscosity is considered.

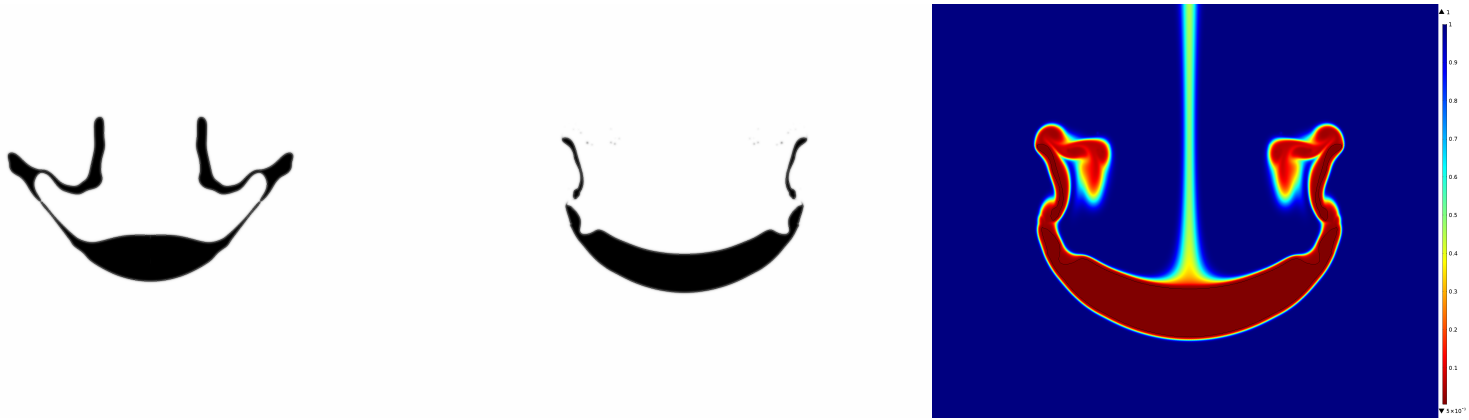


Figure 20: Final form at breaking of the drop for a constant viscosity magma ocean (Left) and a temperature dependent viscosity magma ocean (Middle). On the right, view of the viscosity field (in Pa.s) around the drop in the later case. In this model, $Re = 50$, $Pe = 3330$ and $R_\mu=200$ (simulation #10 in Table 2).

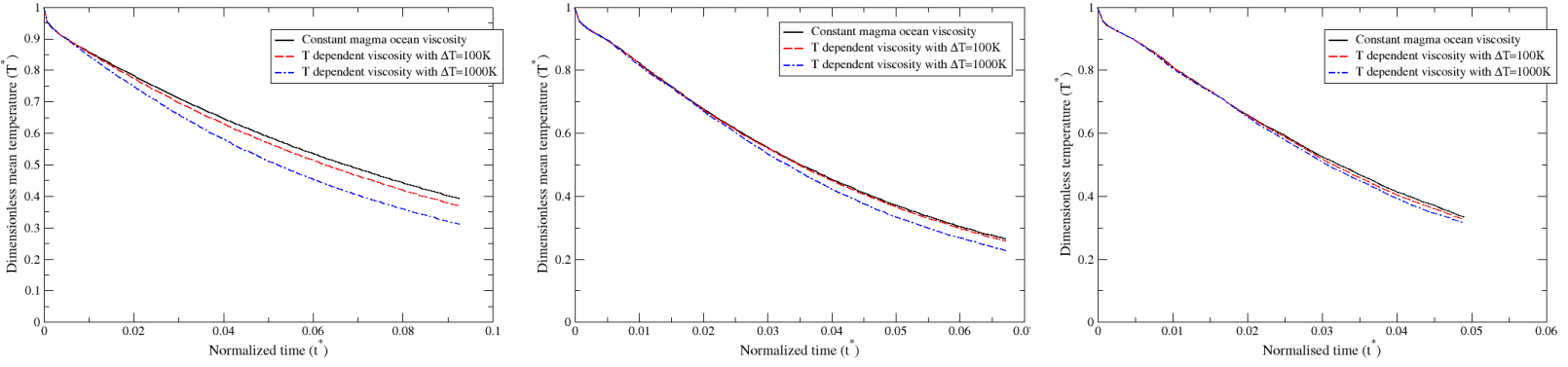


Figure 21: Comparison of the dimensionless temperature as a function of normalized time for a non-deformable drop (Left), a weakly deformable drop (Middle) and a strongly deformable drop (Right). The black lines present the constant magma ocean viscosity. The red (blue) dashed lines present the temperature dependent viscosity with $\Delta T = 100\text{ K}$ ($\Delta T = 1000\text{ K}$).

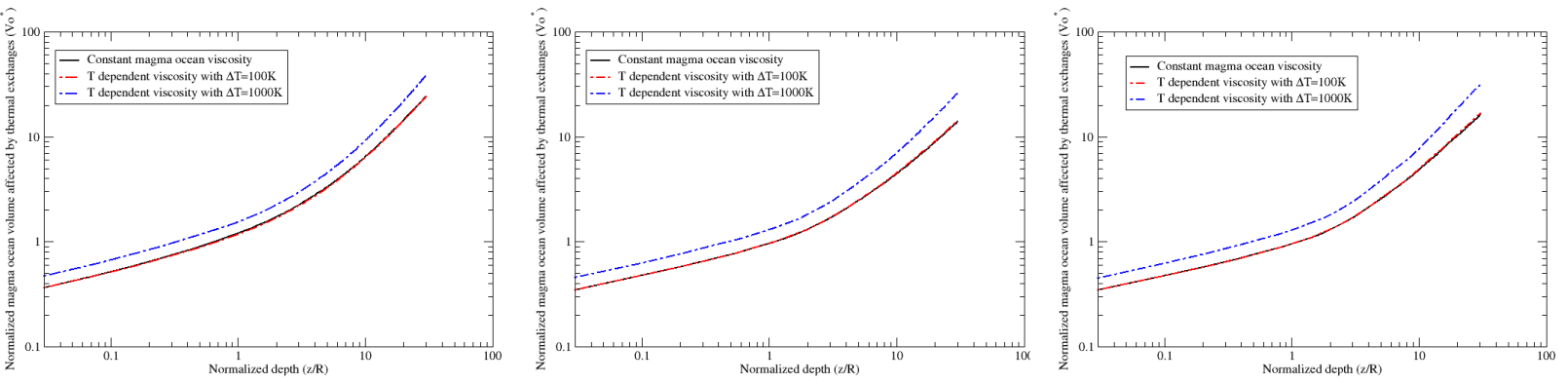


Figure 22: Comparison of normalized heated magma ocean volume as a function of normalized time for a non-deformable drop (Left), a weakly deformable drop (Middle) and a strongly deformable drop (Right). The black lines present the constant magma ocean viscosity. The red (blue) dashed lines present the temperature dependent viscosity with $\Delta T = 100$ K ($\Delta T = 1000$ K).