

Transfer matrix modeling and numerical verification of locally reacting acoustic mosaic

Intro

Method

Results

Conclusion

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Conventional sound absorbers (CSA)

Multilayers

Single layers



The common point is...

Surface homogeneity



Intro

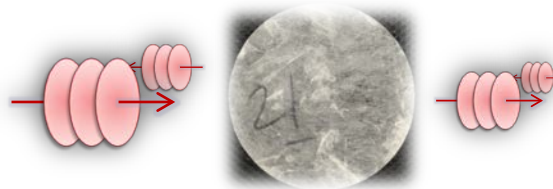
Method

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Modeling conventional sound absorbers (CSA)

Single layer – Transfer Matrix Method (TMM)



$$\begin{pmatrix} p \\ v \end{pmatrix}_{front} = \begin{pmatrix} a & b \\ c & a \end{pmatrix} \begin{pmatrix} p \\ v \end{pmatrix}_{back}$$

Multilayer – TMM in series



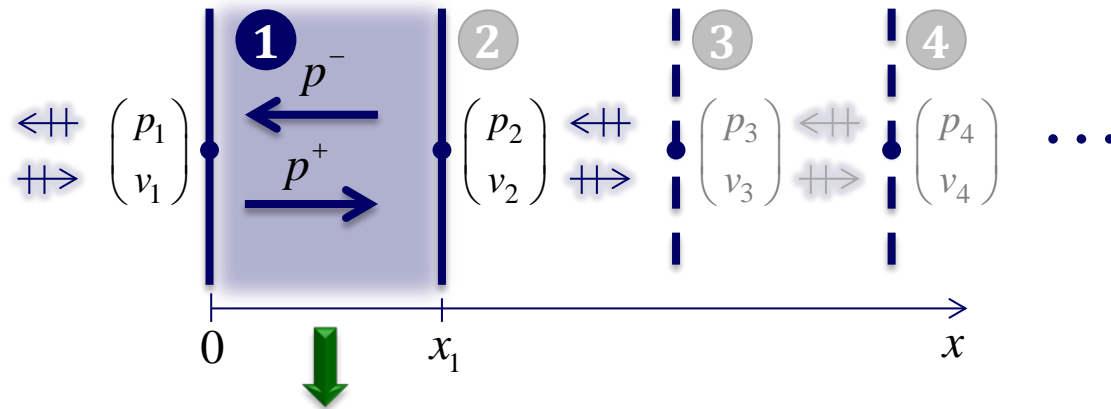
$$\begin{pmatrix} p \\ v \end{pmatrix}_{front} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} p \\ v \end{pmatrix}_{back}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} a & b \\ c & a \end{pmatrix}_1 \begin{pmatrix} a & b \\ c & a \end{pmatrix}_2 \begin{pmatrix} a & b \\ c & a \end{pmatrix}_3 \begin{pmatrix} a & b \\ c & a \end{pmatrix}_4 \begin{pmatrix} a & b \\ c & a \end{pmatrix}_5$$

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Transfer matrix in series

Transmission through a series of layers (multilayer)



$$p(x) = Ae^{-j\tilde{k}_1x} + Be^{j\tilde{k}_1x}$$

$$v(x) = \frac{A}{\tilde{Z}_1}e^{-j\tilde{k}_1x} - \frac{B}{\tilde{Z}_1}e^{j\tilde{k}_1x}$$

\tilde{k}_1 : Complex wave number of medium 1

\tilde{Z}_1 : Characteristic impedance of medium 1

$$p_1 = p(0) = A + B$$

$$p_2 = p(x_1) = Ae^{-j\tilde{k}_1x_1} + Be^{j\tilde{k}_1x_1}$$

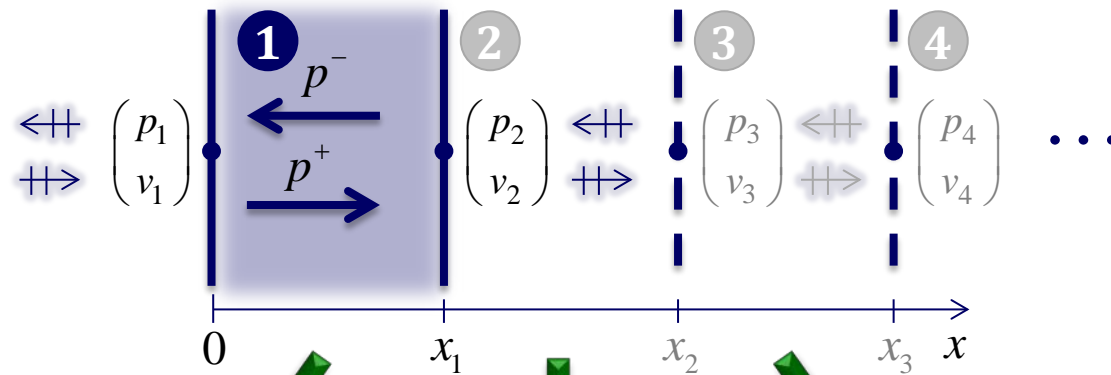
$$v_1 = v(0) = \frac{1}{\tilde{Z}_1}(A - B)$$

$$v_2 = v(x_1) = \frac{A}{\tilde{Z}_1}e^{-j\tilde{k}_1x_1} - \frac{B}{\tilde{Z}_1}e^{j\tilde{k}_1x_1}$$

$$\begin{pmatrix} p_1 \\ v_1 \end{pmatrix} = \begin{pmatrix} \cos \tilde{k}_1 h_1 & j\tilde{Z}_1 \sin \tilde{k}_1 h_1 \\ j\frac{1}{\tilde{Z}_1} \sin \tilde{k}_1 h_1 & \cos \tilde{k}_1 h_1 \end{pmatrix} \begin{pmatrix} p_2 \\ v_2 \end{pmatrix} \rightarrow \begin{pmatrix} p_1 \\ v_1 \end{pmatrix} = \mathbf{T}_1 \begin{pmatrix} p_2 \\ v_2 \end{pmatrix}$$

Transfer matrix in series

Transmission through a series of layers (multilayer)



$$\begin{pmatrix} p_1 \\ v_1 \end{pmatrix} = \mathbf{T}_1 \begin{pmatrix} p_2 \\ v_2 \end{pmatrix}$$

$$\begin{pmatrix} p_2 \\ v_2 \end{pmatrix} = \mathbf{T}_2 \begin{pmatrix} p_3 \\ v_3 \end{pmatrix}$$

$$\begin{pmatrix} p_3 \\ v_3 \end{pmatrix} = \mathbf{T}_3 \begin{pmatrix} p_4 \\ v_4 \end{pmatrix}$$

$$\begin{pmatrix} p_2 \\ v_2 \end{pmatrix} = \mathbf{T}_1 \mathbf{T}_2 \mathbf{T}_3 \begin{pmatrix} p_4 \\ v_4 \end{pmatrix}$$

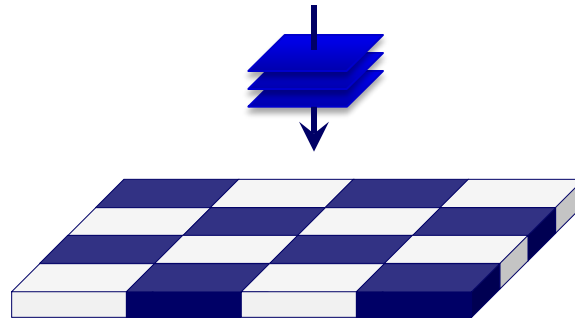
Transfer matrix combined in series

$$\begin{pmatrix} p_2 \\ v_2 \end{pmatrix} = \mathbf{T}_G \begin{pmatrix} p_4 \\ v_4 \end{pmatrix}$$

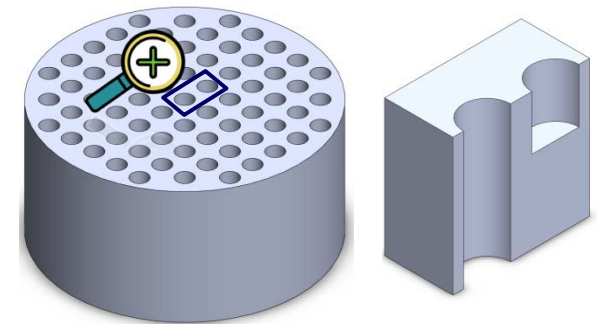
Non-conventional sound absorbers (NCSA)

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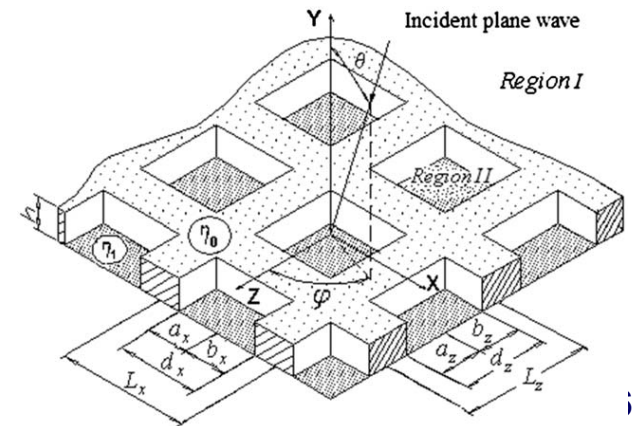
Patchwork



Dead-end surface porosity



Grooved surface



The common point is...



Source : J. Ducourneau et al.

Journal of Sound and Vibration 329 (2010) 2276–2290

Modeling non-conventional sound absorbers (NCSA)

Analytical, Semi-analytical, and FE models

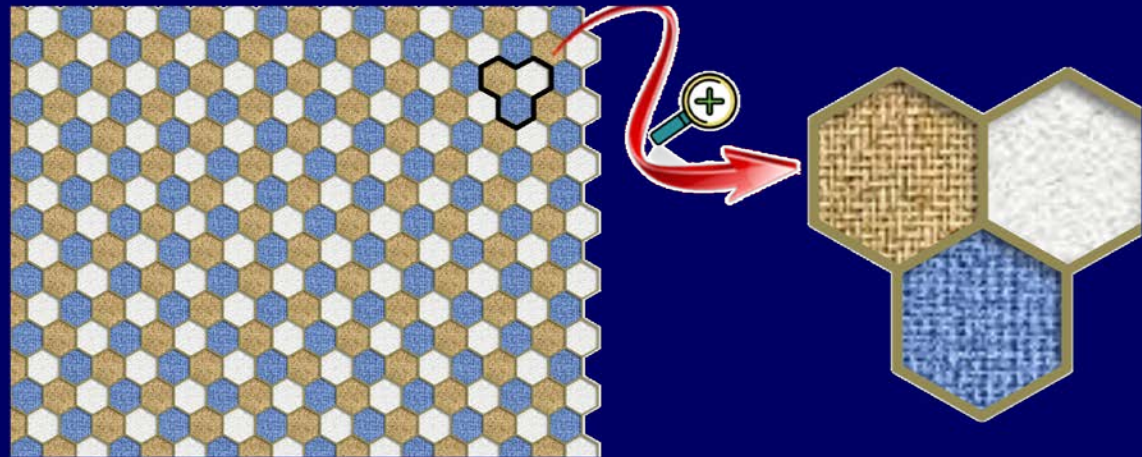
- *Non-propagative static model – Thin patchworks* ($kl \ll 1$)
[Atalla, Panneton, and Allard, *ACUSTICA - Acta acustica* **83**, 891-896 (1997)]
- *Finite element method – Patchworks*
[Atalla, Panneton, and Allard, *ACUSTICA - Acta acustica* **83**, 891-896 (1997)]
- *Wave based method (indirect Trefftz method) - Patchworks*
[Lanoye, Vermerir, Lauriks, Sgard, Desmet, *JASA* **123**, 793-802 (2008)]
- *Waveguide method – Grooved surface*
[J. Ducourneau et al. , *Journal of Sound and Vibration* **329**, 2276–2290 (2010)]
- *Finite element method – Parallel MPP absorbers*
[Wang and Huang, *J. Acoust. Soc. Am.* **130**, 208-218 (2011)]
- *Wave theory, Bruijn's theory – Grooved surface*
[Wang, Leistner and Li, *Applied Acoustics* **79**, 960-968 (2012)]

*No Transfer
Matrix Method*

Objective of the research

Extend the Transfer Matrix Method (TMM) to non-conventional sound absorbers having surface heterogeneities.

For this presentation, we limit to locally reacting acoustic mosaic



Intro

Method

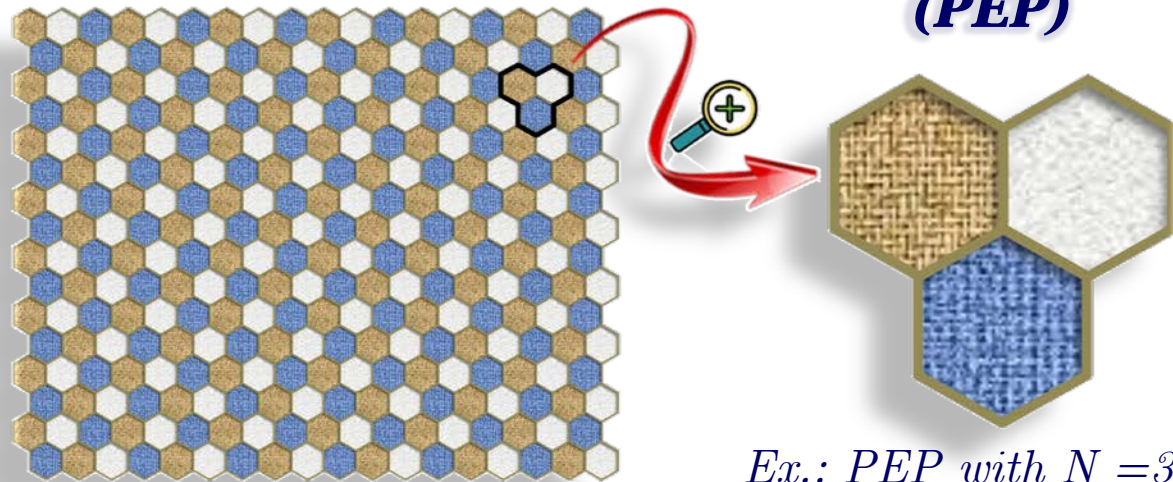
Results

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Transfer Matrix Method

Acoustic Mosaic

Periodic Elementary Patch (PEP)



Ex.: PEP with $N = 3$ cells

Assumptions

- *The walls of a cell are impervious (locally reacting)*
- *Each cell is filled with a sound absorber (single or multilayer)*
- *Sound absorbers are modeled as rigid/limp equivalent fluid*
- *PEP excited by a normally incidence plane wave*
- *Wavelength much larger than the PEP size*
- *The surface ratio of a cell in a PEP is :*

$$r_i = S_i / S_{PEP}$$

Intro

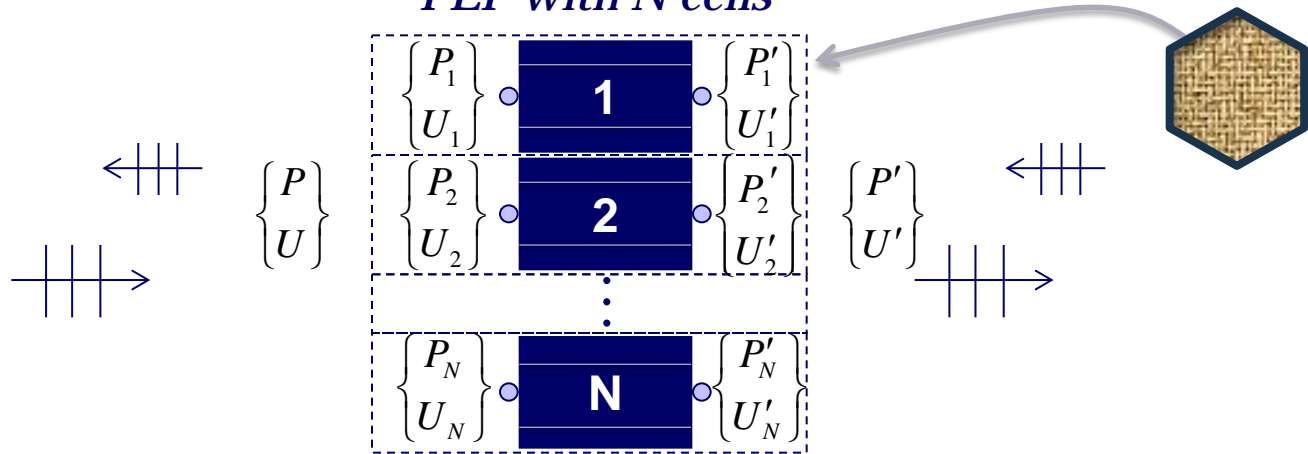
Method

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Transfer Matrix Method

PEP with N cells



$$\begin{Bmatrix} P_i \\ U_i \end{Bmatrix} = \begin{bmatrix} T_{i,11} & T_{i,12} \\ T_{i,21} & T_{i,22} \end{bmatrix} \begin{Bmatrix} P'_i \\ -U'_i \end{Bmatrix}$$

Transfer matrix



$$\begin{Bmatrix} U_i \\ U'_i \end{Bmatrix} = \begin{bmatrix} y_{i,11} & y_{i,12} \\ y_{i,21} & y_{i,22} \end{bmatrix} \begin{Bmatrix} P_i \\ P'_i \end{Bmatrix}$$

Admittance matrix

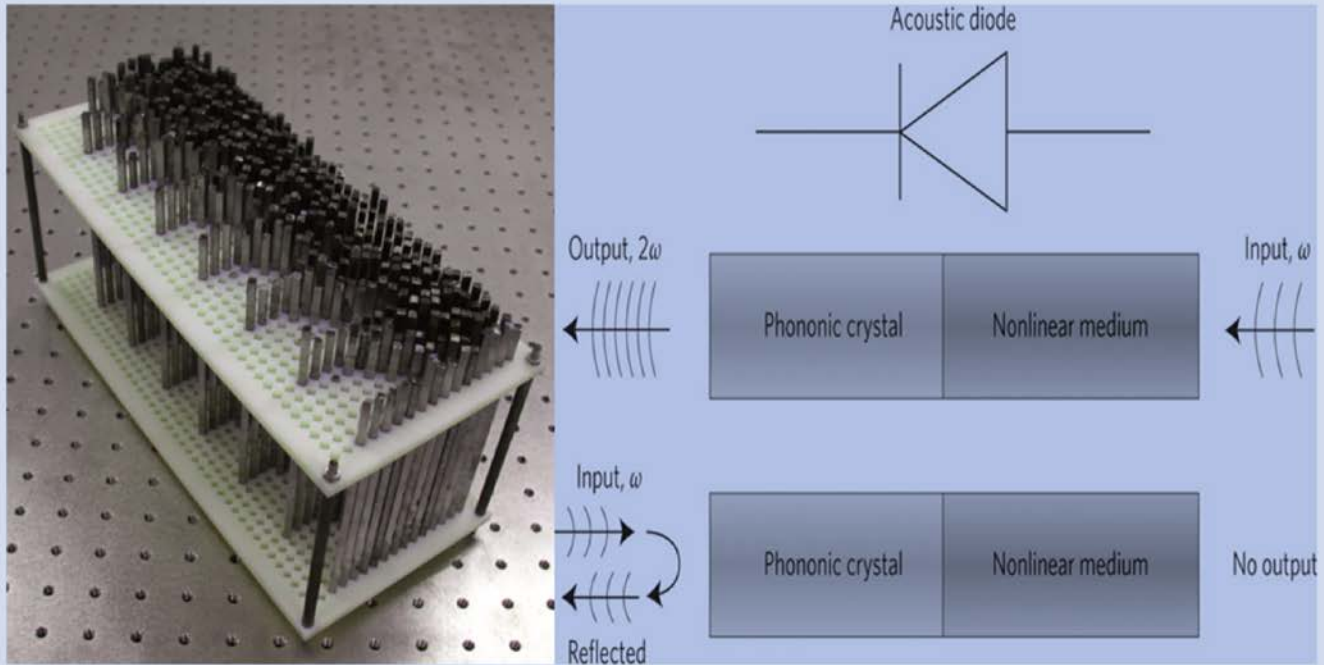
$$y_{i,11} = T_{i,22} / T_{i,12}; \quad y_{i,21} = -1 / T_{i,12}; \quad y_{i,22} = T_{i,11} / T_{i,12};$$

$$y_{i,12} = (T_{i,12}T_{i,21} - T_{i,11}T_{i,22}) / T_{i,12} = -1 / T_{i,12} \text{ (with } |T|=1 \text{ for reciprocity, } y_{i,12} = y_{i,21})$$

Transfer Matrix Method

DED with N cells

*One metamaterial : the acoustic diode
 Non-reciproque material*



rix

$$y_{i,11} = T_{i,22} / T_{i,12}; \quad y_{i,21} = -1 / T_{i,12}; \quad y_{i,22} = T_{i,11} / T_{i,12};$$

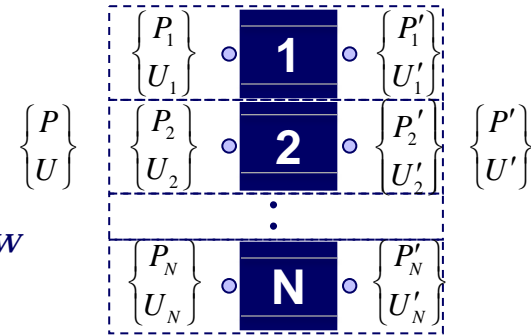
$$y_{i,12} = (T_{i,12}T_{i,21} - T_{i,11}T_{i,22}) / T_{i,12} = -1 / T_{i,12} \quad (\text{with } |T|=1 \text{ for reciprocity, } y_{i,12} = y_{i,21})$$

Transfer Matrix Method

Continuity conditions

$$\begin{cases} U \\ U' \end{cases} = \sum \begin{cases} r_i U_i \\ r_i U'_i \end{cases}$$

Continuity of upstream flow
Continuity of downstream flow



$$\begin{cases} P \\ P' \end{cases} = \begin{cases} P_i \\ P'_i \end{cases}$$

Continuity of upstream pressure
Continuity of downstream pressure for open-end cells

$$\begin{cases} r_i U_i \\ r_i U'_i \end{cases} = r_i \begin{bmatrix} y_{i,11} & y_{i,12} \\ y_{i,21} & y_{i,22} \end{bmatrix} \begin{cases} P_i \\ P'_i \end{cases}$$



$$\begin{cases} U \\ U' \end{cases} = \left(\sum r_i \begin{bmatrix} y_{i,11} & y_{i,12} \\ y_{i,21} & y_{i,22} \end{bmatrix} \right) \begin{cases} P \\ P' \end{cases}$$

Admittance matrix combined in parallel

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Transfer Matrix Method

From admittance to transfer matrix...

$$\begin{Bmatrix} P \\ U \end{Bmatrix} = \mathbf{T}_{\text{PEP}} \begin{Bmatrix} P' \\ -U' \end{Bmatrix}$$

Parallel summation of transfer matrices

$$\mathbf{T}_{\text{PEP}} = \frac{1}{B} \begin{bmatrix} -C & -1 \\ B^2 - AC & -A \end{bmatrix}$$

$$A = \sum_i r_i T_{i,22} T_{i,12}^{-1}$$

$$B = -\sum_i r_i T_{i,12}^{-1} ; \quad C = \sum_i r_i T_{i,11} T_{i,12}^{-1}$$

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Transfer Matrix Method

Particular case of cells with closed end

$$\begin{cases} U \\ U' \end{cases} = \sum \begin{cases} r_i U_i \\ r_i U'_i \end{cases}$$

Continuity of upstream flow

Continuity of downstream flow

$$P_i = P$$

Continuity of upstream pressure

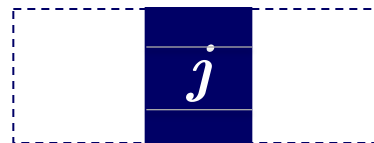
$$P'_j = P'$$

Continuity of downstream pressure for open-end cells

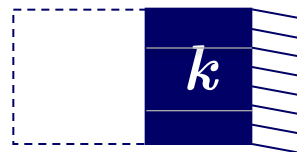
$$P'_k = -\frac{y_{k,21}}{y_{k,22}} P$$

Downstream pressure for cells closed to end

Boundary condition backing a cell



Cell with open end (subscript j)



Cell with closed end (subscript k)

Transfer Matrix Method

General case of open and closed backing...

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$$\begin{Bmatrix} P \\ U \end{Bmatrix} = \mathbf{T}_{\text{PEP}} \begin{Bmatrix} P' \\ -U' \end{Bmatrix}$$

Parallel summation of transfer matrices

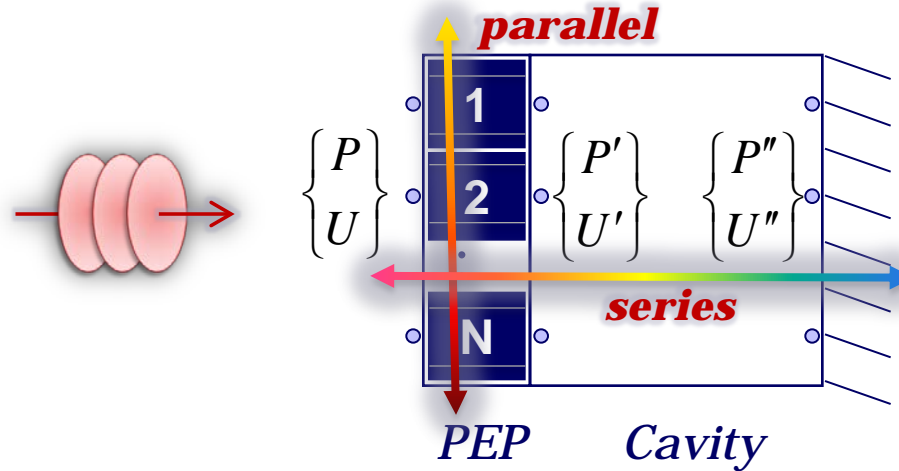
$$\mathbf{T}_{\text{PEP}} = \frac{1}{B} \begin{bmatrix} -C & -1 \\ B^2 - AC & -A \end{bmatrix}$$

$$A = \sum_i r_i T_{i,22} T_{i,12}^{-1} - \sum_k r_k T_{k,11} T_{k,12}^{-1}$$

$$B = -\sum_j r_j T_{j,12}^{-1} ; \quad C = \sum_j r_j T_{j,11} T_{j,12}^{-1}$$

Transfer Matrix Method

Adding a backing layer (ex.: air cavity backing)

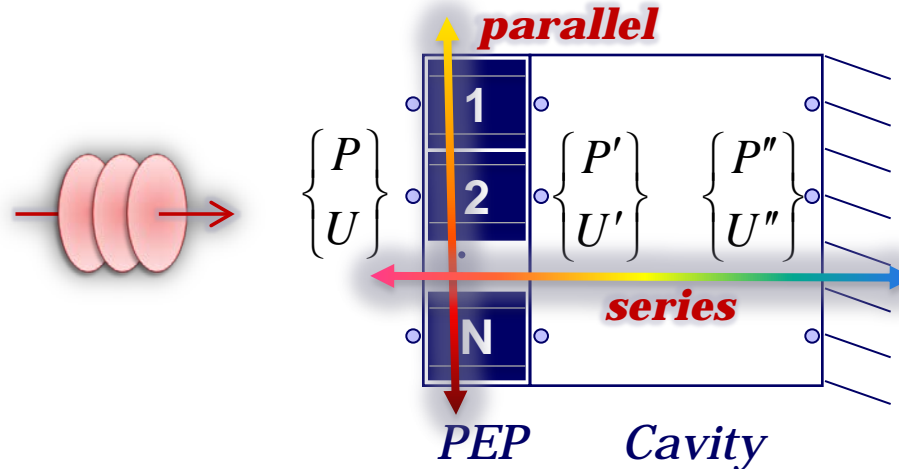


$$\begin{Bmatrix} P' \\ U' \end{Bmatrix} = \mathbf{T}_{cavity} \begin{Bmatrix} P'' \\ -U'' \end{Bmatrix} \quad \text{Transfer matrix for air-cavity}$$

$$\mathbf{T}_{cavity} = \begin{pmatrix} \cos k_0 d & j\rho_0 c_0 \sin k_0 d \\ j \frac{1}{\rho_0 c_0} \sin k_0 d & \cos k_0 d \end{pmatrix}$$

Transfer Matrix Method

Adding a backing layer (ex.: air cavity backing)



$$\begin{Bmatrix} P \\ U \end{Bmatrix} = \mathbf{T}_{\text{Global}} \begin{Bmatrix} P'' \\ -U'' \end{Bmatrix}$$

*Combination of cells
 in parallel and
 in series*

$$\mathbf{T}_{\text{Global}} = \mathbf{T}_{\text{PEP}} \mathbf{T}_{\text{cavity}}$$

Parallel

Series

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Transfer Matrix Method

Acoustical indicators

$$\mathbf{T}_{\text{Global}} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

Sound absorption coefficient

$$\alpha = 1 - \left| \frac{A - \rho c C}{A + \rho c C} \right|^2$$

Sound transmission loss

$$nSTL = 20 \log \left| A + \frac{B}{\rho c} + \rho c C + D \right| - 6 \text{ dB}$$

Finite Element Method

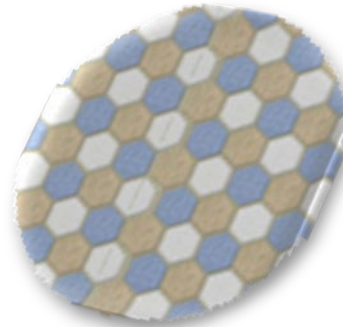
Objective : Validate the developed transfer matrix against FE simulations of impedance tube measurements

Intro

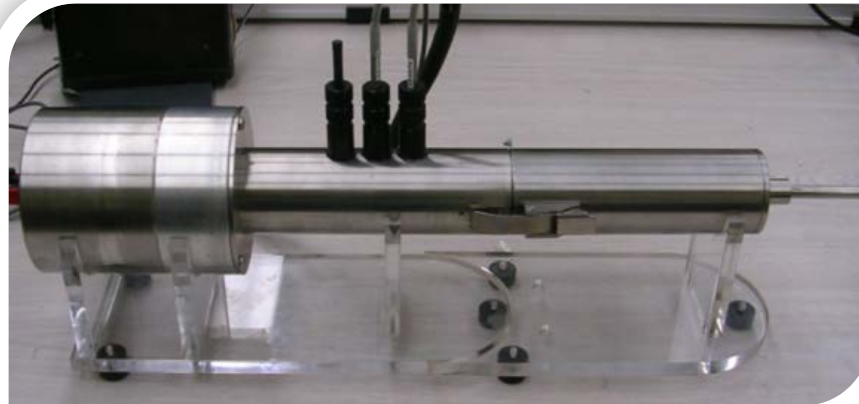
➔ Method

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Conclusion

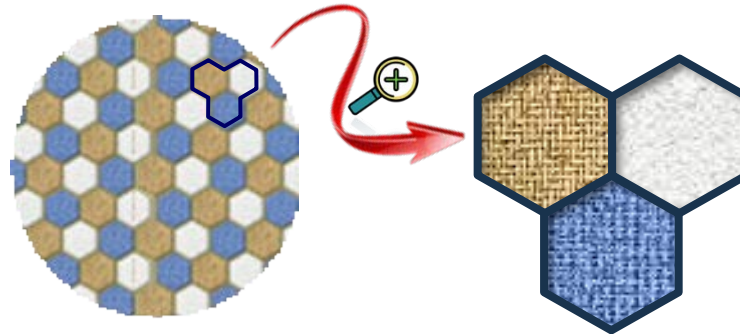


Virtual sample

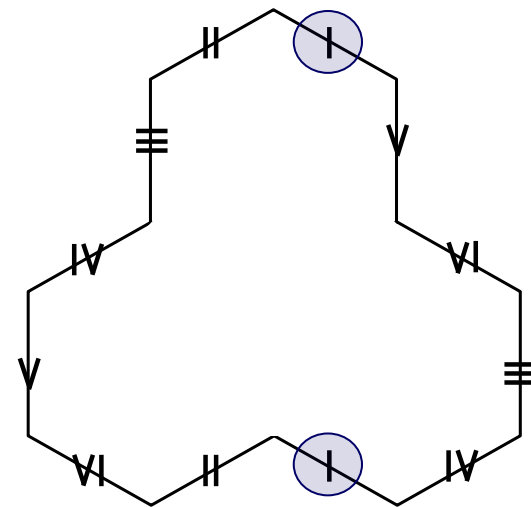


Finite Element Method

FE model of the PEP only



Periodic conditions



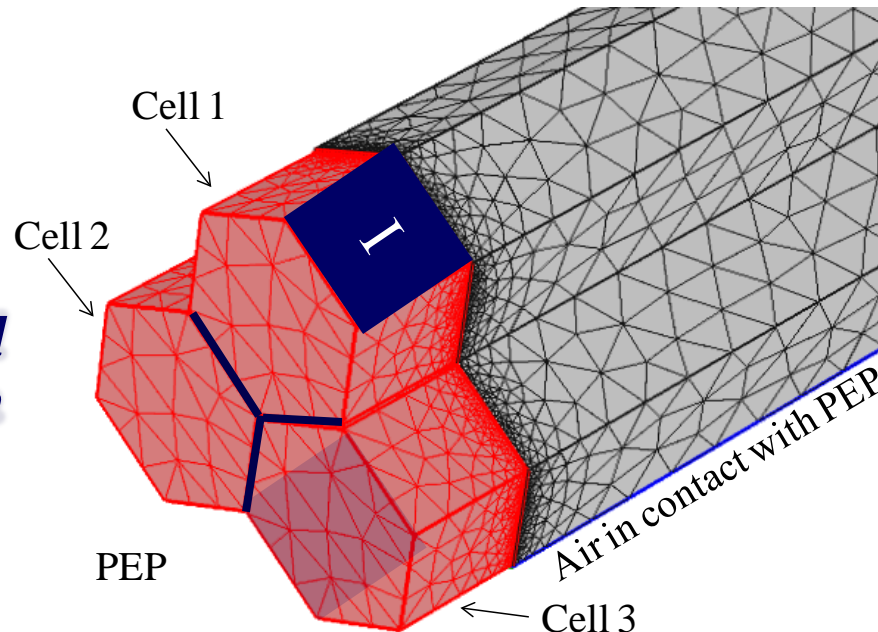
Pairs of periodic surface condition on the air domain in contact with PEP

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Hardwall condition

Finite Element Method

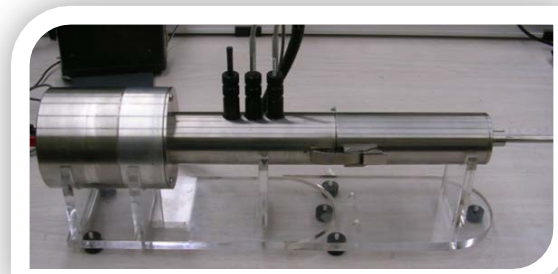
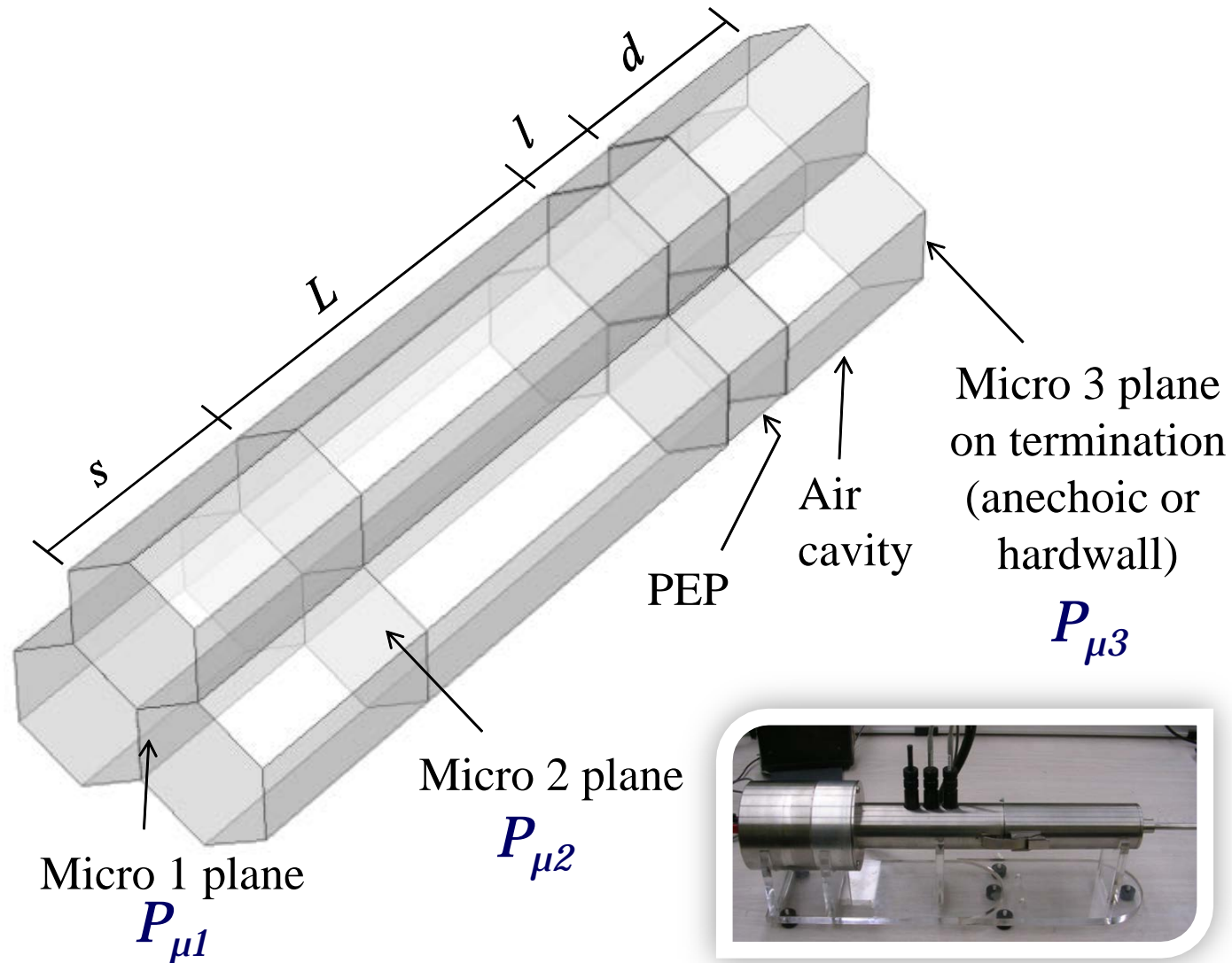
FE model of the PEP in the tube

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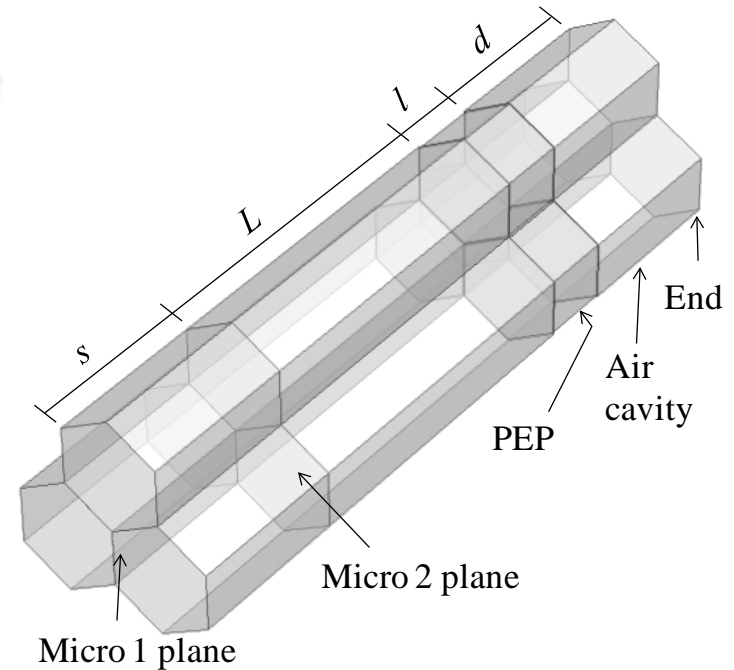
Finite Element Method

Acoustical indicators

Reflection coefficient

$$R = \frac{H_{12}e^{jks} - 1}{1 - H_{12}e^{-jks}} e^{2jkL}$$

$$H_{12} = \frac{P_{\mu 2}}{P_{\mu 1}}$$



Sound absorption coefficient

$$\alpha = 1 - |R|^2$$

Sound transmission loss

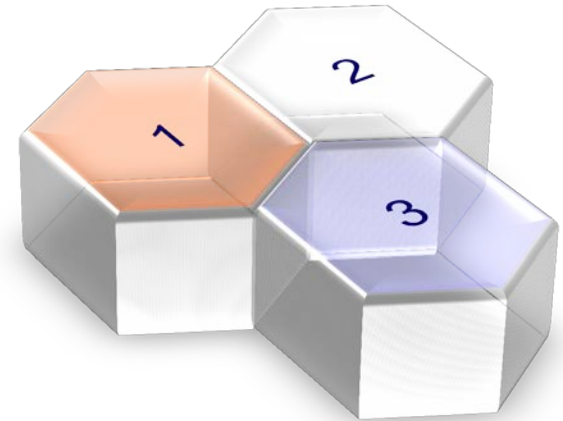
$$nSTL = 20 \log \left| P_{\mu 3} / P_{inc} \right|$$

$$P_{inc} = \frac{P_{\mu 1}}{e^{-jkX_{\mu 1}} + Re^{jkX_{\mu 1}}}$$

Description of sample 1

Honeycomb filled with porous materials

PEP contains: 3 cells
 PEP thickness: 15 mm
 Cell width: 12 mm
 Wall thickness: 0.5 mm



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Material	1	2	3
Open porosity	0.941	0.883	0.765
Flow resistivity (N.s.m ⁻⁴)	6 260	34 403	80 697
Tortuosity	1.35	1.77	1.88
Viscous characteristic length (μm)	125	343	28
Thermal characteristic length (μm)	302	360	355

Description of sample 1

Porous material modeling

- **JCA** model
- Rigid frame
- Equivalent fluid model
- Visco-inertial effects
 - **Johnson et al. (1987)** - $\phi, \sigma, \alpha_\infty, \Lambda$
- Thermal effects
 - **Champoux and Allard (1991)** - ϕ, Λ'

$$\Delta p + \omega^2 \frac{\rho_{eq}}{K_{eq}} p = 0$$

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Transfer matrix for *i*th material

$$\mathbf{T}_i = \begin{pmatrix} \cos k_{eq}l & jZ_{eq} \sin k_{eq}l \\ j \frac{1}{Z_{eq}} \sin k_{eq}l & \cos k_{eq}l \end{pmatrix}_i$$

$$k_{eq} = \omega \sqrt{\frac{\rho_{eq}}{K_{eq}}} \quad Z_{eq} = \sqrt{\rho_{eq} K_{eq}}$$

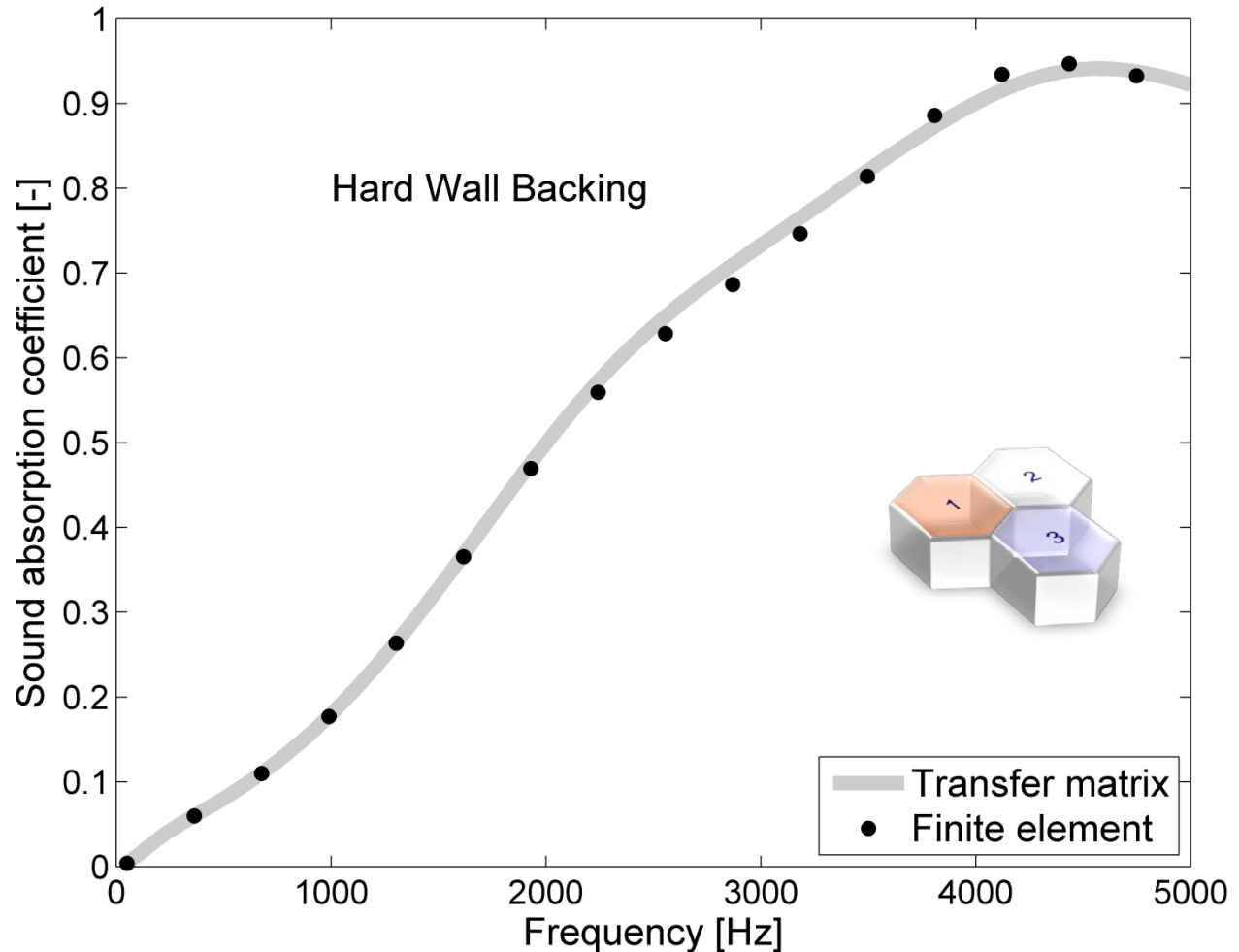
Results on sample 1

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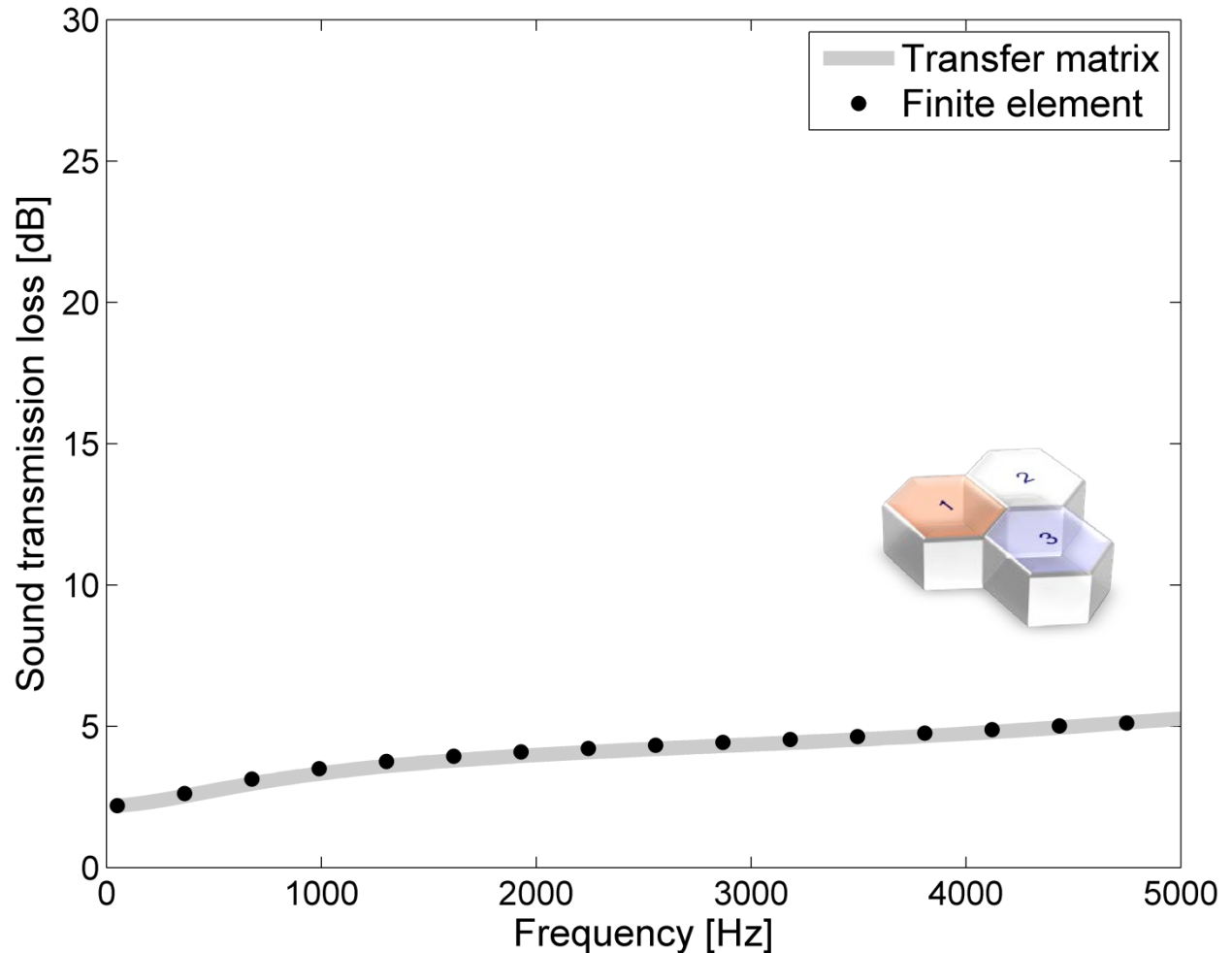
Results on sample 1

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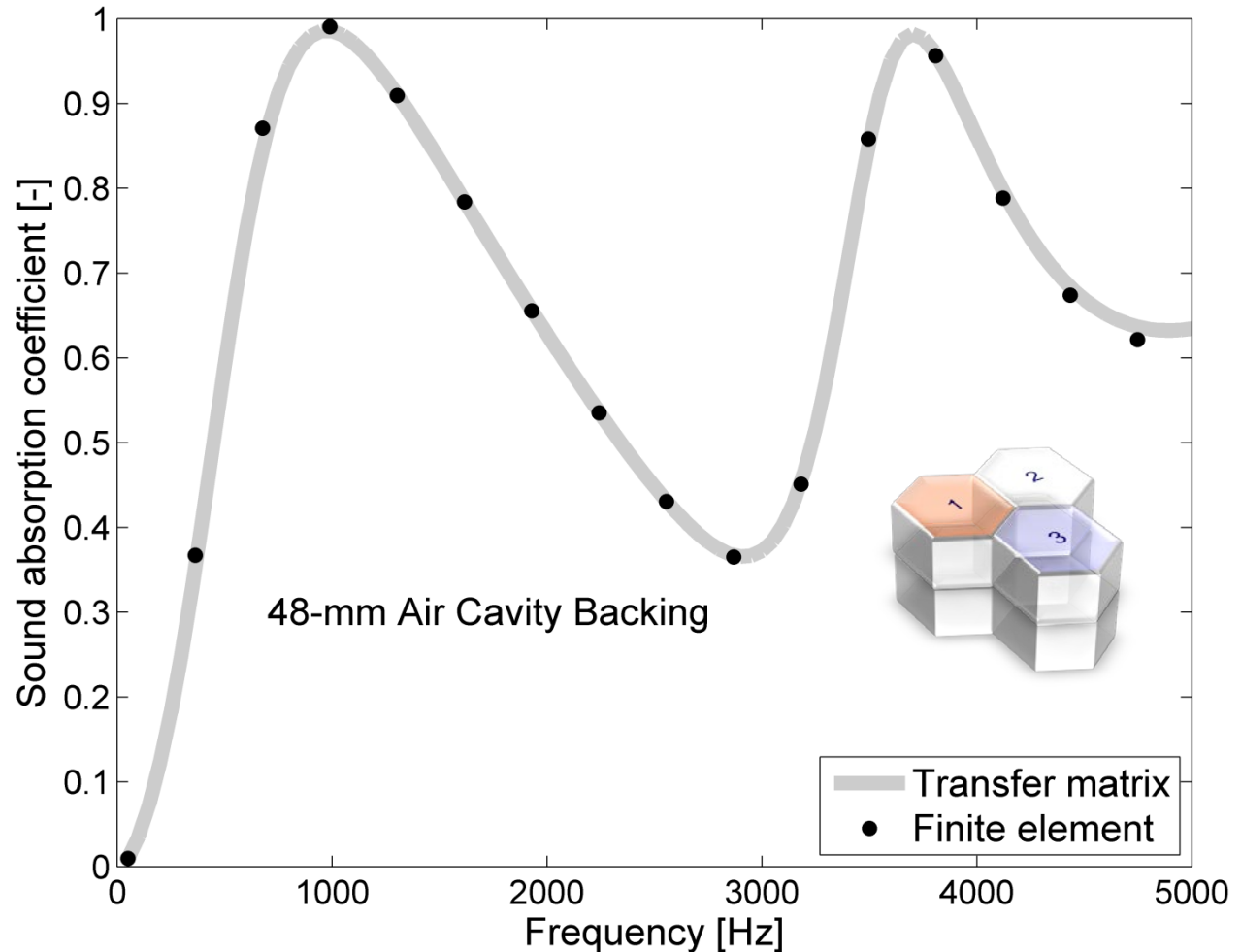
Results on sample 1

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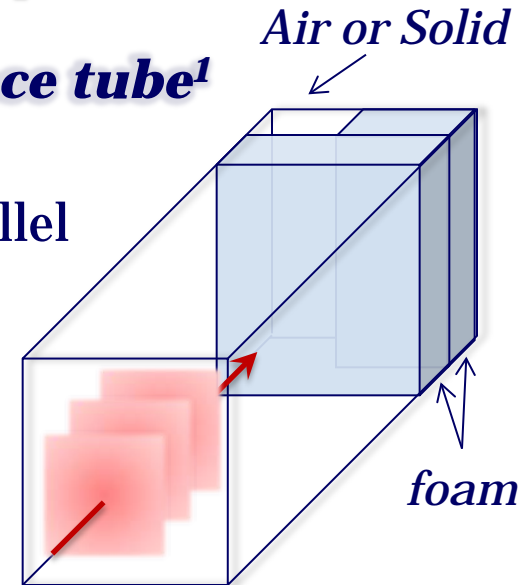
Conclusion



Description of sample 2

Patchwork in square impedance tube¹

Configuration: Series and Parallel
 Tube size : 0.6 x 0.6 m²
 Cutoff frequency: 570 Hz
 End condition: Rigid end
 1st layer thickness: 50 mm
 2nd layer thickness: 50 mm



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Material

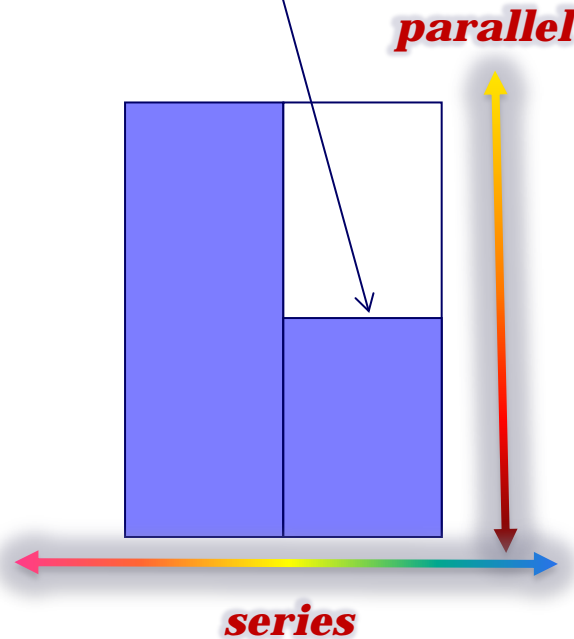
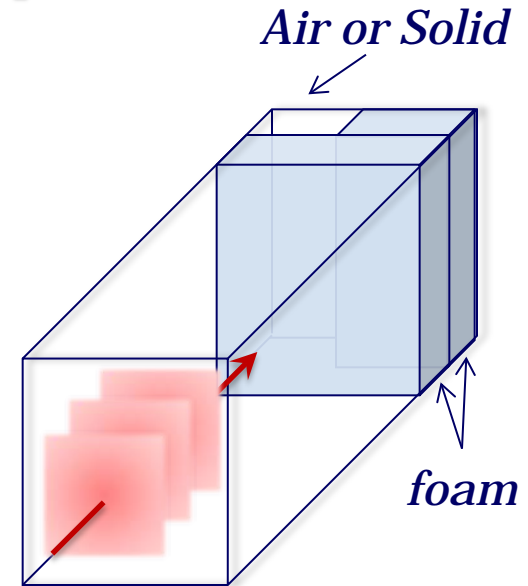
Melamine foam

Open porosity	0.98
Flow resistivity (N.s.m ⁻⁴)	10 000
Tortuosity	1.1
Viscous characteristic length (μm)	100
Thermal characteristic length (μm)	150

Description of sample 2

Special remark

*One assumption not fulfilled:
No impervious wall
between the 2 patches*



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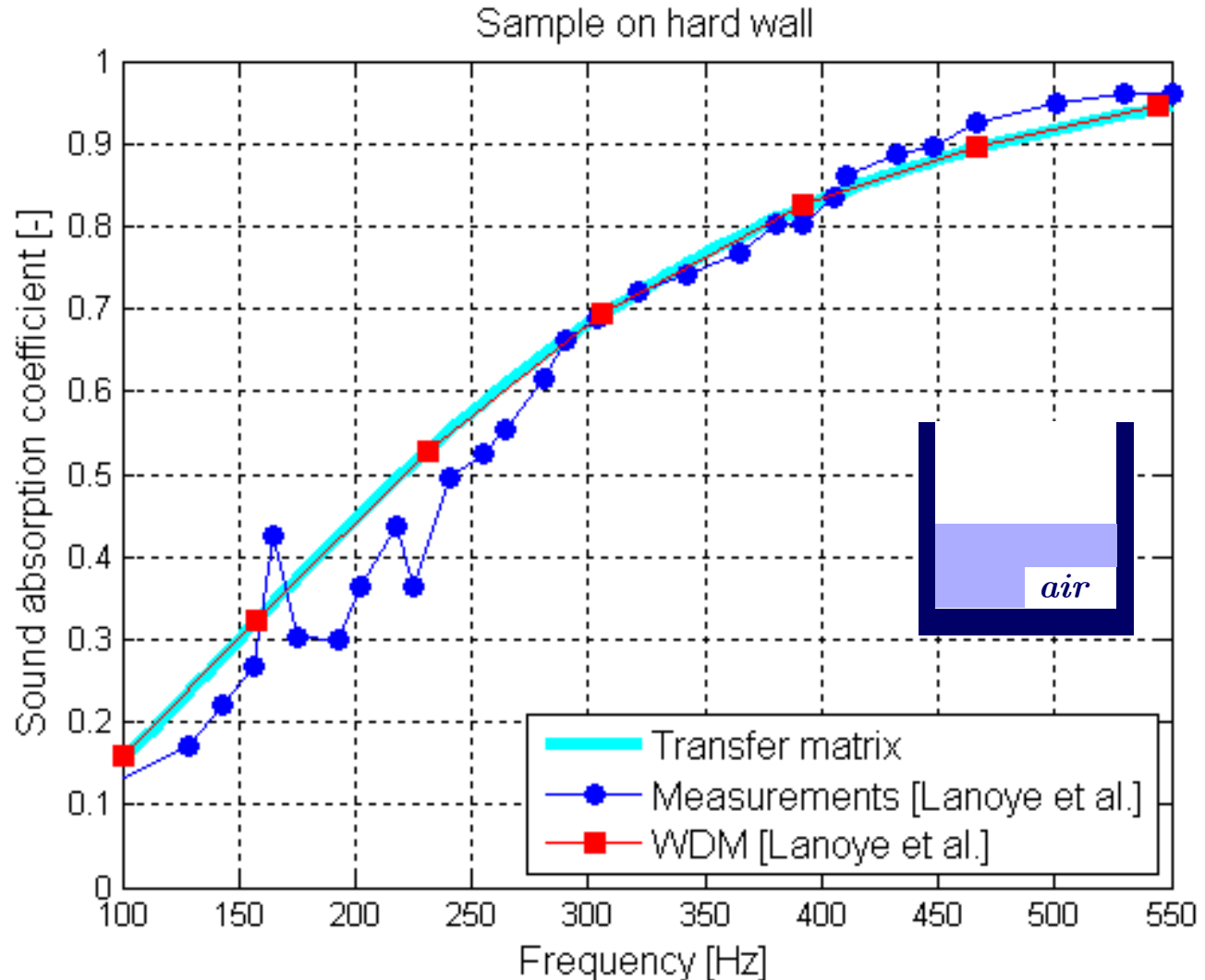
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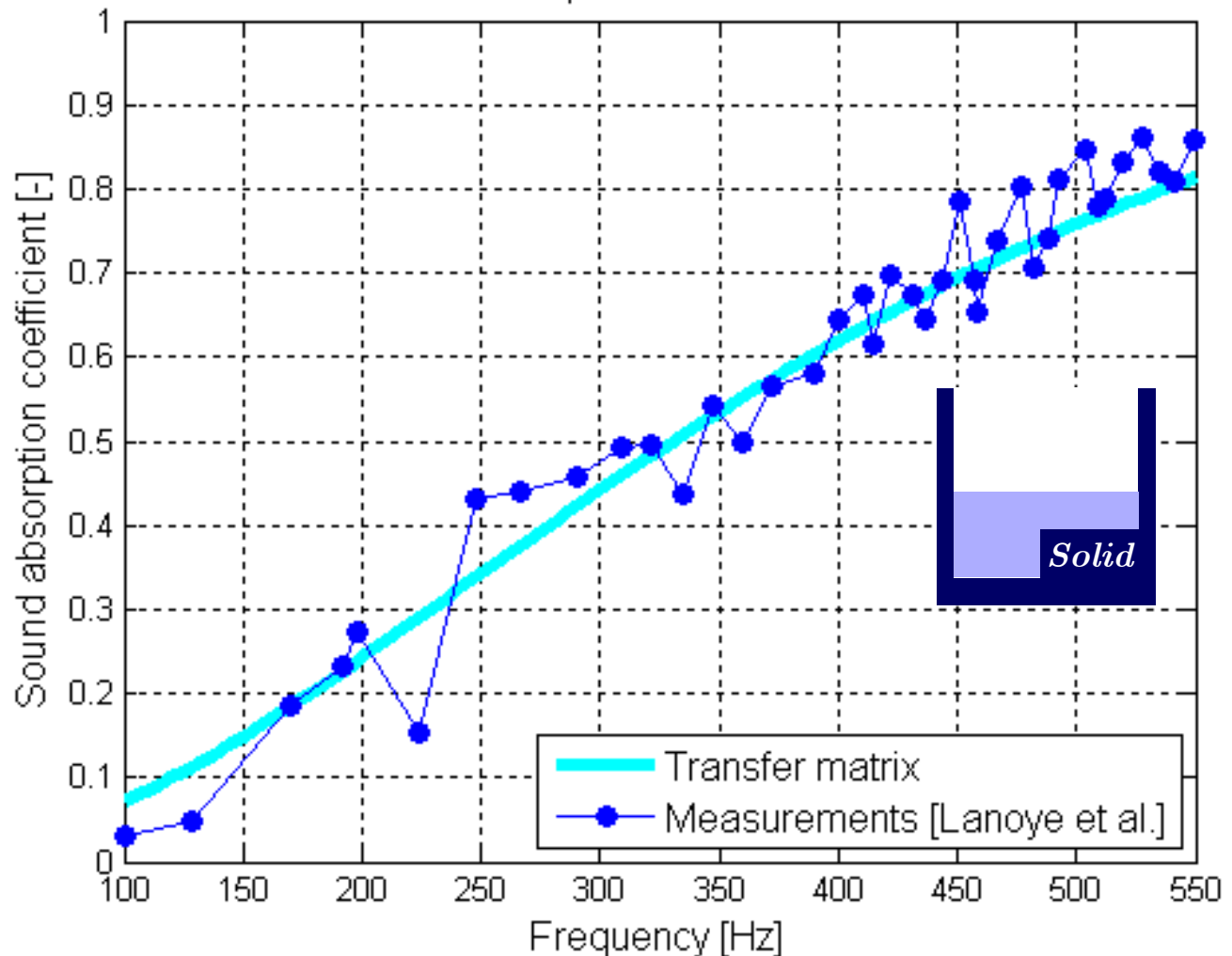
Conclusion

Results on sample 2a



Results on sample 2b

Sample on hard wall



Intro

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Intro

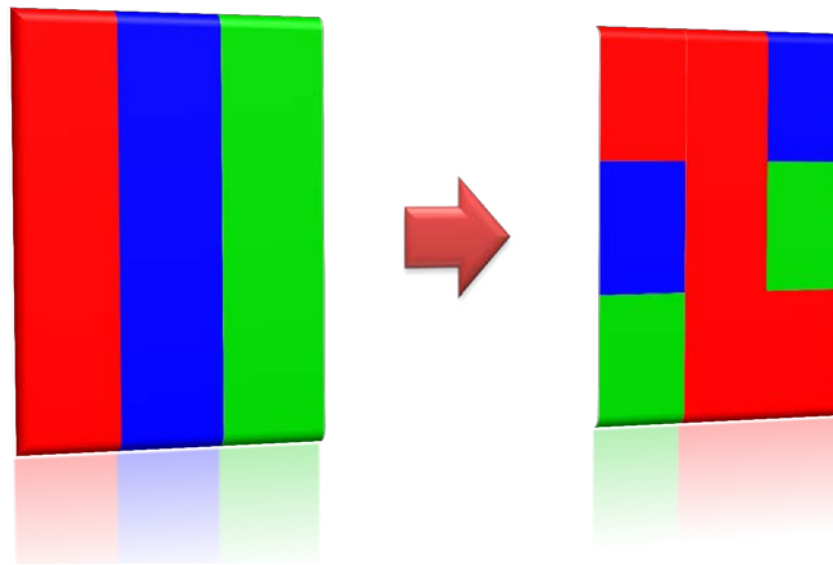
Method

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Conclusion

- Development of **parallel assembling** of Transfer Matrix
- Parallel assembling accounts for parallel patchworks and surface non-homogeneity under **locally reacting** assumption
- Can be assembled **in series** with classical transfer matrix to account for multilayer configurations



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Conclusion

- *Validated for **locally reacting** acoustical mosaic (honeycomb with impervious walls filled with materials)*
- *Validated for non-locally reacting patchworks when **pressure diffusion** between materials is weak (patches with no impervious walls)*

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Future works

- *Test other and more complex configurations*
- *Remove assumptions to enable:*
 - *Excitations different from normal incidence plane wave*
 - *Interactions between cells (multipole matrix) to account for pressure diffusion*
 - *Poroelastic modelling of porous materials*

See recent publications on this topic at:

http://ema.recherche.usherbrooke.ca/?page_id=264