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Abstract

In [7] a Finite Impulse Response (FIR) Youla-Kučera (YK) parametrized adaptive feedforward compensator has been introduced for active vibration control with mechanical coupling. The same configuration can also be used for active noise control with acoustical coupling. The major advantage of this scheme is the possibility to guarantee the stability of the internal positive feedback loop independently of the evolution of the adaptive parameters of the YK filter which will be tuned in order to minimize the residual error. The stability of the global scheme will however depend on the satisfaction of a strictly positive real (SPR) condition. It is shown in this note that a particular choice for the central stabilizing filter used in YK parametrization will drastically simplify the implementation of the algorithm without affecting the performance. This will be illustrated experimentally by the results on an active noise control system with acoustic coupling.

Keywords: active vibration control, active noise control, adaptive feedforward compensation, adaptive control, Youla-Kucera parametrization.

1. Introduction

Adaptive active feedforward compensation of noise and vibrations is widely used when a signal correlated with the disturbance (image of the disturbance) is available. Unfortunately an internal positive coupling between the feedforward compensation and the measurement of the image of the disturbance occurs in most of the applications of adaptive feedforward compensation schemes for active noise and vibration control (ANVC) [8],[3]. This often leads to the instability of the system [5],[4]. Figure 1 gives the basic block diagram of the adaptive feedforward compensation in the presence of the internal positive coupling between the output of the compensator and the measurement of the image of the incoming disturbance. The incoming disturbance propagates through the so called primary path D and its effect is compensated by a secondary source (sound or force) through the secondary path G driven by a feedforward compensator N. The input to the feedforward compensator is the sum of the image of the incoming disturbance and of the internal acoustical or mechanical positive feedback M. This unwanted coupling raises problems in practice and makes the analysis of adaptive algorithms more difficult. The residual noise or acceleration $\chi_0$ is used to emulate the adaptation of the feedforward compensator in order to take into account the unknown characteristics of the disturbance. The dynamics characteristics of the system are almost constant and models of the system are obtained by identification from experimental data.

In [7] a Finite Impulse Response (FIR) Youla-Kučera (YK) parametrized feedforward compensator has been introduced for active vibration control with mechanical coupling. The motivation for the introduction of the FIR Youla-Kučera parametrization was the separation of the stabilization of the “internal” positive feedback loop created by the internal coupling from the adaptation of the FIR YK filter parameters in order to minimize the residual error (acceleration). The same configuration can also be used for active noise control with acoustical coupling. The major advantage of this scheme is the possibility to guarantee the stability of the internal positive feedback loop independently of the evolution of the adaptive parameters of the YK filter. The stability of the global scheme will however depend...
on the satisfaction of a strictly positive real (SPR) condition.

A number of options exist for the design of the central filter which will stabilize the internal positive feedback loop and which will define its closed loop poles. It is shown in this note that a particular choice for the central filter will drastically simplify the implementation of the algorithm without affecting the performance. This will be illustrated experimentally by the results obtained on an active noise control system with acoustic coupling.

While this note tries to be self contained, it uses extensively the notations and the results given in [7].

2. Basic equations and notations

The corresponding block diagram for the adaptive feedforward compensation using FIR YK parametrization of the feedforward compensator is shown in Figure 2. \( \hat{u}(t) \) denotes the effective output provided by the measurement device and which will serve as input to the adaptive feedforward filter \( \hat{N} \). The output of this filter denoted by \( \hat{y}(t) \) is applied to the actuator through an amplifier. The transfer function \( G \) (the secondary path) characterizes the dynamics from the output of the filter \( \hat{N} \) to the residual acceleration or noise measurement (amplifier + actuator + dynamics of the system). The transfer function \( D \) between \( d(t) \) and the measurement of the residual acceleration/noise (in open loop operation) characterizes the primary path. The coupling between the output of the filter and the measurement \( \hat{u}(t) \) through the compensator actuator is denoted by \( M \). As indicated in figure 2 this coupling is a “positive” feedback.

The different blocks of the ANVC system (Figure 2) will be described in this section. The primary (\( D \)), secondary (\( G \)), and reverse (positive coupling) (\( M \)) paths are characterized by the asymptotically stable transfer operators:

\[
X(q^{-1}) = q^{-d_x} B_X(q^{-1}) = q^{-d_x} \frac{b_X q^{-1} + \ldots + b_{n_B} q^{-n_B x}}{1 + a_1 X q^{-1} + \ldots + a_{n_A x} X^{-n_A x}}.
\]

with \( B_X = q^{-d} B_X \) for any \( X \in \{G, M, D\} \). \( \hat{G} = q^{-d_0} \frac{\hat{b}_G}{\hat{a}_G}, \hat{M} = q^{-d_0} \frac{\hat{b}_M}{\hat{a}_M}, \) and \( \hat{D} = q^{-d_0} \frac{\hat{b}_D}{\hat{a}_D} \) denote the identified (estimated) models of \( G, M, \) and \( D \). The system’s order is defined by (the indexes \( G, M, \) and \( D \) have been omitted):

\[
n = \max(n_A, n_B + d).
\]

The optimal feedforward filter (unknown) is defined by

\[
N(q^{-1}) = \frac{R(q^{-1})}{S(q^{-1})}
\]

Using the FIR Youla-Kučera parametrization (Q-parametrization), the filter polynomials \( R(q^{-1}) \) and \( S(q^{-1}) \) get the form

\[
R(q^{-1}) = R_0(q^{-1}) - A_M(q^{-1})Q(q^{-1})
\]

\[
S(q^{-1}) = S_0(q^{-1}) - B_M(q^{-1})Q(q^{-1})
\]

where \( A_M(q^{-1}) \) and \( B_M(q^{-1}) \) denote respectively the denominator and numerator of the positive feedback coupling path. \( S_0(q^{-1}) \) and \( R_0(q^{-1}) \) are the polynomials of the central (stabilizing) filter (\( N_0 = R_0/S_0 \)). The closed loop poles are defined by the roots of:

\[
P(q^{-1}) = A_M(q^{-1})S_0(q^{-1}) - B_M(q^{-1})R_0(q^{-1})
\]

and they remain unchanged independently of the values of the \( Q \) FIR filter. For the purpose of this paper, the optimal polynomial \( Q(q^{-1}) \) (like in [7]) is considered to be of the form

\[
Q(q^{-1}) = q_0 + q_1 q^{-1} + \ldots + q_{n_Q} q^{-n_Q}
\]

The estimated \( Q \) polynomial is denoted by \( \hat{Q}(q^{-1}) \), and when it is a linear filter with constant coefficients or \( \hat{Q}(t, q^{-1}) \) during estimation (adaptation). Similarly the estimated polynomials \( R \) and \( S \) will be denoted \( \hat{R}(q^{-1}) \) and \( \hat{S}(q^{-1}) \).

The \textit{a posteriori} output of the feedforward filter (which is the control signal applied to the secondary path) is denoted by \( \hat{y}(t) \) and is given by:

\[
\hat{y}(t + 1) = -S_0 \hat{y}(t) + R_0 \hat{u}(t + 1) + \hat{Q}(t + 1, q^{-1}) \alpha(t + 1),
\]

where:

\[
\alpha(t + 1) = B_M \hat{y}(t + 1) - A_M \hat{u}(t + 1) = B_M \hat{y}(t) - A_M \hat{u}(t + 1)
\]

is the input to the \( Q \) filter.

3. The adaptation algorithm

The algorithms introduced in [7] can be summarized as follows. The estimated polynomial \( Q \) is given by:

\[
\hat{Q}(q^{-1}) = q_0 + q_1 q^{-1} + \ldots + q_{n_Q} q^{-n_Q}
\]

One defines the optimal parameter vector \( \theta \), the estimated parameter vector \( \hat{\theta} \) and the observation vector \( \phi \) as:

\[
\theta^T = [q_0, q_1, q_2, \ldots, q_{n_Q}]
\]

\[
\hat{\theta}^T = [\hat{q}_0, \hat{q}_1, \hat{q}_2, \ldots, \hat{q}_{n_Q}]
\]

\[
\phi^T(t) = [\alpha(t + 1), \alpha(t), \ldots, \alpha(t - n_Q + 1)]
\]
One defines also a filtered observation vector:

\[ \phi_f(t) = L(q^{-1})\Phi(t) = [\alpha_f(t+1), \alpha_f(t), \ldots, \alpha_f(t-nQ+1)] \]

where

\[ \alpha_f(t+1) = L(q^{-1})\alpha(t+1) \]

and \( L(q^{-1}) \) is an asymptotically stable filter.

The following parameter adaptation algorithm is used:

\[ \hat{\theta}(t+1) = \hat{\theta}(t) + F(t)\Phi(t)v(t+1); \]

\[ v(t+1) = \frac{\nu(t+1)}{1 + \Phi^T(t)F(t)\Phi(t)}; \]

\[ F(t+1) = \frac{1}{\lambda_1(t)} \left[ F(t) - \frac{F(t)\Phi(t)\Phi^T(t)F(t)}{\lambda_1(t)} + \Phi^T(t)F(t)\Phi(t) \right]; \]

\[ 1 \geq \lambda_2(t) > 0; 0 \leq \lambda_2(t) < 2; F(0) > 0 \]

where \( \lambda_1(t) \) and \( \lambda_2(t) \) allow to obtain various time profiles for the matrix adaptation gain \( F(t) \) (see [6]).

The stability condition is

\[ H'(\zeta^{-1}) = H(\zeta^{-1} - \frac{\lambda_2(t)}{2}) \leq \lambda_2 < 2 \]

should be strictly positive real (SPR) \(^2\). For Algorithm II this condition is:

\[ H(q^{-1}) = \frac{A_M(q^{-1})G(q^{-1})}{P(q^{-1})L(q^{-1})}, \Phi = \phi_f \]

and the stability condition is

\[ H'(\zeta^{-1}) = H(\zeta^{-1} - \frac{\lambda_2(t)}{2}), \max [\lambda_2(t)] \leq \lambda_2 < 2 \]

should be SPR and for Algorithm III one has:

\[ H(q^{-1}) = \frac{A_M(q^{-1})G(q^{-1})}{P(q^{-1})L(q^{-1})}, \Phi = \phi_f \]

should be SPR. It is clear from (26) that for any stable \( P \) using Algorithm III one always can satisfy the SPR condition provided that the estimated model is enough good (which is the case in practice). The term "enough good" is interpreted as the condition that the phase difference between the true model \( (A_MG/P) \) and the estimated model \( \hat{M} \hat{G}/\hat{P} \) is between \(-90^0 \) and \(+90^0 \) at all frequencies which assures the satisfaction of the SPR condition for constant scalar adaptation gain. This condition is slightly sharper for the constant trace algorithm. However in many situations, for the Algorithm II the SPR condition will be violated over a certain frequency range and even the averaging arguments [2] may not be applicable for the relaxation of the stability condition. If the objective is only to design a stabilizing central filter (which is generally the case), one can make the choice \( P = \hat{M} \) by choosing \( R_0 = 0; S_0 = 1 (\hat{P} = \hat{M}) \). With this choice, both algorithms become the same and the stability condition will be:

\[ H(q^{-1}) = \frac{G}{\hat{P}}, \max [\lambda_2(t)] \leq \lambda_2 < 2 \]

should be SPR. This condition will be always satisfied provided that one has a good estimation of the model of the secondary path \( \hat{G} \). This leads to a drastic simplification of the central stabilizing filter which using pole placement will have at least \( n_{A_M} + n_{R_0} + d_G \) parameters\(^3\). There is also a drastic simplification in the filter \( L \) by suppressing the term \( \hat{M} \hat{P} / \hat{G} \) which has \( 2n_{A_M} + 1 + n_{B_M} + d_M \) parameters. Since the orders of the models in active vibration and noise control are high, the reduction of the number of parameters is significant.

4. Experimental results

The view of the active noise control test-bench used for experiments is shown in Fig. 3 and its detailed scheme is given in Fig. 4.

\(^2\)For the constant scalar adaptation gain \( \lambda_2 = 0 \) and this is valid for all the subsequent stability conditions

\(^3\)This assumes that the compensation path is asymptotically stable, which indeed is the case in practice

\(^4\)One has to associate a multiplication and an addition for each parameter
The speaker used as the source of disturbances is labeled as 1, while the control speaker is marked as 2. At pipe’s open end, the microphone that measures the system’s output (residual noise $\chi(t)$) is denoted as 3. $s(t)$ is the disturbance. Inside the pipe, close to the source of disturbances, the second microphone, labeled as 4, measures the perturbation’s image, denoted as $u(t)$. $y(t)$ is the control signal. The transfer function between the disturbance’s speaker and the microphone (1→3) is called Global Primary Path, while the transfer function between the control speaker and the microphone (2→3) is denoted Secondary Path. The transfer function between microphones (4→3) is called Primary Path. The internal coupling found between (2→4) is denoted Reverse Path. These marked paths have a double differentiator behavior, since as input we have the voice coil displacement and as output the air acoustical pressure. Speakers and microphones are connected to an xPC Target computer with Simulink Real-time®. A second computer is used for development and operation with Matlab. Taking into account that disturbances up to 400 Hz may need to be attenuated, a sampling frequency $f_s = 2500$ Hz has been chosen ($T_s = 0.0004$ sec). The orders of the identified models are summarized in Table 1. The frequency characteristics of the identified models for the global primary$^5$, secondary and reverse paths are shown in Fig. 5. These characteristics present multiple resonances (low damped complex poles)$^6$ and anti-resonances (low damped complex zeros). One can see that the secondary path has a high gain between 70 to 270 Hz which means that disturbances can be efficiently attenuated in this zone. It is also clear that the reverse path has a significant gain on a large frequency range (up to 400 Hz) so its effect can not be neglected.

A broad band disturbance located between 70 and 270 Hz is considered as an unknown disturbance. Table 2 gives a comparison of the various adaptation algorithms in terms of global attenuation for a FIRYK adaptive compensator with 60 adaptive parameters. Results are given for scalar and matrix adaptation gains. In this case the poles of the internal loop are different from the poles of the $A_M$ (the central controller has 59 parameters resulting from the solution of a Bezout equation). It was observed that the Alg.II is unstable and this can be understood when looking to the phase plot of the estimated $A_M$ given in Figure 6. It can be observed that $A_M$ is not positive real in a

<table>
<thead>
<tr>
<th>Adaptation algorithm</th>
<th>Attenuation [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix (Alg. II/Alg. III)</td>
<td>27.19</td>
</tr>
<tr>
<td>Scalar (Alg. II/Alg. III)</td>
<td>27.17</td>
</tr>
</tbody>
</table>

Table 3: Experimental results for simplified FIRYK 60/0 adaptive compensators (70-270 Hz broad-band disturbance, 180 s experiments).
large frequency range from 110 Hz to 760 Hz \(^7\) and one absolutely needs to use the Algorithm III. Table 3 gives the comparison of the various algorithms as in Table 2 but for the case \(P = AM\) with \(R_0 = 0, S_0 = 1 (\hat{P} = \hat{AM})\). As expected both algorithms will give the same results and the performance is similar to that of the Alg. III in Table 2. In this case 59 parameters in the central filter and 78 parameters in the filter \(L\) have been suppressed. Figure 7 gives the evolution of the residual noise in the case \(P \neq AM\) for a matrix adaptation gain and Figure 8 gives the evolution of the residual noise for the case \(P = AM\) and a matrix adaptation gain. The behaviour is similar for the two cases. Figure 9 gives the power spectral density for the two cases in steady state. In the region of interest (70-270 Hz) they are extremely close.

5. Conclusion

A drastic simplification of the FIR Youla Kucera adaptive feedforward compensator for active control with internal coupling is obtained by choosing the poles of the internal closed loop equal to the poles of the reverse path. The performance obtained on a duct silencer are comparable with those for other choices of the poles of the internal closed loop but with a significant reduction of the algorithm complexity and a simplification of the design of the central controller. This option should also lead to a relaxation of the SPR conditions for combined feedforward/feedback active control schemes [1].

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\(^7\)Averaging can not be used since the region of non positive realness is much larger than the region where \(\hat{AM}\) is SPR even within the range 70 - 270 Hz.

References