



**HAL**  
open science

## Self-consistent quasi-static radial transport during the substorm growth phase

O. Le Contel, R. Pellat, A. Roux

► **To cite this version:**

O. Le Contel, R. Pellat, A. Roux. Self-consistent quasi-static radial transport during the substorm growth phase. *Journal of Geophysical Research Space Physics*, 2000, 105 (A6), pp.12929-12944. 10.1029/1999JA900498 . hal-02550837

**HAL Id: hal-02550837**

**<https://hal.science/hal-02550837>**

Submitted on 11 Jan 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Self-consistent quasi-static radial transport during the substorm growth phase

O. Le Contel

Centre d'Etude des Environnements Terrestre et Planétaires, Vélizy, France

R. Pellat

Centre de Physique Théorique, Ecole Polytechnique, Palaiseau, France

A. Roux

Centre d'Etude des Environnements Terrestre et Planétaires, Vélizy, France

**Abstract.** We develop a self-consistent description of the slowly changing magnetic configuration of the near-Earth plasma sheet (NEPS) during substorm growth phase. This new approach is valid for quasi-static fluctuations  $\omega < k_{\parallel} v_A$  ( $v_A$  being the Alfvén velocity), with characteristic frequency lower than the bounce frequencies of electrons and ions ( $\omega < \omega_{bi}, \omega_{be}$ ), and for spatial scales larger than the ion Larmor radius. The basic equations are obtained from a linearization of the cyclotron and bounce-averaged Vlasov equation, together with Maxwell equations. The Vlasov-Maxwell system of equations is solved for the quasi-dipolar NEPS region. Using a 2-D dipole for the equilibrium, we calculate analytically the perturbed components of the electromagnetic field as a function of an external forcing current. The quasi-neutrality condition (QNC) is solved via an expansion in the small parameter  $T_e/T_i$  ( $T_e/T_i$  is the ratio between the electronic and ionic temperatures). To the lowest order in  $T_e/T_i$ , we find that the enforcement of QNC implies the presence of a global electrostatic potential which is constant for a given magnetic field line but varies across the magnetic field. The corresponding electric field shields the effect of the inductive component of the electric field, thereby producing a partial reduction of the motion that would correspond to the inductive electric field. Furthermore, we show that enforcing the QNC implies a field-aligned potential drop which is computed to the next order in  $T_e/T_i$  in a companion paper [Le Contel *et al.*, this issue]. In the present paper, we show that the direction of the azimuthal electric field varies along the field line, thus the equatorial electric field cannot be mapped onto the ionosphere. Furthermore during the growth phase, the (total) azimuthal electric field is directed eastward, close to the equator, and westward, off-equator. Thus large equatorial pitch angle particles drift tailward, whereas small pitch angle particles drift earthward.

## 1. Introduction

The process(es) by which the plasma is transported in the Earth's magnetosphere is a critical issue, and there is no consensus yet about its exact nature. Earlier models are generally based on simple assumptions: a static magnetospheric electric field (i.e., an electric field derived from a scalar potential), driving a steady earthward "convective" motion (e.g., Cowley and Ashour-Abdalla [1975] or Huang and Birmingham [1994]). Flow velocities deduced from the measurements of coherent

and incoherent scatter radar tend to support the view that there is a regular earthward convective motion notably prior to substorms [e.g., *Forme et al.*, 1998]. In the case of incoherent scatter radar, however, the Earth rotation is used to build maps of the flow velocity in the ionosphere, thus a steady state (over one Earth rotation) is implicitly assumed. However, electric field and/or flow velocity measurements, carried out in the magnetosphere on board several spacecraft, are not easy to reconcile with the simple view of a steady convection. Huang and Frank [1986] carried out a statistical survey of plasma flows, measured on ISEE 1, inside the plasma sheet. In their survey they excluded flow velocities above 150 km/s. Their results show that the average flow is too small to account for the rate

Copyright 2000 by the American Geophysical Union.

Paper number 1999JA900498.  
0148-0227/00/1999JA900498\$09.00

expected by steady convection models. *Baumjohann et al.* [1989] conducted a survey of slow and fast flows from AMPTE/IRM data. While slow flows were observed most of the time, in agreement with *Huang and Frank* [1986], *Baumjohann et al.* [1989], taking advantage of a better time resolution, gave evidence for transient bursts of fast flows, with speeds above 300 km/s, generally directed earthwards (at least inside of  $19 R_E$ ,  $R_E$  being the Earth radius). Conversely, the slow flows have no preferred direction. *Angelopoulos et al.* [1992] further analyzed the fast flow bursts and defined bursty bulk flows (BBFs); these authors suggest that most of the radial transport of the flux could be effected through these short lasting but intense flow bursts. These measurements indicate that there is not a unique regime. These results and other similar results, not described here, have led *Kennel* [1995], to introduce the possibility of a bimodal plasma sheet flow, to stress the difference between the morphology of fast laminar flows and slow turbulent flows. The standard convection models, based on a constant potential electric field, do not describe these complex situations characterized either by a high temporal variability of the fast transient BBFs, or by slow flows with no obvious preferred direction. Thus “one wonders if steady uniform convection has ever been found” [*Kennel*, 1995, p. 22].

In the present paper we do not assume any static convection electric field. We adopt a different view point and investigate the transport under the influence of an externally applied electromagnetic perturbation that mimics the variation induced by the solar wind. What is the difference between a steady convection and a time-dependent transport? In a steady state magnetosphere the electric field can only be electrostatic. Then, the assumption of the absence of a parallel electric field  $E_{\parallel}$  is equivalent to consider the magnetic field lines as equipotentials ( $E_{\parallel} = -\partial\Phi/\partial l = 0 \iff$  the potential electrostatic  $\Phi$  is independent of  $l$  the distance along the magnetic field line). Conversely, in a non-stationary approach,  $E_{\parallel} = -\partial\Phi/\partial l - \partial/\partial t A_{\parallel} = 0 \iff \Phi + \partial/\partial t \int dl A_{\parallel} = \text{const}$ , where  $\delta A_{\parallel}$  is the parallel component of the vector potential. Therefore the absence of parallel electric field does not imply the equipotentiality of the magnetic field lines but only that the variation along the field line of the electrostatic component of the electric field cancels the inductive component. Thus, for a time-dependent perturbation the mapping between the equatorial and the ionospheric electric fields is not granted. Here we investigate the relation between a time-dependent transport and the formation of thin current sheets (TCS) during the substorm growth phase. For the sake of simplicity, in the rest of the paper, we will call “convection” a steady motion driven by a constant (e.g., dawn to dusk) electric field, derived from a constant scalar potential. The term “transport” will be used to describe the effect of time varying perturbations, associated for instance with a growing current in the tail.

The theoretical basis of standard convection models also deserves some discussion. Most of the models rely upon the assumption that convecting particles are moving in a given magnetic field; in other words, no attempt is made to take into account the effect the particles have on the fields; the approach is not self-consistent, which introduces serious limitations on the significance of the results. MHD is a priori more appealing, because it is self-consistent, and provides a simple description of plasma dynamics [*Schindler and Birn*, 1982; *Erickson*, 1992]. The validity of the MHD approach, however, is also subject to restrictions. Indeed, MHD approach is only valid for perturbations whose the frequency is greater than  $k_{\parallel}v_e$  ( $k_{\parallel}$  being the parallel component of the wave vector and  $v_e$  the thermal velocity of the electrons). In a magnetic-mirror geometry as the NEPS, particles are trapped between mirror turning points. Therefore the validity condition of MHD writes  $\omega > \omega_{be} \simeq v_e/L_{\parallel}$  ( $L_{\parallel}$  being the magnetic field length) or equivalently the characteristic timescale  $\tau < \tau_{be}$ , the bounce period of electrons. In the NEPS, at  $7 R_E$ ,  $\tau_b$  is typically 1 s for energetic electrons (1 keV). Thus MHD is not a valid approximation for long-period fluctuations (see, for instance, *Rosenbluth and Varma* [1967], *Rutherford and Frieman* [1968], *Antonson and Lane* [1980], and *Hurricane et al.* [1994]). The bounce resonance is only one of the nonlocal processes that limit the validity of MHD; the magnetic field gradient, the pressure gradient, also introduce nonlocal effects, as will be discussed in the course of the paper. See also discussions on the importance of these resonances by *Chen and Hasegawa* [1988, 1991], *Cheng* [1982], and *Cheng and Johnson* [1999].

Plasma transport can also be achieved via low-frequency hydromagnetic waves, as discussed by *Johnson and Cheng* [1997] and by *Chen* [1999]. These works are based on a quasi-linear formalism applied to the gyrokinetic equations; hence the rate of transport is proportional to the square of the amplitude of these hydromagnetic waves. Observations carried out in the near-Earth plasma sheet (NEPS), for instance, by *GEOS-2* [*Roux et al.*, 1991], show that during the growth phase, the amplitude of hydromagnetic waves is very weak. Thus wave-induced transport is unlikely to play an important role during the growth phase. While periodic oscillations are negligible, one regularly observes a slow change in the magnetic configuration that will be modeled as a quasi-static perturbation. Therefore, in the present paper we describe a linear self-consistent approach to the transport of the plasma in the NEPS, in response to quasi-static variations of the magnetic field. We use an approach developed for electromagnetic perturbations with a frequency lower than the bounce frequency of electrons and ions ( $\omega < \omega_{bi}, \omega_{be}$ ) and for spatial scales larger than the ion Larmor radius ( $k_{\perp}\rho < 1$ ), but we neglect Alfvén waves, assuming  $\omega < k_{\parallel}v_A$ . The present approach is based on the linearized response of the plasma to low frequency elec-

tromagnetic perturbation ( $\omega < \omega_b$ ) obtained by Pellat from the Vlasov equation by assuming conservation of the first adiabatic invariant [Pellat, 1990]. The response of the plasma is completed by Maxwell equations. For long perpendicular wavelength electromagnetic perturbations ( $k_{\perp} \lambda_D \ll 1$ , where  $\lambda_D = \sqrt{\epsilon_0 k_B T / (n_0 q^2)}$  is the Debye length,  $\epsilon_0$  is the vacuum permittivity,  $k_B$  the Boltzmann constant,  $T$  the temperature,  $n_0$  the density, and  $q$  the charge of particles), the Gauss's equation reduces to the QNC. Yet, owing to the low frequency of the perturbations, we need to take into account the existence of nonlocal terms. For instance, in the electrostatic case, for a multipole, the QNC implies the existence of an electrostatic perturbed potential  $\Phi_0$  constant along the field line [Pellat *et al.*, 1994]. This result has been extended to electromagnetic perturbations by Hurricane *et al.* [1995b] for  $\omega > \omega_*$ ,  $\bar{\omega}_d$  ( $\omega_*$  being the diamagnetic drift frequency and  $\bar{\omega}_d$  being the bounce-averaged magnetic drift frequency), and applied to the magnetotail. In section 2.2 we solve the QNC to the lowest order in  $T_e/T_i < 1$  and give a generalization of Hurricane's result valid for perturbations with arbitrary frequencies.

Finally, the system is completed by the parallel and perpendicular projections of the Ampère's law. Unlike substorm injection which is known to be a sudden process ( $\omega \geq \omega_{bi}$ ) with a small spatial scale ( $k_y \rightarrow \infty$ ), the buildup of a tail-like configuration is a slow process ( $\simeq 30$  min) affecting a large fraction of the tail (small  $k_y$ ). Thus the applied electromagnetic perturbation is not considered as being the consequence of a local internal instability (as it is probably the case for breakup). The change from a dipole to a tail-like configuration, instead, is considered as the result of the response of the magnetotail to a quasi-static forcing caused by variations in the solar wind [e.g., Jacquy, 1996]. The full treatment of this problem is very difficult since it would require a full description of the forcing caused by the solar wind, taking into account the boundary conditions imposed at the magnetopause. Moreover, the way the solar wind drives the stretching of the magnetic field lines is still not completely understood. To simplify, we assume that the change of the dipolar field close to the Earth is due to an increase of the westward current farther in the tail [e.g., Jacquy, 1996], neglecting the local electrical currents (low  $\beta$  assumption,  $\beta$  being the ratio between the kinetic pressure and the magnetic pressure). Thus, for the equilibrium, Ampère's law gives  $\nabla \times \vec{B} = \vec{0}$ . Perturbing the equilibrium with an external current located far from the dipolar region, we solve the linearized Ampère's law gives  $\nabla \times \delta \vec{B} = \mu_0 \delta \vec{j}_{ext}$  to obtain the perturbed components of the magnetic field.

The above few lines suggest the following questions, to be discussed in the course of the paper: (1) What are the consequences of enforcing the QNC in a time-dependent situation? (2) What is the role of the time varying electric fields (associated with electromagnetic

perturbations) on the transport of the plasma inside the plasma sheet? (3) Can we map the electric field from the equatorial magnetosphere, down to the ionosphere during the growth phase?

The linearized response of the plasma is described in subsection 2.1 and the QNC is solved in subsection 2.2. In subsection 2.3 we build a Green function and solve the linearized Ampère's equation to obtain the perturbed magnetic field from the external current. In order to allow an analytical approach, a simple magnetic field model, a two-dimensional (2-D) dipole, is used. This model is also presented in subsection 2.3. In section 3 we give the spatial profile of the azimuthal electric field along the field line and show the implications on the transport of the plasma across magnetic field lines during the growth phase. The solution of the QNC, to the first order in  $(T_e/T_i)$ , will be described in a companion paper [Le Contel *et al.*, this issue], together with its consequences, namely the development of a finite parallel electric field.

## 2. Linearized Vlasov-Maxwell System of Equations

### 2.1. Solution of the Bounce-Averaged Vlasov Equation

Assuming that the electromagnetic perturbation is periodic in time ( $t$ ) and in space (across the magnetic field) we take the perturbing electrostatic ( $\delta\Phi$ ) and magnetic vector ( $\delta\vec{A}$ ) potentials as

$$\begin{aligned} \delta\Phi(\vec{r}, t), \delta\vec{A}(\vec{r}, t) \\ = \widehat{\delta\Phi}(\vec{k}_{\perp}, \omega, l), \widehat{\delta\vec{A}}(\vec{k}_{\perp}, \omega, l) \exp \left[ i(\vec{k}_{\perp} \cdot \vec{r}_{\perp} + \omega t) \right], \end{aligned}$$

where  $\vec{r}$  ( $\vec{k}_{\perp}$ ) is the position vector (the wave vector) and  $\perp$  denotes the component perpendicular to the magnetic field. For the sake of simplicity we omit the hat symbol and the exponential factor in the following formulas, then the linearized response  $\delta f$  is given by

$$\begin{aligned} \delta f = q \frac{\partial f_0}{\partial E} \left[ \delta\Phi - u_y \delta A_y \right. \\ \left. + \left( 1 + \frac{\omega_*}{\omega} \right) \lambda e^{-iS} - \left( 1 + \frac{\omega_*}{\omega} \right) g \right], \quad (1) \end{aligned}$$

where  $f_0(E, p_y)$  is the equilibrium distribution function ( $E$  is the particle energy and  $p_y$  the canonical momentum),  $u_y$  is the diamagnetic drift velocity,  $\omega_* = k_y u_y$  is the diamagnetic drift frequency,  $k_y$  is the wavenumber in the  $y$  direction (azimuthal). We work in local field-aligned coordinates defined by the triad of unit vectors:

$$\vec{e}_{\chi} = \frac{\vec{B}}{B}, \quad \vec{e}_{\psi} = \frac{\nabla \psi}{|\nabla \psi|}, \quad \vec{e}_{\eta} = \vec{e}_{\chi} \times \vec{e}_{\psi}. \quad (2)$$

In this frame, the velocity becomes

$$\vec{v} = |v_{\perp}| (\vec{e}_{\psi} \cos \xi + \vec{e}_{\eta} \sin \xi) + v_{\parallel} \vec{e}_{\chi}. \quad (3)$$

To obtain the linear response (1), a change of variables  $(v_\psi, v_y, v_\parallel) \rightarrow (E, \mu, \xi)$  has been made, where  $E = \frac{1}{2}mv_\parallel^2 + \mu B$  is the kinetic energy,  $\mu = \frac{1}{2}mv_\perp^2/B$  is the magnetic moment, and  $\xi$  is the gyrophase angle. The elementary volume in velocity space becomes  $d^3v = \sum_{\sigma=-1,+1} BdEd\mu d\xi/(m^2|v_\parallel|)$ , where  $\sigma = \text{sign}(v_\parallel)$  (for more details, see *Rutherford and Frieman* [1968], *Hurricane* [1994], and *Hurricane et al.* [1995a, b]). In (1) the function  $g$  contains the nonlocal wave-particle interaction. To first order in  $\omega/\omega_b$ , the function  $g$  becomes

$$g = e^{-iS} \left( \frac{\omega}{\omega + \bar{\omega}_d} \bar{H} + \sigma i\omega \int \frac{dl}{|v_\parallel|} \left[ H - \frac{\omega + \omega_d}{\omega + \bar{\omega}_d} \bar{H} \right] \right), \quad (4)$$

$S = k_\perp v_\perp \sin(\alpha_k - \xi)/\Omega$ ,  $\alpha_k = \text{Arctan}(k_\psi/k_y)$ ,  $\Omega$  is the cyclotron frequency,  $\omega_d = k_y v_d$  is the gradient-curvature drift frequency, the upper bar denotes bounce averaging and  $H$  is given by

$$H = J_0 (\delta\Phi - v_d \delta A_y) + \frac{(\omega + \omega_d)\lambda}{\omega} + iJ_1 v_\perp \left( \frac{k_\psi}{k_\perp} \delta A_y - \frac{k_y}{k_\perp} \delta A_\psi \right), \quad (5)$$

where  $\lambda = i\omega \int dl' J_0 \delta A_\parallel$  and  $J_n$  are Bessel functions of argument  $k_\perp |v_\perp|/\Omega$ . In the next section we substitute the linearized solution of the Vlasov equation into the QNC. The bounce-averaged linear solution of Vlasov equation obtained here is similar to those developed and used by different authors [e.g., *Antonsen and Lane*, 1980; *Cheng*, 1982; *Chen and Hasegawa*, 1991].

## 2.2. Quasi-neutrality Equation

In this subsection, we solve the QNC via an expansion in the small parameter  $T_e/T_i$ . The validity of this expansion is suggested by several observations indicating that this ratio is small in the magnetotail. For instance, *Lui et al.* [1992] presented a statistical study of current disruptions from AMPTE/CCE when the spacecraft was in the near-Earth current sheet. They showed that the electron to proton temperature ratio is in the range of 0.11 to 0.57. They pointed out that these values are higher than those reported by *Baumjohann et al.* [1989] based on IRM data. Indeed, *Baumjohann et al.* [1989] obtained average plasma properties, notably an electron to proton temperature ratio in the range: 0.09-0.18. These authors also noticed that this ratio is nearly the same as the one found by *Slavin et al.* [1965] at distances of  $|X|=30-60 R_E$ . More recently, during a dusk-dawn crossing of the near-Earth tail by Geotail, during a relatively quiet period, a ratio around 0.2 was measured [*Frank et al.*, 1996]. It is therefore possible to consider  $T_e/T_i$  as a small parameter over a wide range of radial distances from the Earth, and for different levels of activity.

From the linear response of the plasma (equations (1) to (5)) and assuming that  $f_0$  is a Maxwellian distribution function ( $f_0 = n_0 [m/(2\pi T)]^{3/2} \exp(-(E/T)$ ), the QNC:  $\sum_{j=i,e} q_j \int d^3v \delta f_j \simeq 0$  can be written

$$\sum_{j=i,e} \left( \frac{q_j}{m_j} \right)^2 \frac{1}{T_j} \int \frac{4\pi BdEd\mu}{|v_\parallel|} f_{0j} \left[ \delta\Phi + \left( \frac{\omega + \omega_{*j}}{\omega} \lambda - \frac{\omega + \omega_{*j}}{\omega + \bar{\omega}_d} \bar{H} \right) J_0 \right] \simeq 0, \quad (6)$$

where we have performed the gyrophase integration and summed over streaming and antistreaming velocities. This latter operation cancels out the part of  $\delta f$  that is an odd function of  $\sigma$ . The above relation was derived earlier by *Hurricane et al.* [1995b], where more details are given about the derivation. The gauge  $\delta A_y = 0$  has been chosen. Since we are interested in large-scale perturbations (the growth phase), the usual wavelength ordering  $k_\perp \rho_j \ll 1$  is made. In this limit, the Bessel functions become  $J_0 \simeq 1$  and  $J_1 \simeq k_\perp |v_\perp|/2\Omega$ , and the expression for  $H$  simplifies; we get

$$H = \delta\Phi + \lambda + \Xi, \quad (7)$$

where  $\Xi = \omega_d \lambda/\omega - i\mu k_y \delta A_\psi/q$ . Then, after some algebraic manipulations, the QNC can be written as

$$\begin{aligned} & \sum_{j=i,e} \left( \frac{q_j}{m_j} \right)^2 \frac{1}{T_j} \int \frac{4\pi BdEd\mu}{|v_\parallel|} f_{0j} \left[ (\delta\Phi - \bar{\delta\Phi}) + (\lambda - \bar{\lambda}) \right] \\ &= - \sum_{j=i,e} \left( \frac{q_j}{m_j} \right)^2 \frac{1}{T_j} \int \frac{4\pi BdEd\mu}{|v_\parallel|} f_{0j} \\ & \cdot \left[ \frac{\bar{\omega}_{dj} - \omega_{*j}}{\omega + \bar{\omega}_{dj}} (\bar{\delta\Phi} + \bar{\lambda}) - \frac{\omega + \omega_{*j}}{\omega + \bar{\omega}_{dj}} (\Xi_j) \right], \quad (8) \end{aligned}$$

where the terms  $\omega_{*j}\lambda$  cancel between electrons and ions, because  $u_{yi}/T_i + u_{ye}/T_e = 0$ , [*Hurricane et al.*, 1995b]. Note that the diamagnetic drift frequency and the purely magnetic drift frequency of electrons can easily be related to the corresponding terms for ions:  $\bar{\omega}_{de}^{th}(\alpha) = -T_e/T_i \bar{\omega}_{di}^{th}(\alpha)$  and  $\omega_{*e} = -T_e/T_i \omega_{*i}$  (where  $th$  means thermal quantities and  $\alpha$  denotes the particle pitch angle). The QNC becomes

$$\begin{aligned} & \int \frac{4\pi BdEd\mu}{m_e^2 |v_\parallel|} f_{0e} \left[ (\delta\Phi - \bar{\delta\Phi}) + (\lambda - \bar{\lambda}) \right. \\ & \quad \left. + \frac{T_e}{T_i} ((\delta\Phi - \bar{\delta\Phi}) + (\lambda - \bar{\lambda})) \right] \\ &= \frac{T_e}{T_i} \int \frac{4\pi BdEd\mu}{m_i^2 |v_\parallel|} f_{0i} \left[ \frac{\bar{\omega}_{di} (\bar{\omega}_{di} - \omega_{*i})}{\omega (\omega + \bar{\omega}_{di})} (\bar{\delta\Phi} + \bar{\lambda}) \right. \\ & \quad \left. + \frac{\omega_{*i} - \bar{\omega}_{di}}{\omega + \bar{\omega}_{di}} (\Xi_i) \right]. \quad (9) \end{aligned}$$

Since the right-hand side (RHS) term of (9) is proportional to  $(T_e/T_i)$ , we get to the lowest order in  $(T_e/T_i)$ :

$$\int \frac{4\pi B dE d\mu}{m_e^2 |v_{\parallel}|} f_{0e} \left[ (\delta\Phi - \bar{\delta\Phi}) + (\lambda - \bar{\lambda}) \right] = 0 + O(T_e/T_i). \quad (10)$$

A trivial solution of (10) is

$$\delta\Phi + \lambda = \Phi_0(\psi, y) + O(T_e/T_i), \quad (11)$$

where  $\Phi_0$  is constant for a given magnetic field line. This constant component of the perturbed electrostatic potential is always taken to be equal to zero ( $\delta\Phi + \lambda = 0$ ) in studies based on MHD. An external electrostatic field, modeling the convection, is often added to the inductive part of the electric field in order to better fit the data [e.g., *Sauvaud et al.*, 1996]. In the present paper, we show that the quasi-neutrality over the volume of the flux tube implies that  $\Phi_0$  is different from zero. We compute  $\Phi_0$  in a self-consistent manner, as a function of the electromagnetic perturbation defined by  $\lambda$  and  $\delta B_{\parallel}$ . Integrating the QNC (9) over the volume of the flux tube, we find

$$\int \frac{dl}{B} \left[ \int \frac{4\pi B dE d\mu}{m_i^2 |v_{\parallel}|} f_{0i} \left[ \frac{\bar{\omega}_{di}(\bar{\omega}_{di} - \omega_{*i})}{\omega(\omega + \bar{\omega}_{di})} (\Phi_0) + \frac{\omega_{*i} - \bar{\omega}_{di}}{\omega + \bar{\omega}_{di}} (\bar{\Xi}_i) \right] \right] = 0, \quad (12)$$

where the left-hand side (LHS) term of (9) has vanished thanks to the identity  $\int dl/B \int d^3v (X - \bar{X}) = 0$ , valid for any function  $X(E, \mu, l)$ . Finally,  $\Phi_0$  writes

$$\Phi_0 = \frac{\int \frac{dl}{B} \left[ \int \frac{4\pi B dE d\mu}{m_i^2 |v_{\parallel}|} f_{0i} \left[ \frac{\omega_{*i} - \bar{\omega}_{di}}{\omega + \bar{\omega}_{di}} (\bar{\Xi}_i) \right] \right]}{\int \frac{dl}{B} \left[ \int \frac{4\pi B dE d\mu}{m_i^2 |v_{\parallel}|} f_{0i} \left[ \frac{\bar{\omega}_{di}(\omega_{*i} - \bar{\omega}_{di})}{\omega(\omega + \bar{\omega}_{di})} \right] \right]}. \quad (13)$$

Now, we can calculate the self-consistent perturbed electric field. Taking into account the implications of the QNC (11) to the lowest order in  $T_e/T_i$ , the perturbed electric field  $\delta\vec{E} = -\vec{\nabla}(\delta\Phi) - \partial/\partial t(\delta\vec{A})$ , in the  $y$  direction, becomes (remembering that  $\delta A_y = 0$ ):

$$\delta E_y = -ik_y(\delta\Phi) = -ik_y(\Phi_0 - \lambda). \quad (14)$$

Thus the complete perpendicular electric field associated with the perturbation is the sum of an inductive component ( $\lambda$ ) plus an electrostatic component ( $\Phi_0$ ) determined from the QNC. This electric field will produce a transport of the plasma. Notice that this transport is different from a steady convection; it is associated with an electromagnetic perturbation (see discussion in introduction). The electrostatic component, associated with  $\Phi_0$ , tends to reduce the effect of the inductive component of the electric field  $\lambda$ , thereby producing a partial shielding of the motion that would correspond to the inductive electric field (if it was not shielded). This effect can explain why large bulk flows are not detected in the NEPS during the growth phase.

The expression  $\lambda = i\omega \int^l dl' \delta A_{\parallel}$  shows that the partial derivative of  $\lambda$  with respect to  $l$  is equal to the inductive component of the parallel electric field ( $\partial\lambda/\partial l = \partial\delta A_{\parallel}/\partial t$ ). Thus locally and in the limit  $T_e < T_i$ , (11) implies that the inductive component of the parallel electric field  $\partial\delta A_{\parallel}/\partial t$  is balanced by the parallel gradient of the perturbed electrostatic potential  $\partial\delta\Phi/\partial l$ . Hence (11) is equivalent to the usual MHD approximation, where one assumes the absence of a parallel electric field ( $E_{\parallel} = -\partial/\partial l(\delta\Phi + \lambda) = 0$ ). In the present study, the absence of a parallel electric field is not an assumption but an (approximate) result, obtained by solving the QNC in the limit  $T_e < T_i$ . We show, however, in a companion paper [*Le Contel et al.*, this issue], that the solution of the QNC to the first order in  $(T_e/T_i)$  allows us to compute a finite parallel electric field.

Studying low-frequency perturbations, *Chen and Hasegawa* [1991] considered a magnetospheric plasma consisting of two populations: a core (100 eV) and an energetic component (10 keV). In their work the core population is denser than the energetic population and therefore plays a key role in the QNC. For this population,  $\omega_{be} > \omega > \omega_{bi}$ . They obtained  $\delta\Phi + \lambda = 0$ , i.e., no parallel electric field ( $\delta E_{\parallel} = -\partial_l(\delta\Phi + \lambda) = 0$ ). In the present work carried out for  $\omega_{be}, \omega_{bi} > \omega$ , we also find  $\delta E_{\parallel} \simeq 0$  in the limit  $T_e < T_i$  since we have obtained  $\delta\Phi + \lambda = \Phi_0(\psi, y)$  where  $\Phi_0$  is constant along a field line. However, this constant potential  $\Phi_0$  modifies the perpendicular transport of the plasma as already mentioned.

Finally, one should notice that the perpendicular electric field (14) varies along the field line even when there is no parallel electric field. Thus, in the quasi-static limit, the absence of a parallel electric field does not imply that the equatorial perpendicular electric field can be mapped onto the ionospheric electric field. This is true only in the purely electrostatic case (steady convection); an assumption which is certainly not valid during the growth phase. In the next subsection, we solve the Ampère's law to determine the magnetic field perturbation.

### 2.3. Ampère's Law

Close to the Earth, we can neglect the local electrical currents which corresponds to assuming  $\beta < 1$ . Thus we can approximate the field by a dipole. To allow us to carry out analytical calculations, we use a two-dimensional (2-D) dipole [*Huang and Birmingham*, 1994] to describe the equilibrium magnetic field. Using cylindrical coordinates  $(r, \theta, y)$  where  $\theta$  is the colatitude, the 2-D magnetic field model is defined by

$$\vec{B} = -\frac{\hat{D}}{r^2} (\cos\theta \vec{u}_r + \sin\theta \vec{u}_{\theta}), \quad (15)$$

where  $\hat{D}$  is the dipolar moment. The magnetic field strength is given by

$$B = \frac{B_{eq}}{\sin^2 \theta}, \quad (16)$$

where  $B_{eq} = \hat{D}/L^2$ , is the equatorial magnetic field strength,  $L$  is the equatorial-crossing distance of the relevant field line. For the 2-D dipole the local coordinates become

$$\psi = -\hat{D}/L, \chi = \hat{D} \cot \theta/L. \quad (17)$$

It follows that  $\vec{B} = \nabla\psi \times \vec{e}_y$ , and the magnetic field strength is

$$B = \frac{\psi^2}{\hat{D} \sin^2 [\text{arccot}(-\chi/\psi)]}. \quad (18)$$

The bounce period and the bounce average curvature-gradient magnetic drift velocity are

$$\tau_b = \frac{2\pi L}{v}, \overline{v_d} = \oint \frac{dl}{v_{\parallel}} \overline{v_d} = \frac{-2E}{qLB_{eq}} \vec{e}_y. \quad (19)$$

Then, we perturb the dipolar equilibrium by an external current, flowing in the westward direction and located far in the tail. The linearized Ampère's law becomes

$$\nabla \times \delta \vec{B} = \mu_0 \delta j_{ext} \vec{e}_y. \quad (20)$$

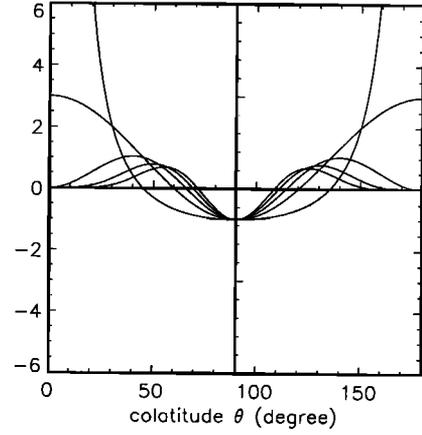
We assume that

$$\delta \vec{B}(\vec{r}, t) = \widehat{\delta B}(\psi, k_y, l, \omega) \exp[i(k_y y + \omega t)],$$

for the sake of simplicity we omit the hat symbol and the exponential factor in the following formulas. The external current  $\delta j_{ext}$  is defined by

$$\delta j_{ext}(L, k_y, \theta, \omega) = \delta j_{eq}(k_y, \omega) \delta(L - L_c) \cdot \sin^{2n} \theta [(2n + 1) \cot^2 \theta - 1]. \quad (21)$$

We have assumed that the current is highly localized in radial distance, and we choose for simplicity a Dirac function  $\delta(L - L_c)$ , where  $L_c$  is the radial location of the forcing current. Therefore we are interested in  $L$  values between  $0 < L < L_c$  where the 2-D dipole assumption is valid. Along the field line, we have chosen a class of forcing current whose the dependence allows us to obtain easily the magnetic field perturbation and corresponds to an increase in the equatorial current as suggested by the observations [e.g., *Sauvaud and Winckler, 1980; Sergeev et al., 1993*]. This class is labeled by an index  $n$ , the larger  $n$ , the more localized is the perturbation close to the magnetic equator (see Figure 1). Comparison between results obtained for various  $n$  gives insight on how sensitive the results are to the  $\theta$  dependence of the forcing current. For  $n = 0$  the perturbed current is divergent at high latitudes but as we will check later on, this divergence does not modify the results because the perturbed components of the electromagnetic field do not diverge. For the sake of simplicity, in the course



**Figure 1.** Variation of the external current  $\delta j_{y_{ext}}(\theta) \propto \sin^{2n} \theta ((2n + 1) \cot^2 \theta - 1)$  versus the colatitude  $\theta$  for  $n = 0, 1, 2, 3, 4$ .

of the paper, we often use the case  $n = 0$  to obtain estimates of the various characteristic quantities.

After some algebra (described in Appendix A) and, in the limit  $|k_y|L > 1$  and  $|k_y(L - L_c)| < 1$ , the perturbed components of the magnetic field write

$$\delta B_{\psi} = -\frac{\mu_0 |k_y| \delta j_{eq}(k_y, \omega)}{2} L_c^2 (\sin^{2n+1} \theta \cos \theta) \quad (22)$$

$$\delta B_{\parallel} = -\frac{\mu_0 \delta j_{eq}(k_y, \omega) L_c^2}{2L} \cdot \sin^{2n} \theta [(2n + 1) - (2n + 2) \sin^2 \theta] \cdot \{H(L - L_c) - H[-(L - L_c)]\} \quad (23)$$

For  $\lambda$ , we obtain (Appendix A)

$$\lambda_n(L, k_y, \theta, \omega) = \lambda_{neq} [c_n(L, k_y) + (\sin^2 \theta)^{n+1}], \quad (24)$$

where we have defined  $c_n(L, k_y) = (n + 1)! / (|k_y(L - L_c)|)^{n+1}$  and  $\lambda_{neq} = 1/[4(n + 1)] \mu_0 (|k_y/k_y| \omega) \delta j_{eq}(k_y, \omega) \cdot L_c^2 L$ . To obtain the real components of the perturbed magnetic field, we have to perform an inverse Fourier transform in time and in  $y$ . We find (see Appendix B)

$$\delta B_{\psi} = \frac{\mu_0}{2} L_c^2 (\sin^{2n+1} \theta \cos \theta) \cdot Pf \left( \int_{-\infty}^{\infty} dy' \frac{\delta j_{eq}(y', t)}{(y - y')^2} \right) \quad (25)$$

$$\delta B_{\parallel} = -\frac{\mu_0 \delta j_{eq}(y, t) L_c^2}{2L} \sin^{2n} \theta \left[ (2n + 1) - (2n + 2) \sin^2 \theta \right] \left\{ H(L - L_c) - H[-(L - L_c)] \right\} \quad (26)$$

Now, we have to specify the variation of the current in the  $y$  direction. During the growth phase, spacecraft ob-

servations close to midnight [McPherron, 1979; Sauvaud and Winckler, 1980; Roux et al., 1991] show that the magnetic field changes from a dipole-like configuration to a tail-like configuration. The equatorial value of the magnetic field decreases, whereas, off-equator, the radial component increases. The duration of this variation is typically  $\simeq 30\text{-}45$  min. Moreover, breakup is usually observed to start close to midnight in a longitudinally narrow sector, while the rest of the magnetotail keeps on stretching [Nagai, 1991]. These observations suggest that while the reconfiguration at breakup is localized in longitude, the formation of the current sheet during the growth phase is more homogeneous in longitude. Thus, while the limit  $k_y \rightarrow \infty$  is adapted to study breakup, the formation of the current sheet can be better described by a finite  $k_y$ . Therefore we consider an external current localized around the noon-midnight meridian, flowing eastward and slowly increasing with the time as

$$\delta j_{eq}(y, t) = \delta j_m \exp\left(-\frac{y^2}{\Delta^2}\right) \exp(\gamma t), \quad (27)$$

where  $\delta j_m$  is the initial magnitude of the current,  $1/\gamma$  is the characteristic time scale of the growth phase, and  $\Delta$  is the characteristic scale along  $y$  where the tail current increases. The complete expression of the external current becomes

$$\begin{aligned} \delta j_{ext}(L, y, \theta, t) &= \delta j_{eq}(y, t) \delta(L - L_c) \\ &\cdot \sin^{2n} \theta \left[ (2n + 1) \cot^2 \theta - 1 \right]. \end{aligned} \quad (28)$$

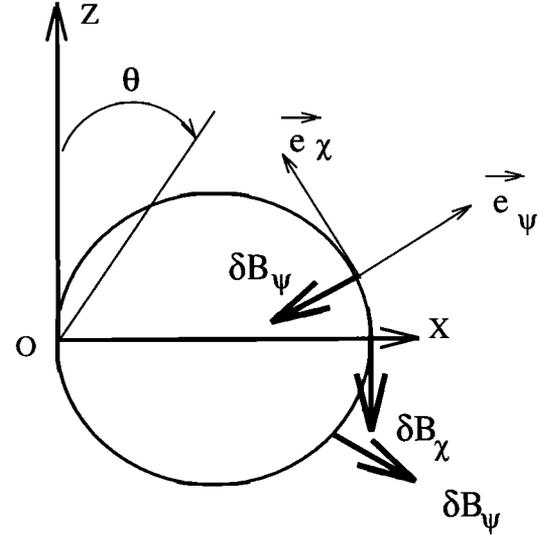
We verify that for  $\theta = \pi/2$  (magnetic equator), the forcing current flows westward as suggested by observations. Then, we can compute the  $\psi$  component of the perturbed magnetic field which gives

$$\begin{aligned} \delta B_\psi &= -\frac{\mu_0 \delta j_m}{\sqrt{\pi}} L_c^2 (\sin^{2n+1} \theta \cos \theta) \\ &\cdot \frac{1}{\Delta} \left\{ 1 + \zeta Pf \left[ \tilde{W}(\zeta) \right] \right\} \end{aligned} \quad (29)$$

where  $\tilde{W}(\zeta) = 1/(\sqrt{\pi}) \int_{-\infty}^{\infty} dV \exp(-V^2)/(V - \zeta)$  is the Fried-Conte function and we have defined  $V = y'/\Delta$  and  $\zeta = y/\Delta$ . Close to midnight,  $\zeta < 1$ , and in this limit, the Fried-Conte function can be approximated by  $Pf(\tilde{W}(\zeta)) \simeq -2\zeta + O(\zeta^3)$  and we obtain

$$\begin{aligned} \delta B_\psi &= -\frac{\mu_0 \delta j_m}{\sqrt{\pi}} L_c^2 (\sin^{2n+1} \theta \cos \theta) \\ &\cdot \frac{1}{\Delta} \left[ 1 - 2 \left( \frac{y}{\Delta} \right)^2 \right]. \end{aligned} \quad (30)$$

One should notice that in the opposite limit  $\zeta > 1$  (far away of the maximum of the current in the  $y$  direction), the expansion of the Fried-Conte function is  $-1/\zeta$  and  $\delta B_\psi = 0$ . Finally, close to midnight ( $\zeta < 1$ ), the two perturbed components of the magnetic field write



**Figure 2.** Schematic diagram of the electromagnetic perturbation applied on a 2-D dipole field to model the change of configuration which occurs during the growth phase. As indicated by the arrows, the magnetic field perturbation tends to produce a tail-like configuration.

$$\begin{aligned} \delta B_\psi &= -\frac{\mu_0 \delta j_m}{\sqrt{\pi}} L_c^2 (\sin^{2n+1} \theta \cos \theta) \\ &\cdot \frac{1}{\Delta} \left[ 1 + 2 \left( \frac{y}{\Delta} \right)^2 \right] \end{aligned} \quad (31)$$

$$\begin{aligned} \delta B_{\parallel} &= -\frac{\mu_0 \delta j_{eq}(y, t)}{2} \frac{L_c^2}{L} \sin^{2n} \theta \left[ (2n + 1) \right. \\ &\quad \left. - (2n + 2) \sin^2 \theta \right] \left\{ H(L - L_c) \right. \\ &\quad \left. - H[-(L - L_c)] \right\}. \end{aligned} \quad (32)$$

We verify that for a forcing current directed westward ( $\delta j_m > 0$ ) at the magnetic equator ( $\theta = \pi/2$ ), the radial component of the equilibrium magnetic field increases off-equator, whereas the equatorial component decreases, which corresponds to observations carried out during the growth phase (see Figure 2).

### 3. Transport of the Plasma

In the previous subsections 2.1, 2.2, and 2.3, we have completely solved the Vlasov-Maxwell system of equations in the quasi-static limit ( $\omega < k_{\parallel} v_A$  and  $\omega < \omega_b$ ). In (5), only the terms  $\overline{\omega_d \lambda} / \omega$  and  $\overline{i \mu k_y \delta A_\psi} / q = -\overline{\delta B_{\parallel}} / q$  appear. It is useful to compare the size of these two terms. Remembering that  $\overline{\omega_d \lambda} / \omega \simeq k_y \overline{v_d} \lambda_{neq} / \omega$  and  $\overline{\mu \delta B_{\parallel}} / q = -\mu B \overline{Y} / q \simeq E[-2(n + 1) \lambda_{neq}] / \omega / \hat{D} / q \simeq 1 / (k_y L) k_y \overline{v_d} \lambda_{neq} / \omega$ , we can conclude that in the limit  $|k_y| L > 1$  we have  $\overline{\mu \delta B_{\parallel}} / q < \overline{\omega_d \lambda} / \omega$ . Thus the term containing  $\delta B_{\parallel}$  can be neglected. In this case the linearized Vlasov equation, which describes the behavior of the plasma, can be simplified, (1) and (4) remains

the same but the expression (5) of  $H$  becomes

$$H = \left( J_0 \delta \Phi + \frac{(\omega + \omega_d)}{\omega} \lambda \right), \quad (33)$$

where  $\delta \Phi$  is given by the QNC (9) that implies  $\delta \Phi = \Phi_0 - \lambda$ , with  $\Phi_0$  given by (13), and  $\lambda$  and  $\delta B_{\parallel}$  given by (24) and (32) (from the Ampère's law solved in the limit  $|k_y|L > 1$  and  $|k_y(L - L_c)| < 1$ ). Now, to obtain the self-consistent perpendicular electric field associated with the magnetic field perturbations, we need to compute the constant part,  $\Phi_0$ , of the perturbed electrostatic potential. Taking into account that  $\mu \delta B_{\parallel} / q \ll \omega_d \lambda / \omega$ , the expression, (13), of  $\Phi_0$  becomes

$$\Phi_0 = \frac{\int \frac{dl}{B} \left\{ \int \frac{4\pi B dE d\mu}{m_i^2 |v_{\parallel}|} f_{0i} \left[ \frac{\omega_{*i} - \bar{\omega}_{d_i}}{\omega + \bar{\omega}_{d_i}} \left( \frac{\omega_d \lambda}{\omega} \right) \right] \right\}}{\int \frac{dl}{B} \left[ \int \frac{4\pi B dE d\mu}{m_i^2 |v_{\parallel}|} f_{0i} \left( \frac{\bar{\omega}_{d_i} (\omega_{*i} - \bar{\omega}_{d_i})}{\omega (\omega + \bar{\omega}_{d_i})} \right) \right]}. \quad (34)$$

From the expression (24) of  $\lambda$  we obtain (see Appendix C)

$$\frac{\bar{\omega}_d \lambda}{\omega} = \frac{\bar{\omega}_d}{\omega} \lambda_{neq} \left[ c_n + 2 \sum_{k=0}^{n+1} (-1)^k C_{n+1}^k \frac{(2k-1)!!}{(2k+2)!!} \cdot \left( 1 - \frac{\mu B_{eq}}{E} \right)^k \left( 1 + k \frac{\mu B_{eq}}{E} \right) \right]. \quad (35)$$

Then, we compute the expression of  $\Phi_0$  (see Appendix D) and obtain

$$\Phi_0 = (c_n + S_n) \lambda_{neq}, \quad (36)$$

where we have defined

$$S_n = \sum_{k=0}^{n+1} (-1)^k C_{n+1}^k \frac{(2k-1)!!}{(2k+2)!!} \sum_{j=0}^k (-1)^j C_k^j \cdot \frac{(2j)!!}{2^{j-1}(j+1)!} \left( 1 + k \frac{(2j+2)(j+3/2)}{(2j+3)(j+2)} \right). \quad (37)$$

Now, from (14), the self-consistent perpendicular electric field writes

$$\delta E_y = -ik_y \lambda_{neq} \left[ S_n - (\sin^2 \theta)^{n+1} \right]. \quad (38)$$

After an inverse Fourier transform, we obtain

$$\delta E_y = \frac{\mu_0 L_c^2 L}{4(n+1)\pi} \frac{1}{\pi} \left[ S_n - (\sin^2 \theta)^{n+1} \right] \cdot \frac{\partial}{\partial t} \left[ Pf \left( \int_{-\infty}^{\infty} dy' \frac{\delta j_{eq}(y', t)}{(y - y')^2} \right) \right]. \quad (39)$$

Taking into account the expression of the current (27), we find

$$\delta E_y = -\frac{\mu_0 \delta j_m \gamma L_c^2 L}{2(n+1)\sqrt{\pi}} \left[ S_n - (\sin^2 \theta)^{n+1} \right] \cdot \frac{1}{\Delta} \left\{ 1 + \zeta Pf \left[ \tilde{W}(\zeta) \right] \right\} \exp(\gamma t). \quad (40)$$

Then, in the limit  $\zeta < 1$  we obtain

$$\delta E_y = \delta E_{L,y,t}(L, y, t) \frac{1}{n+1} \left[ S_n - (\sin^2 \theta)^{n+1} \right], \quad (41)$$

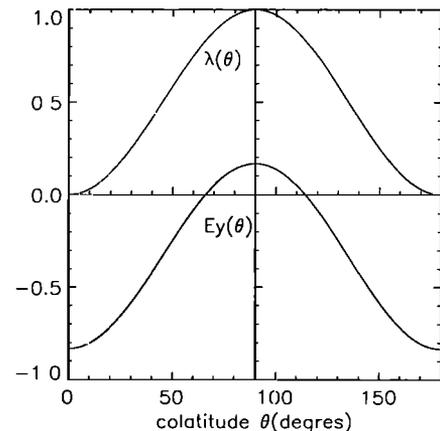
where we have defined

$$\delta E_{L,y,t}(L, y, t) = -\frac{\mu_0 \delta j_m \gamma L_c^2 L}{2\sqrt{\pi}} \frac{1}{\Delta} \left[ 1 - 2 \left( \frac{y}{\Delta} \right)^2 \right] \exp(\gamma t). \quad (42)$$

The colatitude  $\theta$  where the perpendicular electric field changes sign is given by

$$\theta_0 = \arcsin \left( S_n^{\frac{1}{2n+2}} \right). \quad (43)$$

Because  $S_n$  is always smaller than unity, the direction of the perpendicular electric field changes along the field line even in the absence of a parallel electric field (see Figure 3). As noted in section 2.2,  $\delta E_y$  is directed eastward (positive) close to the equator, it is null for  $\theta = \theta_0$ , and it is directed westward (negative) for  $\theta < \theta_0$ . The larger  $n$ , the larger is  $\theta_0$  therefore the region where  $\delta E_y$  is eastward gets thinner (as  $n$  increases). As an example, for  $n = 0$ ,  $S_0 = 5/6$ ,  $\lambda_0 = (c_0 + \sin^2 \theta) \lambda_{0eq}$ ,  $\Phi_0 = (c_0 + 5/6) \lambda_{0eq}$ , and  $\delta E_y = \delta E_m(L, y, t) (5/6 - \sin^2 \theta)$ . The electric field  $\delta E_y$  is directed eastward (positive) close to the equator, it is null for  $\theta = \arcsin(5/6)^{1/2}$  and is directed westward (negative) for  $\theta < \arcsin(5/6)^{1/2}$ . From (42) we can estimate the intensity of the perpendicular electric field during the growth phase. For instance, the characteristic variation of the equatorial magnetic field at the geostationary orbit ( $L_c \simeq L = 6.6 R_E$ ) is of order of 30 nT for a duration of the growth phase of 30 min. We assume that the radial scale of the current sheet is  $\simeq 1 R_E$ ; therefore  $\mu_0 \delta j_m \simeq 30 \times 10^{-9} / (6.4 \times 10^6) \text{T/m}$ . In the  $y$  direction we assume that the spatial scale  $\Delta$  is  $\simeq 4 R_E$ . We obtain for the inductive component of the perpendicular electric field (without the contribu-



**Figure 3.** Variation of  $\lambda$  and  $\delta E_y$  versus the colatitude  $\theta$  for  $n = 0$ .

tion of  $\Phi_0$ ),  $\delta E_{L,y,t} \simeq 2.5$  mV/m which is reduced to 0.4 mV/m at the equator due to the contribution of the electrostatic component  $\Phi_0$  ( $S_n$  in (41)). As we have previously mentioned, the effect of the  $\Phi_0$  is to decrease the magnitude of the total electric field compared to the inductive component. Therefore the plasma transport is also reduced.

Next, we can compute the bounce average electric drift to study the motion of the particles along  $\psi$  as a function of their pitch angle. We obtain (see Appendix E)

$$\begin{aligned} \overline{u_{E_y}} = \frac{1}{\tau_b} \oint \frac{dl}{|v_{\parallel}|} \frac{\delta E_y}{B} = \frac{\delta E_{L,y,t}(L, y, t)}{B_{eq}} \frac{1}{n+1} \\ \cdot \left[ \frac{S_n}{2} \left( 1 + \frac{\mu B_{eq}}{E} \right) - \sum_{k=0}^{n+2} (-1)^k C_{n+2}^k \frac{(2k-1)!!}{(2k)!!} \right. \\ \left. \cdot \sum_{j=0}^{n+2} (-1)^j C_k^j \left( \frac{\mu B_{eq}}{E} \right)^j \right]. \quad (44) \end{aligned}$$

We note that the bounce average electric drift, associated with  $\delta E_y$ , depends on the magnetic moment. To simplify, we can consider two extreme cases: equatorial pitch angle particles of  $90^\circ$  ( $\mu B_{eq}/E \simeq 1$ ) and equatorial pitch angle particles of  $0^\circ$  ( $\mu B_{eq}/E \simeq 0$ ). We obtain

$$\overline{u_{E_y}} \simeq \begin{cases} \delta E_{L,y,t}/B_{eq} \left( S_n/2 - \sum_{k=0}^{n+2} (-1)^k \right. \\ \left. \cdot C_{n+2}^k (2k-1)!!/(2k)!! \right), \alpha_{eq} = 0^\circ, \\ \delta E_{L,y,t}/B_{eq} (S_n - 1), \alpha_{eq} = 90^\circ, \end{cases} \quad (45)$$

where  $\delta E_{L,y,t}$ , given by (42) is always negative close to the midnight meridian ( $y < \Delta$ ). Since  $S_n$  is always smaller than unity the bounce average electric drift of  $90^\circ$  particles and that of  $0^\circ$  particles have opposite directions. The  $90^\circ$  particles drift tailward, whereas  $0^\circ$  particles drift earthward, during the magnetic field line stretching. When  $n$  increases, the perturbation is more and more localized close to the equator and  $S_n$  decreases. Therefore, from (45) we deduce that  $90^\circ$  particles drift more and more tailward, whereas  $0^\circ$  particles remain almost at rest. Again for  $n = 0$ , we find

$$\overline{u_{E_y}} \simeq \begin{cases} 1/24(\delta E_{L,y,t}(L, y, t)/B_{eq}), \alpha_{eq} = 0^\circ, \\ -1/6(\delta E_{L,y,t}(L, y, t)/B_{eq}), \alpha_{eq} = 90^\circ. \end{cases}$$

Thus the correct treatment of the QNC implies a perpendicular motion in response to a quasi-static electromagnetic perturbation ( $\omega < k_{\parallel} v_A$  and  $\omega < \omega_b$ ). Yet, because the perpendicular electric field direction varies with the position along the field line, the bounce-averaged motion is different for different pitch angles. Ninety degrees pitch angle particles, which mirror close to the equator drift tailward while zero degree parti-

cles drift earthward. This result is very different from the results of *Huang and Birmingham* [1994], who consider a static magnetic field and impose an electrostatic field to ensure the convection of the plasma toward the Earth. In the present work, the transport is due to the response of the plasma to the quasi-static perturbation and to the necessity of enforcing the quasi-neutrality.

## 4. Conclusion

In the present paper, we have given a self-consistent description of the quasi-static transport of the plasma, during the growth phase, in response to an external forcing. The full linearized Vlasov-Maxwell system of equations has been solved for quasi-static electromagnetic perturbations, satisfying  $\omega < k_{\parallel} v_A$  and  $\omega < \omega_b$ . In order to get a simple equilibrium the pressure gradient has been assumed to be small. Thus the local current is small, and the local perturbation of the magnetic field is due to currents flowing farther in the tail. From Ampère's law we have obtained the perturbed components of the electric and magnetic fields as functions of the external forcing current. For the sake of simplicity, this current is assumed to be localized close to the magnetic equator and around the noon-midnight meridian. It flows in the east-west direction, as expected during the growth phase. Using a 2-D dipole model to describe the region close to the Earth (the NEPS), we have built a Green function to relate the fields in the NEPS with this external driving current. Thus the solutions for the fields, in the plasma sheet, have been obtained as the products of the Green function by the forcing current. Using the linear bounce-averaged solution of the Vlasov equation obtained by *Pellat* [1990], the QNC has been solved. To the lowest order in  $T_e/T_i$  ( $T_e/T_i < 1$ ), we found the following:

1. The QNC imposes the existence of a component  $\Phi_0$ , given by (13), of the perturbed electrostatic potential. This component  $\Phi_0$  is constant along the field line and varies in the azimuthal direction, thereby contributing to the azimuthal electric field. This electric field tends to reduce the effect of the inductive component of the electric field, which explains why no large bulk flows are associated with large timescale electromagnetic perturbation ( $\tau > \tau_b$ ) like the growth phase. Unlike what is done for the particle test and MHD approaches, in the present paper, the electrostatic component of the azimuthal electric field is not assumed; it is determined, in a self-consistent manner, by the response of the plasma and related to an externally applied electromagnetic perturbation. We point out that the existence of the component  $\Phi_0$  is a purely kinetic effect occurring for  $\omega < \omega_b$ . In a forthcoming paper we will show that the component  $\Phi_0$  exists as long as  $\omega < \omega_{be}$ . In all cases it cannot be described by MHD.

2. The total azimuthal electric field (14), which is the sum of these two components, varies in amplitude and direction, as a function of the position along the

field line. The changes in amplitude and in direction of the azimuthal electric field implies a bounce-averaged transport of the particles (44) that strongly depends on the pitch angle.

3. The parallel electric field is null to the order  $T_e/T_i < 1$  (see (11)), therefore the residual parallel electric field should be calculated from the QNC developed to the order  $T_e/T_i$ . This calculation is presented in a companion paper.

Notice that the full Vlasov-Maxwell system of equations has been solved only in the quasi-static limit, notably the solutions of Ampère's law, (31) and (32), are valid only in this limit. However, the results obtained to the lowest order in  $T_e/T_i$  from the QNC (summarized above) are, also basically valid for low-frequency perturbations  $\omega < \omega_b$ . Furthermore, for the sake of simplicity and because in the NEPS the effect of magnetic drift are expected to be more important than the finite larmor radius effects, the present calculations have been performed in the long wave length limit. However, we can easily include these effects if necessary since they are retained in the expression of the perturbed distribution function (5).

For the magnetic field variations corresponding to the growth phase, namely for a slow decrease (increase) of the component of the magnetic field, perpendicular to the equatorial plane (radial), the azimuthal electric field is explicitly computed. It is found to be directed eastward, close to the equator and westward off-equator. As a consequence, during the growth phase, large equatorial pitch angle particles, which mirror near the equator, drift tailward, whereas small equatorial pitch angle particles, which mirror far away from the equator, drift earthward. Furthermore, this result suggests that the mapping between the perpendicular electric field in the equatorial region and the electric field in the ionosphere is not simple; a self-consistent approach, including non-local kinetic effects associated with the bounce motions of particles, is needed to sort out the consequences of a time-dependent transport. The corresponding characteristic azimuthal electric field is of order of 0.4 mV/m at the magnetic equator. In order to test these theoretical results it would be necessary to have electric field and/or plasma flow data organized as a function of the distance from the center of the current sheet, during the growth phase.

## Appendix A: Solution of the Linearized Ampère's Law

Using the local field-aligned coordinates and with the gauge  $\delta A_y = 0$ , the curl of  $\vec{B}$  gives

$$\vec{\nabla} \times \delta \vec{B} = \left\{ \frac{\partial}{\partial y} \left( -\frac{\partial \delta A_\psi}{\partial y} \right) - \frac{1}{JB} \frac{\partial}{\partial \chi} \left[ \frac{1}{J} \left( \frac{\partial}{\partial \chi} \left( \frac{\delta A_\psi}{B} \right) - \frac{\partial}{\partial \psi} (JB \delta A_{\parallel}) \right) \right] \right\} \vec{e}_\psi$$

$$\begin{aligned} & + \frac{1}{J} \left\{ \frac{\partial}{\partial \chi} \left( \frac{1}{B} \frac{\partial \delta A_{\parallel}}{\partial y} \right) - \frac{\partial}{\partial \psi} \left[ JB \left( -\frac{\partial \delta A_\psi}{\partial y} \right) \right] \right\} \vec{e}_y \\ & + \left\{ B \frac{\partial}{\partial \psi} \left[ \frac{1}{J} \left( \frac{\partial}{\partial \chi} \left( \frac{\delta A_\psi}{B} \right) - \frac{\partial}{\partial \psi} (JB \delta A_{\parallel}) \right) \right] - \frac{\partial}{\partial y} \left( -\frac{\partial \delta A_{\parallel}}{\partial y} \right) \right\} \vec{e}_{\parallel}, \end{aligned} \quad (\text{A1})$$

where  $J$  is the Jacobian of the change of coordinates between the cartesian and the local frame (see also the Appendix A of *Hurricane et al.* [1995b]). We assume that

$$\widehat{\delta \vec{A}}(\vec{r}, t) = \widehat{\delta \vec{A}}(\psi, k_y, l, \omega) \exp[i(k_y y + \omega t)],$$

for the sake of simplicity we omit the hat symbol and the exponential factor in the following formulas. Defining two variables  $X = k_y \lambda / \omega$  and  $\tilde{Y} = ik_y \delta A_\psi / B$  as *Bernstein et al.* [1958], interchanging the partial derivatives,  $\partial_\psi (JB \partial_l) = JB \partial_l \partial_\psi$ , (see *Hurricane et al.* [1995b] for details about the interchange of the partial derivatives) when necessary, the Ampère's law writes

$$-ik_y \tilde{Y} B - \frac{1}{ik_y} \frac{\partial}{\partial l} \left( B \frac{\partial \tilde{Y}}{\partial l} - B \frac{\partial^2 X}{\partial l \partial \psi} \right) = 0, \quad (\text{A2})$$

$$B \frac{\partial}{\partial l} \left( \frac{1}{B} \frac{\partial X}{\partial l} \right) + \frac{1}{J} \frac{\partial}{\partial \psi} (JB^2 \tilde{Y}) = \mu_0 \delta j_{yext}, \quad (\text{A3})$$

$$\frac{B}{ik_y} \frac{\partial}{\partial \psi} \left( B \frac{\partial \tilde{Y}}{\partial l} - B \frac{\partial^2 X}{\partial l \partial \psi} \right) - ik_y \frac{\partial X}{\partial l} = 0. \quad (\text{A4})$$

One can show that (A4) can be obtained from (A2) and (A3); therefore the system is reduced to these two latter equations. Inserting (A2) to (A3), we obtain

$$B \frac{\partial}{\partial l} \left( \frac{1}{B} \frac{\partial X}{\partial l} \right) + \frac{1}{Jk_y^2} \frac{\partial}{\partial \psi} \left[ JB \frac{\partial}{\partial l} \left( B \frac{\partial \tilde{Y}}{\partial l} - B \frac{\partial^2 X}{\partial l \partial \psi} \right) \right] = \mu_0 \delta j_{yext}. \quad (\text{A5})$$

Again, we interchange the partial derivatives ( $\partial_\psi (JB \partial_l) = JB \partial_l \partial_\psi$ ), divide by  $B$  and integrate along the field line which yields

$$\left( \frac{1}{B} \frac{\partial X}{\partial l} \right) + \frac{1}{k_y^2} \frac{\partial}{\partial \psi} \left( B \frac{\partial \tilde{Y}}{\partial l} - B \frac{\partial^2 X}{\partial l \partial \psi} \right) = \mu_0 \int \frac{dl}{B} \delta j_{yext}. \quad (\text{A6})$$

Therefore we have to solve the following system of equations:

$$\tilde{\gamma} = \frac{1}{k_y^2 B} \frac{\partial}{\partial l} \left( B \frac{\partial \tilde{Y}}{\partial l} - B \frac{\partial^2 X}{\partial l \partial \psi} \right), \quad (\text{A7})$$

$$\begin{aligned} \left( \frac{1}{B} \frac{\partial X}{\partial l} \right) + \frac{1}{k_y^2} \frac{\partial}{\partial \psi} \left( B \frac{\partial \tilde{Y}}{\partial l} - B \frac{\partial^2 X}{\partial l \partial \psi} \right) \\ = \mu_0 \int \frac{dl}{B} \delta j_{y \text{ ext}}. \end{aligned} \quad (\text{A8})$$

To go further, we assume a priori that  $\partial \tilde{Y} / \partial l < \partial^2 X / \partial l \partial \psi$  (and will check it afterwards) which corresponds to assume that the variation of  $\delta B_{\parallel}$  along the field line is smaller than the variation of  $\delta B_{\psi}$  in the radial direction. This assumption is well adapted to the choice of an external current highly localized in the radial direction. The system becomes

$$\begin{aligned} \tilde{Y} = -\frac{1}{k_y^2 B} \frac{\partial}{\partial l} \left( B \frac{\partial^2 X}{\partial l \partial \psi} \right), \quad (\text{A9}) \\ \left( \frac{1}{B} \frac{\partial X}{\partial l} \right) - \frac{1}{k_y^2} \frac{\partial}{\partial \psi} \left( B \frac{\partial^2 X}{\partial l \partial \psi} \right) \\ = \mu_0 \int \frac{dl}{B} \delta j_{y \text{ ext}}. \end{aligned} \quad (\text{A10})$$

Defining a new variable,  $U = \partial X / \partial l$ , (A10) becomes

$$\begin{aligned} \frac{\partial^2 U}{\partial \psi^2} + \frac{\partial \ln JB^2}{\partial \psi} \frac{\partial U}{\partial \psi} - \frac{k_y^2}{B^2} U \left[ 1 \right. \\ \left. - \frac{B}{k_y^2} \frac{\partial}{\partial \psi} \left( \frac{1}{J} \frac{\partial JB}{\partial \psi} \right) \right] = -\frac{\mu_0 k_y^2}{B} \int \frac{dl}{B} \delta j_{y \text{ ext}}. \end{aligned} \quad (\text{A11})$$

Assuming that the perturbation varies faster in the  $\psi$  direction than the equilibrium, the second term of (A11) can be neglected. Moreover, in the limit  $|k_y|L > 1$  the term between parenthesis is equivalent to unity. Therefore (A11) can be rewritten in a classical form of a linear second-order differential equation:

$$\frac{\partial^2 U}{\partial \psi^2} - \frac{k_y^2}{B^2} U = S(\psi, k_y, l, \omega), \quad (\text{A12})$$

where  $S(\psi, k_y, l, \omega) = -(\mu_0 k_y^2 / B) \int [dl / B \delta j_{\text{ext}}(\psi, k_y, l, \omega)]$  is the forcing term. To solve (A12), we have to build a Green function from the solutions of the homogeneous equation;  $U_1 = \exp(|k_y| \int d\psi / B)$  and  $U_2 = \exp(-|k_y| \int d\psi / B)$  [see, e.g., *Zwillinger*, 1989]. Taking into account the properties of the 2-D dipole model it is more convenient to use the variables  $L$  and  $\theta$  defined by (17). While  $L$  and  $\theta$  are not strictly speaking independent variables, we can consider them as such in the limit where we neglect the variations of the equilibrium compared to those of the perturbation, these variables allow us to easily solve (A12), which becomes

$$\frac{\partial^2 U}{\partial L^2} - (k_y^2 \sin^4 \theta) U = S(L, k_y, \theta, \omega), \quad (\text{A13})$$

where  $S(L, k_y, \theta, \omega) = \mu_0 k_y^2 L \sin^2 \theta \int d\theta \sin^2 \theta \delta j_{\text{ext}}(L, k_y, \theta, \omega)$ . Now, the solutions of the homogeneous equation become  $U_1 = \exp|k_y|L \sin^2 \theta$  and  $U_2 = \exp -|k_y|L \sin^2 \theta$ . The Green function is defined by

$$G(L, L_0) = \begin{cases} \frac{U_1(L)U_2(L_0)}{W(L_0)} \\ = \frac{\exp(|k_y|(L-L_0)\sin^2 \theta)}{W(L_0)}, & 0 \leq L \leq L_0; \\ \frac{U_1(L_0)U_2(L)}{W(L_0)} \\ = \frac{\exp(-|k_y|(L-L_0)\sin^2 \theta)}{W(L_0)}, & L_0 \leq L < \infty, \end{cases}$$

where  $W(L_0) = -2|k_y| \sin^2 \theta$ , is the Wronskian of  $U_1$  and  $U_2$ . The Green function becomes

$$G(L, L_0) = -\frac{\exp(-|k_y|(L-L_0)\sin^2 \theta)}{2|k_y|\sin^2 \theta}. \quad (\text{A14})$$

The complete solution of (A13) writes (omitting to specify all the dependences)

$$U(L) = \int_0^\infty dL_0 G(L, L_0) S(L_0), \quad (\text{A15})$$

which yields (specifying all the spatiotemporal dependences)

$$\begin{aligned} U(L, \theta, k_y, \omega) \\ = -\frac{\mu_0 |k_y|}{2} \int_0^\infty dL_0 L_0 \exp(-|k_y|(L-L_0)\sin^2 \theta) \\ \cdot \int d\theta \sin^2 \theta \delta j_{y \text{ ext}}(L_0, \theta, k_y, \omega). \end{aligned} \quad (\text{A16})$$

Taking into account the expression (21) of the external current, the complete solution for this class of current is

$$\begin{aligned} U_n(L, k_y, \theta, \omega) = -\frac{\mu_0 |k_y| \delta j_{\text{eq}}(k_y, \omega)}{2} L_c^2 \\ \cdot \exp\left(-|k_y|(L-L_c)\sin^2 \theta\right) \\ \cdot (\sin^{2n+1} \theta \cos \theta + C(k_y, \psi)), \end{aligned} \quad (\text{A17})$$

where we have used the integral  $\int d\theta \sin^{2n+2} \theta [2(n+1) \cdot \cot^2 \theta - 1] = \sin^{2n+1} \theta \cos \theta + C(k_y, \psi)$ . We impose  $U = \partial X / \partial l = \delta B_{\psi} = 0$  at the equator ( $\theta = \pi/2$ ) which implies  $C = 0$ . From (A17) we obtain

$$\begin{aligned} X_n(L, k_y, \theta, \omega) = \frac{\mu_0 |k_y| \delta j_{\text{eq}}(k_y, \omega)}{4} L_c^2 L \\ \cdot \int du u^n \exp(Zu), \end{aligned} \quad (\text{A18})$$

where we have defined  $u = \sin^2 \theta$  and  $Z = -|k_y|(L-L_c)$ . The solution becomes

$$\begin{aligned} X_n(L, k_y, \theta, \omega) = \frac{\mu_0 |k_y| \delta j_{\text{eq}}(k_y, \omega)}{4} L_c^2 L \left( \frac{u^n}{Z} \right. \\ \left. + \sum_{k=1}^n (-1)^k n(n-1)\dots(n-k+1) \frac{u^{n-k}}{Z^{k+1}} \right) \exp(Zu), \end{aligned} \quad (\text{A19})$$

where we have used the integral  $\int du u^n \exp(Zu) = [u^n/Z + \sum_{k=1}^n (-1)^k n(n-1) \dots (n-k+1) u^{n-k}/Z^{k+1}] \cdot \exp(Zu)$ . Now, we can solve the second equation (A9) of the system to obtain  $\tilde{Y}$ . With the variables  $L$  and  $\theta$ , and in the limit  $|k_y|L > 1$ , (A9) becomes

$$\tilde{Y} = \frac{1}{k_y^2 B L} \frac{\partial}{\partial \theta} \left( B \frac{L^2}{D} \frac{\partial U}{\partial L} \right). \quad (\text{A20})$$

After some algebra, we obtain

$$\begin{aligned} \tilde{Y} = & \frac{\mu_0 \delta j_{eq}(k_y, \omega)}{2} L_c^2 \frac{L}{D} u^{n+1} \left( (2n+1) - (2n+2)u \right. \\ & \left. + 2Zu(1-u) \right) \left\{ H(L-L_c) \right. \\ & \left. - H[-(L-L_c)] \right\} \exp(Zu), \quad (\text{A21}) \end{aligned}$$

where  $H$  stands for the Heaviside function. As claimed above, we verify a posteriori that the assumption  $\partial \tilde{Y}/\partial l < \partial^2 X/\partial l \partial \psi$  is valid. Indeed, we have

$$\frac{\partial \tilde{Y}/\partial l}{\partial^2 X/\partial l \partial \psi} \simeq \frac{\partial \tilde{Y}/\partial l}{\partial U/\partial \psi} \simeq \frac{1}{k_y^2 L^2}. \quad (\text{A22})$$

Thus the above assumption is valid in the limit  $|k_y|L > 1$ . Then, we take the limit  $|Z| = |k_y(L-L_c)| < 1$ , which implies that the wave length of the perturbation is larger than the distance between the current and the location where the solution is calculated. In this limit, one can show that  $X$  writes

$$\begin{aligned} X_n(L, k_y, \theta, \omega) \simeq & \frac{\mu_0 |k_y| \delta j_{eq}(k_y, \omega)}{4} L_c^2 L \left( \frac{n!}{Z^{n+1}} \right. \\ & \left. + \frac{u^{n+1}}{n+1} + O(Z) \right). \quad (\text{A23}) \end{aligned}$$

Finally, we obtain for  $\lambda$

$$\begin{aligned} \lambda_n(L, k_y, \theta, \omega) \simeq & \lambda_{neq} \left[ c_n(L, k_y) + (\sin^2 \theta)^{n+1} \right. \\ & \left. + O(-|k_y(L-L_c)|) \right], \quad (\text{A24}) \end{aligned}$$

where we have defined  $c_n(L, k_y) = (n+1)!/(-|k_y(L-L_c)|)^{n+1}$  and  $\lambda_{neq} = 1/[4(n+1)]\mu_0(|k_y|/k_y)\omega\delta j_{eq}(k_y, \omega)L_c^2 L$ . The second variable  $\tilde{Y}$  becomes

$$\begin{aligned} \tilde{Y} = & 2(n+1)\lambda_{neq} \frac{k_y}{|k_y|\omega D} \sin^{2n+2} \theta \left( (2n+1) \right. \\ & \left. - (2n+2) \sin^2 \theta \right) \left\{ H(L-L_c) \right. \\ & \left. - H[-(L-L_c)] \right\}. \quad (\text{A25}) \end{aligned}$$

Therefore, in the limit  $|k_y|L > 1$  and  $|k_y(L-L_c)| < 1$  the perturbed components of the magnetic field write

$$\begin{aligned} \delta B_\psi = U_n(L, \theta) = & -\frac{\mu_0 |k_y| \delta j_{eq}(k_y, \omega)}{2} \\ & \cdot L_c^2 (\sin^{2n+1} \theta \cos \theta) \quad (\text{A26}) \end{aligned}$$

$$\begin{aligned} \delta B_{||} = -B\tilde{Y} = & -\frac{\mu_0 \delta j_{eq}(k_y, \omega)}{2} \frac{L_c^2}{L} \sin^{2n} \theta \left( (2n+1) \right. \\ & \left. - (2n+2) \sin^2 \theta \right) \left\{ H(L-L_c) \right. \\ & \left. - H[-(L-L_c)] \right\} \quad (\text{A27}) \end{aligned}$$

## Appendix B: Inversion of the Fourier Transform

In this appendix we do not omit the Fourier notations; therefore we have

$$\widehat{U} = \frac{\partial \widehat{X}}{\partial l} = \widehat{\delta B_\psi}, \quad (\text{B1})$$

and the real component  $\delta B_\psi$  writes

$$\begin{aligned} \delta B_\psi(L, y, \theta, t) = & \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{+\infty} dk_y \widehat{\delta B_\psi}(L, k_y, \theta, \omega) \\ & \cdot \exp[i(k_y y + \omega t)], \quad (\text{B2}) \end{aligned}$$

$$\begin{aligned} \delta B_\psi(L, y, \theta, t) = & -\frac{\mu_0}{2} f(\theta) \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega \int_{-\infty}^{+\infty} dk_y |k_y| \\ & \cdot \widehat{\delta j_{eq}}(k_y, \omega) \exp[i(k_y y + \omega t)], \quad (\text{B3}) \end{aligned}$$

where  $f(\theta) = L_c^2 \sin^{2n+1} \theta \cos \theta$ . The  $\omega$  integration is straightforward and gives

$$\begin{aligned} \delta B_\psi(L, y, \theta, t) = & -\frac{\mu_0}{2\sqrt{2\pi}} f(\theta) \int_{-\infty}^{+\infty} dk_y |k_y| \\ & \widehat{\delta j_{eq}}(k_y, t) \exp(ik_y y). \quad (\text{B4}) \end{aligned}$$

For the  $k_y$  integration we note that

$$\begin{aligned} |k_y| = k_y \text{sign}(k_y) = & \frac{ik_y}{\pi} FT \left( Pf \left( \frac{1}{y} \right) \right) \frac{1}{\pi} \\ & \cdot FT \left\{ \frac{\partial}{\partial y} \left[ Pf \left( \frac{1}{y} \right) \right] \right\}, \quad (\text{B5}) \end{aligned}$$

where  $FT$  denotes a one dimension Fourier transform and  $Pf$  the Cauchy principal value. The perturbed magnetic field component becomes

$$\begin{aligned} \delta B_\psi = & -\frac{\mu_0}{2} f(\theta) \int_{-\infty}^{+\infty} dk_y \left( \frac{1}{\pi} FT \left\{ \frac{\partial}{\partial y} \left[ Pf \left( \frac{1}{y} \right) \right] \right\} \right) \\ & \cdot FT(\delta j_{eq}(y, t)) \exp(ik_y y). \quad (\text{B6}) \end{aligned}$$

Moreover, the convolution theorem gives

$$\widehat{f * g} = \widehat{f} \cdot \widehat{g} \iff f * g = FT^{-1}(\widehat{f} \cdot \widehat{g}). \quad (\text{B7})$$

Therefore we obtain

$$\begin{aligned} \delta B_\psi = & -\frac{\mu_0}{2\pi} f(\theta) \int_{-\infty}^{+\infty} dy' \delta j_{eq}(y', t) \\ & \cdot Pf \left( -\frac{1}{(y-y')^2} \right). \quad (\text{B8}) \end{aligned}$$

Finally, we can write

$$\delta B_\psi = \frac{\mu_0}{2\pi} f(\theta) P f \left( \int_{-\infty}^{+\infty} dy' \frac{\delta j_{eq}(y', t)}{(y - y')^2} \right). \quad (\text{B9})$$

### Appendix C: Nonlocal Terms

Here we compute the first nonlocal term  $\overline{\omega_d \lambda} / \omega$  of  $\delta f$ . In the field-aligned coordinates the expression of the magnetic curvature-gradient drift is [Hurricane *et al.*, 1995b]

$$v_d = -\frac{mv_{\parallel}}{qJB} \frac{\partial}{\partial \psi} (JBv_{\parallel}). \quad (\text{C1})$$

After the  $\psi$  derivative we obtain

$$v_d = -\frac{1}{q} \frac{\partial}{\partial \psi} \left( \frac{1}{2} mv_{\parallel}^2 \right) - \frac{mv_{\parallel}^2}{q} \frac{\partial}{\partial \psi} (\ln JB),$$

replacing  $1/2mv_{\parallel}^2$  by  $E - \mu B$  and taking into account the conservation of the kinetic energy  $E$  (static magnetic field and absence of electrostatic field) and conservation of the magnetic moment  $\mu$ , in the adiabatic regime, we obtain

$$v_d = \frac{1}{q} \mu \frac{\partial B}{\partial \psi} + \frac{mv_{\parallel}^2}{qB} \frac{\partial B}{\partial \psi},$$

where  $J = 1/B^2$ . Indeed, from the definition of the Jacobian  $J = (1/B^2) \exp \left( -\int d\psi (\nabla \times \vec{B}) \cdot \vec{e}_y / B^2 \right)$  [Bernstein *et al.*, 1958], we have

$$\frac{\partial \ln J}{\partial \psi} = -2 \frac{\partial \ln B}{\partial \psi} - \frac{\mu_0 p'}{B^2},$$

which can be approximated by

$$\frac{\partial \ln J}{\partial \psi} \simeq -\frac{2}{LB} - \frac{2\mu_0 p}{L_p B^3},$$

where  $L_p$  is the scale length of the pressure gradient and  $L$  is the scale length of the magnetic field gradient. Noticing that  $\beta = 2\mu_0 p / B^2$ , the above relation becomes

$$\frac{\partial \ln J}{\partial \psi} \simeq -\frac{2}{LB} \left( 1 + \frac{1}{4} \beta \frac{L}{L_p} \right).$$

In the limit  $\beta \ll 1$  even if the scale length of the pressure gradient is small ( $L/L_p \gg 1$ ), we can still get  $\beta L/L_p \ll 1$  and write

$$\frac{\partial \ln J}{\partial \psi} \simeq -\frac{2}{B} \frac{\partial B}{\partial \psi}, \quad (\text{C2})$$

which gives  $J = 1/B^2$ . Now, we have to compute the  $\psi$  derivative of  $B$ , keeping  $\chi$  constant. From (18), we obtain

$$\frac{\partial B}{\partial \psi} = -\frac{2}{L}. \quad (\text{C3})$$

The purely magnetic drift becomes

$$v_d = -\frac{2}{qL} \left[ \mu + \frac{mv_{\parallel}^2}{B} \right]. \quad (\text{C4})$$

The bounce average of the product  $v_d \lambda$  gives

$$\overline{\omega_d \lambda} = \frac{1}{\tau_b} \oint \frac{dl}{|v_{\parallel}|} \left[ -\frac{2k_y}{qL} \left( \mu + \frac{mv_{\parallel}^2}{B} \right) \right] \lambda. \quad (\text{C5})$$

With the definition of  $\lambda$  given in (24) we obtain

$$\overline{\omega_d \lambda} = \overline{\omega_d} c_n \lambda_{neq} - \frac{1}{\tau_b} \oint \frac{dl}{|v_{\parallel}|} \left[ -\frac{2k_y}{qL} \left( \mu + \frac{mv_{\parallel}^2}{B} \right) (\sin^2 \theta)^{n+1} \right] \lambda_{neq}. \quad (\text{C6})$$

In (C6),  $T_1$ , the first term of the expression between bracket becomes

$$T_1 = -\frac{2k_y}{qL} \mu \lambda_{neq} \frac{1}{\tau_b} \oint \frac{dl}{|v_{\parallel}|} (\sin^2 \theta)^{n+1}, \quad (\text{C7})$$

The parallel velocity is given by

$$|v_{\parallel}| = \sqrt{\frac{2}{m} (E - \mu B)}, \quad (\text{C8})$$

and from (16) we obtain

$$|v_{\parallel}| = \sqrt{\frac{2E}{m}} \sqrt{\left( 1 - \frac{\mu B_{eq}}{E \sin^2 \theta} \right)}. \quad (\text{C9})$$

Then, the term  $T_1$  becomes

$$T_1 = \frac{8k_y}{qv} \mu \lambda_{neq} \frac{1}{\tau_b} \int_{\frac{\pi}{2}}^{\theta_m} \frac{d\theta (\sin^2 \theta)^{n+1}}{\left( 1 - \frac{\mu B_{eq}}{E \sin^2 \theta} \right)^{1/2}}, \quad (\text{C10})$$

where  $v$  is the velocity,  $\theta_m$  is the colatitude of the mirror point and we have used that  $l = L(\pi/2 - \theta)$  so  $dl = -Ld\theta$ . Taking  $Z = \cos \theta / A$  with  $A = 1 - \mu B_{eq} / E$ , we obtain

$$T_1 = -\frac{8k_y}{qv} \mu \lambda_{neq} \frac{4}{\tau_r} \int_0^1 \frac{dZ}{(1 - Z^2)^{1/2}} (1 - AZ^2)^{n+1}. \quad (\text{C11})$$

Using the binomial formula, we obtain

$$T_1 = -\frac{8k_y}{qv} \mu \lambda_{neq} \frac{4}{\tau_r} \int_0^1 \frac{dZ}{(1 - Z^2)^{1/2}} \cdot \sum_{k=0}^{n+1} (-1)^k C_{n+1}^k A^k Z^{2k}, \quad (\text{C12})$$

$$T_1 = -\frac{8k_y}{qv} \mu \lambda_{neq} \sum_{k=0}^{n+1} (-1)^k C_{n+1}^k A^k \cdot \frac{4}{\tau_r} \int_0^1 \frac{dZ}{(1 - Z^2)^{1/2}} Z^{2k}. \quad (\text{C13})$$

We give the following relation

$$\int_0^1 \frac{dZ Z^{2k}}{(1-Z^2)^{1/2}} = \frac{\pi (2k-1)!!}{2 (2k)!!}, k \geq 0, \quad (C14)$$

and perform the  $Z$  integration

$$T_1 = -\frac{2k_y}{qL} \mu \lambda_{neq} \sum_{k=0}^{n+1} (-1)^k C_{n+1}^k A^k \frac{(2k-1)!!}{(2k)!!}. \quad (C15)$$

Following the same approach, the second term  $T_2$  of (C6) gives

$$T_2 = \frac{1}{\tau_b} \oint \frac{dl}{|v_{\parallel}|} \left[ -\frac{2k_y}{qL} \left( \frac{mv_{\parallel}^2}{B} \right) \right] (\sin^2 \theta)^{n+1}, \quad (C16)$$

$$T_2 = -\frac{2k_y E}{qL B_{eq}} \lambda_{neq} A \sum_{k=0}^{n+1} (-1)^k C_{n+1}^k A^k \cdot \frac{(2k-1)!!}{(2k)!!} \left( \frac{1}{k+1} \right). \quad (C17)$$

Summing all terms, we obtain

$$\overline{\omega_d \lambda} = \overline{\omega_d} \lambda_{neq} \left[ c_n + 2 \sum_{k=0}^{n+1} (-1)^k C_{n+1}^k \frac{(2k-1)!!}{(2k+2)!!} \cdot \left( 1 - \frac{\mu B_{eq}}{E} \right)^k \left( 1 + k \frac{\mu B_{eq}}{E} \right) \right], \quad (C18)$$

where  $\overline{\omega_d} = -2k_y E / (qL B_{eq})$ .

### Appendix D: Calculation of the Constant Part $\Phi_0$ of the Perturbed Electrostatic Potential

Thanks to the simplicity of the 2-D dipole, we are able to compute completely  $\Phi_0$ . Remembering that in the limit  $|k_y|L \gg 1$ ,  $\Phi_0$  writes

$$\Phi_0 = -\frac{\int \frac{dl}{B} \left[ \int \frac{4\pi B dE d\mu}{|v_{\parallel}|} f_{0i} \left( \frac{\omega_{\lambda i} - \overline{\omega}_{di}}{\omega + \overline{\omega}_{di}} \left( \frac{\omega_{di} \lambda}{\omega} \right) \right) \right]}{\int \frac{dl}{B} \left[ \int \frac{4\pi B dE d\mu}{|v_{\parallel}|} f_{0i} \left( \frac{\overline{\omega}_{di} (\omega_{\lambda i} - \overline{\omega}_{di})}{\omega (\omega + \overline{\omega}_{di})} \right) \right]}. \quad (D1)$$

Using (C18), the numerator noted  $N$  of (D1) becomes

$$N = \int \frac{dl}{B} \left\{ \int \frac{4\pi B dE d\mu}{|v_{\parallel}|} f_{0i} \left[ \frac{\overline{\omega}_{di} (\omega_{\lambda i} - \overline{\omega}_{di})}{\omega (\omega + \overline{\omega}_{di})} \right] \cdot \lambda_{neq} \left( c_n + 2 \sum_{k=0}^{n+1} (-1)^k C_{n+1}^k \frac{(2k-1)!!}{(2k+2)!!} \cdot \left( 1 - \frac{\mu B_{eq}}{E} \right)^k \left( 1 + k \frac{\mu B_{eq}}{E} \right) \right) \right\}.$$

Using again the binomial formula, we can write

$$N = \int \frac{dl}{B} \left\{ \int 4\pi dE f_{0i} \left[ \frac{\overline{\omega}_{di} (\omega_{\lambda i} - \overline{\omega}_{di})}{\omega (\omega + \overline{\omega}_{di})} \right] \cdot \lambda_{neq} \left( c_n + 2 \sum_{k=0}^{n+1} (-1)^k C_{n+1}^k \frac{(2k-1)!!}{(2k+2)!!} \right) \right\}.$$

$$\cdot \sum_{j=0}^k (-1)^j C_k^j \left( \frac{\mu B_{eq}}{E} \right)^j \left( 1 + k \frac{\mu B_{eq}}{E} \right) \right\}.$$

We give the following integral

$$\int_0^{E/B} \frac{d\mu}{|v_{\parallel}|} \mu^n = \sqrt{2m} \frac{(2n)!!}{(2n+1)!!} \frac{E^{n+1/2}}{B^{n+1}}, \quad (D2)$$

and we perform the  $\mu$  integration (remember that for the 2-D dipole  $\overline{\omega}_{di}$  is independent of  $\mu$ ):

$$N = \int \frac{dl}{B} \left\{ \int 4\pi dE f_{0i} \sqrt{2m_i} E^{1/2} \left[ \frac{\overline{\omega}_{di} (\omega_{\lambda i} - \overline{\omega}_{di})}{\omega (\omega + \overline{\omega}_{di})} \right] \cdot \lambda_{neq} \left( c_n + 2 \sum_{k=0}^{n+1} (-1)^k C_{n+1}^k \frac{(2k-1)!!}{(2k+2)!!} \sum_{j=0}^k (-1)^j C_k^j \cdot \left( \frac{B_{eq}}{E} \right)^j \frac{(2j)!!}{(2j+1)!!} \left( 1 + k \frac{(2j+2) B_{eq}}{(2j+3) B} \right) \right) \right\}.$$

The denominator  $D$  of D1 gives

$$D = \int \frac{dl}{B} \left[ \int \frac{4\pi B dE d\mu}{|v_{\parallel}|} f_{0i} \left( \frac{\overline{\omega}_{di} (\overline{\omega}_{di} - \omega_{\lambda i})}{\omega (\omega + \overline{\omega}_{di})} \right) \right] = \int \frac{dl}{B} \left[ \int 4\pi dE f_{0i} \sqrt{2m_i} E^{1/2} \cdot \left( \frac{\overline{\omega}_{di} (\overline{\omega}_{di} - \omega_{\lambda i})}{\omega (\omega + \overline{\omega}_{di})} \right) \right]. \quad (D3)$$

The terms being integrated over the energy vanish between the numerator  $N$  and the denominator  $D$ , we obtain

$$\Phi_0 = \int \frac{dl}{B} \left[ \lambda_{neq} \left( c_n + 2 \sum_{k=0}^{n+1} (-1)^k C_{n+1}^k \frac{(2k-1)!!}{(2k+2)!!} \cdot \sum_{j=0}^k (-1)^j C_k^j \left( \frac{B_{eq}}{E} \right)^j \frac{(2j)!!}{(2j+1)!!} \cdot \left( 1 + k \frac{(2j+2) B_{eq}}{(2j+3) B} \right) \right) \right] / \int \frac{dl}{B}.$$

Remember that  $B = B_{eq} / \sin^2 \theta$ , we perform an  $l$  integration and we obtain :

$$\Phi_0 = \int \frac{dl}{B} \left\{ \lambda_{neq} \left[ c_n + \frac{4}{\pi} \sum_{k=0}^{n+1} (-1)^k C_{n+1}^k \frac{(2k-1)!!}{(2k+2)!!} \cdot \sum_{j=0}^k (-1)^j C_k^j \frac{(2j)!!}{(2j+1)!!} \pi^{1/2} \frac{\Gamma(j+3/2)}{\Gamma(j+2)} \cdot \left( 1 + k \frac{(2j+2)(j+3/2)}{(2j+3)(j+2)} \right) \right] \right\} / \int \frac{dl}{B}.$$

where we have used the following integral:

$$\int_0^{\pi} d\theta \sin^p \theta = \pi^{1/2} \frac{\Gamma((p+1)/2)}{\Gamma(p/2+1)}, \quad (D6)$$

where  $\Gamma$  is the classical Gamma function ( $\Gamma(n+1/2) = (2n-1)!! \pi^{1/2} / 2^n$ ). Finally, we obtain

$$\Phi_0 = \lambda_{neq} (c_n + S_n), \quad (D7)$$

where we have defined

$$S_n = \sum_{k=0}^{n+1} (-1)^k C_{n+1}^k \frac{(2k-1)!!}{(2k+2)!!} \cdot \sum_{j=0}^k (-1)^j C_k^j \frac{(2j)!!}{2^{j-1}(j+1)!} \left(1 + k \frac{j+1}{j+2}\right). \quad (\text{D8})$$

## Appendix E: Calculation of the Bounce-Averaged $(\delta \vec{E}_y \times \vec{B})/B^2$ Drift

Taking into account the expression (41) of the perpendicular electric field  $\delta E_y$ , the bounce-averaged electric drift writes

$$\overline{u_{E_y}} = \frac{1}{\tau_b} \oint \frac{dl}{|v_{\parallel}|} \frac{\delta E_y}{B} = -\frac{4L}{v\tau_b} \frac{\delta E_{L,y,t}(L,y,t)}{B_{eq}} \frac{1}{n+1} \cdot \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{d\theta}{\left(1 - \frac{\mu B_{eq}}{E \sin^2 \theta}\right)^{1/2}} \left(S_n - (\sin^2 \theta)^{n+1}\right) \sin^2 \theta.$$

Using the same change of variables as in Appendix C, we obtain

$$\overline{u_{E_y}} = \frac{4L}{v\tau_b} \frac{\delta E_{L,y,t}(L,y,t)}{B_{eq}} \frac{1}{n+1} \cdot \int_0^1 \frac{dZ}{(1-Z^2)} \left(S_n(1-AZ^2) - (1-AZ^2)^{n+2}\right).$$

Again, we use the binomial formula and perform the  $Z$  integration (see Appendix C), we get

$$\overline{u_{E_y}} = \frac{4L}{v\tau_b} \frac{\delta E_{L,y,t}(L,y,t)}{B_{eq}} \frac{1}{n+1} \cdot \left(S_n \frac{\pi}{2} \left(1 - \frac{A}{2}\right) - \sum_{k=0}^{n+2} (-1)^k C_{n+2}^k A^k \frac{(2k-1)!!}{(2k+2)!!} \frac{\pi}{2}\right).$$

Then, we substitute the expression of  $\tau_b = 2\pi L/v$  and  $A = (1 - \mu B_{eq}/E)$  and obtain

$$\overline{u_{E_y}} = \frac{\delta E_{L,y,t}(L,y,t)}{B_{eq}} \frac{1}{n+1} \left[ \frac{S_n}{2} \left(1 + \frac{\mu B_{eq}}{E}\right) - \sum_{k=0}^{n+2} (-1)^k C_{n+2}^k \frac{(2k-1)!!}{(2k+2)!!} \sum_{j=0}^k (-1)^j C_k^j \left(\frac{\mu B_{eq}}{E}\right)^j \right].$$

**Acknowledgments.** We thank G. Belmont and D. Le Quéau for useful discussions. This work was partially supported by University of Versailles St Quentin and by a post-doctoral fellowship funded by CNES.

Michel Blanc thanks Liu Chen and another referee for their assistance in evaluating this paper.

## References

- Angelopoulos, V., W. Baumjohann, C. F. Kenel, F. V. Coroniti, M. G. Kivelson, R. Pellat, R. J. Walker, H. Lühr, and G. Paschmann, Bursty bulk flows in the inner central plasma sheet, *J. Geophys. Res.*, **97**, 4027, 1992.
- Antonsen, T. M., Jr., and B. Lane, Kinetic equations for low frequency instabilities in inhomogeneous plasmas, *Phys. Fluids*, **23**, 1205, 1980.
- Baumjohann, W., G. Paschmann, and C. A. Cattell, Average plasma properties in the central plasma sheet, *J. Geophys. Res.*, **94**, 6597, 1989.
- Bernstein, I. B., E. A. Frieman, M. D. Kruskal, and R. M. Kulsrud, An energy principle for hydromagnetic stability, *Proc. Roy. Soc. London, Ser. A*, **244**, 17, 1958.
- Chen, L., Theory of plasma transport induced by low-frequency hydromagnetic waves, *J. Geophys. Res.*, **104**, 2421, 1999.
- Chen, L., and A. Hasegawa, On magnetospheric hydromagnetic waves excited by energetic ring-current particles, *J. Geophys. Res.*, **93**, 8763, 1988.
- Chen, L., and A. Hasegawa, Kinetic theory of geomagnetic pulsations, 1, Internal excitations by energetic particles, *J. Geophys. Res.*, **96**, 1503, 1991.
- Cheng, C. Z., Kinetic theory of collisionless ballooning modes, *Phys. Fluids*, **25**, 1020, 1982.
- Cheng, C. Z., and J. R. Johnson, A kinetic-fluid model, *J. Geophys. Res.*, **104**, 413, 1999.
- Cowley, S. W. H., and M. Ashour-Abdalla, Adiabatic plasma convection in a dipole field: variation of plasma bulk parameters with  $l$ , *Planet. Space Sci.*, **23**, 1527, 1975.
- Erickson, G. M., A quasi-static magnetospheric convection model in two dimensions, *J. Geophys. Res.*, **97**, 6505, 1992.
- Forme, F., D. Fontaine, and M. A. L. Persson, Large perpendicular velocity fluctuations observed in the topside ionosphere, *J. Geophys. Res.*, **103**, 4001, 1998.
- Frank, L. A., W. R. Paterson, K. L. Ackerson, S. Kokubun, and T. Yamamoto, Plasma velocity distributions in the near-Earth plasma sheet: A first look with the Geotail spacecraft, *J. Geophys. Res.*, **101**, 10,627, 1996.
- Huang, C. Y., and L. A. Frank, A statistical study of the central plasma sheet: Implications for substorm models, *Geophys. Res. Lett.*, **13**, 652, 1986.
- Huang, T. S., and T. J. Birmingham, Kinetic and thermodynamic properties of a convecting plasma in a two dimensional dipole field, *J. Geophys. Res.*, **99**, 17,295, 1994.
- Hurricane, O. A., The kinetic theory and stability of a stochastic plasma with respect to low frequency perturbations and magnetospheric convection, Ph.D. thesis, Univ. of Calif., Los Angeles, 1994.
- Hurricane, O. A., R. Pellat, and F. V. Coroniti, The kinetic response of a stochastic plasma to low frequency perturbations, *Geophys. Res. Lett.*, **21**, 253, 1994.
- Hurricane, O. A., R. Pellat, and F. V. Coroniti, The stability of a stochastic plasma with respect to low frequency perturbations, *Phys. Plasmas*, **2**, 289, 1995a.
- Hurricane, O. A., R. Pellat, and F. V. Coroniti, A new approach to low-frequency "mhd-like" waves in magnetospheric plasmas, *J. Geophys. Res.*, **100**, 19,421, 1995b.
- Jacquey, C., Time-variation of the large-scale tail magnetic field prior to substorm related to solar wind changes, in *Third International Conference on Substorms*, edited by E. J. Rolfe and B. Kaldeich, p. 295, ESA, Noordwijk, The Netherlands, 1996.
- Johnson, J. R., and C. Z. Cheng, Kinetic alfvén waves and plasma transport at the magnetopause, *Geophys. Res. Lett.*, **104**, 413, 1997.
- Kennel, C. F., *Convection and Substorms*, Oxford Univ. Press, New York, 1995.
- Le Contel, O., R. Pellat, and A. Roux, Self-consistent quasi-static parallel electric field associated with substorm growth phase, *J. Geophys. Res.*, this issue.
- Lui, A. T., et al., Current disruption in the near-Earth neutral sheet region, *J. Geophys. Res.*, **97**, 1461, 1992.
- McPherron, R. L., Magnetospheric substorms, *Rev. Geophys.*, **17**, 657, 1979.

- Nagai, T., An empirical model of substorm-related magnetic field variations at synchronous orbit, in *Magnetospheric Substorms, Geophys. Monogr. Ser.*, edited by J. R. Kan, T. R. Potemra, S. Kokobun, and T. Iijima, vol. 64, p. 91, AGU Washington, D. C., 1991.
- Pellat, R., Une nouvelle approche de la reconnexion magnétique: Sous-orages magnétosphériques - vents stellaires, *C. R. Acad. Sci.*, 311, 1706, 1990.
- Pellat, R., O. A. Hurricane, and F. V. Coroniti, Multipole stability revisited, *Phys. Plasmas*, 1, 3502, 1994.
- Rosenbluth, M. N., and R. K. Varma, Approximate equations for plasmas in mirror machines, *Nucl. Fusion*, 7, 33, 1967.
- Roux, A., et al., Plasma sheet instability related to the westward traveling surge, *J. Geophys. Res.*, 96, 17,697, 1991.
- Rutherford, P. H., and E. A. Frieman, Drift instabilities in general magnetic field configurations, *Phys. Fluids*, 11, 569, 1968.
- Sauvaud, J.-A., and J. R. Winckler, Dynamics of plasma, energetic particles, and fields near synchronous orbit in the nighttime sector during magnetospheric substorms, *J. Geophys. Res.*, 85, 2043, 1980.
- Sauvaud, J.-A., T. Beutier, and D. Delcourt, On the origin of flux dropouts near geosynchronous orbit during the growth phase of substorms, 1, Betatron effects, *J. Geophys. Res.*, 101, 19,911, 1996.
- Schindler, K., and J. Birn, A theory of the substorm mechanism, *J. Geophys. Res.*, 87, 2263, 1982.
- Sergeev, V. A., D. G. Mitchell, C. T. Russel, and D. J. Williams, Structure of the tail plasma/current sheet at  $\simeq 11 R_E$  and its changes in the course of a substorm, *J. Geophys. Res.*, 98, 17,345, 1993.
- Slavin, J. A., E. J. Smith, D. G. Sibeck, D. N. Baker, R. D. Zwickl, and S.-I. Akasofu, An ISEE 3 study of average and substorm conditions in the distant tail, *J. Geophys. Res.*, 90, 10,875, 1965.
- Zwillinger, D., *Handbook of Differential Equations*, Harcourt Brace Jovanovich, Boston, Mass., 1989.

---

O. Le Contel and A. Roux, Centre d'Etude des Environnements Terrestre et Planétaires, 10-12 avenue de l'Europe, F-78140 Vélizy, France. (olivier.lecontel@cetp.ipsl.fr)

R. Pellat, Centre de Physique Théorique, Ecole Polytechnique, F-91128, Palaiseau, France.

(Received June 10, 1998; revised September 3, 1999; accepted November 30, 1999.)