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# Comment on “Do magnetospheric shear Alfvén waves generate sufficient electron energy flux to power the aurora?”

by C. E. Watt and R. Rankin

F. Mottez<sup>1</sup>

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## 1. Introduction

[1] *Watt and Rankin* [2010] used a 1-D self-consistent numerical code to investigate whether electron acceleration by shear Alfvén waves in the plasma sheet boundary layer is sufficient to cause auroral brightening in the ionosphere. Their numerical simulation code is based on a series of equations first derived by *Thompson and Lysak* [1996] in a 2-D geometrical context and adapted to a 1-D ( $z$  dependent) geometry. In 2-D, the directions of variation are  $x$  and  $z$  (parallel to the ambient magnetic field  $B_0$ ), and the computed magnetic field (the perturbed one) is supposed to be perpendicular to the  $z$  axis. Then the perturbed electromagnetic field can be written in terms of the electric potential  $\Phi$ , and of the parallel component  $\mathcal{A}_{\parallel}$  of the vector potential.

[2] In the 1-D simulations of *Watt and Rankin* [2010], the variations along the vertical  $z$  axis are expressed through partial derivatives. The perpendicular direction  $x$  is actually not a direction of invariance, because we are interested in the behavior of an oblique shear Alfvén wave. The wave is assumed with a single perpendicular wave number  $k_{\perp}$  that depends on  $z$ . The scalar and vector potentials are written

$$\begin{aligned} \mathcal{A}_{\parallel}(z, x, t) &= A_{\parallel}(z, t) \exp [ik_{\perp}(z)x] \\ \Phi(z, x, t) &= \phi(z, t) \exp [ik_{\perp}(z)x]. \end{aligned} \quad (1)$$

A gauge equation is used, and, according to *Watt and Rankin* [2010], for a 1-D geometry, it is

$$\frac{\partial \phi}{\partial t} + \frac{v_A^2}{k_{\perp}^2} \frac{\partial A_{\parallel} k_{\perp}^2}{\partial z} = 0. \quad (2)$$

In the present comment, it is argued that the gauge equation (2) is not correct and the concept is not appropriate for a 1-D geometry with a perpendicular wave number that depends on the altitude  $z$ .

## 2. The Gauge in 2-D Geometry

[3] Let us see how this gauge is derived in a 2-D context (with SI units). One of the equations used to build the gauge is the Maxwell-Ampere equation projected on the perpendicular  $x$  direction

$$\frac{\partial^2 \mathcal{A}_{\parallel}}{\partial z \partial x} = \mu_0 j_{\perp}. \quad (3)$$

This is the reduced Maxwell-Ampere equation because the displacement current has been neglected (which supposes  $v_A \ll c$ ). It is also assumed, as in the derivation of the linearized equations of the inertial Alfvén wave [*Goertz*, 1984], that the perpendicular current is carried only by the ions and that their motion is caused by the polarization drift

$$u_x = \frac{m_i}{eB_0} \frac{\partial^2 \Phi}{\partial t \partial x}, \quad (4)$$

where  $u_x$  is the mean ion velocity in the  $x$  direction. When equations (3) and (4) are combined, we find

$$\frac{\partial}{\partial x} \left[ v_A^2 \frac{\partial \mathcal{A}_{\parallel}}{\partial z} + \frac{\partial \Phi}{\partial t} \right] = 0. \quad (5)$$

One of the solutions is

$$v_A^2 \frac{\partial \mathcal{A}_{\parallel}}{\partial z} + \frac{\partial \Phi}{\partial t} = 0. \quad (6)$$

When the displacement current is kept in the Maxwell-Ampere equation, we have instead

$$\frac{c^2 v_A^2}{c^2 + v_A^2} \frac{\partial \mathcal{A}_{\parallel}}{\partial z} + \frac{\partial \Phi}{\partial t} = 0. \quad (7)$$

This is the gauge derived by *Thompson and Lysak* [1996], and used by them and other authors in 1-D numerical simulations. The gauge equation (6) is pertinent as long as  $v_A \ll c$ . This gauge is based on a characteristic property of inertial Alfvén waves and on the choice of a particular (and simple) first integral of the second order differential equation (5).

## 3. The 1-D Case

[4] When the Ansatz equation (1) is used, equation (5) becomes

$$ik_{\perp} \frac{\partial \mathcal{A}_{\parallel}}{\partial z} + i \left( \frac{dk_{\perp}}{dz} \right) A_{\parallel} - \left( \frac{dk_{\perp}}{dz} \right) k_{\perp} x A_{\parallel} + i \frac{1}{v_A^2} k_{\perp} \frac{\partial \phi}{\partial t} = 0. \quad (8)$$

This is not equivalent to equation (2) (unless one formally replaces  $k_{\perp} x$  with  $-i$  in the third term). This equation is not

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<sup>1</sup>LUTH, Observatoire de Paris, CNRS, Université Paris Diderot, Meudon, France.

Corresponding author: F. Mottez, LUTH, Observatoire de Paris, CNRS, Université Paris Diderot, 5 Place Jules Janssen, 92190 Meudon, France. (fabrice.mottez@obspm.fr)

even compatible with a 1-D geometry, because the second coordinate  $x$  appears explicitly. Of course, it would be possible to solve the problem for  $x = 0$ , but in that case, we should use the gauge

$$\frac{\partial \phi}{\partial t} + \frac{v_A^2}{k_\perp} \frac{\partial A_\parallel k_\perp}{\partial z} \quad (9)$$

that is different from equation (2). But even in that case, the solution would be valid only for the peculiar value  $x = 0$ , from which it would be difficult to derive general results about electron acceleration over the whole auroral arc. When  $k_\perp$  depends on the altitude  $z$ , it is therefore better not to carry 1-D simulations using this formalism.

#### 4. Discussion

[5] Another equation is also used in the same context by *Watt et al.* [2005] and *Watt and Rankin* [2008],

$$\frac{\partial \phi}{\partial t} + v_A^2 \frac{\partial A_\parallel}{\partial z}. \quad (10)$$

This gauge equation is correct because  $k_\perp$  is considered as independent of the altitude  $z$ .

[6] Actually, even when  $k_\perp$  varies, there is a way of managing 1-D simulations. *Watt and Rankin* [2010] state that  $k_\perp$  is scaled along the simulation domain according to the dipolar configuration of the magnetic field. The dipole coordinates  $v$ ,  $\phi$ , and  $\mu$  are then appropriate. A magnetic field line is labeled by  $v$  and  $\phi$ , and  $\mu$  is the coordinate along the magnetic field line. If we suppose that

$$A_\mu = A_\parallel(\mu) \exp[ik_\perp v], \quad (11)$$

where  $k_\perp$  is a constant, the perpendicular wavelength varies proportionally to the distance between the magnetic field lines. Dipole coordinates are used in *Lysak* [2004] and *Lysak et al.* [2013], as well as the low-frequency approximation is used (equivalent to equation (4) combined with the Maxwell-Ampere equation). In *Lysak* [2004], the electromagnetic field is computed without the help of the scalar and vector potentials, and no gauge is necessary. But it can be shown that the gauge would be equivalent to

$$\frac{1}{v_A^2} \frac{\partial^2 \Phi}{\partial t \partial v} + \frac{1}{h^2} \frac{\partial}{\partial \mu} \left[ h^2 B^2 \frac{\partial}{\partial v} \left( \frac{A_\mu}{B^2} \right) \right] = 0, \quad (12)$$

where  $h = r \sin \theta$ ,  $v = R_E \sin^2 \theta / r$ , and  $\mu = R_E^2 \cos \theta / r^2$ , and  $r$ ,  $\theta$ , and  $\phi$  are the usual spherical coordinates (*R. Lysak*, private communication, 2013).

[7] A useful approximation is also given by quasi-dipolar coordinates that, for low altitudes, mimic correctly the dipolar scaling [*Lysak and Song*, 2002]. They can be defined as the orthogonal system  $v = x/\sqrt{B}$ ,  $\phi = y/\sqrt{B}$ , and  $\mu = z$ , where  $z$  is the altitude and the modulus of the magnetic field  $B \propto r^{-3}$ . In that case,

$$\frac{1}{v_A^2} \frac{\partial^2 \Phi}{\partial t \partial v} + \frac{\partial^2 A_\mu}{\partial v \partial \mu} = 0. \quad (13)$$

This is formally very similar to equation (5). With the perpendicular wave vector scaled according to equation (11), the gauge is simply equation (9).

[8] It should be emphasized that in spite of the interest of 1-D models, a single perpendicular wave vector does not really correspond to the output of 2-D simulations (see, for instance, [*Swift*, 2007]). Moreover, observations have shown that the plasma density is not uniform in the  $x$  direction either. As shown by *Génot et al.* [1999], the transverse gradient of Alfvén velocity (along the direction  $x$ ) is a source of parallel accelerating electric field that is complementary to the parallel density gradient. In their section “Discussion,” *Watt and Rankin* [2010] write that it is not clear how magnetospheric Shear Alfvén waves develop the short perpendicular scales required to accelerate auroral electrons. Actually, the transverse gradient of Alfvén velocity (along the direction  $x$ ) is a *cause* of perpendicular wave structure [*Génot et al.* 2001a, 2001b], and the electron acceleration can be studied in this context [*Génot et al.*, 2004]. And here again, it appears that the transverse structure of the wave cannot be simply represented with a single transverse wave number.

[9] Independently of the geometry of the system, one must also be careful about the plasma regime. The gauge derivation is based on the hypothesis of cold ions, that is closely related to the property of inertial Alfvén waves (through equation 4). The typical transverse scale allowing an associated parallel electric field is  $k_\perp c/\omega_{pe} \sim 1$ , where  $c/\omega_{pe}$  is the electron inertial length. With a warmer plasma, parallel electric fields can be associated to kinetic Alfvén waves, where  $k_\perp \rho_i \sim 1$  where  $\rho_i$  is the ion Larmor radius [*Hasegawa and Mima*, 1978]. The transition between the inertial and the kinetic regime depends on the ratio  $\beta m_i/m_e$ , where  $m_i$  and  $m_e$  are the ion and electron masses, and  $\beta$  is the ratio of the plasma to magnetic pressures. This ratio is smaller than one in the inertial regime where the gauge is relevant, and larger in the kinetic regime, where equations (6) and (7) are not proven.

[10] The study by *Watt and Rankin* [2010] covers the range  $4 - 7 R_E$  of radial distances for a magnetic field line reaching the equatorial plasma sheet boundary layer at  $9 R_E$ . If it is well established that low altitude auroral field lines correspond to the inertial regime, it is not true near the plasma sheet. It is questionable whether the inertial regime approximation required in the derivation of the gauge equations (6) and (7) is still appropriate at these distances from the Earth.

#### 5. Conclusion

[11] The gauge given in equation (7) is appropriate to study the acceleration of electrons by inertial Alfvén waves in a 2-D geometry. It can be simplified into equation (6) if  $v_A \ll c$ . This equation has been used in the 1-D case for Alfvén waves having a unique perpendicular wave number  $k_\perp$  that is independent of the altitude  $z$ . This is correct. But when the transverse wave number is a function  $k_\perp(z)$  of  $z$ , the gauge equation (2) used in *Watt and Rankin* [2010] is not correct. As shown with equation (8), we find instead that the coordinate  $x$  is an explicit parameter of the gauge equation, making 1-D calculations impossible.

[12] This conclusion does not mean that the work of *Watt and Rankin* [2010] should be discarded. Indeed, this paper contains a lot of significant physics regarding auroral

electron acceleration. But equation (2) should better not be used anymore, as well as the hypothesis of a  $z$  dependent perpendicular wave vector  $k_{\perp}(z)$  in 1-D simulations.

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