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AN IMPROVED GREENOFDM SCHEME FOR PAPR REDUCTION

A PREPRINT

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ABSTRACT

This paper presents an improvement to a recent Peak-to-Average-Power-Ratio (PAPR) reduction technique in Orthogonal Frequency Division Multiplexing (OFDM) modulation, the GreenOFDM. This technique, which is inspired by the well-known PAPR reduction technique SeLected Mapping (SLM), generates several waveform candidates from a certain amount of Inverse Fast Fourier Transforms (IFFT) and selects the one with the lowest PAPR for the transmission of the actual OFDM symbol. In a previous work it was shown that, for the same amount of IFFTs (U), the PAPR reduction capabilities of GreenOFDM were better than those of SLM-OFDM because more candidates were available for the former ($U^2/4$) than for the latter (U). In this work, a simple way to increase the number of candidates to U^2 instead of $U^2/4$ is presented, which further reduces the PAPR (or reduces by a factor of 2 the number of IFFTs to compute for the same PAPR performance).

Keywords OFDM · PAPR · Selected Mapping · GreenOFDM

1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a widely used modulation technique for wireless communication systems thanks to its high spectral efficiency and robustness against frequency-selective fading.

However, a well-known major drawback of OFDM is its high power consumption due to its high Peak-to-Average Power Ratio (PAPR), which requires highly linear amplifiers operating with a large back-off to avoid nonlinear distortion of the transmitted signal. Several techniques for PAPR reduction have been proposed [1]. The SeLected Mapping OFDM (SLM-OFDM) [2][3] is a well known one and has very recently been improved into the so-called GreenOFDM technique [4].

The baseline of SLM-OFDM is to generate a number of different OFDM symbol waveform candidates with the same data and to select the waveform with the lowest PAPR for transmission. The more candidates are generated, the lower the PAPR. GreenOFDM is a SLM-OFDM variant that increases the number of candidates in a square-scale while keeping the same number of required IFFTs as in conventional SLM-OFDM. The result is, for a given number of computed IFFTs, a better PAPR reduction for GreenOFDM as compared to SLM-OFDM.

In this paper we present a simple yet efficient way to obtain a higher number of candidates without increasing the number of IFFTs we need to compute. The proposed method differs from other GreenOFDM improvements [5, 6] where the problem is not attacked at the root. The number of candidates must be increased in order to improve the PAPR reduction performance without increasing the number of IFFTs to compute.

This document is organized as follows: First, a brief OFDM and PAPR description is given. Then, SLM-OFDM and GreenOFDM algorithms are succinctly presented before introducing the improved version. In the last part, computer simulations of the PAPR distribution are presented to clearly demonstrate the improvement of the proposed technique and finally conclusions are drawn.

2 OFDM and PAPR

OFDM is a multi-carrier modulation technique in which every time-domain symbol carries the sum of N overlapping orthogonal subcarriers mapped by $\{A_k\}$ symbols (e.g. Quadrature Amplitude Modulation: QAM symbols) with $0 \leq k \leq N - 1$.

The OFDM base-band time-domain symbol is obtained by applying the Inverse Discrete Fourier Transform (IDFT)¹ to the frequency-domain inputs $\{A_k\}$. The digital time-domain symbols are often oversampled in order not to miss out any peak that might appear in the analog and hence obtain a well-approximated digital version. A LN points IFFT over

$$\{X_k\} = \{A_0 A_1 \cdots A_{\frac{N}{2}-1} \underbrace{0 \cdots 0}_{(L-1) \cdot N} A_{\frac{N}{2}} A_{\frac{N}{2}+1} \cdots A_{N-1}\}$$

achieves the oversampling, where L is the oversampling factor. The OFDM base-band time-domain symbol is hence given by:

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{LN-1} X_k \cdot \exp \left\{ i2\pi \frac{kn}{LN} \right\} \quad (1)$$

with $0 \leq n \leq LN - 1$ and $i = \sqrt{-1}$. Generally an oversampling factor $L = 4$ is applied since it is enough to well approximate the analog symbols. Higher values will only add a higher computational complexity without substantial gain in the approximation.

The PAPR of the oversampled OFDM symbol is given by:

$$\text{PAPR} = \frac{\max_{0 \leq n \leq LN-1} \{|x[n]|^2\}}{\mathbb{E}\{|x[n]|^2\}} \quad (2)$$

where $\mathbb{E}\{\cdot\}$ is the expectation and $\mathbb{E}\{|x[n]|^2\} = \mathbb{E}\{|X_k|^2\}$ for X_k i.i.d. random variables ($\{X_k\} \in 2^m$ -QAM with $n = 2m, m \in \mathbb{N}^+$). The PAPR of OFDM symbols is high because the numerator of Eq. (2) is high. This is because *sometimes* many modulated subcarriers $X_k \cdot \exp \left\{ i2\pi \frac{kn}{LN} \right\}$ add themselves in-phase generating high peaks. Furthermore, since the PAPR is a random variable it can be characterized by its Complementary Cumulative Distribution Function (CCDF), i.e. the probability that the PAPR of an OFDM symbol exceeds a predetermined threshold γ . A semi-empirical approximation is given by [7]:

$$\text{CCDF}_{\text{OFDM}}(\gamma) \approx 1 - (1 - \exp\{-\gamma\})^{2.8N} \quad (3)$$

3 From SLM-OFDM to GreenOFDM

In conventional SLM-OFDM, U different symbol waveforms are generated from the same data set $\{X_k\}$. To do so, the input data set $\{X_k\}$ is repeated U times and each copy is multiplied element-wise by pseudo-random sequences where the coefficients have unitary norm $\phi_{u,k}$, $0 \leq u \leq U - 1$; i.e. $X_{u,k} = X_k \cdot \phi_{u,k}$ with $|\phi_{u,k}| = 1$.

It is noteworthy to mention that in the following, U represents the total number of computed IFFTs as well as the total number of pseudo-random sequences.

Then, the time-domain symbol waveforms $x_u[n]$ are computed by zero-padded LN -IFFTs with inputs $\{X_{u,k}\}$. The waveform that exhibits the lowest PAPR among the $C_{\text{SLM}} = U$ candidates is selected for transmission. The SLM-OFDM algorithm is summarized in Algorithm 1.

The CCDF of PAPR of conventional SLM-OFDM is well approximated by [3]:

$$\text{CCDF}_{\text{SLM}}(\gamma) \approx \left(1 - (1 - \exp\{-\gamma\})^{2.8N} \right)^{C_{\text{SLM}}} \quad (4)$$

The function $\text{CCDF}_{\text{SLM}}(\text{PAPR})$ decreases when the number of candidates $C_{\text{SLM}} = U$ increases. However, higher values of C_{SLM} lead to larger values of the computational burden because of the required number of IFFTs to compute.

In order to encompass this conventional SLM limitation, the key idea of GreenOFDM is to generate the candidates by adding (with a normalisation factor $\sqrt{2}$) two coupleable IFFT-based waveforms.

¹Digitally implemented with an Inverse Fast Fourier Transform (IFFT).

Algorithm 1 The SLM-OFDM algorithm.

Require: $\{X_k\}, \{\phi_{u,k}\}$ with $\phi_{u,k} \in \{\pm 1\}$
 $\text{minPAPR} \leftarrow +\infty;$
for ($u \leftarrow 0$ **to** $U - 1$) **do**
 $\{X_{u,k}\} \leftarrow \{X_k \cdot \phi_{u,k}\};$
 $\{x_u[n]\} \leftarrow \text{IFFT}\{X_{u,k}\};$
if ($\text{PAPR}\{x_u[n]\} < \text{minPAPR}$) **then**
 $\text{minPAPR} \leftarrow \text{PAPR}\{x_u[n]\};$
 $\{x_{\bar{u}}[n]\} \leftarrow \{x_u[n]\};$
end if
end for
SEND $\{x_{\bar{u}}[n]\};$

This allows for the generation of more candidates C_{Green} corresponding to all the possible combinations, whilst maintaining the same PAPR reduction behaviour of SLM-OFDM with respect to the number of candidates (i.e. equation (4) with C_{Green} instead of C_{SLM}).

The GreenOFDM technique generates $C_{\text{Green}} = U^2/4$ OFDM symbols candidates by first splitting the U set of copied and randomized data $\{X_{u,k}\}$ into two groups: a first one represented by the index g_1 , with $0 \leq g_1 \leq \frac{U}{2} - 1$ and a second one represented by the index g_2 , with $\frac{U}{2} \leq g_2 \leq U - 1$. Then, the first group waveforms $\{x_{g_1}[n]\}$ is obtained, like in SLM-OFDM, through IFFTs over $\{X_{g_1,k} = X_k \cdot \phi_{g_1,k}\}$ (with $\{\phi_{g_1,k}\} = \{\phi_{u,k}\}$, $u < \frac{U}{2}$ and $\phi_{u,k} \in \{\pm 1\}$).

Similarly, the second group waveforms $\{x_{g_2}[n]\}$ is obtained through IFFTs over $\{X_{g_2,k} = X_k \cdot \phi_{g_2,k}\}$ (with $\{\phi_{g_2,k}\} = \{i \times \phi_{u,k}\}$, $u \geq \frac{U}{2}$ and $\phi_{u,k} \in \{\pm 1\}$). As it can be seen, the difference between the two groups of waveforms is the fact that the sequences $\{\phi_{g_2,k}\}$ use pure imaginary unitary values: $\{\pm i\}$ whereas the sequences $\{\phi_{g_1,k}\}$ use pure real unitary values: $\{\pm 1\}$. In this way, individual elements of the two sequences are orthogonal in order not to cancel out some subcarriers after adding the two IFFT outputs.

The last part of the GreenOFDM method, which depicts the way the $\frac{U^2}{4}$ waveform candidates are obtained, is described in Algorithm 2.

Algorithm 2 The GreenOFDM algorithm.

Require: $\{x_{g_1}[n]\} = \text{IFFT}\{X_k \cdot \phi_{g_1,k}\}, \{x_{g_2}[n]\} = \text{IFFT}\{X_k \cdot i \cdot \phi_{g_2,k}\}$
 $\text{minPAPR} \leftarrow +\infty;$
for ($g_1 \leftarrow 0$ **to** $\frac{U}{2} - 1$) **do**
for ($g_2 \leftarrow \frac{U}{2}$ **to** $U - 1$) **do**
 $\{x_{g_1,g_2}[n]\} \leftarrow \frac{\{x_{g_1}[n] + x_{g_2}[n]\}}{\sqrt{2}};$
if ($\text{PAPR}\{x_{g_1,g_2}[n]\} < \text{minPAPR}$) **then**
 $\text{minPAPR} \leftarrow \text{PAPR}\{x_{g_1,g_2}[n]\};$
 $\{x_{\bar{g}_1,\bar{g}_2}[n]\} \leftarrow \{x_{g_1,g_2}[n]\};$
end if
end for
end for
SEND $\{x_{\bar{g}_1,\bar{g}_2}[n]\};$

4 An improved GreenOFDM version

In order to increase the total number of candidates without increasing the number of IFFTs to compute, it is important to note that the waveforms from the first group $\{x_{g_1}[n]\} = \{x_u[n]\}$ with $u < \frac{U}{2}$ and that the waveforms from the second group $\{x_{g_2}[n]\} = i \times \{x_u[n]\}$ with $u \geq \frac{U}{2}$.

Indeed, due to the linearity of the IFFT, the $U/2$ complex waveforms of the second group are equal to the $U/2$ complex waveforms $\{x_u[n]\}$, only modified by a factor i . So, the second group could be obtained from the IFFT outputs $\{x_u[n]\}$ by only shifting the complex values $(\Re + i\Im)$ into $(-\Im + i\Re)$, eliminating the need to compute the IFFTs again.

The improved GreenOFDM method is described in Algorithm 3. As it can be observed, the first and second groups are redefined. The first group contains the U IFFTs (the equivalent to the SLM-OFDM generated candidates), and the second group is trivially generated by exploiting the linearity of the IFFT.

Algorithm 3 The improved GreenOFDM algorithm.

```

Require:  $\{x_u[n]\} = \text{IFFT}\{X_k \cdot \phi_{u,k}\}$ 
minPAPR  $\leftarrow +\infty$ ;
for ( $u_1 \leftarrow 0$  to  $U - 1$ ) do
  for ( $u_2 \leftarrow 0$  to  $U - 1$ ) do
    if  $u_1 == u_2$  then
       $\{x_{u_1, u_2}[n]\} \leftarrow \{x_{u_1}[n]\}$ ;
    else
       $\{x_{u_1, u_2}[n]\} \leftarrow \frac{\{x_{u_1}[n] + i \times x_{u_2}[n]\}}{\sqrt{2}}$ ;
    end if
    if ( $\text{PAPR}\{x_{u_1, u_2}[n]\} < \text{minPAPR}$ ) then
      minPAPR  $\leftarrow \text{PAPR}\{x_{u_1, u_2}[n]\}$ ;
       $\{x_{\bar{u}_1, \bar{u}_2}[n]\} \leftarrow \{x_{u_1, u_2}[n]\}$ ;
    end if
  end for
end for
SEND  $\{x_{\bar{u}_1, \bar{u}_2}[n]\}$ ;

```

In the particular case $u_1 = u_2$, there is no need to perform the additions $\{i \cdot x_{u_1}[n] + x_{u_2}[n]\}$ because it does not change the PAPR of the initial waveform $\{x_{u_1}[n]\}$. In the proposed method, these extra operations are avoided in that particular case ($u_1 = u_2$) by choosing the candidate directly from the IFFT output, i.e. we choose the candidate to be equal to $\{x_{u_1}[n]\}$.

The number of candidates that are generated in the proposed method, here after referred to as the *GreenOFDMv2*, is equal to the number of times the inner loop is run. It corresponds to $C_{\text{Greenv2}} = U^2$. This means that we have 4 times more candidate waveforms as compared to the initial version of the GreenOFDM by computing the same number of IFFTs, hence a better PAPR reduction is expected.

5 Simulations

Computer simulations were carried out to compare the CCDF of PAPR of GreenOFDM and the improved version proposed in this work.

Results for 10^5 symbols with $N = 64$ QPSK-modulated subcarriers, with the oversampling factor $L = 4$ and $U = 16$ IFFTs for the SLM-OFDM, GreenOFDM and the GreenOFDMv2, are depicted in Fig.1 where the dashed lines correspond to the approximated expressions of the CCDF in Eq.(3) and Eq.(4) and the marks correspond to the simulation results.

The simulation results are well approximated by the CCDFs expressions for OFDM, SLM-OFDM (with $C_{\text{SLM}} = U$), GreenOFDM (by replacing C_{SLM} by $C_{\text{Green}} = U^2/4$) and GreenOFDMv2 (by replacing C_{SLM} by $C_{\text{Green}} = U^2$).

Moreover, it can be seen that the best performance for a fixed probability (CCDF(γ)) is obtained for the GreenOFDMv2 method as compared to SLM-OFDM and the previously proposed GreenOFDM, with a gain of 1,2 dB and 0,5 dB respectively for a CCDF(γ) = 10^{-3} (the gain versus conventional OFDM is about 5 dB). This is because of the increase in the number of candidates (U^2 for GreenOFDMv2 rather than U for SLM-OFDM or $\frac{U^2}{4}$ for GreenOFDM).

Different values of N and U were also simulated to highlight the advantage of the proposed method in different configurations. To simplify the representation, the value of γ is calculated against the number of sub-carriers and the number of candidates for a fixed value of CCDF(γ) = p . Eqs.(3)&(4) were reformulated in order to compare the approximated expressions with the simulations. In the case of OFDM, Eq.(3) was reformulated as:

$$\gamma \approx -\log\left(1 - (1 - p)^{\frac{1}{2.8N}}\right) \quad (5)$$

and for the other methods, Eq.(3) was reformulated as:

$$\gamma \approx -\log\left(1 - \left(1 - p^{\frac{1}{C}}\right)^{\frac{1}{2.8N}}\right) \quad (6)$$

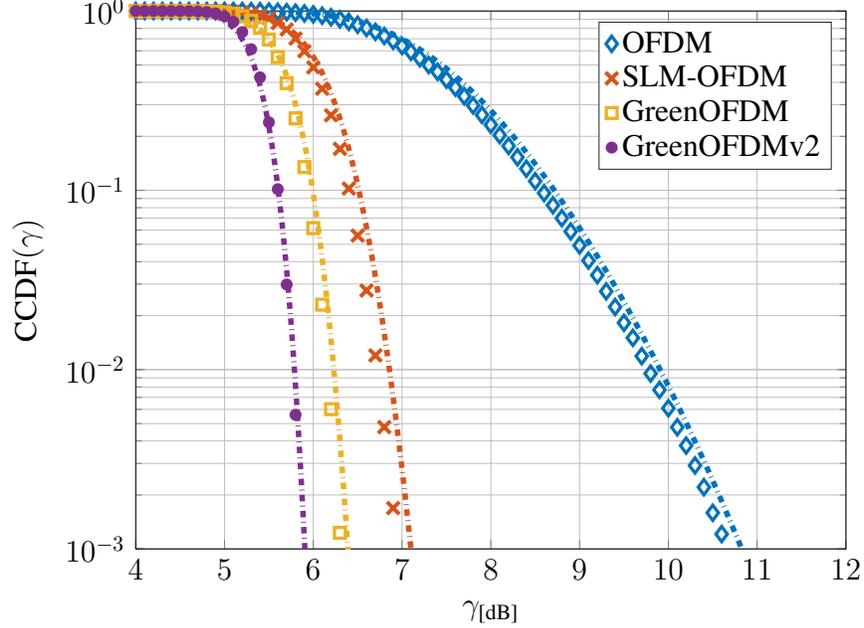


Figure 1: The CCDF of PAPR for conventional OFDM, SLM-OFDM, GreenOFDM and GreenOFDMv2 with the same parameter $U = 16$ and for $N = 64$ subcarriers per symbol.

where $C = U$ for SLM-OFDM, $C = \frac{U^2}{4}$ for GreenOFDM and $C = U^2$ for the proposed method.

Results are summarized in Fig.2 where the performances for different values of U and N are plotted at a fixed probability $\text{CCDF}(\gamma) = 10^{-3}$. For OFDM, SLM-OFDM, GreenOFDM and the proposed method, the dots represent the simulation results and the dashed lines correspond to the approximated expressions in (5) and (6).

In all the cases, the value of $\gamma[\text{dB}]$ is always lower for the proposed method as compared to the SLM-OFDM and the GreenOFDM. However, the difference with the original version of GreenOFDM reduces as U increases.

It can also be noted that, for all the tested values of N and U , the CCDF threshold γ obtained by simulation is very close to the one obtained from Eq.(6). It means that the PAPR performance of the proposed technique is also well deterministically linked to the number of candidates and can be accurately predicted by the use of the Eqs.(4) and (6).

Another way to interpret the results is in terms of the reduced number of IFFTs needed to attain the same performance (hence the same number of available candidates).

To facilitate the observation of this fact, Fig.3 regroups the different configurations with equivalent performances where for all the cases, the proposed method requires the smallest amount of IFFTs to compute, leading to a reduction factor of 2 with respect to the initial GreenOFDM method.

Therefore, the proposed method contributes to the reduction of the implementation complexity and can be adapted to implement the methods proposed in [8] for further computational complexity reductions.

The drawback, as for all the SLM-based methods, is the need of a Side Information (SI) in order to correctly de-randomize the received data symbols on the receiver-side.

6 Conclusion

The proposed method adds a simple yet efficient improvement to the GreenOFDM to reduce the PAPR by increasing the number of available candidates by a factor of 4 without increasing the number of IFFTs to compute.

Alternatively, the same PAPR can be obtained by using only half the number of IFFTs in comparison to the first version of the GreenOFDM method. Finally, the strong complexity reduction compared to the original Selected Mapping Method, renders GreenOFDM a serious candidate for the future energy efficient OFDM systems.

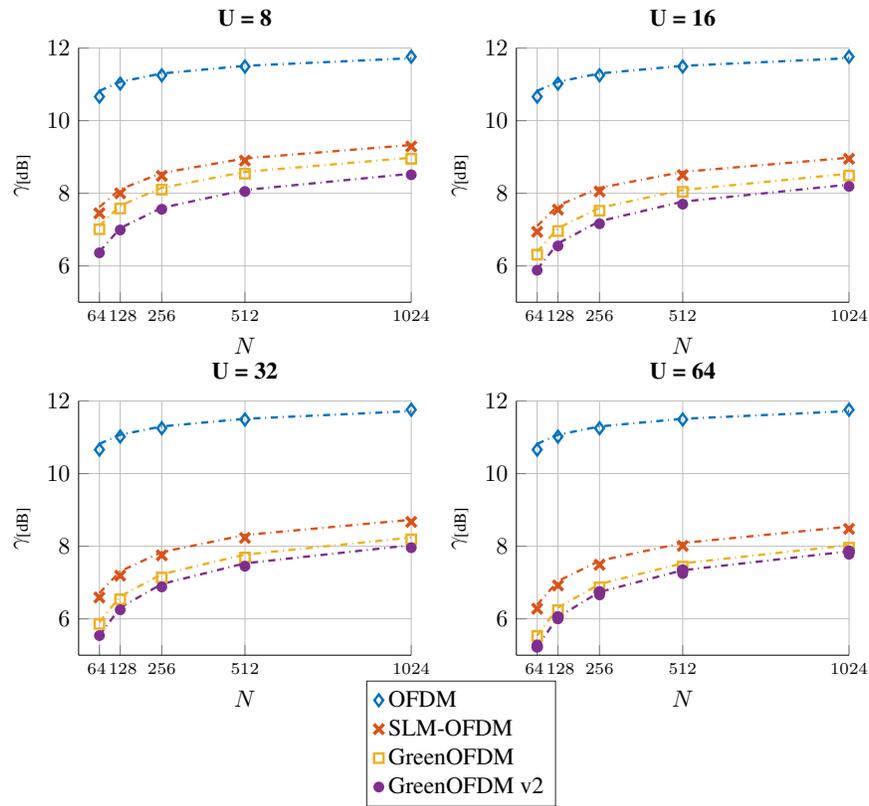


Figure 2: The value of γ [dB] to attain $\text{CCDF}(\gamma) = 10^{-3}$ for different values of U and N for OFDM, SLM-OFDM, GreenOFDM and GreenOFDMv2.

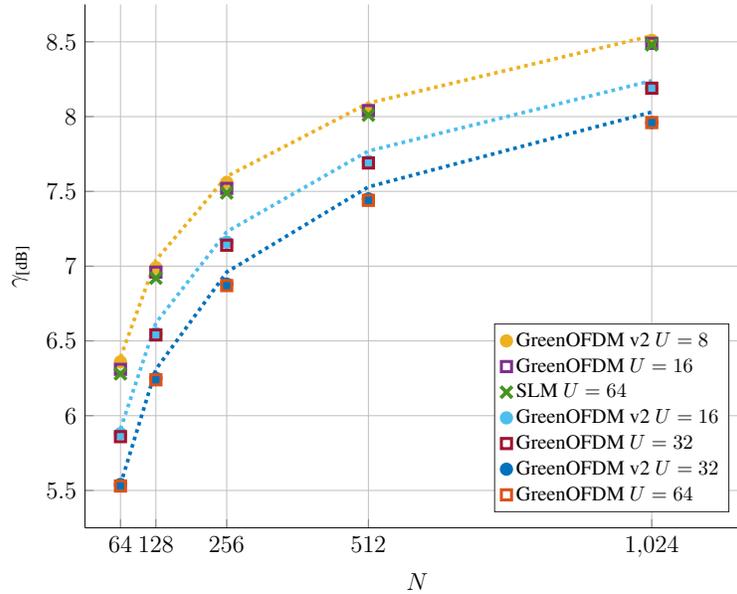


Figure 3: The value of γ [dB] to attain $\text{CCDF}(\gamma) = 10^{-3}$ for different values of U and N for SLM-OFDM, GreenOFDM and GreenOFDMv2 obtained by simulation. Approximated expressions in Eq.(6) are also plotted in dashed lines for GreenOFDMv2.

Further studies are needed to propose efficient blind methods adapted to the presented technique to recover the data on the receiver side without the need of implicit side-information.

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