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Minimizing the makespan for a UET bipartite graph on a single processor with an integer precedence delay

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Abstract

We consider a set of tasks of unit execution times and a bipartite precedence delays graph with a positive precedence delay d : an arc (i, j) of this graph means that j can be executed at least d time units after the completion time of i . The problem is to sequence the tasks in order to minimize the makespan.

Firstly, we prove that the associated decision problem is NP-complete. Then, we provide a non trivial polynomial time algorithm if the degree of every tasks from one of the two sets is 2. Lastly, we give an approximation algorithm with ratio $\frac{3}{2}$.

1 Introduction

Single and multiprocessors scheduling problems have been extensively studied in the literature [16]. Scheduling problems with precedence delays arise independently in several important applications and many theoretical studies were devoted to these problems : this class of problems was considered for resource-constrained scheduling problem [3, 13]. It was also studied as a relaxation for the job-shop problem [1, 8]. For computer systems, it corresponds to the basic pipelines scheduling problems [15, 20].

An instance of a scheduling problem with precedence delays is usually defined by a set of tasks $T = \{1, \dots, n\}$ with durations $p_i, i \in T$, an oriented precedence graph $G = (T, E)$ and integer delays $d_{ij} \geq 0, (i, j) \in E$. For

Table 1: Complexity results

NP-Hard Problem	Reference
1 chains, $d_{ij} = d C_{\max}$	Wikum et al.[23]
1 prec, $d_{ij} = d, p_i = 1 C_{\max}$	Leung et al.[18]

every arc $(i, j) \in E$, task j can be executed at least d_{ij} time units after the completion time of i . The number of processors is limited. The problem is to find a schedule minimizing the makespan, or other regular criteria. Using standard notations [16], the minimization of the makespan is denoted by $P|prec, d_{ij}|C_{\max}$.

In this paper, we suppose that the graph G is bipartite : T is split into two sets X and Y and every arc $(i, j) \in E$ verifies $i \in X$ and $j \in Y$. We also consider that there is only one processor, the duration of tasks is one and that the delay is the same for every arc. This problem is noted $1|bipartite, d_{ij} = d, p_i = 1|C_{max}$. The decision problem associated is called SEQUENCING WITH DELAYS and is defined as :

- Instance : A bipartite oriented graph $G = (X \cup Y, E)$, a positive delay d and a deadline D .
- Question : is there a solution to the sequencing problem with a delay d and a makespan smaller than or equal to D ?

We prove in section 2 that $1|bipartite, d_{ij} = d, p_i = 1|C_{max}$ is NP-Hard. The complexity of this problem was a challenging question since several authors proved the NP-Hardness of more general instances of this problem as shown in the table 1. In section 3, we prove that if the degree of every task in X is 2, then the problem is polynomial and we provide a greedy algorithm to solve it.

Several authors have adapted the classical polynomial algorithms for m processors and particular graphs structures to a sequencing problem with a unique delay as shown in the table 2. Note that Bampis [2] proved that $P|bipartite, p_i = 1|C_{max}$ is NP-Hard, but his transformation doesn't seem to be easily extended to our problem.

Wikum et al. [23] also proved several complexity results, polynomial special cases and approximation algorithms for unusual particular classes of graphs (in fact, subclasses of trees). Munier and Sourd proved that $1|chains, d_{ij} = d, p_i = p|C_{\max}$ is polynomial. Lastly, Engels et al.[9] have

Table 2: Polynomial special cases

Polynomial Problem	Reference	Comments
$1 \text{tree}, d_{ij} = d, p_i = 1 C_{max}$	Bruno et al.[6]	Based on [14]
$1 \text{prec}, d_{ij} = 1, p_i = 1 C_{max}$	Leung et al. [18]	Based on [7]
$1 \text{interval orders}, d_{ij} = d, p_i = 1 C_{max}$	Leung et al.[18]	Based on [21]

developed a polynomial algorithm for $P|\text{tree}, d_{ij} \leq D, p_i = 1|C_{max}$ if D is a constant value.

At last, there are some approximation algorithms for problems with delays : Graham's list scheduling algorithm [11] was extended to $P|\text{prec. delays}, d_{ij} = k, p_j = 1|C_{max}$ to give a worst-case performance ratio of $2 - 1/(m(k + 1))$ [15, 20]. This result was extended by Munier et al. [19] to $P|\text{prec. delays}, d_{ij}|C_{max}$. Bernstein and Gerner [5] study the performance ratio of the Coffman-Graham algorithm for $P|\text{prec. delays}, d_{ij} = d, p_i = 1|C_{max}$ and slightly improve it in [4]. Schuurman [22] developed a polynomial approximation scheme for a particular class of precedence constraints. We prove in section 4 that the bound 2 of Graham's list algorithm may be achieved in the worst case for $1|\text{bipartite}, d_{ij} = d, p_i = 1|C_{max}$ and we develop a simple algorithm with worst case performance ratio equal to $3/2$ for this problem.

2 Complexity of the problem

Let us consider a non oriented graph $G = (V, E)$ and an ordering L of the vertices of G (ie, a one-to-one function $L : V \rightarrow \{1, \dots, |V|\}$). For all integer $i \in \{1, \dots, |V|\}$, the set $V_L(i) \subset V$ is :

$$V_L(i) = \{v \in V, L(v) \leq i \text{ and } \exists u \in V, \{v, u\} \in E \text{ and } L(u) > i\}$$

VERTEX SEPARATION is then defined as :

- Instance : A non oriented graph $G = (V, E)$ and a positive integer K .
- Question : Is there an ordering L of the vertices of G such that, for all $i \in \{1, \dots, |V|\}$, $|V_L(i)| \leq K$?

This problem is proved to be NP-complete in [17]. For the following, our proofs will be more elegant if we consider the converse ordering of the

tasks. Let $n = |V|$. If we set, $\forall v \in V, L'(v) = n - L(v), j = n - i + 1$ and $B_{L'}(j) = V_L(i)$, we get for every value $j \in \{1, \dots, n\}$:

$$B_{L'}(j) = \{v \in V, L'(v) > j \text{ and } \exists u \in V, \{v, u\} \in E \text{ and } L'(u) \leq j\}$$

So, the equivalent INVERSE VERTEX SEPARATION problem may be defined as :

- Instance : A non oriented graph $G = (V, E)$ and a positive integer K .
- Question : Is there an ordering L of the vertices of G such that, for all $i \in \{1, \dots, |V|\}, |B_L(i)| \leq K$?

We prove the following theorem :

Theorem 2.1. *There exists a polynomial transformation f from INVERSE VERTEX SEPARATION to SEQUENCING WITH DELAYS.*

Proof. Let I be an instance of INVERSE VERTEX SEPARATION. The associated instance $f(I)$ is given by a bipartite graph $G' = (X \cup Y, E')$, a delay d and a deadline D defined as :

1. To any vertex $v \in V$ is associated two elements $x_v \in X$ and $y_v \in Y$ and an arc $(x_v, y_v) \in E'$.
2. To any edge $\{u, v\} \in E$ is associated the arcs (x_u, y_v) and (x_v, y_u) in E' .
3. The delay is $d = n - 1 - K$ and the deadline $D = 2n$.

f can be clearly computed in polynomial time (see an example figure 1).

Let us suppose that L is a solution to the instance I . Then, we build a solution to $f(I)$ as follows :

1. Tasks from Y are executed between time n and $2n$ following L : they are executed from $y_{L^{-1}(1)}$ to $y_{L^{-1}(n)}$.
2. Let us define the partition $P_i, i = 1 \dots n$ of X as :

$$P_i = \{x_{L^{-1}(i)}\} \cup \{x_u, u \in B_L(i)\} - \bigcup_{j=1}^{i-1} P_j$$

Tasks from X are executed between 0 and n following $P_1 \dots P_n$.

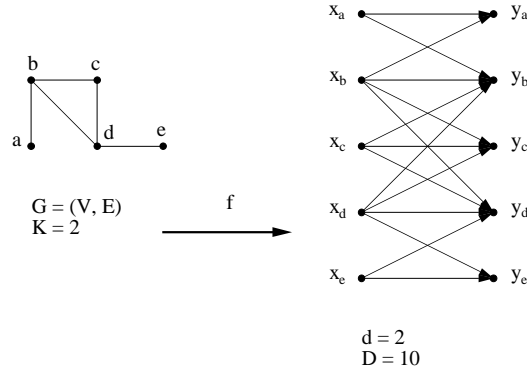


Figure 1: Example of transformation f

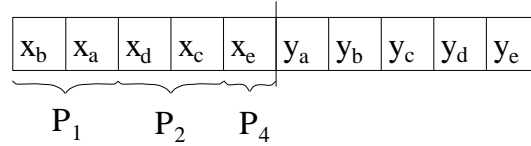


Figure 2: The schedule associated with L

For example, if we consider the order defined by $L(a) = 1$, $L(b) = 2$, $L(c) = 3$, $L(d) = 4$ and $L(e) = 5$, the sets P_i , $i = 1 \dots 5$, are defined by $P_1 = \{x_a, x_b\}$, $P_2 = \{x_c, x_d\}$, $P_3 = \emptyset$, $P_4 = \{x_e\}$ and $P_5 = \emptyset$. Figure 2 shows the corresponding solution for $f(I)$ for our example.

We have to prove now that this schedule fulfill all the precedence delays of G' . Let us consider the task $y_{L^{-1}(i)}$, $i = 1 \dots n$. We must show that all its predecessors in G' are completed at time $(n + i - 1) - d = K + i$.

1. We claim that all the predecessors of $y_{L^{-1}(i)}$ in G' are in $\bigcup_{j=1}^i P_j$. Indeed, $x_{L^{-1}(i)} \in P_j$, $j \leq i$ by construction.

The other predecessors of $y_{L^{-1}(i)}$ are vertices x_v with v adjacent to $u = L^{-1}(i)$ in G . Now, if $L(v) < L(u)$, then $x_v \in P_k$ with $k \leq L(v)$. Otherwise, $v \in B_L(i)$ so $x_v \in P_k$ with $k \leq L(u)$.

2. We show that $|\bigcup_{j=1}^i P_j| \leq K + i$. Indeed, this set is composed by : [1] i tasks $x_{L^{-1}(j)}$, $j = 1 \dots i$, and [2] tasks x_u with $L(u) > i$, so $u \in B_L(i)$.

So, we built a solution to the instance $f(I)$.

Now, let us consider that we have a solution to $f(I)$. Since the graph G' is bipartite, we can exchange the tasks such that tasks from X are all completed before the first task from Y . We build an order L from tasks in Y such that, $\forall i \in \{1, \dots, n\}$, $L^{-1}(i)$ is the task $u \in V$ such that y_u is executed at time $n + i - 1$. Then, we must prove that, $\forall i \in \{1, \dots, n\}$, $|B_L(i)| \leq K$.

Let consider $i \in \{1, \dots, n\}$. Tasks executed during the interval $[0, K + i)$ can be decomposed into [1] $x_{L^{-1}(1)} \dots x_{L^{-1}(i)}$ and [2] A set Q_i of K other tasks from $X \cup Y$.

Let be $v \in B_L(i)$. We claim that $x_v \in Q_i$. Indeed, we get that $L(v) > i$ and there exists $u \in V$ with $L(u) \leq i$ and $\{u, v\} \in E$. By definition of G' , we have then $(x_v, y_u) \in E$, so $x_v \in Q_i$.

We deduce that $|B_L(i)| \leq |Q_i| = K$. □

Corollary 2.2. $1|bipartite, d_{ij} = d, p_i = 1|C_{max}$ is NP-Hard.

3 A polynomial special case

Let us consider a non oriented connected graph $G = (V, E)$ without loops (*i.e.* without edges $\{u, u\}$, $u \in V$) and an ordering L of the vertices. We set $|V| = n$. $\forall i \in \{1, \dots, n\}$, we define the sequences $E_L(i)$ by :

$$E_L(i) = \{\{u, v\} \in E, L(u) \leq i\}$$

$E_L(i)$ is the set of edges adjacent to at least one vertices in $\{L^{-1}(1), \dots, L^{-1}(i)\}$.

We define the problem MIN ADJACENT SET LINEAR ORDERING by :

- Instance : A non oriented graph $G = (V, E)$ without loops and a positive integer K .
- Question : Is there an ordering L of the vertices of G such that, for all $i \in \{1, \dots, |V|\}$, $|E_L(i)| \leq K + i$?

Notice that the formulation of this problem is quite similar to MIN-CUT LINEAR ARRANGEMENT [10], which is NP-complete. In the following, we consider the subproblem Π of SEQUENCING WITH DELAYS with the restriction that the degree of every vertex from X is exactly 2.

Theorem 3.1. *There exists a polynomial transformation from Π to MIN ADJACENT SET LINEAR ORDERING*

Proof. Let us consider an instance I of Π given by a bipartite graph $G = (X \cup Y, E)$, a delay d and a deadline D . We build an instance $f(I)$ of MIN ADJACENT SET LINEAR ORDERING as follows :

- $G' = (Y, E')$. For every $x \in X$ with (x, y_1) and $(x, y_2) \in E$ is associated an edge $e_x = \{y_1, y_2\}$ in E' .
- the value $K = D - d - |Y| - 1$.

f can be computed in polynomial time. We prove now that f is a polynomial transformation (see figure 3 for an example)

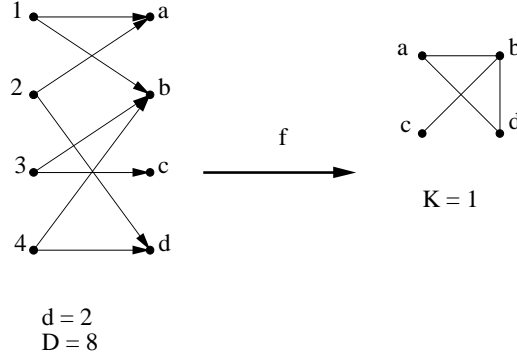


Figure 3: Example of transformation f

Let us suppose that a solution to I is given. Then, without loosing generality, we can suppose that the tasks from X are performed during $[0, \dots, |X|)$ and tasks from Y during $[D - |Y|, \dots, D)$. We build a linear ordering L following the sequencing order of tasks $Y : \forall i \in \{1, \dots, |Y|\}$, $L(i)$ is the i th task of Y in the schedule.

$\forall i \in \{1, \dots, |Y|\}$, let be $t = D - |Y| + (i - 1) = K + i + d$ the starting time of the task $L^{-1}(i)$ from Y . At time $t - d = K + i$, all the predecessors of $L^{-1}(1), \dots, L^{-1}(i)$ must be completed. Now, for every edge $e_x \in E_L(i)$ is associated exactly one of those predecessors. So, $|E_L(i)| \leq K + i$.

Conversely, let us suppose that a solution to $f(\Pi)$ is given. Then, we perform tasks from Y following L during the interval $[D - |Y|, \dots, D)$. We define then the following sequence $X_i \subset X$:

1. $X_1 = \{x \in X, e_x \in E_L(1)\}$,
2. $\forall i = 2, \dots, n, X_i = \{x \in X, e_x \in E_L(i)\} - \bigcup_{j=1}^i X_j$.

Notice that, by construction that, $\forall i \in \{1, \dots, n\}, \bigcup_{j=1}^i X_j = \{e_x \in E_L(i)\}$. Tasks of X are performed during $[0, \dots, |X|)$ following $X_1, X_2 \dots X_n$. Every task from $\bigcup_{j=1}^i X_j$ is then completed at time $K + i$ (see figure 4 for the corresponding schedule).

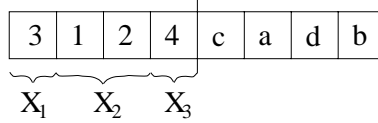


Figure 4: A corresponding schedule

We must prove that the delays constraints are fulfilled : let us consider the task $y = (L^{-1}(i))$. For every task $x \in \Gamma^{-1}(y)$ is associated $e_x \in E_L(i)$. So, $x \in \bigcup_{j=1}^i X_j$ and is completed at time $K + i$. Since y is performed at time $t = D - |Y| + i - 1$, we get :

$$t - (K + i) = D - |Y| + i - 1 - (K + i) = d$$

So, the delays are fulfilled. \square

Theorem 3.2. *Let us consider an instance I of MIN ADJACENT SET LINEAR ORDERING given by a graph $G = (V, E)$ and an integer $K > 0$. A necessary and sufficient condition for the existence of a solution is that*

$$|E| \leq K + |V| - 1$$

Proof. The condition is necessary : since the graph G is connected without loops, every linear ordering L verifies $E_L(n - 1) = E$. So, if L verifies the condition, we get the condition of the theorem.

The condition is sufficient : let us consider a linear ordering L and a family of graph $G_i, i = 0, \dots, n$ defined such that,

- $G_0 = G$,
- $\forall i = 1, \dots, n$, we choose a vertex u in the subgraph $G_{i-1} = (V - \{L^{-1}(1), \dots, L^{-1}(i - 1)\}, E)$ with a minimum degree in G_{i-1} and we set $L(u) = i$.
- $G_n = \emptyset$.

We note E_i the edges of G_i . Notice that, $\forall i = 1, \dots, n$, the two sets $E_L(i)$ and E_i are a partition of E .

We prove by contradiction that the linear ordering L is a solution to MIN ADJACENT SET LINEAR ORDERING.

- Let us suppose that $|E_L(1)| \geq K + 2$, then the degree of any vertex in G is greater than or equal to $K + 2$. So, $2|E| \geq |V|(K + 2)$. By hypothesis, we get $2K + 2|V| - 2 \geq K|V| + 2|V|$, so $K(2 - |V|) \geq 2$. Since $K > 0$, we get that $|V| < 2$, so $|V| = 1$. In this case, we get $|E_L(1)| = |E| = 0$, which contradicts $|E| \geq K + 2$.
- Now, let us suppose that, for $i < n - 2, \forall j \in \{1, \dots, i\}, |E_L(j)| \leq K + j$ and that $|E_L(i + 1)| \geq (i + 1) + K + 1$. For every vertex $u \in G_i$, we set $d_{G_i}(u)$ the degree of u in G_i .

The total number of edges verifies $|E| = |E_L(i + 1)| + |E_{i+1}|$.

1. By hypothesis, $|E_L(i + 1)| \geq (i + 1) + K + 1$.
2. By definition of the sequences $G_i, |E_{i+1}| = |E_i| - d_{G_i}(L^{-1}(i + 1))$. Since $u = L^{-1}(i + 1)$ is the vertex of G_i with a minimum degree, the number of arcs of G_i verifies

$$2|E_i| \geq (n - i)d_{G_i}(L^{-1}(i + 1))$$

So,

$$|E_{i+1}| \geq \frac{1}{2}(n - i)d_{G_i}(L^{-1}(i + 1)) - d_{G_i}(L^{-1}(i + 1))$$

We show that $d_{G_i}(L^{-1}(i + 1)) \geq 2$. Indeed, let us denote by $e(k) = \{L^{-1}(i + 1), L^{-1}(k)\}$ an edge of G adjacent to $L^{-1}(i + 1)$. Then, we get easily that $E_L(i + 1) - E_L(i) = \{e(k) \in G_i\}$, so

$$d_{G_i}(L^{-1}(i + 1)) = |E_L(i + 1)| - |E_L(i)| \geq (i + 1) + K + 1 - (K + i) = 2$$

We deduce that

$$|E_{i+1}| \geq \frac{n - i - 2}{2}d_{G_i}(L^{-1}(i + 1)) \geq n - i - 2$$

So, the total number of edges of G verifies :

$$|E| = |E_L(i + 1)| + |E_{i+1}| \geq (i + 1) + K + 1 + n - i - 2 = |V| + K$$

which contradicts the hypothesis of the theorem. \square

Notice that this proof is constructive : if the condition of the theorem is fulfilled, one can easily implements a greedy polynomial algorithm to build a linear ordering.

Corollary 3.3. Π is polynomial.

If we heavily sort the the vertices at each step of the algorithm, the complexity of the algorithm will be bounded by $O(n^2 \log n + m)$.

4 An Approximation algorithm

In this section, we consider the analysis of the performances of two approximation algorithms.

The first one is the classical Graham list scheduling algorithm [12]. At each time t , a schedulable task is chosen to be performed without any priority rule. For the bipartite graph $G = (X \cup Y, E)$, it consists on performing tasks from X in any order and tasks from Y as soon as possible. Several authors show that the performance ratio of this algorithm is upper bounded asymptotically by 2 [15, 20, 19]. We prove here that this bound is reached for bipartite graphs :

Theorem 4.1. *The performance ratio of a list scheduling for a bipartite graph tends asymptotically to 2.*

Proof. Let us consider a value $d > 0$ and a bipartite graph $G = (X \cup Y, E)$ with $X = \{a_1, \dots, a_d\} \cup \{b\}$, $Y = \{c\}$ and $E = \{(b, c)\}$. In the worst case for the Graham list scheduling algorithm, tasks $\{a_1, \dots, a_d\}$ are performed first. We get then a schedule of length $l_1 = 2d + 2$.

Now, we can get a schedule without idle slots if we perform b first. The length of this second schedule is then $l_2 = d + 2$.

The performance ratio is then bounded by : $r = \frac{2d+2}{d+2} = 2 - \frac{2}{d+2} \rightarrow_{d \rightarrow \infty} 2$. \square

We present now a slightly better approximation algorithm : let us suppose that $G = (X \cup Y, E)$ with $|X| = n$, $|Y| = m$ and $n \geq m$. In the opposite, we modify the orientation of the edges and we consider the graph $G' = (Y \cup X, E')$. We can get a feasible schedule for G by considering the inverse order of a schedule for G' .

Let us consider the set X_1 of tasks from X with a strictly positive out-degree (*i.e.*, X_1 is the set of X with at least one successor in Y). The idea is to apply a list scheduling algorithm which performs tasks from X_1 before those from $X_2 = X - X_1$.

We denote by C_{opt} (resp. C_H) the makespan of an optimal schedule (resp. a schedule obtained using this algorithm). We set $|X_i| = n_i, i = 1, 2$ and $p = \max(0, d + 1 - n_2 - m)$. We prove the following upper bound on C_{opt} :

Lemma 4.2. $C_{opt} \geq n + m + p$.

Proof. The last task of X_1 is performed at time $t \geq n_1$ and has at least one successor in Y , so $C_{opt} \geq n_1 + d + 1$. Now, if $p = d + 1 - n_2 - m$,

$n + m + p = n + m + d + 1 - n_2 - m = n_1 + d + 1$ and the inequality is true. Otherwise, $p = 0$ and we get obviously $C_{opt} \geq n + m$. \square

Theorem 4.3. *The performance ratio of this algorithm is bounded by $\frac{3}{2}$.*

Proof. We denote by \mathcal{I} the idle slots of the schedule obtained by our algorithm. We get, using the previous lemma :

$$C_H = n + m + |\mathcal{I}| \leq C_{opt} + (|\mathcal{I}| - p)$$

1. If $|\mathcal{I}| \leq p$, we get the theorem.
2. Let us assume now that $|\mathcal{I}| > p$. We build a subset $\mathcal{I}_p \subset \mathcal{I}$ by removing from \mathcal{I} the p th first idle slots in our schedule. Let be an element $k \in \mathcal{I}_p$ and $t(k)$ the time of this idle slot.

Clearly, by definition of \mathcal{I}_p , $t(k) \geq p + n$. Moreover, there is at least one task from $y \in Y$ performed after $t(k)$ such that y is not ready at time $t(k)$, so $t(k) \leq n_1 + d$. We get

$$|\mathcal{I}| - p = |\mathcal{I}_p| \leq n_1 + d - (p + n)$$

Then,

$$|\mathcal{I}| - p = |\mathcal{I}_p| \leq d - n_2 - \max(0, d + 1 - n_2 - m)$$

We deduce that

$$|\mathcal{I}_p| \leq \min(d - n_2, m - 1)$$

So, $|\mathcal{I}_p| \leq |Y|$.

Now, the inequality between C_H and C_{opt} becomes :

$$C_H \leq C_{opt} + |\mathcal{I}_p| \leq C_{opt} + |Y|$$

Since $|Y| \leq |X|$, we get that $|Y| \leq \frac{1}{2}(|X| + |Y|) \leq \frac{1}{2}C_{opt}$ and we get the theorem. \square

We can prove that the bound $\frac{3}{2}$ is asymptotically tight : indeed, let us consider an integer $n > 0$ and the bipartite graph $G = (X \cup Y, E)$ with $X = \{x_1, \dots, x_n\}$, $Y = \{y_1, \dots, y_n\}$ and the arcs $E = \{(x_i, y_j), 1 \leq j \leq i \leq n\}$. We set $d = n - 1$. Note that $|X| = n = |Y|$.

If we perform task from X such that $t(x_i) = i - 1, i = 1, \dots, n$, then tasks from Y can't be performed before $n + d - 1$. So, we get a makespan $L_1 = 3n - 2$.

Now, if we perform task from from X such that $t(x_i) = n - i, i = 1, \dots, n$, then we get a schedule without idle slots with makespan $L_2 = 2n$.

So, we get $\frac{L_1}{L_2} \rightarrow_{n \rightarrow +\infty} \frac{3}{2}$.

5 Conclusions

Several new questions arise from the results presented here :

- In order to study the borderline between NP-complete and polynomial problems, the complexity of the problem with a bipartite graph where the degree of vertices from X does not exceed 3 is an interesting problem.
- The existence of better approximation algorithms is also an interesting question.

References

- [1] E. Balas, J. K. Lenstra, and A. Vazacopoulos. The one machine problem with delayed precedence constraints and its use in job-shop scheduling. *Management Science*, 41:94 – 109, 1995.
- [2] E. Bampis. The complexity of short schedules for uet bipartite graphs. *RAIRO Operations Research*, 33:367 – 370, 1999.
- [3] M. Bartusch, R. H. Möhring, and F. J. Radermacher. Scheduling project networks with resource constraints and time windows. *Annals of Operations Research*, 16:201 – 240, 1988.
- [4] D. Bernstein. An improved approximation algorithm for scheduling pipelined machines. In *Int. Conf. on Parallel Processing*, volume 1, pages 430 – 433, 1988.
- [5] D. Bernstein and I. Gertner. Scheduling expressions on a pipelined processor with a maximum delay of one cycle. *ACM Transactions on Programming Languages and Systems*, 11:57 – 66, 1989.
- [6] J. Bruno, J. W. Jones, and K. So. Deterministic scheduling with pipelined processors. *IEEE Transactions on Computers*, C-29:308 – 316, 1980.
- [7] E.G. Coffman and R.L. Graham. Optimal scheduling for two processor systems. *Acta Informatica*, 1:200 – 213, 1972.
- [8] S. Dauzère-Peres and J.-B. Lasserre. A modified shifting bottleneck procedure for job-shop scheduling. *Int. J. Production Research*, 31:923 – 932, 1993.

- [9] D.W Engels, J. Feldman, D.R Karger, and M. Ruhl. Parallel processor scheduling with delay constraints. Rapport Interne MIT.
- [10] M. R. Garey and D. S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. Freeman, San Francisco, 1979.
- [11] R. L. Graham. Bounds for certain multiprocessing anomalies. *Bell System Tech. J.*, 45:1563 – 1581, 1966.
- [12] R. L. Graham. Bounds on the performance of scheduling algorithms. In E. G. Coffman, editor, *Computer and Job-shop Scheduling Theory*. John Wiley Ltd., 1976.
- [13] W. Herroelen and E. Demeulemeester. Recent advances in branch-and-bound procedures for resource-constrained project scheduling problems. In P. Chrétienne, E. G. Coffman Jr., J. K. Lenstra, and Z. Liu, editors, *Scheduling Theory and its Applications*, chapter 12, pages 259 – 276. John Wiley & Sons, 1995.
- [14] T.C. Hu. Parallel sequencing and assembly lines problems. *Operations Research*, 9:841 – 848, 1961.
- [15] E. Lawler, J. K. Lenstra, C. Martel, B. Simons, and L. Stockmeyer. Pipeline scheduling: A survey. Technical Report RJ 5738 (57717), IBM Research Division, San Jose, California, 1987.
- [16] E. L. Lawler, J. K. Lenstra, A. H. G. Rinnooy Kan, and D. B. Shmoys. Sequencing and scheduling: Algorithms and complexity. In S. C. Graves, A. H. G. Rinnooy Kan, and P. H. Zipkin, editors, *Logistics of Production and Inventory*, volume 4 of *Handbooks in Operations Research and Management Science*, chapter 9, pages 445 – 522. North-Holland, Amsterdam, The Netherlands, 1993.
- [17] T. Lengauer. Black-white pebbles and graph separation. *Acta Informatica*, 16:465 – 475, 1981.
- [18] J. Y.-T. Leung, O. Vornberger, and J. Witthoff. On some variants of the bandwidth minimization problem. *SIAM J. Computing*, 13:650 – 667, 1984.
- [19] A. Munier, M. Queyranne, and A. S. Schulz. Approximation bounds for a general class of precedence constrained parallel machine scheduling problems. Number 1412 in Lecture Notes in Computer Science, pages

367 – 382. Springer, 1998. Proceedings of the 6th International IPCO Conference.

- [20] K. W. Palem and B. Simons. Scheduling time critical instructions on risc machines. In *Proceedings of the 17th Annual Symposium on Principles of Programming Languages*, pages 270 – 280, 1990.
- [21] C.H Papadimitriou and M. Yannakakis. Scheduling interval-ordered tasks. *SIAM Journal on Computing*, 8(3), 1979.
- [22] P. Schuurman. A fully polynomial approximation scheme for a scheduling problem withintree-type precedence delays. *Operations Research Letters*, 23:9 – 11, 1998.
- [23] E. D. Wikum, D. C. Llewellyn, and G. L. Nemhauser. One-machine generalized precedence constrained scheduling problems. *Operations Research Letters*, 16:87 – 89, 1994.